EN.530.212 MECHANICAL ENGINEERING DYNAMICS LABORATORY WATER BOTTLE ROCKET SIMULATION PROJECT

Submitted by Katharine Priu, Zi Chang Rao

TA: Nathaniel Davenport, Lab section 3 (Wednesday 4:00-6:00pm)

INTRODUCTION & PROJECT SUMMARY

In this project, we developed a physics-based numerical simulation for the flight of a water bottle rocket. The simulation is capable of estimating thrust duration, maximum thrust force, and maximum altitude using only ullage pressure and the initial mass of the water as input parameters. Accuracy of the simulation was evaluated using root-mean squared error of actual maximum height from predicted maximum height. Excluding otherliers, this value was 11.7ft.

The simulation was coded in MATLAB and built around a variation of the rocket equation presented in Hibbeler (2016), which took into account the effect of drag and variable thrust. Euler's method was used to numerically integrate predicted acceleration values to obtain velocity and displacement estimations. Key assumptions were that both the water and the pressurized air inside the rocket behaved adiabatically--that is--no heat transferred occurred, and the expansion and flow were lossless.

MATERIALS

The following materials were used for experimentally validating our simulation:

- Bottle Rocket kit (commercially available through Amazon)
- Bluetooth load cell & altimeter
- Digital scale, tape measure, and micrometer
- Barometer
- DeWalt electric driver and compressor attachment

The simulation itself was coded in Matlab R2019b, and requires no external add-ons or the installation of third-party libraries to use. There is a crude built-in command line interface for the input of simulation parameters (ullage pressure and inputted water mass).

METHODS

Before testing the actual rockets, we made predictions about the rocket's behavior and wrote a preliminary Matlab program that calculated the expected maximum height given the starting ullage pressure and water mass. We started by figuring out what forces would be acting on our rocket: gravity, drag, and thrust. We then derived equations to relate these forces to the rocket's height (see *Theoretical Calculations*).

We next performed several launches using a commercially-available bottle rocket kit so that we can evaluate the validity of our simulation. The launches were done outside and away from others for safety. We launched the rockets six times using six different initial amounts of water. To get the rocket ready for launch, we poured the water into the rocket and then taped the altimeter to the rocket's nose. The altimeter sat in a styrofoam hemisphere to prevent it from being damaged. This was necessary because the rocket had no recovery systems, so it would land by slamming nose-first into the ground. Once the rocket was ready for launch, we found the initial water mass using the scale. To launch the rocket, we connected the compressor to the opening in the bottle and increased the pressure using a drill-mounted air compressor until the rocket launched. The compressor included a pressure gauge which we filmed in slow-motion during the launch to obtain necessary pressure data for the validation of our simulation.

Using the experimental data, we were able to test our predictions and alter our model in order to output values closer to that measured by experiments. We continued this iterative process until we were left with a model that could accurately predict maximum height of a rocket, as well as other factors.

MEASUREMENTS

Prior to beginning any modelling (or experimentation), the following values of the bottle rocket were measured using the materials described in the *Materials* section. The simulation needed these values as input constants.

Table 1: Initial Measurement of Bottle Rocket

Measurement	**Amount**
Diameter of bottle	85 mm
Diameter of nozzle	21.5 mm
Length of rocket body	24 cm
Dry mass of rocket	76 g
Mass required to fill bottle completely	1238 g
Volume of rocket	0.001242 m^3

Ambient pressure	101300 Pa
Ambient temperature during launches	282.4 K

There aren't many additional comments we want to make about these measurements, except that the volume of the rocket was determined by filling the rocket with water and dividing the increase in mass by the water's density, which rendered it perhaps not as accurate as the other measurements listed above.

THEORETICAL CALCULATIONS

I. Free Body Diagrams

Before diving into calculations, let us first review the free-body-diagrams (FBDs) of the problem. For the water bottle rocket, we identified three key phases of its launch and travel, based on the three different sets of forces the rocket will be experiencing. The first of these phases is the thrust-phase, where the rocket is accelerating upwards due material exiting from its nozzle. The FBD of the thrust phase is as follows:

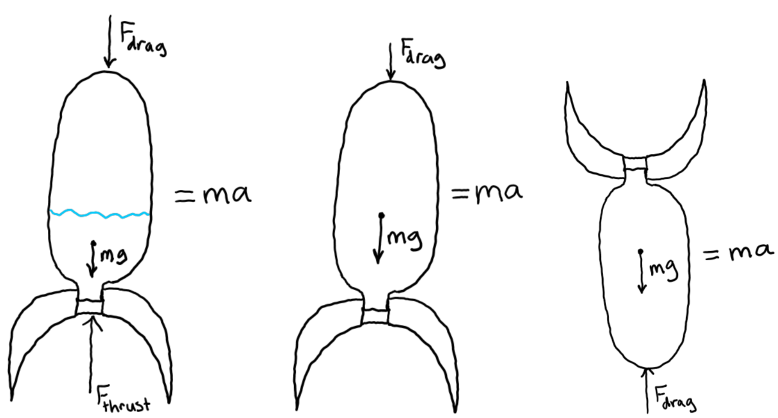

Figure 1. FBD of water bottle rocket rocket during the thrust phase (left), ascent phase (middle), and free fall phase (right).

The thrust phase can be further broken into the water-impulse phase, and the air impulse phase. The naming of the phases indicates the dominant material exiting the nozzle.

The second phase is the upward-trending phase of the rocket. This occurs after the thrust phase, where the motion of the rocket is dominated by drag and gravity, as opposed to by thrust. Here, drag and gravity have the same sense: they both act to reduce the speed of the rocket.

