Eclipse, Kliezl P. Exercise 10.1-3

Q: Consider the linear programming problem

minimize $c1x + c2y$

subject to $x + y \geq 4$

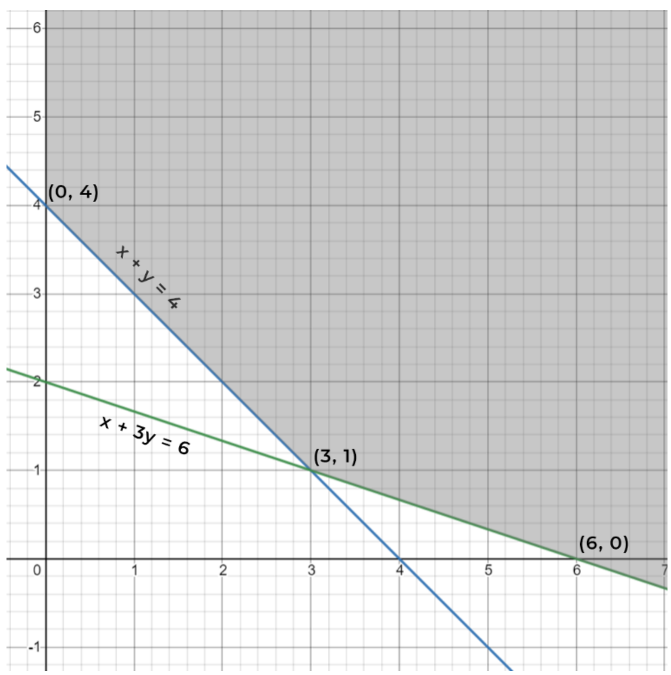
$x + 3y \geq 6$

$x \geq 0, y \geq 0$

where $c1$ and $c2$ are some real numbers not both equal to zero.

a. Give an example of the coefficient values c1 and c2 for which the problem has a unique optimal solution.
b. Give an example of the coefficient values c1 and c2 for which the problem has infinitely many optimal solutions.
c. Give an example of the coefficient values c1 and c2 for which the problem does not have an optimal solution.

A:

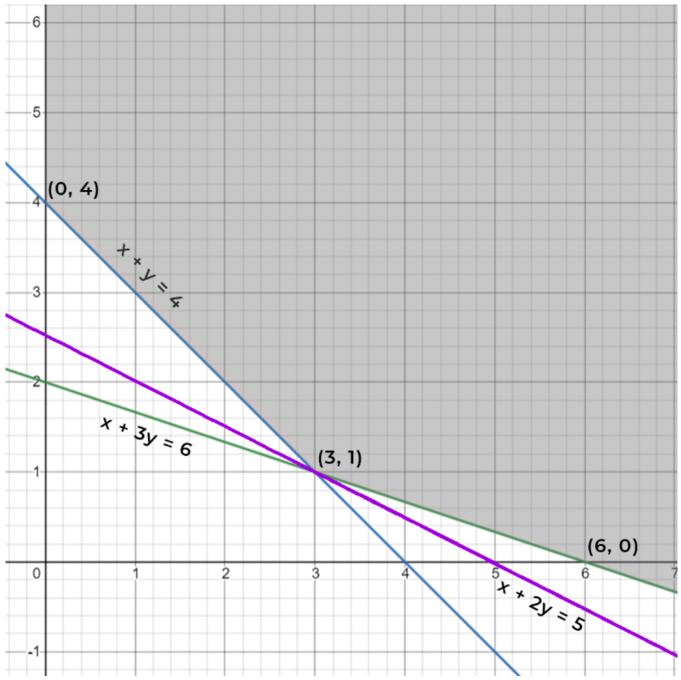


a. Any line with slope between the slope of lines $x + y = 4$ and x + 3y = 6 will hit the point (3, 1) as the only minimum solution.

x + y = 4
x + 3y = 6

2x + 4y = 10
x + 2y = 5

Example coefficients can be c1 = 1 and c2 = 2:



b. There are four possible situations for which the problem will have infinitely many solutions:

1. Let c1 = c and c2 = 0, where c > 0
2. Let c1 = c2 = c, where c > 0
3. Let c1 = c and c2 = 3c, where c > 0
4. Let c1 = 0 and c2 = c, where c > 0

Lines with these coefficients will contain a line segment that will be a part of the feasible region's boundary.

c. There is no possible coefficient value c1 and c2 for which will have no optimal solution since determining whether a problem has an optimal solution or not depends on the given constraints instead of the given objective function.