My Blackjack Strategy

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Abstract

Presentation and analysis via probability of ruin of my personal Blackjack betting strategy.

1 Introduction

I was playing blackjack on my phone and made a sizable return (of fake money) using some basic card counting and a betting strategy of my own invention. I didn't believe that the card counting would work on online apps because I thought that played cards would be replaced and shuffled back into the deck, as is typically the case with e-casinos. I also lacked confidence in my own card counting abilities. Given my success, I (1) have a solid betting strategy (2) am skilled with basic card counting or (3) am lucky. With a little math, we can examine the first claim: "I have a solid betting strategy."

1.1 My Strategy

With initial wealth W, bet $B < W$ in the first hand and after any hand where I win. If I lose, double the bet amount (bet $2B$, then $4B$ if I lose again, then $8B$ if I lose again, etc.). Don't change the bet amount after a push.

2 Definitions

Let p be the probability of losing a hand of blackjack. Define the following 2 sequences of random variables, $\vec{W}, \vec{T}$ as

$$\forall i \in \{1 : k\}, T_i \sim \texttt{Poisson}\left(\frac{1}{1-p}\right)$$

$$T_0 \coloneqq 0$$

$$W_0 \coloneqq W$$

$$\forall k \in \{1 : n\}, W_{T_{k-1}+T_k} = W + kB$$

Note that k counts the number of hands won (k starts at 0 and is incremented by 1 whenever I win). T_i is a random variable of the amount of hands between winning hands plus 1. T_i samples from a Poisson distribution with parameter $\frac{1}{1-p}$, the expected number of hands between winning hands plus 1. $W_{T_{k-1}+T_k}$ is the cumulative wealth immediately after winning hand k.

3 Probability of Ruin

All gambles in math – from casino gambling to insurance solvency – must be accompanied by their corresponding *probabilities of ruin*, the chance of losing W. Define the probability of ruin at winning hand k to be the first time the losses exceed the cumulative winnings plus initial wealth, namely

$$P_{k+1} := \mathbb{P}\Bigg(W_{T_{k-1}+T_k} < \sum_{i=1}^{T_{k+1}} 2^{i-1} B \Bigg)$$

Recall that bets are doubled after losing hands, so the total amount that can be lost between winning hands is $\sum_{i=1}^{T_{k+1}} 2^{i-1}B$. Let's simplify the expression for P_{k+1}.

$$P_{k+1} = \mathbb{P}\Bigg(\frac{W}{B} + k < \sum_{i=1}^{T_{k+1}} 2^{i-1} \Bigg)$$

Now, we can convert both sides of the inequality to binary numbers to note that the the right-hand side of the inequality is equivalent to appending a binary 1 to the summed quantity for every unit T_k. Therefore, T_k must be some power of 2 that is greater than the largest power of 2 in the binary decomposition of the left-side of the inequality. In other words,

$$P_{k+1} = \mathbb{P}\Bigg(\log_2 \left(\frac{W}{B} + k \right) < T_{k+1} \Bigg)$$

We can arrive at the same conclusion more concretely by simply taking the log base-2 of both sides of the inequality and noting the monotonicity of the logarithm as follows

$$\log_2 \left(\frac{W}{B} + k \right) < \log_2 \left(\sum_{i=1}^{T_{k+1}} 2^{i-1} \right) < \log_2 \left(1 + \sum_{i=1}^{T_{k+1}} 2^{i-1} \right) = T_{k+1}$$
$$\Rightarrow P_{k+1} = \mathbb{P}\Bigg(\log_2 \left(\frac{W}{B} + k \right) < T_{k+1} \Bigg)$$

Recall that T_k is a Poisson random variable. In either case, we arrive at the complement of the Poisson CDF.

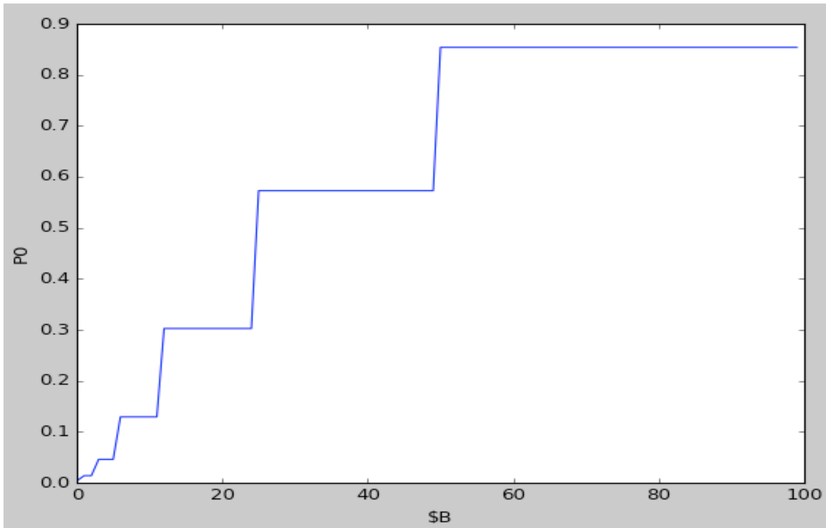
$$P_{k+1} = 1 - F_{\texttt{Poisson}}\Bigg(\log_2 \left(\frac{W}{B} + k \right) \Bigg)$$

With this equation, we clearly see the indirect relationship between P_{k+1} and k, the number of hands won, which coincides with our intuition – with more winnings on hand, more money can be lost without reaching ruin. With this in mind, we can take the maximum value of P_k to inform lower bound on the expected number of winning hands hit before ruin is reached, namely

$$\frac{1}{P_1} = \frac{1}{1 - F_{\texttt{Poisson}}\Big(\log_2 \left(\frac{W}{B} \right) \Big)}$$

Here is a table of data for various values of B for $W = \$100$.

$\$B$	P_0	$\frac{1}{P_0}$
1	.0037	271.9
2,3	.0139	71.71
4-6	.0460	21.75
7-12	.1293	7.736
13-25	.3025	3.306
26-50	.5728	1.745
51-100	.8538	1.171



4 Conclusion

If $W = \$100, B = \$30, p = 0.48$, then $P_0 \approx 0.5728$ and $\frac{1}{P_0} \approx 3.306$. Thus, with this strategy, a player is expected to lose everything, including W after at least 3 winning hands. If a player backs out at 3 hands after implementing this strategy, they'll walk away with $W + 3B$ total wealth.

This seems like a solid betting strategy to me. I would still advise players to learn how to count cards and consider showering themselves with luck.