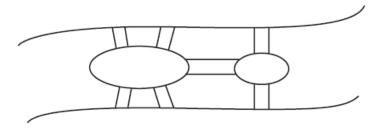
Eclipse, Kliezl P. Exercise 1.3 – 4

Konigsberg bridges

Q: The Konigsberg bridge puzzle is universally accepted as the problem that gave birth to graph theory. It was solved by the great Swiss-born mathematician Leonhard Euler (1707–1783). The problem asked whether one could, in a single stroll, cross all seven bridges of the city of Konigsberg exactly once and return to a starting point. Following is a sketch of the river with its two islands and seven bridges:

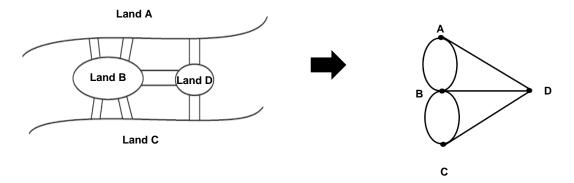


a. State the problem as a graph problem.

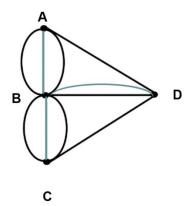
b. Does this problem have a solution? If you believe it does, draw such a stroll; if you believe it does not, explain why and indicate the smallest number of new bridges that would be required to make such a stroll possible.

A:

- a. The problem is if one can cross all seven bridges exactly once in a single stroll and return to a starting point. This means if the map is seen as a graph, one should find a closed trail (a trail with the initial vertex as the final vertex as well, and no repeating edges) which includes all the seven edges.
- b. This problem does not have a solution unless each vertex has an even degree. If the given scenario can be illustrated as an Eulerian graph (a connected graph containing a closed trail that includes every edge and has vertices with even degree), then the problem can be solved. So far, all the vertices have an odd degree Points A, C, and D have a degree of 3 while Point B has a degree of 5. If there were THREE additional edges that will make all four vertices have an even degree (i.e., Edge AB, Edge BC, and Edge BD), a stroll is possible.



Additional Edges:



Stroll:

