Truss Design & Load Test
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Section 05
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Background and Objectives

In this lab, the students will construct a truss out of popsicle sticks and then load it to failure. They will calculate the tension and compression forces for each member of the truss and attempt to predict where the truss will fail when loaded. Understanding how a truss will react to a load is important because trusses are commonly used due to their high strength in comparison to their weight.

During this lab, students will see how their real truss reacts and compare those results to their theoretical predictions. This will give them practice in analyzing trusses and predicting failure. They will see how wood behaves in both tension and compression, which will give them a better understanding of why certain materials are preferred when under tension, while others are used in compression. They will also see the other factors that one needs to consider when building a truss, such as the strength of the joints.

Experimental Methods and Analysis

Part 1: Building the Truss

The first part of this experiment was constructing a truss using popsicle sticks and wood glue. Knowing that wood is stronger under compression than tension, the author chose to build a Howe truss. In a Howe truss, the diagonals of the truss are in compression, while the vertical beams are in tension. The author cut the popsicle sticks so that they fit together, creating stronger joints. The author also reinforced many of the joints with extra popsicle stick pieces. This increased the strength of the members by decreasing the length of the sinusoidal bending wave. Force calculations were done on each member of the truss using the method of joints (see Appendix A), and then those forces were used to calculate the capacity of each member (see Appendix B). After performing the capacity calculations, the weakest members were thickened with extra popsicle sticks and the calculations were done again.

The equilibrium equations used in the method of joints are

$$\sum F_x = 0 \tag{1}$$

$$\sum F_y = 0 \tag{2}$$

where $\sum F_x$ is the sum of the forces in the horizontal direction (in N), and $\sum F_y$ is the sum of the forces in the vertical direction (in N).

The tensile strength of a beam is defined as

$$T = \sigma_T \cdot A \tag{3}$$

where T is the tensile strength (in N), σ_T is the allowable tensile strength (in MPa), and A is the cross-sectional area (in m^2).

The compressive strength of a beam with moment resisting joints is defined as

$$C = \frac{4\pi^2 EI}{L^2} \tag{4}$$

where C is the Euler Buckling Strength (in N), E is the Modulus of Elasticity of the material (in MPa), I is the moment of inertia (in m^4), and L is the length of the popsicle stick between ends (in m).

