Eclipse, Kliezl P. Exercise 2.2 – 2

Q: Use the informal definitions of O, Θ, and Ω to determine whether the following assertions are true or false.

a. $n(n + 1)/2 \in O(n^3)$
b. $n(n + 1)/2 \in O(n^2)$
c. $n(n + 1)/2 \in \Theta(n^3)$
d. $n(n + 1)/2 \in \Omega(n)$

A:

a. True
   Quadratic function $n(n + 1)/2$ has a lower order of growth than cubic polynomial function n^3
b. True
   Quadratic function $n(n + 1)/2$ has the same order of growth as n^2
c. False
   Quadratic function $n(n + 1)/2$ does not have the same order of growth as cubic polynomial function n^3
d. True
   Quadratic function $n(n + 1)/2$ has a higher order of growth than a linear function n

Eclipse, Kliezl P. Exercise 2.2 – 2

Q: Use the informal definitions of O, Θ, and Ω to determine whether the following assertions are true or false.

e. $n(n + 1)/2 \in O(n^3)$
f. $n(n + 1)/2 \in O(n^2)$
g. $n(n + 1)/2 \in \Theta(n^3)$
h. $n(n + 1)/2 \in \Omega(n)$

A:

e. True

$Let\ c = 1\ and\ k = 2$

$f(n) = \frac{n(n+1)}{2}$

$g(n) = n^3$

$\frac{n(n+1)}{2} \leq c \cdot n^3\ whenever\ n \geq 2$

$\frac{n(n+1)}{2} \leq n^3\ whenever\ n \geq 2$

$\frac{2(2+1)}{2} \leq 8 \quad \rightarrow \quad \frac{6}{2} \leq 8 \quad \rightarrow \quad 3 \leq 8 \qquad TRUE$

f. True

$Let\ c = 1\ and\ k = 2$

$f(n) = \frac{n(n+1)}{2}$

$g(n) = n^2$

$\frac{n(n+1)}{2} \leq c \cdot n^2\ whenever\ n \geq k$

$\frac{n(n+1)}{2} \leq n^2\ whenever\ n \geq 2$

$\frac{2(2+1)}{2} \leq 4 \quad \rightarrow \quad \frac{6}{2} \leq 4 \quad \rightarrow \quad 3 \leq 4 \qquad TRUE$

g. False

h. True

$Let\ c = 1\ and\ k = 1$

$f(n) = \frac{n(n+1)}{2}$

$g(n) = n$

$\frac{n(n+1)}{2} \geq c \cdot n\ whenever\ n \geq k$

$\frac{n(n+1)}{2} \geq n\ whenever\ n \geq 1$

$\frac{1(1+1)}{2} \geq 1 \quad \rightarrow \quad \frac{2}{2} \geq 1 \quad \rightarrow \quad 1 \geq 1 \qquad TRUE$