The third phase is the freefall phase, or the descent phase. We make a distinction between the freefall phase and the ascent phase because here, the drag force has the same sense as gravity, which needs to be accounted for in the simulation.

II. Equations and Derivations.

In this section, we will discuss the equations used in our simulation and where the equations originated from. We will state the assumptions we made when choosing and deriving equations with brief discussions of the validity of these assumptions.

First, we must list equations from which elementary properties of our bottle can be calculated. The first of which is the cross-sectional area of the bottle, which we found using the following:

$$A_{top} = \pi \frac{d_b^2}{4} \tag{1}$$

where d_b is the diameter of the bottle. The identical equation was used to compute the cross-sectional area of the nozzle, or, the exit of the bottle:

$$A_e = \pi \frac{d_e^2}{4} \tag{2}$$

where A_e is the cross sectional area of the exit of the bottle and d_e is the exit diameter.

The velocity of the bottle we will determine using numerical integration, and follows directly from the its definition: $v = \int_{t0}^{t1} v_{dot} dt$.

$$\therefore \quad v(t + dt) = v(t) + v_{dot}(t + dt) \times dt \tag{3}$$

where v is the velocity, t is the current timestep, t+dt is the next time step, dt is the change in time, and v_{dot} is the acceleration of the bottle. In practice, dt was set to some small rational number, such as 0.0001.

The vertical displacement of the bottle also follows directly from its definition of $x = \int_{t0}^{t1} v dt$:

$$x(t + dt) = x(t) + v(t + dt) \times dt \tag{4}$$

where x is the vertical displacement. The reader should note that, in the MATLAB script (attached in the Appendix and submitted as a separate .m file), vertical displacement was denoted using the variable d. (Not to be confused with d_e, the diameter of the nozzle exit).

The drag force on the bottle, $F_d(t)$, was calculated using the following expression:

$$F_d(t) = c_d A_{top} \frac{\rho v(t)^2}{2} \tag{5}$$

where c_d is the drag coefficient, A_{top} is the cross-sectional area of the top of the rocket, $v(t)$ is the velocity of the rocket with respect to ground, and ρ is the density of the outside air. We have already discussed how $v(t)$ will be calculated. There will be a more in-depth discussion on the procurement of C_d, the drag coefficient, in the *Discussion* section of this report.

The force of gravity on the bottle was calculated using the following equation:

$$F_g = m_{total} g \tag{6}$$

where m_{total} is the total mass of the rocket and the water and g is the acceleration due to gravity. The equation, once again, follows directly from fundamental laws. It should be noted that m_{total} is not a fixed quantity, and changes with respect to time.

Our expression for the exit velocity of the water followed directly from Bernoulli's equation for incompressible, lossless flow, which stated the following:

$$P_1 + 0.5 \times \rho u_1^2 + \rho g h_1 = P_2 + 0.5 \times \rho u_2^2 + \rho g h_2 \tag{7}$$

Here, u is fluid velocity, P is pressure, and h is the height with respect to any frame. The subscripts indicate different spatial points in the flow where the properties u, P, and h are measured. Note that u, in this case, is measured with respect to the bottle, i.e. it follows a reference frame centered at the rocket. Let subscript 1 denote the water-air boundary and let subscript 2 denote the conditions immediately following the exit of the nozzle, and set $h_2 = 0$ (since we are using the bottle as our reference frame anyways). We make the assumption that $u_1 = 0$ simply because we would be left with an under constrained system if u_1 is also undetermined. We further assume that $h_1 \approx h_2$ because the differences in gravitational energies of the water mere centimeters apart will contribute negligibly to its energy density when compared with the energy difference caused by the pressure gradient. If we further assume a quasi-static mechanical equilibrium at the water-gas boundary and between water and the atmosphere at the nozzle exit, we can state $P_1 = P_{gas}$ and $P_2 = P_0$. The equation was then rearranged into the following form to facilitate the calculation of u_2, which was needed for computing the thrust force.

$$u(t) = \sqrt{\frac{|2(P_{gas}(t) - P_o)|}{\rho_{water}}} \tag{8}$$

where u is the exit velocity, P_{gas} is the pressure of the air inside the bottle, P_o is the atmospheric pressure, and ρ_{water} is the density of water, which we took to be 1000 kg/m^3. The absolute value was put into place to prevent Matlab from taking the square root of a negative number, which may happen near water depletion for the water-impulse phase.

We found the mass flow rate of the water exiting the bottle using the following equation:

$$c(t) = A_e \times \rho_{water} \times u(t), \tag{9}$$

where c is the mass flow rate of the water. This equation simply originated from mass conservation: the mass flow rate of an object with density ρ across a cross-sectional area A with velocity u must be the product of those three quantities.

The thrust force on the rocket during the water expulsion phase was as follows:

$$F_{thrust} = u(t) \times c(t), \tag{10}$$

which follows directly from the rocket equation, as presented in Hibbeler (2016). This equation was derived using the law of conservation of momentum

$$F_{thrust} = m_{dot}v + v_{dot}m \,, \tag{11}$$

where F_{thrust} is the force caused by the thrust. Since we are assuming that the water is leaving the rocket at a constant velocity (at least during narrow time slices, and due to the constant shape of the nozzle), this equation can be simplified to

$$F_{thrust} = m_{dot}v \,, \tag{12}$$

where m_{dot} is the mass flow rate of water out of the rocket and v is the velocity of the water exiting our rocket, measured with respect to the rocket. Using our notation, we recover (10) as shown above.