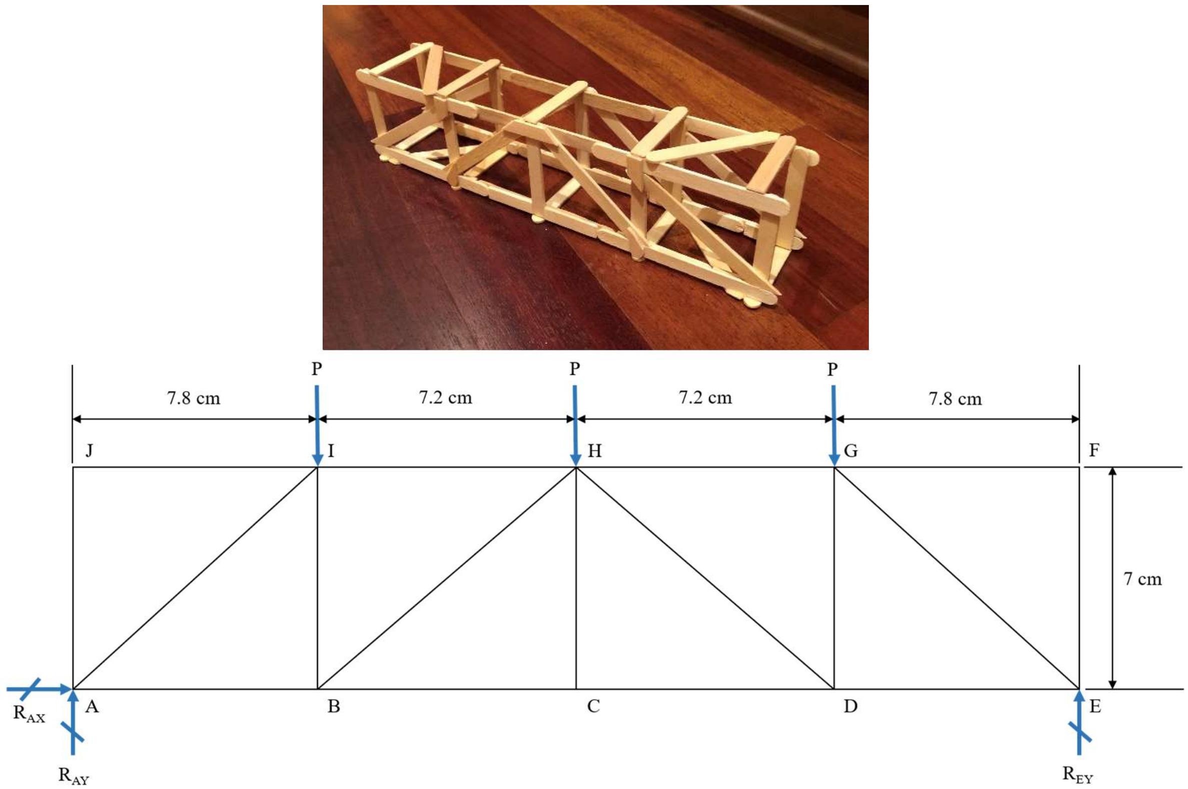


Figure 1. The completed popsicle stick truss (top) and the two-dimensional free body diagram of one side truss (bottom)

Table 1. Analysis results obtained for truss in Figure 1.

Truss Member	Force (Newtons)	Capacity (N)
AB	1.67P (T)	147.5
AI	2.25P (C)	188
AJ	0	N/A
BC	2.18P (T)	147.5
BH	0.717P (C)	205
BI	0.5P (T)	69.8
CD	2.18P (T)	147.5
CH	0	N/A
DE	1.67P (T)	147.5
DG	0.5P (T)	69.8
DH	0.717P (C)	205
EF	0	N/A
EG	2.25P (C)	188
FG	0	N/A
GH	1.67P (C)	398
HI	1.67P (C)	398
IJ	0	N/A

Part 2: Loading the Truss

After its completion, the bridge was weighed and its weight was recorded. Pictures were also taken from many different angles for later use and analysis.

The bridge was placed between two large blocks of wood, with one end of the bridge resting on each block. The apparatus was set up far enough away from the wall and on a carpeted floor so that nothing would be damaged when truss failed and the load fell (see Figure 2 below). Closed toed shoes were worn throughout the lab, and students were careful to keep their feet and hands away from heavy loads that could fall.

To ensure loading only occurred at the joints, the first weight placed on the bridge was a smaller block of wood that lay across only the joints. The bridge was then loaded with 4.5kg, 2.3kg, and 1.1kg weights until it failed. After placing each weight, the bridge was inspected for any signs of beginning failure. Pictures and video were taken of the bridge throughout to analyze the loading and eventual failure. Once the bridge failed, the maximum amount of weight it held was counted and recorded.

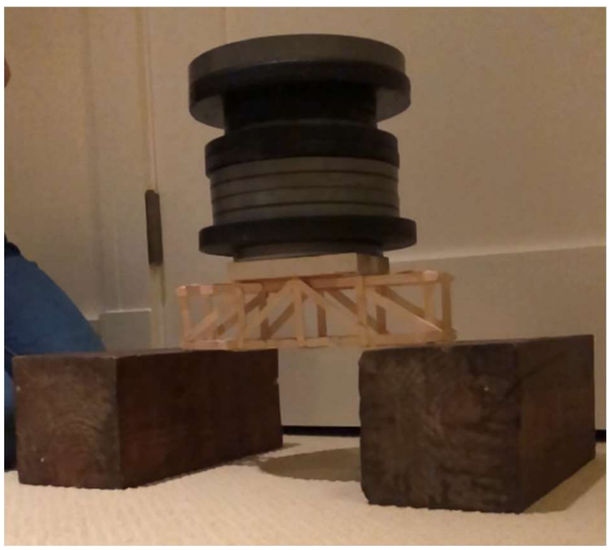


Figure 2. The bridge and testing apparatus with a load applied

Once the data was collected, the total weight that the truss held was used to calculate P.

