Katharine Priu, Sarah Pardee, Elayna Williams

530.216 Mechanics Based Design Laboratory

Crane Design Project Report

I. Final Crane Description

A. Power Transmission and Lifting System

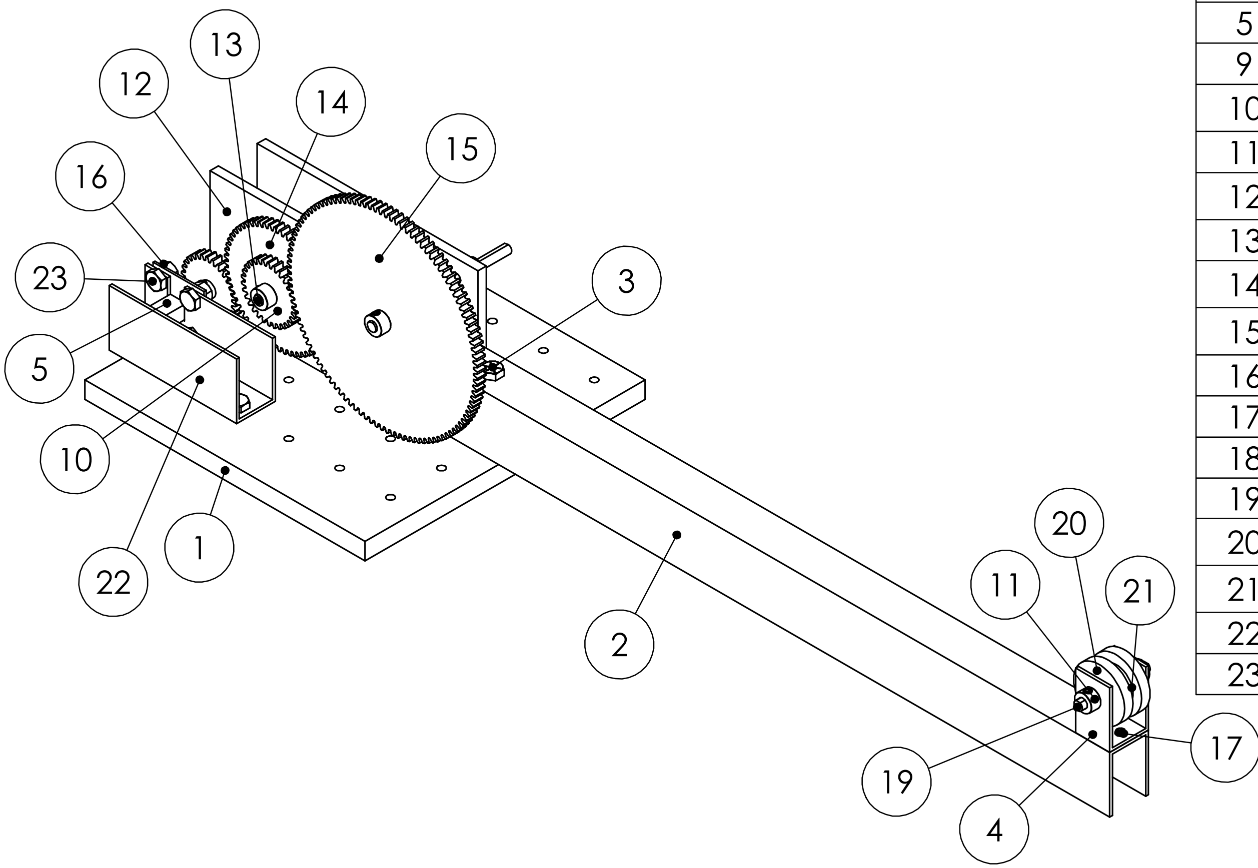
In order to transmit power from the motor to the lifting mechanism, the motor is connected to a gear and pulley system. The motor is mounted to a 3.75” long PVC U-Channel designed to hold the motor. The motor’s coupler is attached to a gear shaft that passes through a 30-tooth gear connecting to a 60-tooth gear. This gear shares its shaft with another 30-tooth gear that leads to a 110-tooth gear. Each of these gears are 0.25” thick, have a pitch of 20, are made of acrylic, and have a pressure angle of 14.5 degrees. The pitch diameters of the 30, 60, and 110- tooth gears are 1.5”, 3.0”, and 5.5” respectively. This gear system fulfills the necessary gear ratio, 22:3, for the crane to lift the weight. The lifting wire is attached to the 110-tooth gear’s shaft and passes over the length of the boom. It then is fed through a pulley made up of two 0.25” thick acrylic circles with a diameter of 1.50” that sandwich a circle with a 1.0” diameter and same thickness. A shaft passes through the pulley and is mounted to a PVC U-bracket at the end of the boom.

B. Boom and Structural Support System

The main component of this crane design is a rectangular U-shaped boom made out of PVC plastic material. The boom is cut to 24.75 inches long, with 6.75 inches on one end screwed down to the crane mounting plate. There is one screw securing the boom on either side of the mounting plate, with 6 inches in between each screw. The base of the cross-sectional area of the boom is 1.14 inches and the height is 1.65 inches; the thickness of the boom walls is 0.08 inches. It is oriented “upside-down,” with the open end facing downwards. The transmission system is attached to two gear plates made out of acrylic on each side of the boom. The shafts that hold these plates and gears in place are positioned above and through the boom (2 shafts above and 2 shafts through), helping to secure it onto the mounting plate. All shafts are made out of 1045 carbon steel. The crane mounting plate is positioned on a table top that is 36 inches above the ground on which the weight sits, and 22 inches above the hook to which this crane will latch onto. The weight is also a horizontal distance of 18 inches from the edge of the mounting plate.

II. CAD Drawings

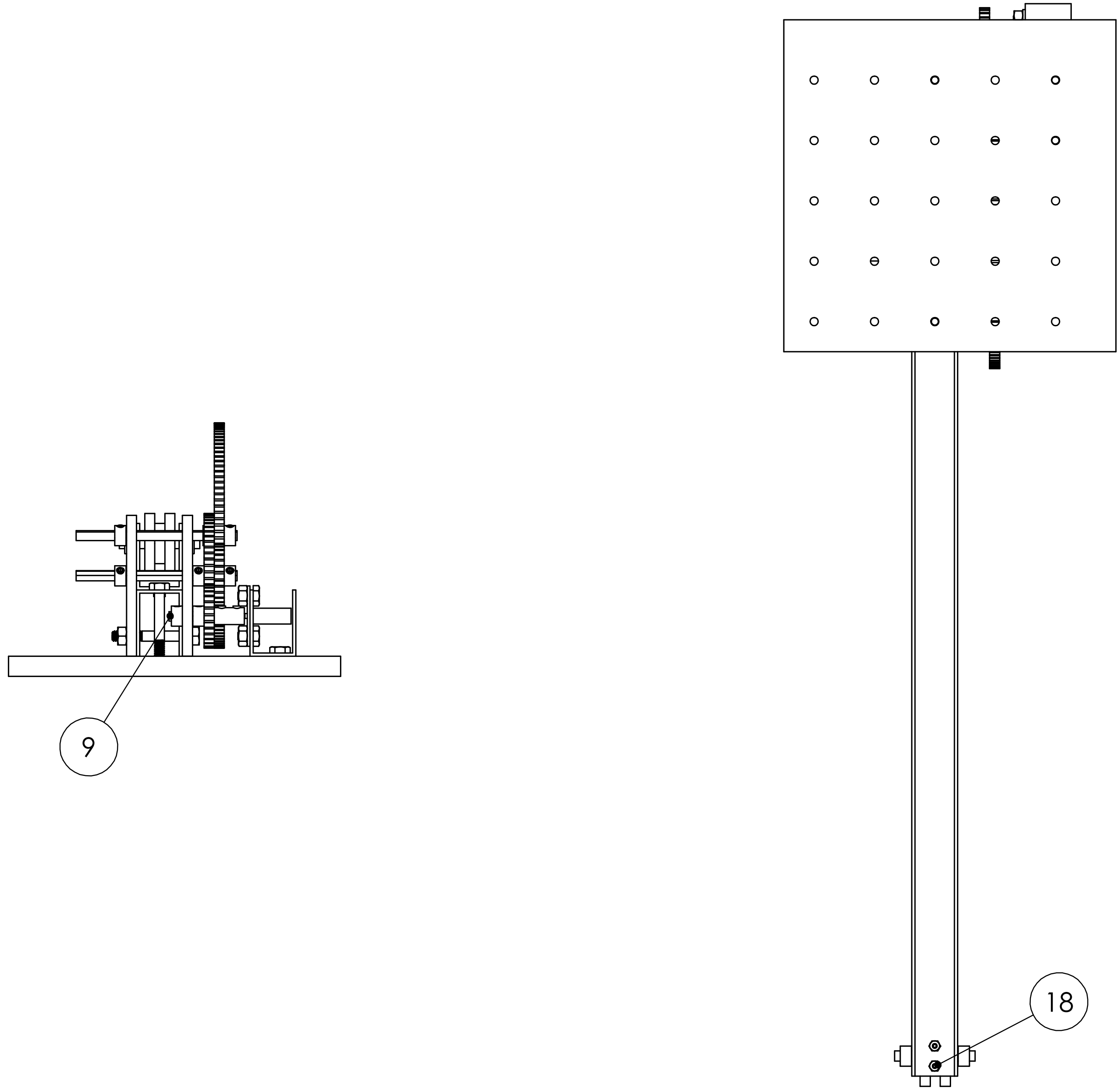
1. Crane Assembly
2. 30 Tooth Gear
3. 60 Tooth Gear
4. 110 Tooth Gear
5. Acrylic Gearplate
6. Boom
7. 3.75 in PVC
8. U-Bracket
9. Pulley Inner
10. Pulley Outer
11. 1.5 in Shaft
12. 2 in Shaft
13. 4 in Shaft