The mass of the water is determined using the following equation:

$$m_{water}(t + dt) = m_{water}(t) - c(t) \times dt \,, \tag{13}$$

where m_{water} is the mass of the water. We subtract from the current mass the mass efflux of water, which is calculated by multiplying our mass flow rate with our dt to often the water mass at the next time step. Again, this equation is nothing more than a restatement of mass conservation for a control volume.

We used the following equation to calculate the volume of the air inside the bottle:

$$V_{air}(t) = V_{rocket} - \frac{m_{water}(t)}{\rho_{water}} \,. \tag{14}$$

Here, $V_{air}(t)$ describes the volume of air inside the bottle at time t. We simply subtract the volume of water remaining in the rocket from the total volume of the rocket to obtain the above expression. It's worth noting that, once all the water has been expelled, this equation reduces to $V_{air}(t) = V_{rocket}$.

The density of the air inside the bottle is found using the following:

$$\rho_{air}(t) = \frac{m_{air}(t)}{V_{air}(t)}\,, \tag{15}$$

where ρ_{air} is the density of the air. This simply followed from the definition of density.

The air pressure of the gas during the water-impulse phase was estimated by assuming that the air expands adiabatically during this phase. According to Wheeler (2002), the rate of convective heat transfer between the surroundings and the material inside the bottle is on the order of seconds, while the thrust phase of both water and air are on the order of deci-seconds. This suggests that it is permissible to assume adiabatic expansion. We will comment on the validity of this assumption further in the discussion section. Knowing this, we can write the following expression for P_{gas}

$$P_{gas}(t + dt) = P_{gas}(0) \times \left(\frac{\rho_{air}(t)}{\rho_{air}(0)}\right)^{\gamma}\,, \tag{16}$$

which can be derived from thermodynamic principles for isentropic expansion. These principles are outside the scope of this report and a derivation will not be shown. Here, $P_{gas}(0)$ is the original internal pressure of the air and $\rho_{air}(0)$ is the original density of the air. “Original” here refers to at time t=0. We used this to calculate the air pressure of the gas inside the rocket during the water-impulse portion of the thrust phase.

The air pressure of the gas during the air-impulse phase was estimated using the following expression:

$$P_{gas}(t + dt) = \frac{m_{air}(t+dt) \times R_{air} \times T}{V_{rocket}}\,, \tag{17}$$

where R_{air} is the ideal gas constant for air (we used a value of 287m^2kgs^-2K^-1mol^-1) and T is the ambient temperature. For this phase, we are assuming that the air is behaving like an ideal gas. This assumption is valid because air’s critical pressure is much higher than P_0, and its critical temperature is orders of magnitude beyond what can be possibly achieved in a PET soda bottle on a Wednesday afternoon. Regardless, the air-impulse phase is very short. Removing the consideration of air impulse only affects our results negligibly (see *Discussion*). Note that equations (9) through (12) were still used to estimate the thrust from the air, since the equations themselves applied to any material exiting a control volume, and is not restricted to only liquids.

Last, the air pressure of the gas inside the rocket during free-fall phase was set as equal to the atmospheric pressure. The value of this parameter has no numerical significance at this point since we no longer need it to calculate thrust, so we just set it to be equal to P_0. We did this because the air should, at this point, be in mechanical equilibrium with its surroundings.

We found the rocket’s acceleration using Newton’s second law

$$\Sigma F = ma$$

$$\Sigma F = F_{thrust} - F_g - F_d$$

$$\therefore \quad a = v_{dot} = \tfrac{1}{m}(F_{thrust} - F_g - F_d) \tag{18}$$

These equations hold true even during the free-fall phase of the rocket, where F_{thrust} is zero, and where F_d will be opposite in sense to F_g.

III. Predicted Values

In this section, we will present the predictions for maximum height, thrust duration, maximum thrust, and various other values using the model we described in the previous section.

Table 2: Predicted Values for Maximum Height (ft) vs Ullage Pressure and Initial Water Mass

Launch Pressure (psi)	**Water Mass (g)**	**Maximum Height (ft)**	**Thrust Duration**	**Pred max thrust**
33	890	53.18	0.332	334.58
32	380	101.96	0.043	322.59
31	216	90.2	0.024	303.46
27.5	508	86.12	0.075	236.62
28	320	88.63	0.041	246.51
29	500	91.8	0.069	263.68

Where the thrust duration refers to the thrust duration of the water-impulse phase. As expected, the model predicted a greater maximum height for greater launch pressures, assuming near-constant water mass (compare the data from row 2 to that of row 5 for an example). The launch pressures used were reflective of the actual launch pressures from experiments.

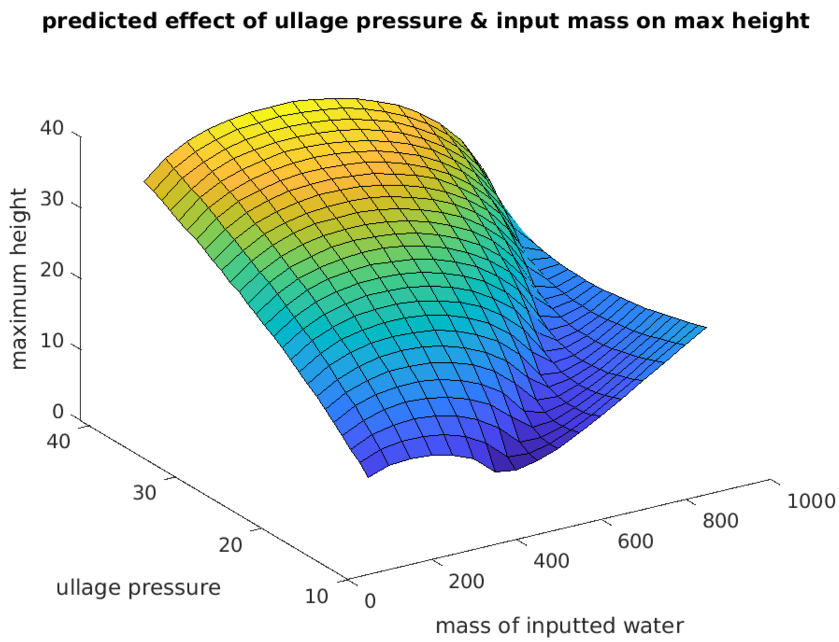

Figure 2: Height Surface of Simulated Bottle Rocket

From this plot, the highly non-linear relationship between max height, mass of inputted water, and ullage pressure can be visualized. This plot offers a good way to visualize the effect of changing parameters of the simulation can have on the maximum height of the rocket. For example, decreasing the drag on the rocket will shift the plot up:

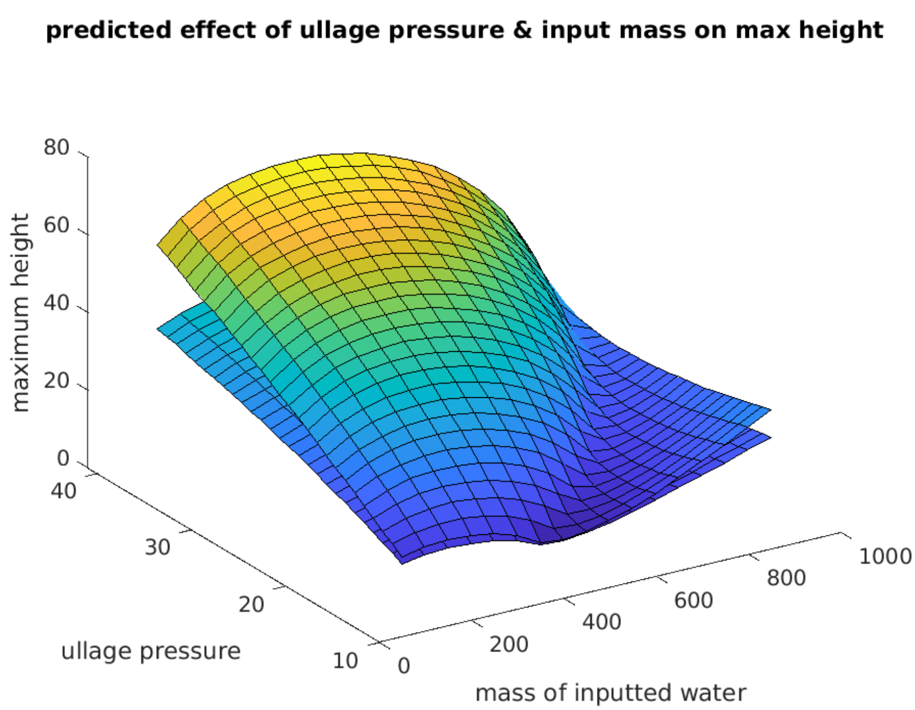

Figure 3: Effect of Decreasing Drag on Height Surface

We can also see the effects of changing the density of the propellant, or increasing the ambient temperature, or other such parameters. However, it should be noted that, since our model represents reality rather poorly, the conclusions drawn from this model may not be applicable to real life. However, several interesting insights can still be gleaned from it. For example, regardless of the pressure, the optimum mass of inputted water is around 450g. However, it is worth noting (as we will mention in detail later) that the launch pressure and the mass of inputted water is not decoupled from each other (at least, according to experiments).

In addition to plotting the maximum height versus input parameters, the simulation is also capable of generating displacement v time and thrust v time plots for individual simulations. Below, we give some example plots of displacement vs time, as well as thrust vs time, for the rocket. These plots were generated using the built-in Matlab plot functions.

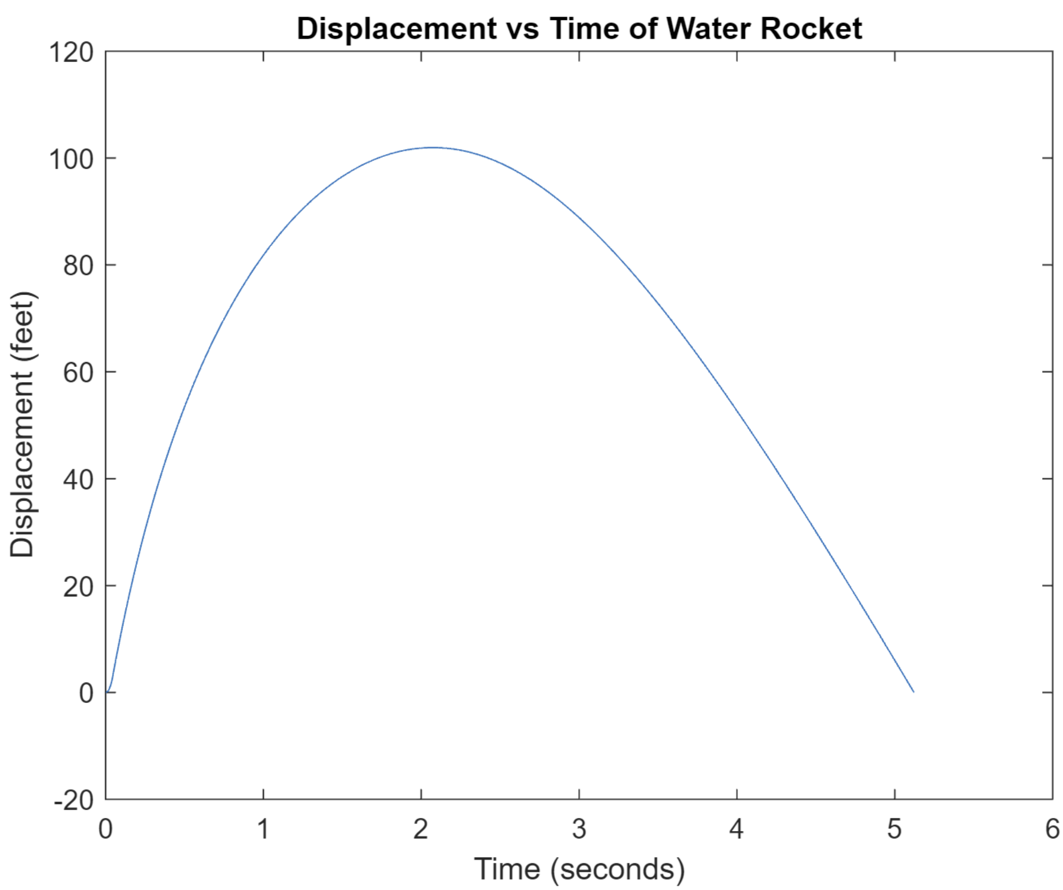

Figure 4. The predicted displacement vs time plot for trial 2

The displacement vs time plot is shown above. It is interesting to observe how this plot does not behave like a perfect parabola, as displacement time plots should if the acceleration is constant. We can see that the initial slope for the displacement is quite large, and that the rocket actually reaches its maximum height before the halfway time (from landing). Even though this is only a predicted simulation result, this is intuitive, and matches what we expect to happen (for comparison of displacement - time curves, see *Discussion*). This is because we expect displacement to increase rapidly while thrust is being provided, and expect it to decrease slowly during free fall phase due to the presence of drag.

We can see hints of the rocket beginning to reach terminal velocity when the slope of the displacement-time graph approaches a flat time past t=4. It is interesting that terminal velocity is reached so soon; however, we suspect this happens because we set a large drag coefficient for the rocket.

The following figure plots thrust versus time, from t=0 to the end of the thrust phase.

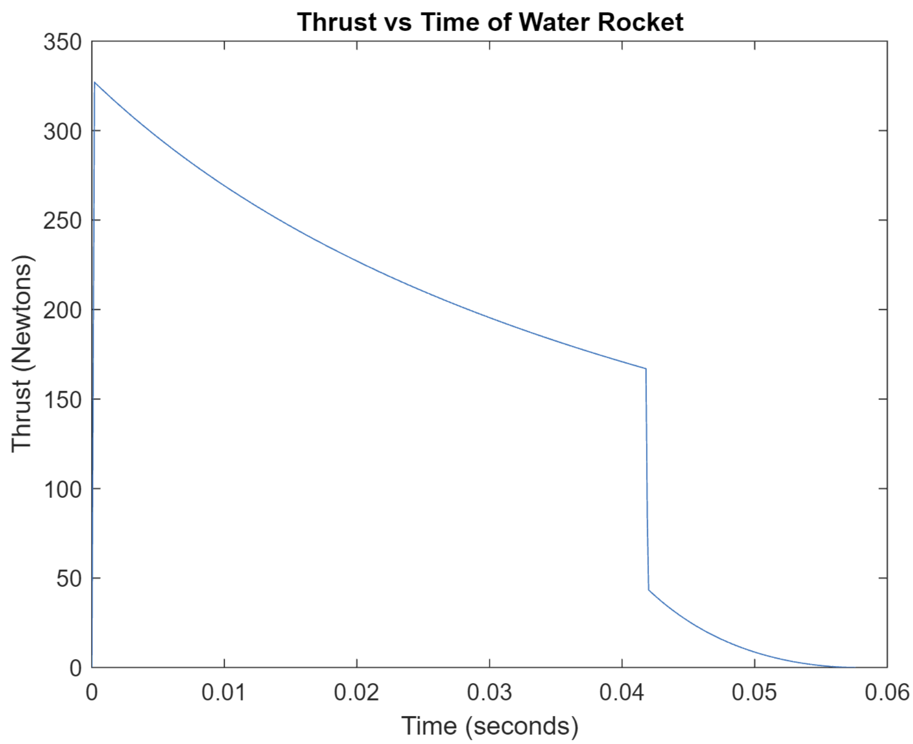

Figure 5. The predicted thrust vs time plot for trial 2.

We can see a clear transition from the water-impulse phase to the air-impulse phase for this plot. This plot also provides an excellent visualization for how important the water-impulse phase is compared to the air-impulse phase. Not only are the raw thrust force (y-axis) values lower for the air-impulse phase, we can also see that the air-impulse phase lasted for much shorter. Recalling the definition of impulse,

$$I = \int_0^t F\,dt$$

We see that the air-impulse phase, due to the factors mentioned above, imparts much less impulse onto the rocket than the water-impulse phase. This means that the thrust from pressurized air probably does not matter much and does not contribute much to the overall momentum change of the rocket.

We can also make these plots for multiple simulated runs while varying certain parameters on the runs, just to see the effect varying these parameters may have. For example, in the following plot, we vary the launch pressure while holding mass constant and we graphed the resulting displacement vs time values:

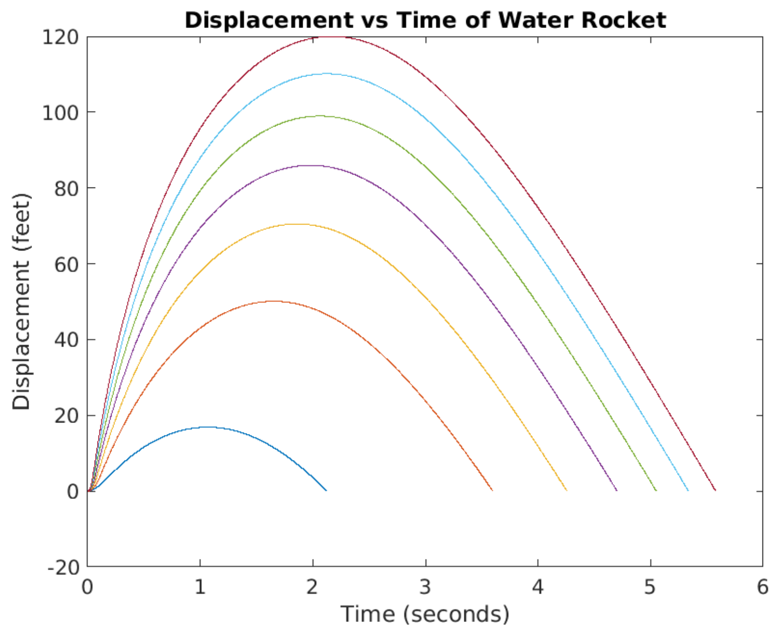

Figure 6: Displacement vs Time for Multiple Launch Pressures

We can see each line as representing a vertical slice of the height surface (generated previously) which is perpendicular to the mass axis. Doing this allows us to conclude what we already know: that increasing the launch pressure increases the maximum height travelled by the rocket.

We can also plot the effect of pressure and maximum thrust force. Examining the area under these curves, we can see that an increase in pressure always leads to an increase in impulse. This can be understood in terms of energy: increasing pressure (while keeping the air at the same volume) always increases the amount of energy stored in the enthalpy of the gas).

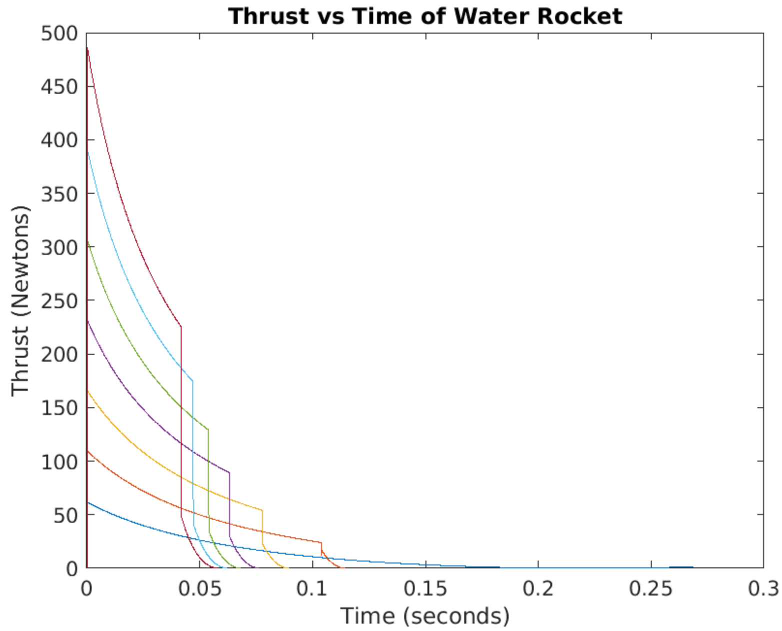

Figure 7: Thrust vs Time While Ullage Pressure is Varied

The concludes our examination of the theoretical predictions of our model. We found that the relationship between maximum height, ullage pressure, and inputted water mass is highly non-linear. We discussed and analyzed displacement vs time graphs and thrust vs time graphs for the simulated model. It is now time to turn our attention to viewing the experimental results.

EXPERIMENTAL RESULTS

In this section, we will present the experimental results obtained. To see how these data were obtained, please review the *Method* section, where our experimental procedure was detailed.

The following table of values was generated from the set of raw values provided by the rocket's on-board altimeter, pressure gauge readings, and measurements from digital scale:

Table 3: Experimental Measurements Comparing Max Height, Ullage Pressure, and Water mass

Launch Pressure (psi)	**Water Mass (g)**	**Maximum Height**
33	890	13 ft = 3.96 m
32	380	96 ft = 29.3 m
31	216	81 ft = 24.7 m

27.5	508	69 ft = 21.0 m
28	320	81 ft = 24.7 m
29	500	79 ft = 24.1 m

For preliminary data analysis, since the initial conditions of each launch varied by quite a lot (perhaps, in hindsight, it would have been wiser to keep inputted water mass constant), computing values such as mean height reached or the standard deviation of heights would have been meaningless. Instead, to gain a big-picture view of our data, we opted to compute the covariance and the correlation coefficients between launch pressure, water mass, and maximum height:

Table 4: Covariance and Correlation Measurements

	Covariance	**Correlation Coefficient**
P vs H	12.5	0.8357885862
M vs H	-356.16	-0.372929222
P vs M	-94.4	-0.4927715081

The above statistics were computed while excluding the outlier measurement (the one where we added 890g of water). We can see that there tends to be a positive relationship between ullage pressure and height, as expected from both the simulation and just from intuition. The relationship between input mass and max height is interesting. Even after excluding our outlier measurement, the relationship is slightly negative. We wanted to see to what extent the input mass affected pressure, since the relationship between pressure is more directly related to max height. Computing the correlation coefficient, we see that, indeed, there is a negative relationship between inputted water mass and ullage pressure.

