Eclipse, Kliezl P. Exercise 2.4 – 1

Q: Solve the following recurrence relations.

a. $x(n) = x(n - 1) + 5$ for $n > 1$, $x(1) = 0$

b. $x(n) = 3x(n - 1)$ for $n > 1$, $x(1) = 4$

c. $x(n) = x(n - 1) + n$ for $n > 0$, $x(0) = 0$

d. $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$ (solve for $n = 2^k$)

e. $x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$ (solve for $n = 3^k$)

A:

a. $x(n) = x(n - 1) + 5$ for $n > 1$, $x(1) = 0$

$x(n - 1) = x(n - 2) + 5$

$x(n - 2) = x(n - 3) + 5$

$x(n) = (x (n - 2) + 5) + 5$

$x(n) = ((x(n - 3) + 5) + 5) + 5$

$x(n) = x(n - 3) + (5 + 5 + 5)$

$x(n) = x(n - y) + (5 * y)$

Let $y = n - 1$

$x(n) = x(n - (n - 1)) + (5 * (n - 1))$

$x(n) = x(n - n + 1) + 5(n - 1)$

$x(n) = x(1) + 5(n - 1)$

$x(n) = 0 + 5(n - 1)$

$\boxed{x(n) = 5(n-1) = 5n - 5}$

b. $x(n) = 3x(n - 1)$ for $n > 1$, $x(1) = 4$

$x(n - 1) = 3x(n - 2)$

$x(n - 2) = 3x(n - 3)$

$x(n) = 3(3(3x(n - 3)))$

$x(n) = 3^y x(n - y)$

Let $y = n - 1$

$x(n) = 3^{n-1}x(n - (n - 1))$

$x(n) = 3^{n-1} x(n - n + 1)$

$x(n) = 3^{n-1} x(1)$

$\boxed{x(n) = 3^{n-1}(4)}$

c. $x(n) = x(n - 1) + n$ for $n > 0$, $x(0) = 0$

$x(n - 1) = x(n - 2) + n$

$x(n - 2) = x(n - 3) + n$

$x(1) = x(0) + n$
$x(1) = 0 + n = n$

$x(n) = ((x(n - 3) + n) + n) + n$
$x(n) = x(n – 3) + 3*n$
$x(n) = x(n – y) + yn$

Let $y = n – 1$

$x(n) = x(n – (n – 1)) + (n – 1)n$
$x(n) = x(1) + (n – 1)n$
$x(n) = n + n(n – 1)$
$x(n) = n + n^2 – n$
$\boxed{x(n) = n^2}$

d. $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$ (solve for $n = 2^k$)
$x(2^k) = x(2^k/2) + 2^k$
$x(2^k) = x(2^{k-1}) + 2^k$
$x(2^{k-1}) = x(2^{k-2}) + 2^{k-1}$
$x(2^{k-2}) = x(2^{k-3}) + 2^{k-2}$
$x(2^k) = x(2^{k-3}) + 2^{k-2} + 2^{k-1} + 2^k$
$x(2^k) = x(2^{k-y}) + 2^{k-y+1} + 2^{k-y+2} + \ldots + 2^k$

Let $y = k$
$x(2^k) = x(2^{k-k}) + 2^{k-k+1} + 2^{k-k+2} + \ldots + 2^k$
$x(2^k) = x(2^0) + 2^1 + 2^2 + \ldots + 2^k$
$x(2^k) = x(1) + 2^1 + 2^2 + \ldots + 2^k$
$x(2^k) = 1 + 2^1 + 2^2 + \ldots + 2^k$
$x(2^k) = 2^{k+1} – 1$
$x(2^k) = 2(2^k) – 1$
$\boxed{x(n) = 2n – 1}$

e. $x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$ (solve for $n = 3^k$)
$x(3^k) = x(3^k/3) + 1$
$x(3^k) = x(3^{k-1}) + 1$
$x(3^{k-1}) = x(3^{k-2}) + 1$
$x(3^{k-2}) = x(3^{k-3}) + 1$
$x(3^k) = x(3^{k-3}) + 1 + 1 + 1$
$x(3^k) = x(3^{k-y}) + y$

Let $y = k$
$x(3^k) = x(3^{k-k}) + k$
$x(3^k) = x(1) + k$
$x(3^k) = 1 + k$
$x(3^k) = 1 + \log_3 3^k$
$x(3^k) = 1 + \log_3 3^k$
$\boxed{x(n) = 1 + \log_3 n}$