ITEM NO.	PART NAME	DRAWING NUMBER	QTY.
1	METAL PLATE	N/A	1
2	BOOM	6	1
3	1/4-20 BOLT 2 IN	N/A	4
4	U-BRACKET	8	1
5	GEARMOTOR	N/A	1
9	2 IN 1/4 DIA SHAFT	12	1
10	30 TOOTH GEAR	2	2
11	1/4 SHAFT COLLAR	N/A	10
12	ACRYLIC GEARPLATE	5	2
13	4 IN 1/4 DIA SHAFT	13	2
14	60 TOOTH GEAR	3	1
15	110 TOOTH GEAR	4	1
16	1/4-20 NUT	N/A	6
17	4-40 BOLT 1/4 IN	N/A	2
18	4-40 NUT	N/A	2
19	1.5 IN 1/4 DIA SHAFT	11	1
20	PULLEY OUTER	10	2
21	PULLEY INNER	9	1
22	3.75 IN PVC	7	1
23	1/4-20 BOLT 3/8 IN	N/A	6

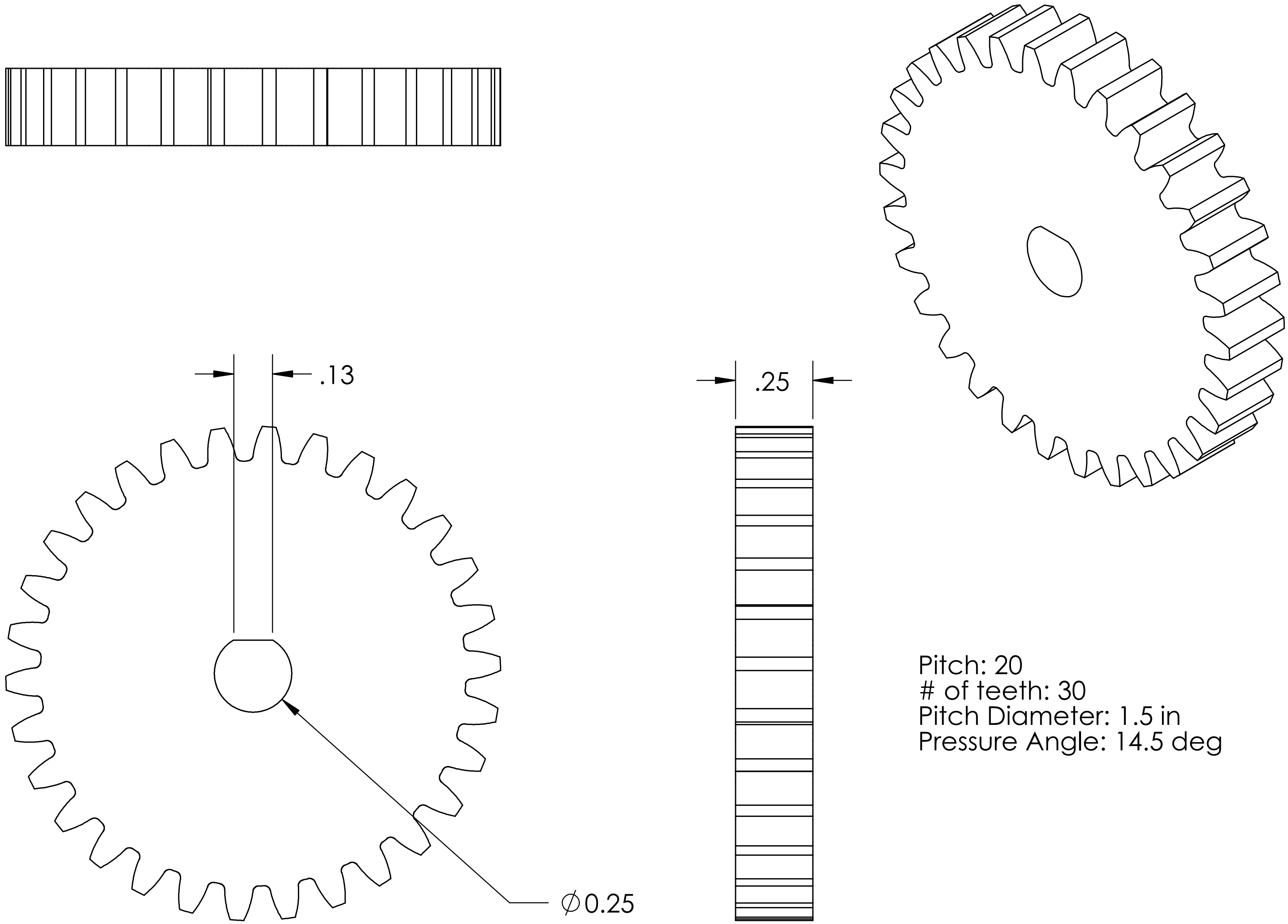
530.216 MBD LAB	TITLE: Crane Assembly	UNITS: INCHES	SCALE: 1:4	MATERIAL:	QTY. REQ.: 1
	TEAM: Craniacs	TEAM MEMBERS: Priu, Pardee, Williams	DATE: 4/24/21	DWG. NO.: 1A	REV. NO.:

5 4 3 2 1



530.216 MBD LAB	TITLE Crane Assembly		UNITS INCHES	SCALE 1:4	MATERIAL	QTY. REQ. 1
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	DATE 4/25/21	DWG. NO. 1B	REV. NO.	

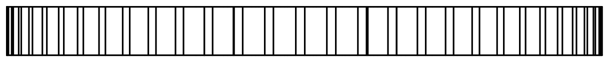
5 4 3 2 1

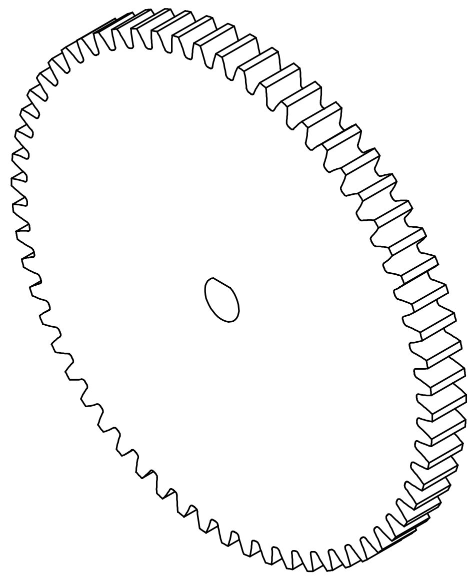


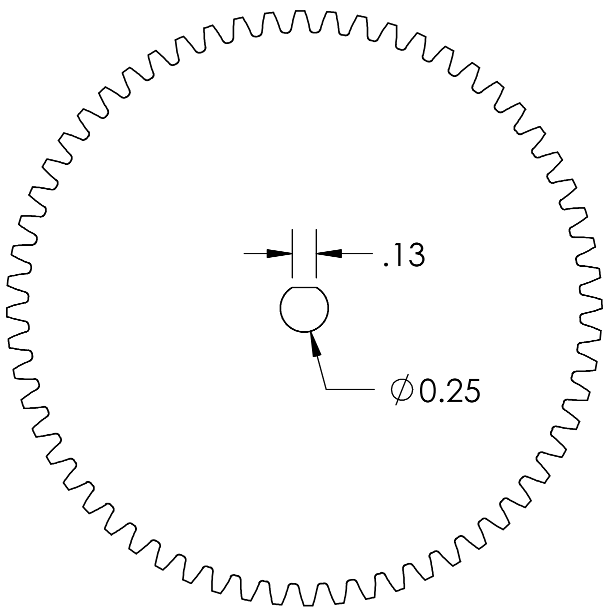
Pitch: 20
# of teeth: 30
Pitch Diameter: 1.5 in
Pressure Angle: 14.5 deg

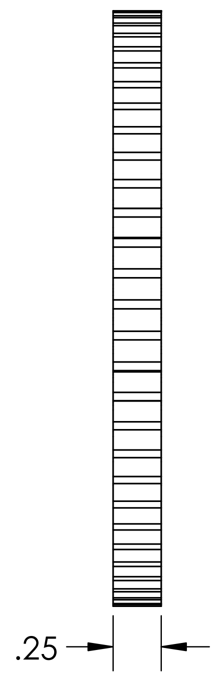
530.216 MBD LAB	TITLE: 30 Tooth Gear	UNITS: INCHES	SCALE: 2:1	MATERIAL: Acrylic	QTY. REQ.: 2
	TEAM: Craniacs	TEAM MEMBERS: Priu, Pardee, Williams	DATE: 4/2/21	DWG. NO.: 2	REV. NO.:

5 4 3 2 1



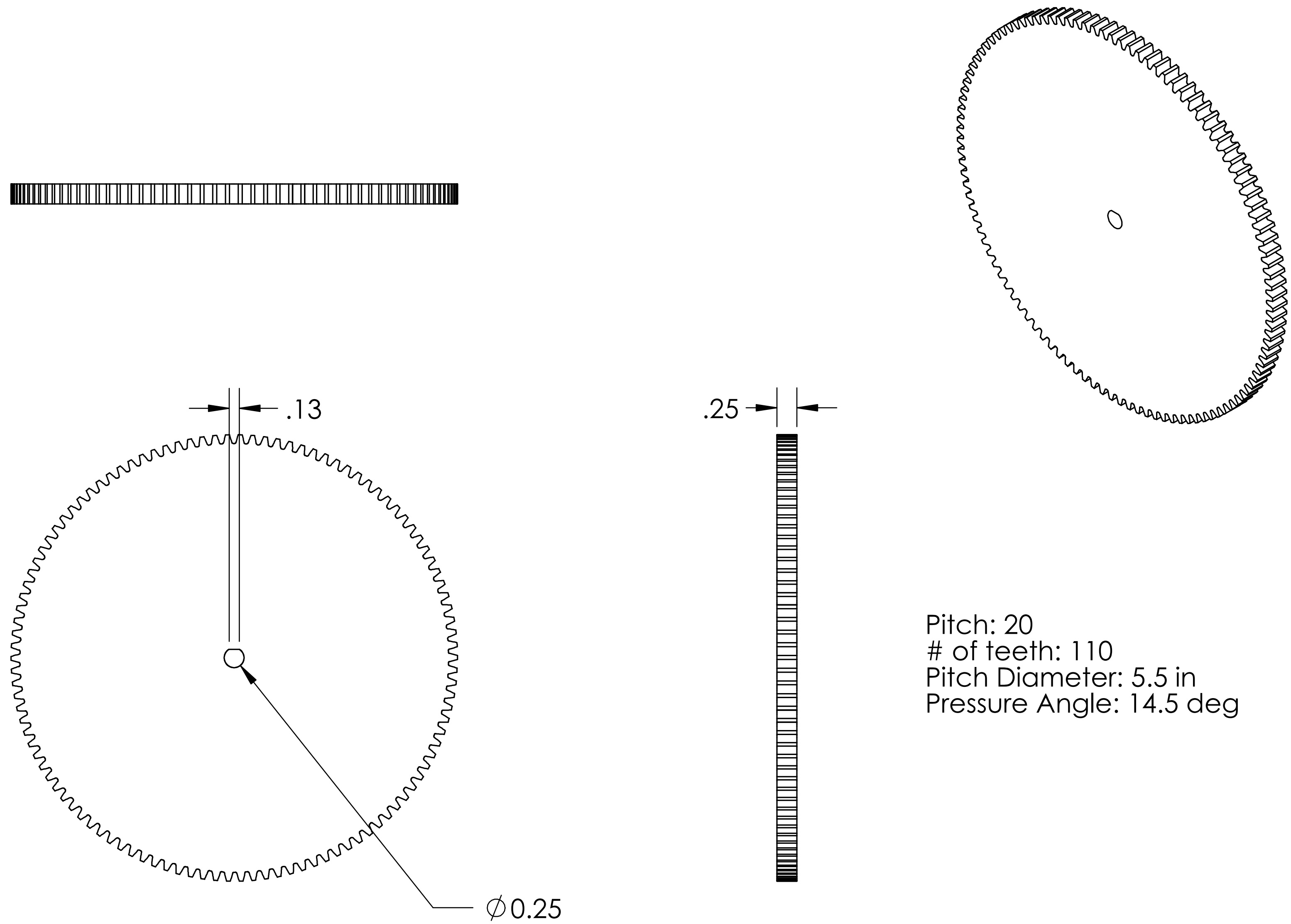






Pitch: 20
# of teeth: 60
Pitch diameter: 3.0 in
Pressure angle: 14.5 deg

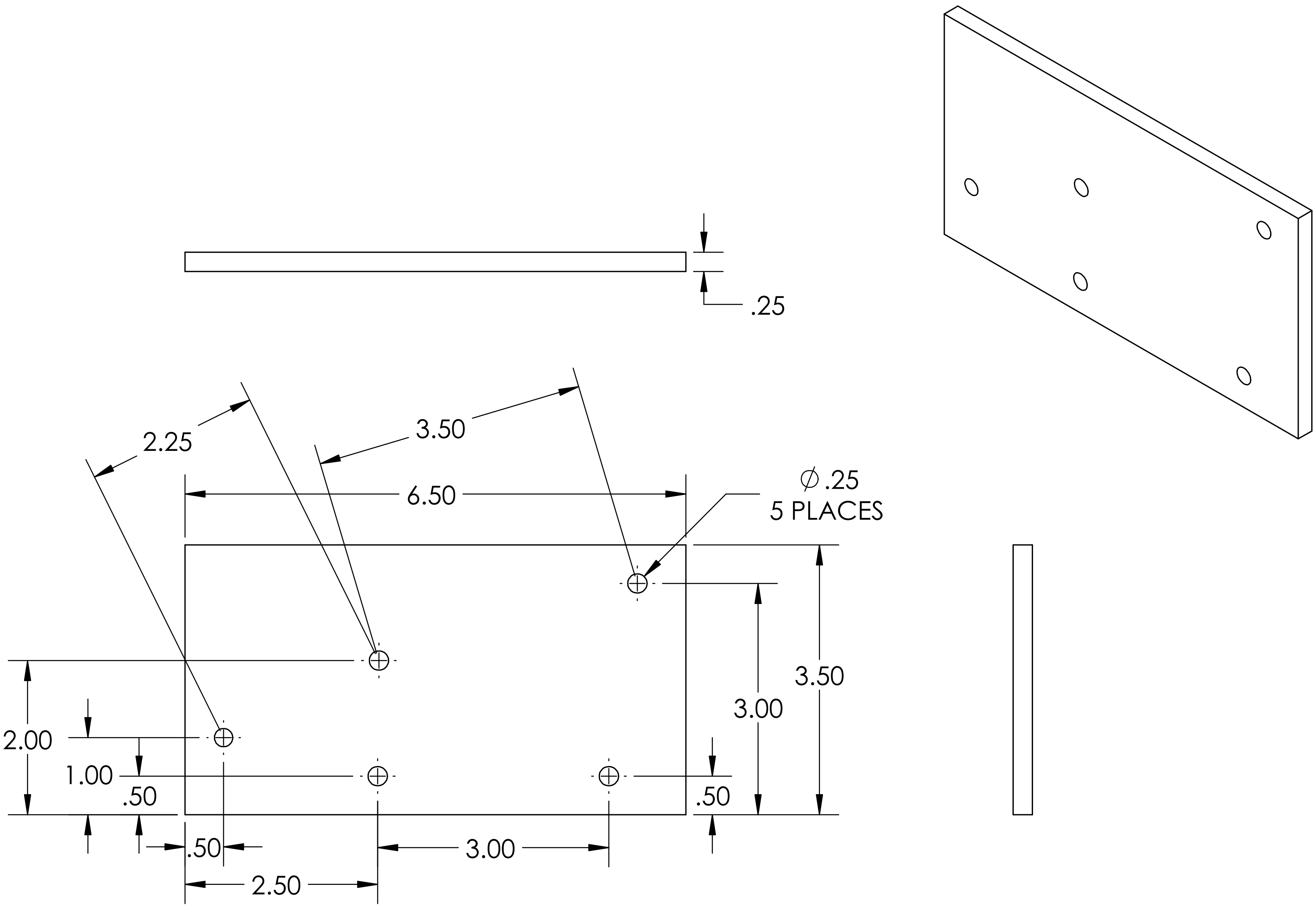
530.216 MBD LAB	TITLE: 60 Tooth Gear	UNITS: INCHES	SCALE: 1:1	MATERIAL: Acrylic	QTY. REQ.: 1
	TEAM: Craniacs	TEAM MEMBERS: Priu, Pardee, Williams	DATE: 4/2/21	DWG. NO.: 3	REV. NO.:



Pitch: 20
# of teeth: 110
Pitch Diameter: 5.5 in
Pressure Angle: 14.5 deg

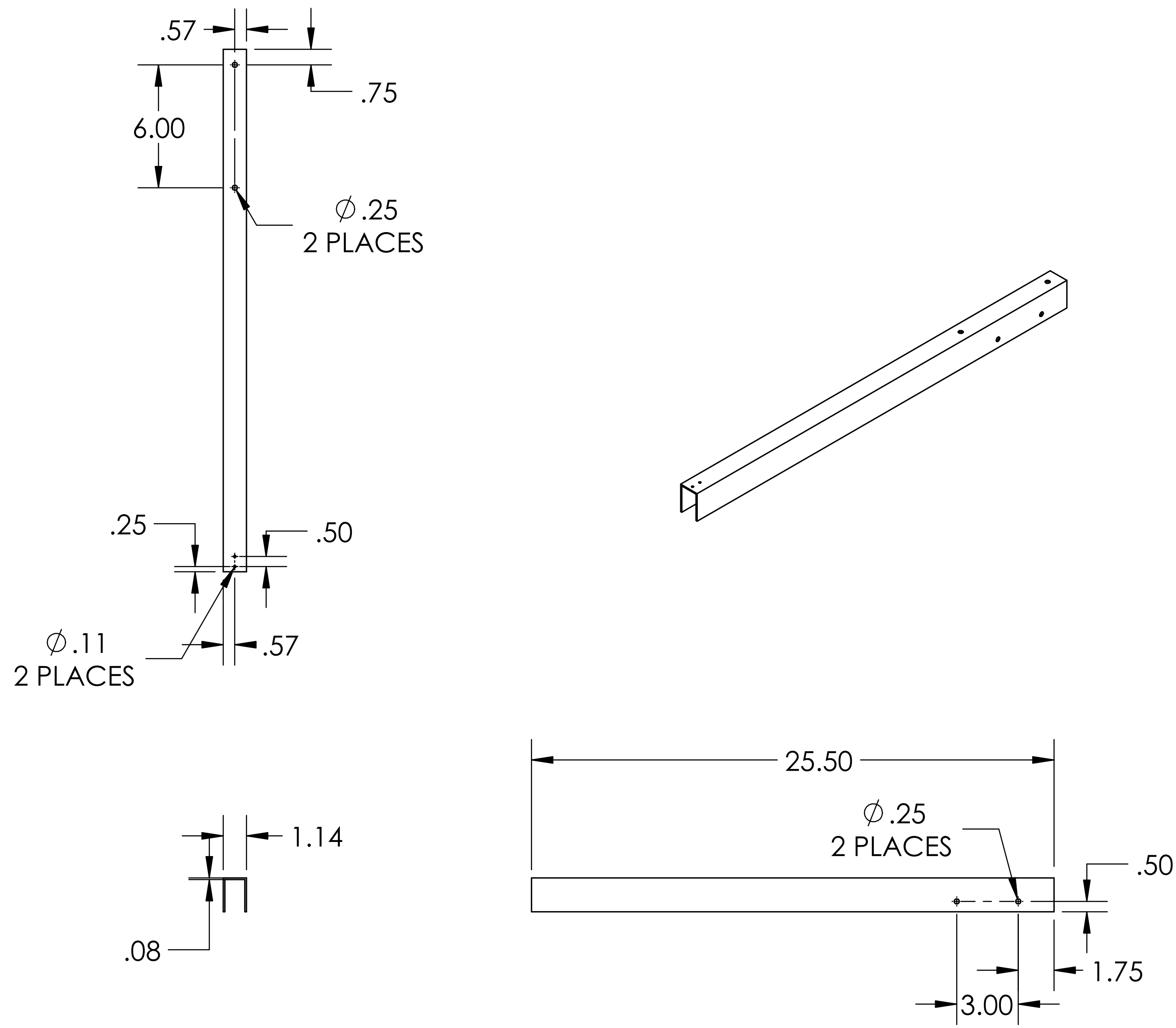
530.216 MBD LAB	TITLE 110 Tooth Gear	UNITS INCHES	SCALE 1:2	MATERIAL Acrylic	QTY. REQ. 1
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	DATE 4/21/21	DWG. NO. 4	REV. NO.

5 4 3 2 1



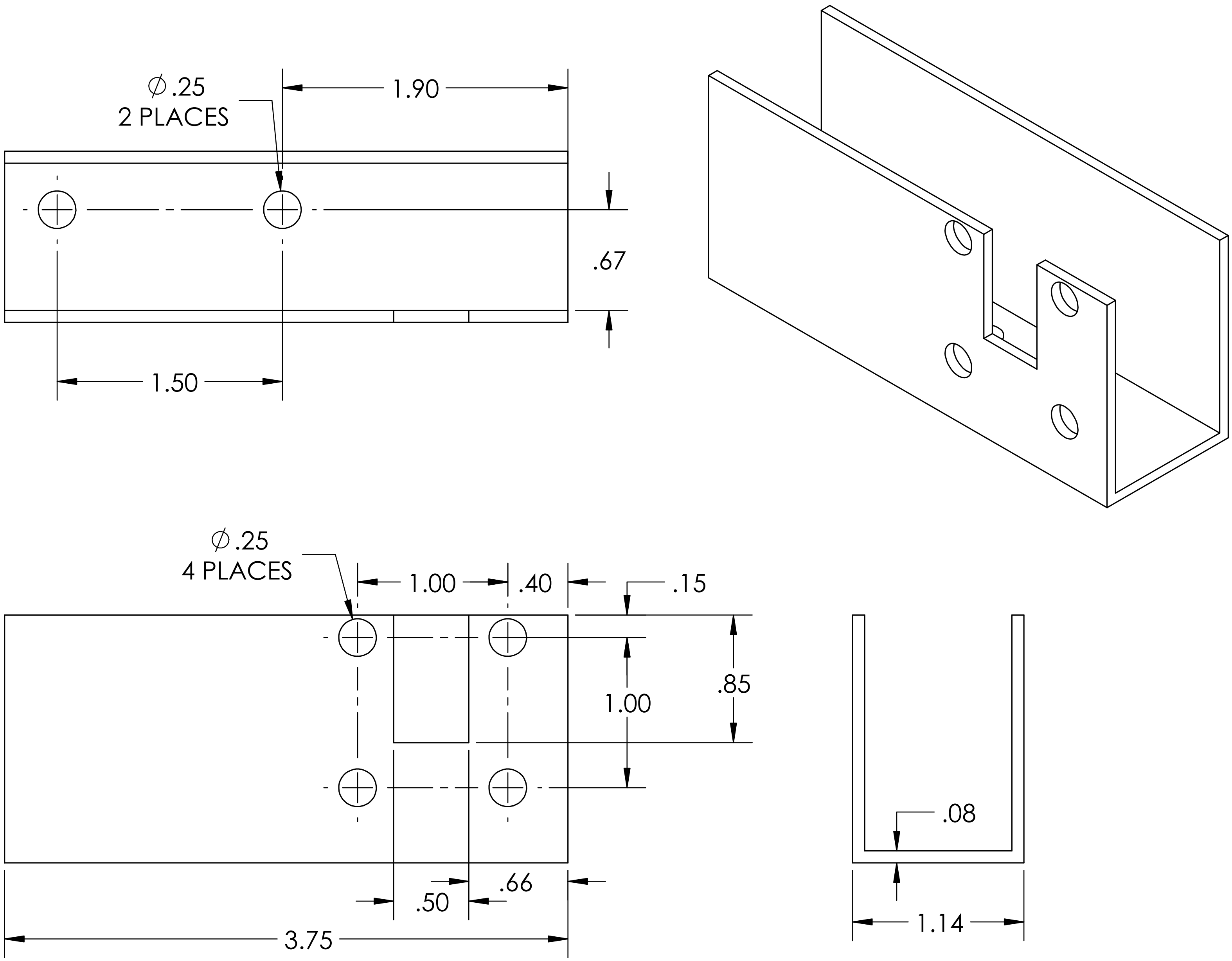
530.216 MBD LAB	TITLE Acrylic Gearplate	UNITS INCHES	SCALE 1:2	MATERIAL Acrylic	QTY. REQ. 2
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	DATE 4/2/21	DWG. NO. 5	REV. NO.

5 4 3 2 1



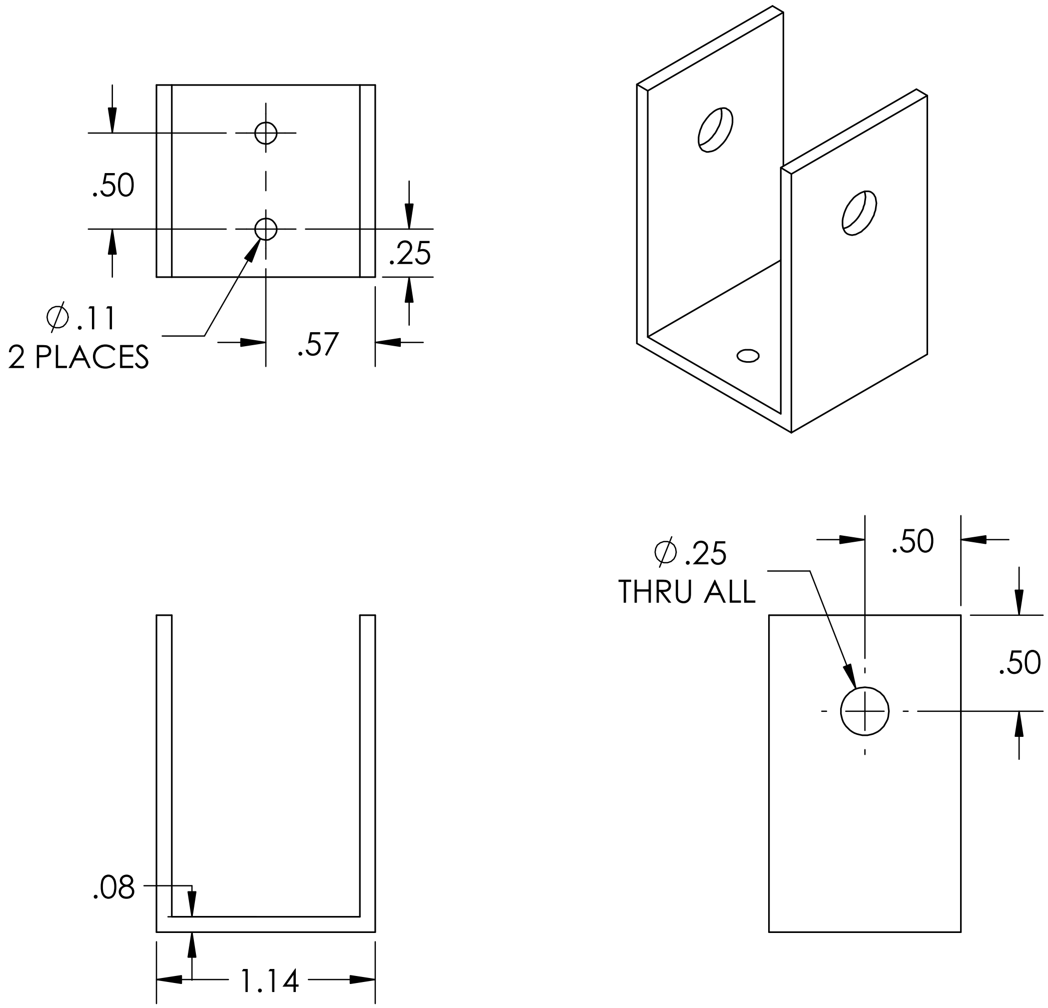
530.216 MBD LAB	TITLE Boom		UNITS INCHES	SCALE 1:8	MATERIAL PVC		QTY. REQ. 1
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams			DATE 4/2/21	DWG. NO. 6	REV. NO.