The above data suggests that increasing the mass tends to decrease the ullage pressure. This is a factor that our simulation did not really take into account: we failed to see how these two variables may be coupled together, and that the input water mass may have an effect on the ullage pressure.

DISCUSSION

In this section, we will compare the simulated and actual data and discuss the accuracy of our model. We will restate the key assumptions involved in our simulation and re-evaluate their validity in light of experimental data.

Throughout both our simulation and experimentation, we saw that increasing the mass of inputted water doesn't always lead to an increase in height. Our launch experiments validated this prediction made by the original simulation, as, according to table 4, a negative relationship exists between inputted water mass and the correlation coefficient of maximum height.

Our assumptions were also not completely valid. We assumed that the water went down as a level surface. In reality, this was not the case, as the water did not flow smoothly out of the bottle, and was trading places with air. This also completely negated the adiabatic expansion assumption we had for the ullage gas (since the control volume now has mass flow rate of gas as well as water in the water-impulse phase).

We also assumed that the rocket went straight up, which also was not the case. It would tilt slightly, which increased the air resistance and caused the thrust vector to not be perpendicular to the ground. Both of these factors would lower the experimental maximum height of the rocket. In our simulation, we compensated for this by increasing the drag, but it's also likely that the thrust of the water was less than predicted due to extra losses from sources such as friction and turbulence.

As seen in the table below, for every close prediction, there was also a prediction that was extremely off from what actually happened. Overall, there were many factors that our simple model did not consider, such as wind, frictional losses, and the tilting of the rocket, that led to inaccurate predictions.

Table 5: Height Surface with Data

Experimental Height (ft)	Calculated Height (ft)
13	53.18
96	101.96
81	90.2
69	86.12
81	88.63
79	91.8

In our first test, we filled the rocket with too much water, and it was unable to fly in the way our simulation expected it to. A majority of the water was not expelled, and the rocket was weighed down and did not go very high. Our simulation assumes that the user will not overfill the rocket. This caused our prediction to be quite different from our experimental value.

In trials 2, 3, 5, and 6, our calculated height was close to our experimental height. Our calculated height was slightly higher each time, which could have been due to the fact that our rocket didn't fly perfectly straight, the mass of the load cell may not have been evenly distributed in the nose, or the rocket could have had some rotation after launch. These discrepancies also could have been caused by wind. Trial 4 was less accurate than 2, 3, 5, and 6, but still much better than trial 1. There could have been a particularly large gust of wind during that test, which would have caused it to be more inaccurate. The rocket's positioning on the cork could have also affected the launch angle.

To further compare model and experimental data, we overlaid the displacement vs time plots from the experimental predicted runs (using the same launch pressure and inputted water mass). Since we performed multiple runs, we will not list graphs for each of them. However, each graph looks similar in the sense that both are approximate parabolas, and the parabola for the experimental run will always be lower and flatter than the parabola for the actual run.

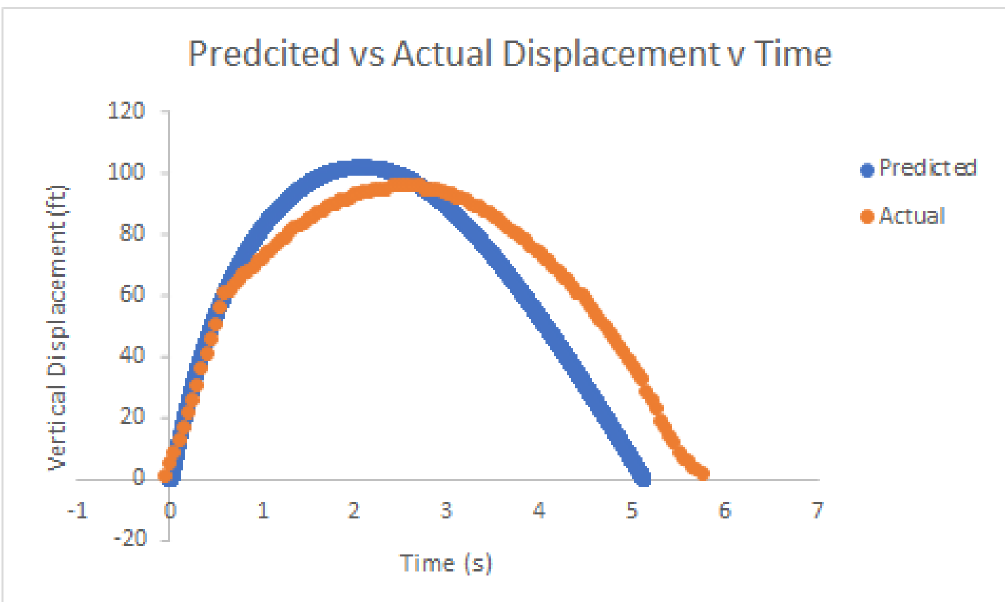

Figure 8: Comparison of Displacement v Time plots for Experimental Data and Generated Data

We tried to overlay the data points from the actual experiment on top of the height surface generated with our model. However, this was rather useless, as the surface is 3D and it's hard to convey how close or far these points are away from the surface.