For the truss shown in Figure 1, P is given by

$$P = \frac{F_{total}}{6} \tag{5}$$

where P is the force applied at each joint (in N), F_{total} is the total force held by the bridge (in N), and 6 is the number of joints that the load was applied to (3 joints on each side of the 2-sided bridge).

The weight of the bridge and the total weight that the truss held was used to calculate the efficiency of the bridge.

The efficiency is given by

$$\eta = \frac{F_{total}}{W} \tag{6}$$

where η is the efficiency and W is the weight of the bridge (in N).

Results

The bridge remained steady throughout the duration of the test. As weights were added, the truss members did not bend enough to be noticed visually. Until the bridge failed, there were no cracking sounds in between load additions. After the final weight was added, the top plane of the bridge swayed slightly before straightening back out again. About a minute passed while the author was photographing the bridge, and right before the author loaded another weight onto the bridge, the weight finally caused the top plane to shift past its point of return, and the bridge collapsed. In the slow motion video take by the author, the bridge could be seen folding and flattening similar to collapsing a cardboard box. The bridge failed at a load of 311 N (meaning each truss held 155.5 N) due to Lateral Instability I. The remains of the bridge (see *Figure 3*) showed that no damage was done to the wooden beams themselves, since none of the popsicle sticks were snapped or bent, even after the weights fell on top of them. Only the glue joints gave out as the top plane rotated.

Figure 3. The truss post-failure after the weights had been removed from the scene

Using equation *6*, the efficiency of the bridge was calculated to be 567, which means that the bridge held 567 times its own weight.

Discussion

The truss bridge shown in *Figure 1* had a predicted capacity of 406 N. The ultimate failure load of the bridge was considerably less than the predicted capacity, which shows that the analysis method used had some limitations. It did not consider any lateral motion that the bridge may undergo, or any bending that the members could experience. It also did not account for the strength (or lack of strength) of the glue joints.

Table 2. Comparison between ultimate failure load and predicted capacity for each member

Truss Member	**Ultimate Failure Load (Newtons)**	**Predicted Capacity (N)**
AB	86.8 (T)	147.5
AI	116.6 (C)	188
AJ	0	N/A
BC	113.0 (T)	147.5
BH	37.1 (C)	205
BI	25.9 (T)	69.8
CD	113.0 (T)	147.5
CH	0	N/A
DE	86.6 (T)	147.5
DG	25.9 (T)	69.8
DH	37.1 (C)	205
EF	0	N/A
EG	116.6 (C)	188
FG	0	N/A
GH	86.6 (C)	398
HI	86.6 (C)	398
IJ	0	N/A

Table 2 shows that most of the individual members held much less than their predicted capacities. Due to other factors that our calculations did not consider, such as bending and lateral motion, none of the members reached their predicted capacity before failure. The highest difference between predicted capacity and ultimate load is in the compression members. Since wood is stronger in compression, it makes sense that the tension members would be closer to their predicted capacity, since they were expected to fail first.

One source of error could be in the placement of the weights on the bridge. Although the author tried to center the weights on the bridge, any slight shift from the center could have caused the bridge to fail sooner. If the weights were not centered, one side of the bridge might have experienced more forces than the other, leading to premature failure. Another source of error is that the length of the members was calculated as if the joints were perfect pin joints. In reality, the lengths of the members that were unsupported were shorter than our calculated lengths. This is because at the joints, popsicle sticks overlapped, which increased the support at the ends of the members. Our predicted capacities for the members would have been higher had we considered this factor.

If the author were to construct another bridge, she would add portal bracing to counteract the moment that led to the first bridge's eventual failure. This would allow the bridge to hold significantly more weight, since no other parts of the bridge were visibly failing when it experienced the lateral motion. The author would also remove the zero force members on the ends of the bridge and use those popsicle sticks to strengthen the bridge elsewhere. They could be used for the portal bracing, and if more were left over, they could also be used to increase the bracing in the tension members. They could also be removed entirely, which would increase the efficiency of the bridge by lowering its weight and therefore increasing its strength to weight ratio.

This design held more weight than the other designs viewed during lab. This design was a deck truss, while the others were through trusses. Loading the weight on top of the bridge allowed it to hold more. While the other bridges were loaded on a single joint, this design spread the load over three joints on each truss, which distributed the weight more evenly. This truss also had its joints reinforced, which helped it resist moments at the joints and bending along the members.

Trusses are found everywhere: in bridges, buildings, and countless other structures. Being able to accurately predict the strength of these trusses allows engineers to ensure that their designs will not fail under their intended load. This experiment shows that when designing structures with trusses, more factors must be considered than only looking at the tension and compression forces in each member. They would have to also consider the strength of their joints, lateral motion, bending, and buckling, among other factors. Knowing the properties of the materials they are using is also important. Although this bridge did not fail due to compression or tension, our calculations showed that wood is much better under compression than tension, which affects the way the bridge should be built. If building with wood, one would design their bridge to be mainly in compression, while if using a material such as steel they would design the members to be in tension. One could also combine wood and steel to create a bridge that had strong members in both tension and compression.

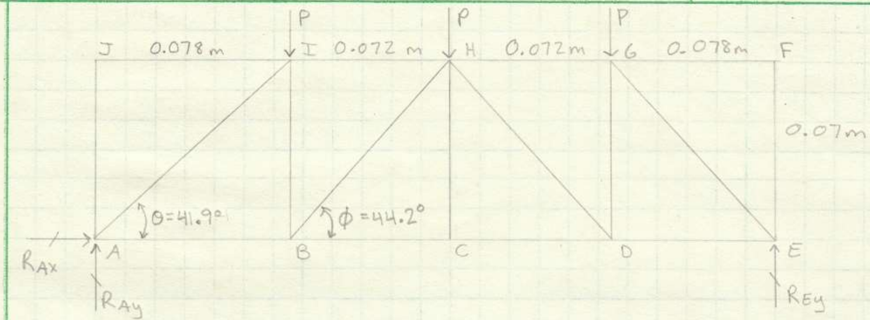
Conclusion

This lab was performed to analyze truss behavior and compare real life results to theoretical predictions. It allowed the students to see the different factors that one must consider when designing a truss. During the experiment, the truss bridge shown in *Figure 1* was loaded to failure. The students observed what occurred in the truss both before and after failure. The truss in *Figure 1* failed due to lateral instability after holding a mass of 31.7kg. The bridge was expected to hold less than calculated because there are factors that were not considered when doing the calculations for this lab that led the bridge to being weaker than predicted. Therefore, it can be concluded that the results from this lab were valid because the truss bridge held less weight than predicted through the calculations. These results showed that engineers must use more analyses than just the method of joints when building trusses, and that they must not only consider tension and compression forces in the members, but also factors such as bending and moments. This experiment met the objectives, as the students were able to analyze the truss both before and after failure and compare their theoretical results to what the truss actually held.

Appendix

APPENDIX A: METHOD OF JOINTS CALCULATIONS

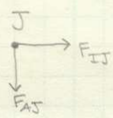
Katharine Priu	560.211/Truss Lab	Method of Joints Calculations	1/1



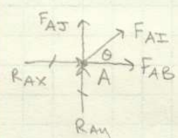
$$\Sigma F_x = 0 = R_{Ax}$$

$$\circlearrowleft \Sigma M_A = -P(0.078m) - P(0.15m) - P(0.222m) + R_{Ey}(0.3m) = 0$$
$$(0.3m)R_{EY} = (0.45m)P$$
$$R_{EY} = 1.5P$$

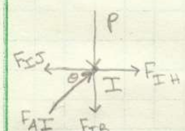
$$\Sigma F_y = 0 = R_{Ay} + R_{EY} - 3P = R_{Ay} + 1.5P - 3P$$
$$R_{Ay} = 1.5P$$



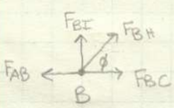
$$\Sigma F_x = 0 = F_{IJ}$$
$$\Sigma F_y = 0 = F_{AJ}$$



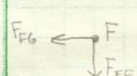
$$\Sigma F_y = R_{Ay} + \cancel{F_{AJ}}^{0} + F_{AI}\sin\theta = 0$$
$$0.668 F_{AI} = -1.5P$$
$$F_{AI} = -2.25P$$
$$\Sigma F_x = 0 = \cancel{R_{Ax}}^{0} + F_{AB} + F_{AI}\cos\theta$$
$$F_{AB} = 2.25P\cos\theta = 1.67P$$



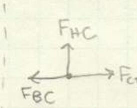
$$\Sigma F_y = 0 = -P - F_{IB} + F_{AI}\sin\theta$$
$$F_{IB} = -P + 1.5P = 0.5P$$
$$\Sigma F_x = -\cancel{F_{IJ}}^{0} + F_{IH} + F_{AI}\cos\theta = 0$$
$$F_{IH} = -1.67P$$



$$\Sigma F_y = 0 = F_{BI} + F_{BH}\sin\phi$$
$$F_{BH}\sin\phi = -0.5P \qquad F_{BH} = -0.717P$$
$$\Sigma F_x = 0 = F_{BC} + F_{BH}\cos\phi - F_{AB}$$
$$F_{BC} = 1.67P - (-0.514P) = 2.18P$$



$$\Sigma F_y = 0 = F_{EF}$$
$$\Sigma F_x = 0 = F_{FG}$$



$$\Sigma F_y = 0 = F_{HC}$$

Bridge is symmetric about HC:

$F_{DE} = F_{AB}$
$F_{EG} = F_{AI}$
$F_{DG} = F_{BI}$
$F_{GH} = F_{IH}$
$F_{DH} = F_{BH}$
$F_{DC} = F_{BC}$

Table A1. Results from Method of Joints Calculations

Truss Member	**Force (Newtons)**
AB	1.67P (T)
AI	2.25P (C)
AJ	0
BC	2.18P (T)
BH	0.717P (C)
BI	0.5P (T)
CD	2.18P (T)
CH	0
DE	1.67P (T)
DG	0.5P (T)
DH	0.717P (C)
EF	0
EG	2.25P (C)
FG	0
GH	1.67P (C)
HI	1.67P (C)
IJ	0

APPENDIX B: STRENGTH AND EXPECTED FAILURE CALCULATIONS

Katharine Priu | 560.211/Truss Lab | Strength Analysis

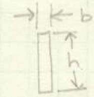
Tensile Strength

$T = \sigma_T \cdot A$

$\sigma_T = 3.5\ MPa = 3.5 \times 10^6\ Pa$

$A = b \cdot h = 1.995 \times 10^{-5} m^2$

$T = 69.8\ N$



$h = 0.00212 m$

$b = 0.00941 m$

Compressive Strength

$C = \frac{4\pi^2 EI}{L^2}$ — Due to moment resisting joints

$E = 7000\ MPa = 7 \times 10^9\ Pa$

$I = \frac{1}{12} bh^3 = 7.472 \times 10^{-12} m^4$

$L_{AI} = L_{EG} = 0.1048 m$

$L_{BH} = L_{DH} = 0.1004 m$

$L_{HI} = L_{GH} = 0.072 m$

$C_{AI} = C_{EG} = 188.0 N$

$C_{BH} = C_{DH} = 205\ N$

$C_{HI} = C_{GH} = 398\ N$

For members BC + CD (double layered) + AB + DE

$b = 0.00448$

$T = \sigma_T \cdot h \cdot b = 147.5 N$

Failure Analysis

AB, BI, DE, DG will fail at 69.8 N

$F_{AB} = F_{DE} = 1.67P = 69.8N$

$P_{failure, AB} = 41.8 N$

⇒ Changed AB + DE to double

$1.67P = 147.5 N$

$P_{failure, AB} = 88.3 N$

$F_{BI} = F_{DG} = 0.5P = 69.8N$

$P_{failure, BI} = 139.6 N$

BC, CD will fail at 147.5N

$F_{BC} = F_{CD} = 2.18P = 147.5N$

$P_{failure, BC} = 67.7 N$

AI, EG will fail at 188 N

$F_{AI} = F_{EG} = 2.25P = 188 N$

$P_{failure, AI} = 83.6N$

BH, DH will fail at 205 N

$F_{BH} = F_{DH} = 0.717P = 205\ N$

$P_{failure, BH} = 286\ N$

HI, GH will fail at 398 N

$F_{HI} = F_{GH} = 1.67P = 398\ N$

$P_{failure, HI} = 238\ N$

I predict that my bridge will fail at members BC and CD under a load of $P = 67.7N$.
This would be when the bridge has a total load of 406.2 N applied to the top.