5 4 3 2 1



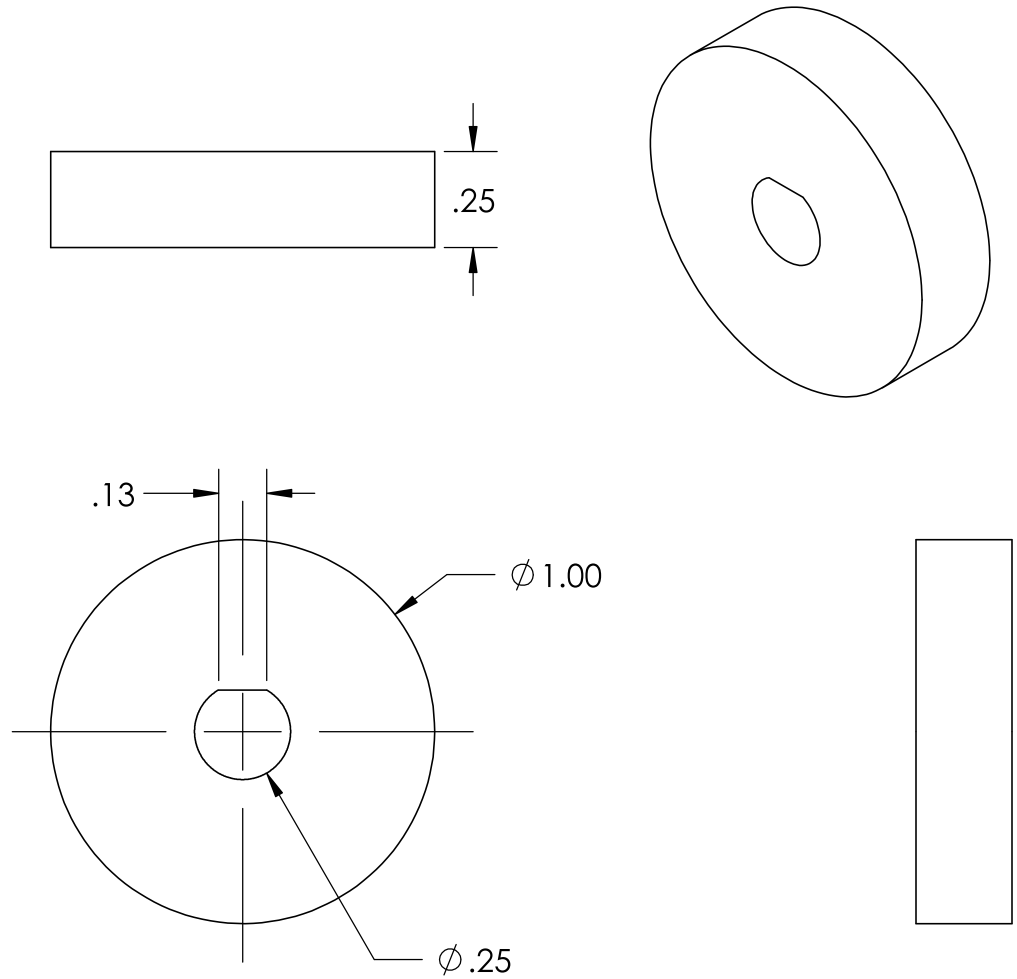
530.216 MBD LAB	TITLE 3.75 in PVC	UNITS INCHES	SCALE 1:1	MATERIAL PVC	QTY. REQ. 1
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	DATE 4/2/21	DWG. NO. 7	REV. NO.

5 4 3 2 1



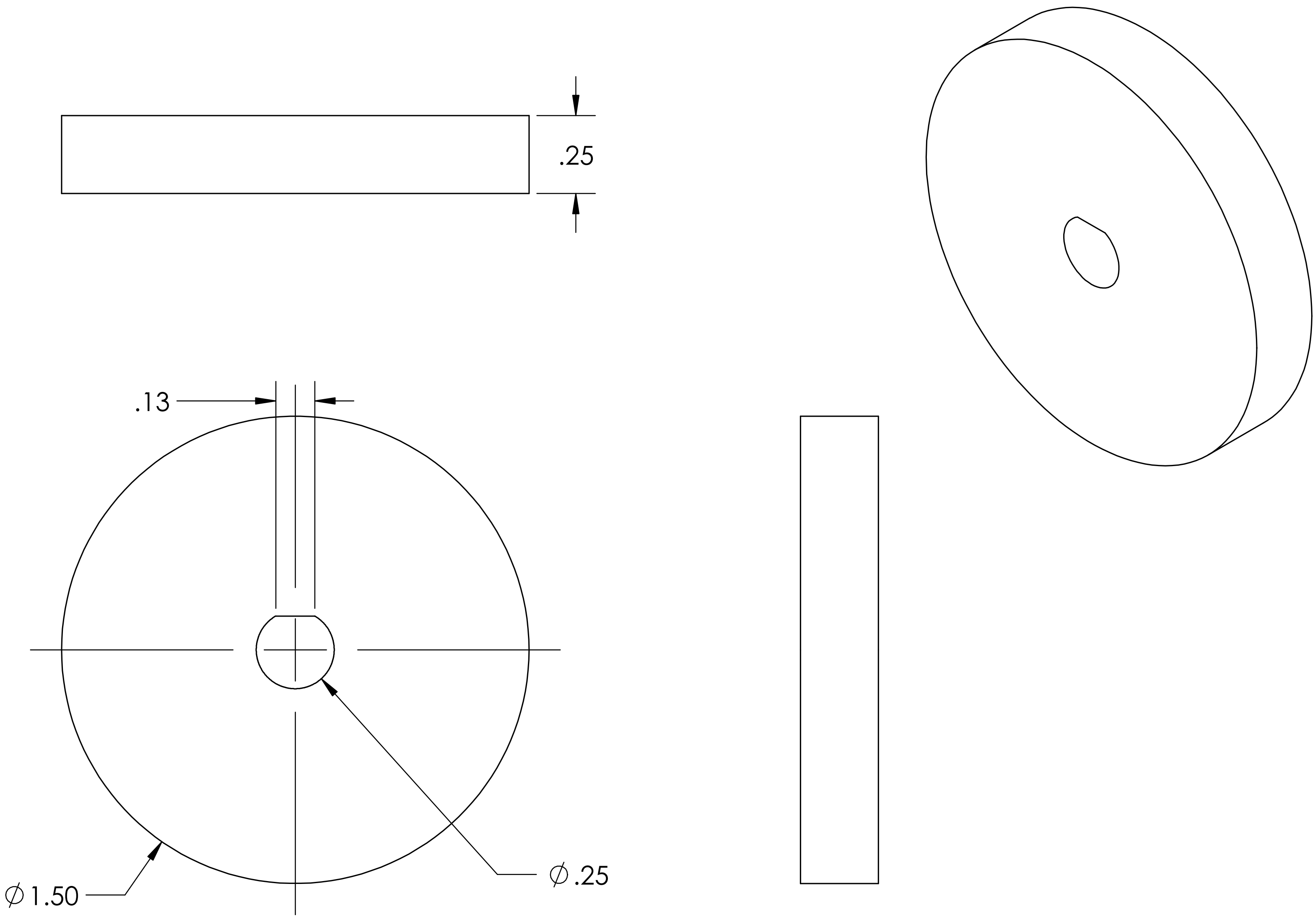
530.216 MBD LAB	TITLE: U-Bracket	UNITS: INCHES	SCALE: 1:1	MATERIAL: PVC	QTY. REQ.: 1
	TEAM: Craniacs	TEAM MEMBERS: Priu, Pardee, Williams	DATE: 4/2/21	DWG. NO.: 8	REV. NO.:

5 4 3 2 1

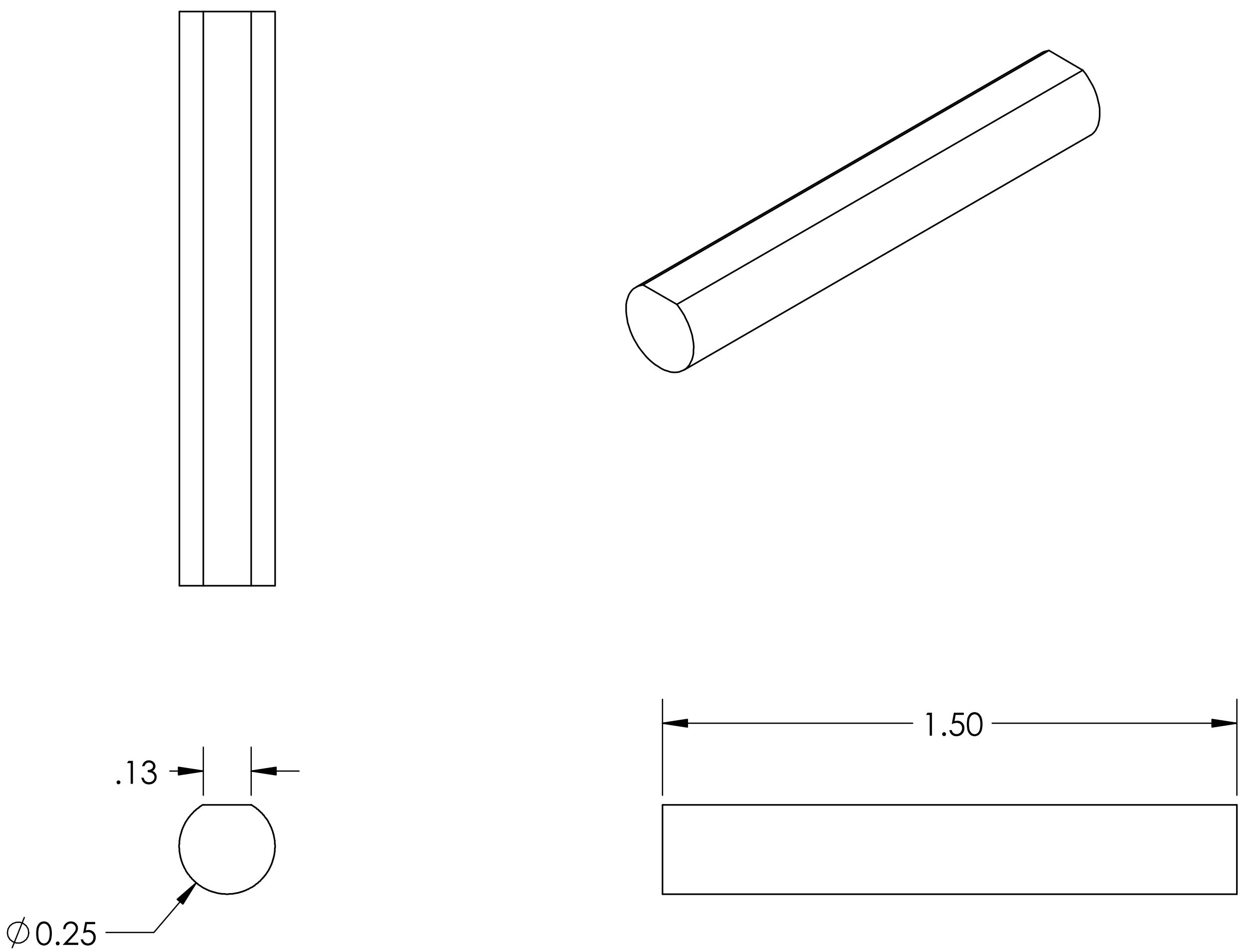


530.216 MBD LAB	TITLE Pulley Inner	UNITS INCHES	SCALE 2:1	MATERIAL Acrylic	QTY. REQ. 1
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	DATE 4/2/21	DWG. NO. 9	REV. NO.

5 4 3 2 1

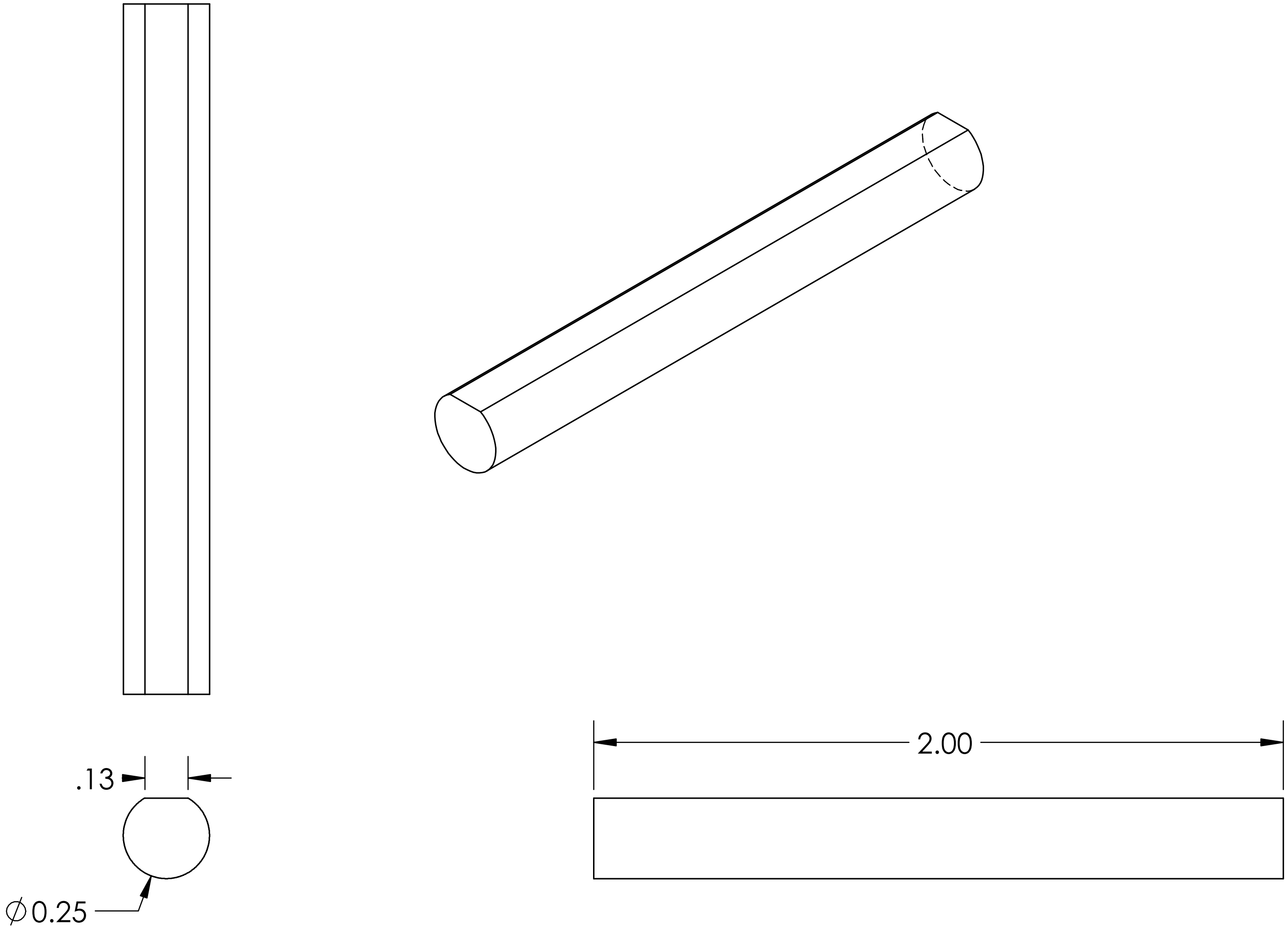


530.216 MBD LAB	TITLE Pulley Outer		UNITS INCHES	SCALE 2:1	MATERIAL Acrylic	QTY. REQ. 2
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	DATE 4/2/21		DWG. NO. 10	REV. NO.



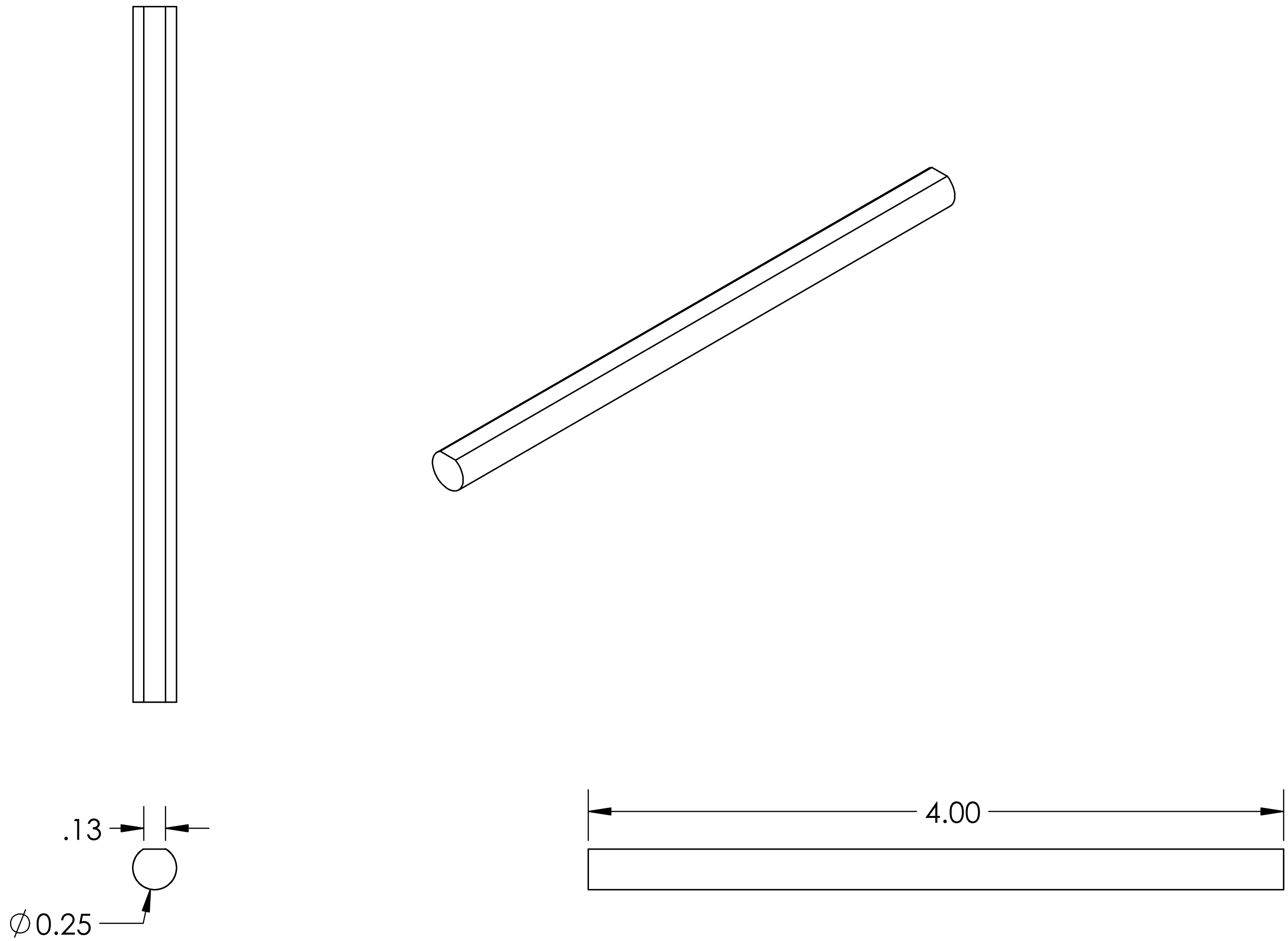
530.216 MBD LAB	TITLE: 1.5 in Shaft	UNITS: INCHES	SCALE: 2:1	MATERIAL: 1045 Carbon Steel	QTY. REQ.: 1
	TEAM: Craniacs	TEAM MEMBERS: Priu, Pardee, Williams	DATE: 4/2/21	DWG. NO.: 11	REV. NO.:

5 4 3 2 1



530.216 MBD LAB	TITLE: 2 in Shaft	UNITS: INCHES	SCALE: 2:1	MATERIAL: 1045 Carbon Steel	QTY. REQ.: 1
	TEAM: Craniacs	TEAM MEMBERS: Priu, Pardee, Williams	DATE: 4/2/21	DWG. NO.: 12	REV. NO.:

5 4 3 2 1



530.216 MBD LAB	TITLE 4 in Shaft	UNITS INCHES	SCALE 1:1	MATERIAL 1045 Carbon Steel	QTY. REQ. 2
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	DATE 4/2/21	DWG. NO. 13	REV. NO.

5 4 3 2 1

III. Structural Design Factor Explanations

We chose a design factor of 1.74 for the bending of the boom. We calculated this design factor using the five areas outlined in the *Design Factors* document:

- $DF_{material} = 1.1$
  - We obtained the Young's Modulus of the boom from the material properties listed for the part on McMaster Carr's website.
    - https://www.mcmaster.com/85065K46/
- $DF_{stress} = 1.2$
  - The boom will have the weight of the 10lb weight pulling down on its end, but also the weight of the pulley used to direct the string, the hook used to hold the weight, and the string used to pull the hook.
- $DF_{geometry} = 1.0$
  - According to the material properties listed on McMaster Carr's website, the tolerances for this U-Channel are tight. The tolerance for the wall thickness is +/- 0.01", the tolerance for the width is +/- 0.114", and the tolerance for the height is +/- 0.165".
- $DF_{failure\ analysis} = 1.1$
  - Our failure analysis was derived for our specific beam configuration.
- $DF_{reliability} = 1.2$
  - The reliability of our boom should be average.

IV. Transmission Design Factor Explanations

Using the five areas outlined in the *Design Factors* document, we chose a design factor of 2.2 for the transmission system.

- $DF_{friction} = 1.4$
  - We are unsure exactly how much friction will be present.
- $DF_{motor} = 1.1$
  - We obtained the specifications for this motor from the manufacturer.
- $DF_{load} = 1.2$
  - The load placed at the end of the beam could be slightly greater than 10 lbs if the weight of the hook is not considered part of the 10 lbs, or if the weight is greater than 10 lbs due to manufacturing tolerances.
- $DF_{reliability} = 1.2$
  - The reliability of our boom should be average.

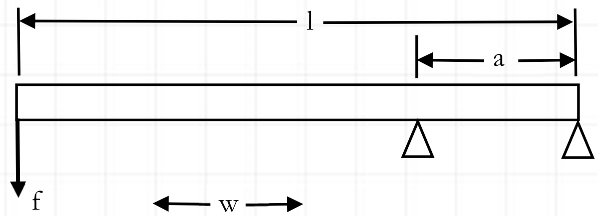
V. Design Calculations

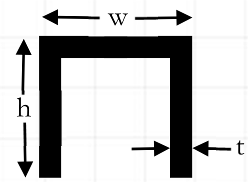
1. Boom Deflection
2. Maximum Bending Stress
3. Gear Ratio Calculations
4. Stress Concentration
5. Gear Tooth Bending Stress Analysis

JOHNS HOPKINS UNIVERSITY DEPT OF MECHANICAL ENGINEERING 530.216 MBD LAB	CALCULATION Boom Deflection		PAGE 1
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	OF 1

Knowns:

- Young's Modulus, E = 435 ksi (https://www.mcmaster.com/85065K46/)
- Thickness of boom walls, t = 0.08"
- Height of boom, h = 1.65"
- Width of boom, w = 1.14"
- Length of boom, l = 24.75"
- Distance between boom supports, a = 6"
- Weight of weight, f = 10 lb





Design factor: 1.74

This calculation will determine how much the boom will deflect given the known conditions and design factor. It must deflect less than 0.5".

Assumptions:

- Material properties given for the boom are accurate
- Uniform bending of boom
- Force from weight is straight down (weight isn't moving)

Calculations:

Cross sectional area of boom, A

$$A = 2ht + t(w - 2t)$$
$$A = 2 \times 1.65 \times 0.08 + 0.08(1.14 - 2 \times 0.08)$$
$$A = 0.3424\ in^2$$

Moment of Inertia of boom, I_y

$$I_y = \frac{t^3(w - 2t)}{3} + \frac{2th^3}{3}$$
$$I_y = \frac{0.08^3(1.14 - 2 \times 0.08)}{3} + \frac{2 \times 0.08 \times 1.65^3}{3}$$
$$I_y = 0.2397\ in^4$$

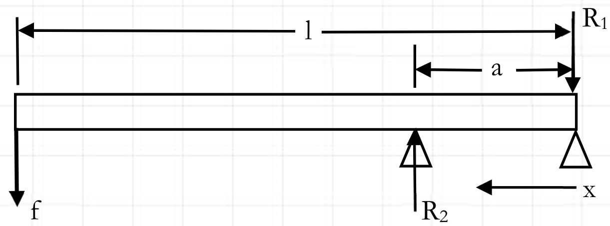
Deflection of boom, y

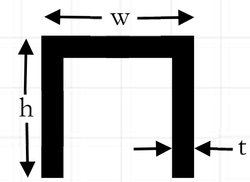
$$y = \frac{l(-1.74f(l - a)^2)}{3EI_y}$$
$$y = \frac{24.75(\text{-}1.74 \times 10\text{lb} \times (24.75" - 6")^2)}{3 \times 435000psi \times 0.2397in^4}$$
$$\boldsymbol{y = -0.4838\ in}$$

JOHNS HOPKINS UNIVERSITY DEPT OF MECHANICAL ENGINEERING 530.216 MBD LAB	CALCULATION Maximum Bending Stress		PAGE 1
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	OF 1

Knowns:

- Thickness of boom walls, t = 0.08”
- Height of boom, h = 1.65”
- Width of boom, w = 1.14”
- Length of boom, l = 24.75”
- Distance between boom supports, a = 6”
- Weight of weight, f = 10 lb
- I_y = 0.2397 in^4 (from previous calculations)
- Cross sectional area, A = 0.3424 in^2 (from previous calculations)
- Tensile yield strength, S_y = 7000 psi (https://www.mcmaster.com/85065K46/)





Design factor: 1.74

This calculation will determine the maximum bending stress in the boom.

Assumptions:

- Material properties given for the boom are accurate
- Force from weight is straight down (weight isn’t moving)

Calculations:

Distance between neutral axis and top of beam, y

$$y = \frac{1}{A}\left(\frac{t^2(w-2t)}{2} + th^2\right)$$
$$y = \frac{1}{0.3424\ in^2}\left(\frac{0.08^2(1.14\text{-}2\times0.08)}{2} + 0.08 \times 1.65^2\right)$$
$$y = 0.645\ in$$

Reaction force at end of beam, R_1

$$R_1 = \frac{fl}{a} - f$$
$$R_1 = \frac{10lb \times 24.75"}{6"} - 10lb$$
$$R_1 = 31.25\ lb$$

Maximum stress in boom, σ_{max} (occurs at x = a)

$$\sigma_{max} = DF \times \frac{-My}{I_y} = DF \times \frac{R_1 ay}{I_y}$$
$$\sigma_{max} = 1.74 \times \frac{31.25lb \times 6" \times 0.645"}{0.2397in^4}$$
$$\boldsymbol{\sigma_{max} = 877.9\ psi}$$

Since σ_{max} is less than S_y, the boom will not fail due to bending stress.

Knowns:

- Optimal Motor Torque, T = 6 oz-in
- Shaft Diameter, d = 0.25"
- Weight of weight, f = 10 lb

Design factor: 2.2

This calculation will determine the gear ratio needed for the transmission system to be able to lift the weight.

Assumptions:

- Motor properties given by manufacturer are accurate
- Friction included in the design factor

Calculations:

Amount of force that the motor can lift on its own, F

$$T = F\frac{d}{2}$$
$$6\ oz - in = F(0.125")$$
$$F = 48\ oz$$

Torque needed to lift the weight, τ

$$\tau = f \times DF \times \frac{d}{2}$$
$$\tau = 10\ lb \times 2.2 \times 0.125"$$
$$\tau = 2.75\ lb - in = 44\ oz - in$$

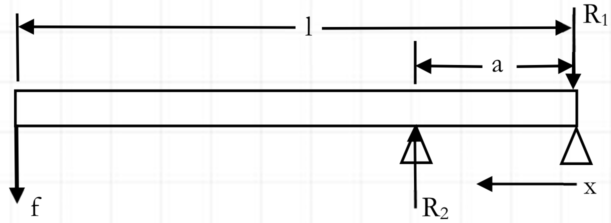
Required gear ratio, G

$$\tau = GT$$
$$44\ oz - in = G(6\ oz - in)$$
$$\boldsymbol{G = 22{:}\,3}$$

To fit this gear ratio, we have our motor go to a 30-tooth gear which connects to a 60-tooth gear, and on the 60-tooth gear's shaft is another 30-tooth gear which connects to a 110-tooth gear.

JOHNS HOPKINS UNIVERSITY DEPT OF MECHANICAL ENGINEERING 530.216 MBD LAB	CALCULATION Stress Concentration		PAGE 1
	TEAM Craniacs	TEAM MEMBERS Priu, Pardee, Williams	OF 1

Knowns:



- Thickness of boom walls, t = 0.08”
- Height of boom, h = 1.65”
- Width of boom, w = 1.14”
- Length of boom, l = 24.75”
- Distance between boom supports, a = 6”
- Weight of weight, f = 10 lb
- I_y = 0.2397 in^4 (from previous calculations)
- R_1 = 31.25 lb (from previous calculations)
- Stress at x = a, σ_a = 877.9 psi (from previous calculations)
- Diameter of screw hole, d = 0.25”
- Tensile yield strength, S_y = 7000 psi (https://www.mcmaster.com/85065K46/)

Design factor: 1.74

This calculation will determine the stress at the screw at x = a, which is a point of stress concentration.

Assumptions:

- Material properties given for the boom are accurate
- Force from weight is straight down (weight isn't moving)

Calculations:

Stress Concentration Factor, K_t

$$\frac{d}{w} = \frac{0.25"}{1.14"} = 0.219$$
$$\frac{d}{t} = \frac{0.25"}{0.08"} = 3.125$$

From chart A-15-2, $K_t \sim 1.6$

Maximum stress at x = a, σ_A

$$\sigma_A = K_t\sigma_a = 1.6 \times 877.9$$
$$\boldsymbol{\sigma_A = 1404.6\ psi}$$

σ_A is less than the yield strength, S_y = 7000 psi, so the beam does not fail.

Knowns:

- Pitch, P = 20 teeth/in
- Motor torque, T = 6 oz-in
- Number of teeth for each gear, N_{30} = 30 teeth, N_{60} = 60 teeth, N_{110} = 110 teeth
- Pitch diameter for each gear, d_{30} = 1.5 in, d_{60} = 3 in, d_{110} = 5.5 in
- Face width, b = 1 in
- Pressure angle, φ = 14.5°
- Lewis Form Factor for each gear, Y_{30} = 0.318, Y_{60} = 0.355, Y_{110} = 0.370 (https://www.engineersedge.com/gears/lewis-factor.htm)
- Yield strength of acrylic, S_y = 10000 psi (https://www.mcmaster.com/8505K734/)

Design factor: 2.2

This calculation will determine the bending stress on the teeth of each gear.

Assumptions:

- Motor properties given by manufacturer are accurate
- Material properties given for the acrylic gears are accurate

Calculations:

First, the 30-tooth gear that is connected to the motor shaft (gear 30a)

Tangential force, F_{30a}^{t}

$$F_{30a}^{t} = \frac{T}{\frac{1}{2}d_{30}} = \frac{6\ oz - in}{0.5(1.5\ in)} = 8\ oz$$

Bending stress in 30 tooth gear, $\sigma_{30a}^{bending}$

$$\sigma_{30a}^{bending} = DF \times \frac{F_{30a}^{t}P}{bY_{30}} = 2.2 \times \frac{8\ oz\left(20\,\frac{teeth}{in}\right)}{1\ in\ (0.318)} \times \frac{1\ lb}{16\ oz}$$
$$\boldsymbol{\sigma_{30a}^{bending} = 69.19\ psi}$$

$\sigma_{30a}^{bending}$ is less than S_y, so the gear does not fail.

Next, the 60-tooth gear

The 60-tooth gear experiences the same magnitude of bending stress as the 30(a)-tooth gear:

$$\boldsymbol{\sigma_{60}^{bending} = 69.19\ psi}$$

Next, the 30-tooth gear that shares a shaft with the 60-tooth gear (gear 30b)

We can find the torque for the 30-tooth gear using the result for the stress in the 60-tooth gear

$$\sigma_{60}^{bending} = DF \times \frac{\frac{T_{60}}{\frac{1}{2}d_{60}} P}{bY_{60}}$$

$$T_{60} = \frac{\sigma_{60}^{bending} bY_{60} d_{60}}{DF \times 2P}$$

$$T_{60} = \frac{(69.19\, psi)(1\, in)(0.355)(3\, in)}{2.2(2)(20\frac{teeth}{in})} = 0.837\, lb - in$$

Tangential force, F_{30b}^{t}

$$F_{30b}^{t} = \frac{T}{\frac{1}{2}d_{30}} = \frac{0.837\, lb - in}{0.5(1.5\, in)} = 1.116\, lb$$

Bending stress in 30 tooth gear, $\sigma_{30b}^{bending}$

$$\sigma_{30b}^{bending} = DF \times \frac{F_{30b}^{t} P}{bY_{30}} = 2.2 \times \frac{1.116\, lb\left(20\frac{teeth}{in}\right)}{1\, in\,(0.318)}$$

$$\boldsymbol{\sigma_{30b}^{bending} = 154.4\, psi}$$

$\sigma_{30b}^{bending}$ is less than S_y, so the gear does not fail.

Finally, the 110-tooth gear

The 110-tooth gear experiences the same magnitude of bending stress as the 30(b)-tooth gear:

$$\boldsymbol{\sigma_{110}^{bending} = 154.4\, psi}$$

None of the gear tooth bending stresses exceed the yield strength, S_y, so our gears will not fail.

VI. Cost Analysis

The total cost of the crane, excluding the gearmotor assembly, was $42.82. A breakdown of each item used in the crane and its cost can be seen in *Table 1* below.

Part	Part Number	Quantity Used	McMaster Price ($)	McMaster Quantity	Total Cost ($)
Boom	85065K46	26.5 (in)	24.43	60 (in)	10.79
1/4-20 Bolts (2")	91257A550	4	10	50	0.80
1/4-20 Nuts	90490A029	2	2.48	100	0.05
1/4-20 Bolts (3/8")	92240A535	6	3.94	50	0.47
4-40 Bolts (1/4")	90316A211	2	4.73	100	0.09
4-40 Nuts	90480A005	2	0.89	100	0.02
12" D-Shaft	8632T139	1	9.74	1	9.74
Shaft Collars	6432K12	9	1.12	1	10.08
1/4"x12"x12" Acrylic	8505K734	1	10.78	1	10.78
				Total cost:	42.82

Table 1. The breakdown of the cost of each part used to construct the crane. The McMaster Price is the price that McMaster-Carr charges for the specified McMaster Quantity of each item (for example, McMaster sells 50 ¼-20 bolts with a length of 2" for $10). Quantity Used is the amount of each part that we used on our crane.

VII. Team Member Contributions

Elayna Williams - In the initial planning process for this design, I performed calculations for the problem set to be compared with other group members. Once we reached an agreement on how to perform these calculations, I researched some different plastic materials that could be used to make up the boom. I designed a hollow rectangular boom with specific dimensions and material properties found on the McMaster-Carr website. I also sketched a 4-pulley transmission that we were looking into utilizing. Lastly, I assisted in performing the actual deflection and bending stress calculations for our boom and its specific properties. In this report, I wrote the boom and structural support system description, and reviewed all other parts.

Katharine Priu - I worked on the original design for the U-channel boom design and the gear transmission system. I did some of the beam stress and deflection calculations when prototyping the original design. Since we chose the designs that I prototyped as our final design, I already had the design factors done from the prototyping process, and I updated them as our assumptions changed. I also did the gear ratio and stress calculations because they were part of the gear design, and I worked on the final stress concentration calculations. I created the CAD parts and the assembly, as well as the drawings, and I did the cost analysis because it went along with the CAD.

Sara Pardee - I did calculations for the problem set, which we reviewed as a group. I designed an acrylic, tube shaped boom that I presented to the team. I chose the dimensions and materials by referring to McMaster-Carr, and later looked into supporting these decisions with calculations. I sketched a three pulley transmission system that we discussed when brainstorming potential designs. I performed calculations for the deflection, bending stress, and torque values. We then reviewed and compared our answers for these calculations. For the report, I wrote the power transmission and lifting mechanism description, and reviewed the other sections.

We did most of the design planning and decision making as a group. Our team met often to select the materials, boom shape and dimensions, and the transmission design together. After individual members of the group did the appropriate calculations for the design, we came together to review them with each other. In addition, every member reviewed one another’s work to catch any errors or ask questions about our design and planning.