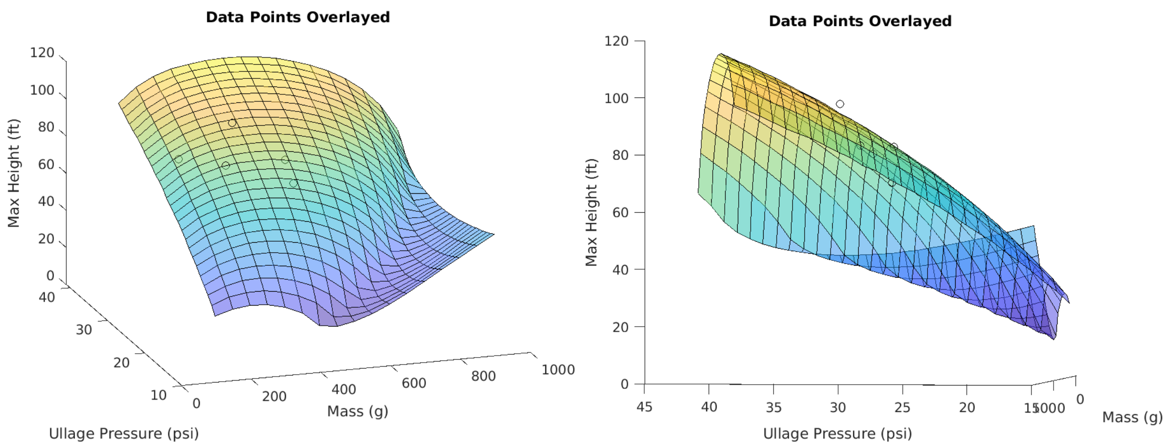

Figure 9: Overlay of Data Points on Height Surface

Regardless, despite how wrong our assumptions (of adiabatic expansion and lossless, incompressible flow) are, our model seemed to generate passable data that at least followed the same trends as the experimental results.

CONCLUSION

In this project, we developed a physics-based numerical simulation for the flight of a water bottle rocket. Accuracy of the simulation was evaluated using root-mean squared error of actual maximum height from predicted maximum height. Excluding outliers, this value was 11.7ft. Key assumptions were that both the water and the pressurized air inside the rocket behaved adiabatically--that is--no heat transferred occurred, and the expansion and flow were lossless. Through experimental validation, these assumptions were found to be false. Experimental data for maximum height was consistently lower than that of the predicted values, suggesting the presence of several sources of unaccounted losses.

Sources

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Appendix

When we initially approached the problem, we were mostly focused on calculating the external forces on the rocket (gravity, drag, and thrust). We weren't considering any other factors, such as the adiabatic expansion of the gas. We treated the air as an ideal gas and assumed ideal conditions for everything. We worked on summing the forces and relating them to the change in momentum of the rocket, similar to some of the launch problems in the variable mass section of the textbook.

When we compared our experimental values to those predicted by our simulation, our rocket flew more than twice as high as our original predictions, so we knew that we needed to make major changes. We then accounted for the adiabatic expansion of the air inside the bottle (Wheeler), which caused our predictions to be close to twice as high as the actual. This made slightly more sense, though, because it was much more likely that we would forget to account for something that would prevent the rocket from flying higher. We then changed the drag coefficient from 0.3 to 0.74, which gave us values much closer to the results we were expecting. We had originally found 0.3 from multiple sources on the internet created by people who had also worked on bottle rocket projects, but we did not find this value to be accurate for our rocket. The rocket is shaped like a cylinder with a length to diameter ratio of 2.82. According to the table in Heddleson's document, the drag coefficient of a cylinder with those dimensions is 0.74. Since the bottle doesn't fly perfectly straight, isn't a streamlined shape, and had the load cell duct taped to the front, it makes sense that the drag coefficient would be closer to 0.74 than 0.3.

If we were to do the project again, we would like to be able to use the wind tunnel to calculate the drag coefficient. We went back and forth between a few different options, and we would have liked to know what the actual drag coefficient is. If we knew that, we may have been able to make more accurate predictions. If it did turn out to be lower than the coefficient we used in our final revision, we would have known that there were other factors that we may have left out and needed to consider.

We were mostly satisfied with our results. The majority of them were close to the experimental results, but we would have liked to get them a bit closer. We may have been able to be more accurate had we considered factors such as the energy lost due to friction as the rocket leaves the cork at the start of the launch. Also as mentioned above, we would have liked to have more confirmation that changing our drag coefficient was the solution.

We learned a lot about the different factors that can affect the rocket's flight. For example, when we started the project we had no idea that we would need to consider that the air in the rocket may not behave like an ideal gas. We also got experience calculating forces on a body that had other forces in addition to the thrust, since our homework problems were more simple and didn't involve forces such as drag. We learned about drag coefficients and how they are calculated. Before this project, we had taken them as given to us and not understood where they actually came from.

From a project management standpoint, we think we did this pretty well. We had our first model ready to go in the middle of March, well before the first day of gathering data. We met frequently and prototyped quickly in order to be ready for the first launch. Further weeks were spent refining the model (and, as mentioned above, tuning the drag coefficient). We were not stressed for time, and at no point were we scrambling to finish the project.

Fun fact page: (yes we may delete this page when we turn it in)

What else has a coefficient of ~0.8? A bus!

As we can see, the bus is roughly a similar shape to the rocket, which makes sense that they would have a similar drag coefficient.