Ensuring Efficient Convergence to a Given Stationary Distribution – An Overview

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April 17, 2018

Abstract

How may we find a transition matrix that guarantees the long-run convergence of a Markov Chain to a given stationary distribution? Solving for this (usually) undetermined system is non-trivial and presents unique computational challenges. We begin to tackle this issue by reviewing Markov Chains and related definitions. Eight different of methods of directly solving for a transition matrix are presented along with their limitations. Relaxations of the two assumptions - the Identityless and Independence Assumptions - underlying these direct methods are considered. Two methods of generating a Mass Matrix - the transition matrix underlying hops between entire population states - are described while developing the notion of successively-bounded weak compositions and describing an algorithm for their exhaustive generation. Applications of some methods are provided with respect to optimizing firm profit via optimally distributing workers among wage bracket and optimizing measures of national wealth via manipulation of class distribution, economic inequality and immigration policy.

1 Introduction

Many systems of a finite state space that evolve in finite time can be modelled as *Markov Chains*. If we let a Markov Chain run for a large number of timesteps, then, while certain conditions are met, we will eventually arrive at a *stationary distribution*, π, which is a vector listing the probabilities of ending up in any of the states.

Markov Chains can be completely described by their *transition matrix*, P, which is a matrix listing the probabilities of hopping from one state to any other state in a single timestep. P may be likened to a set of policies that govern how the Markov system evolves. Normally, we derive π from P – we derive the long-run behavior from the policies we implement. In my thesis, I go in the opposite direction; I attempt to find algorithms that tell us what policies we must implement to achieve a certain goal.

2 Applications

Here, we list a small set of situations coupled with questions that are answered by my thesis. In general, my thesis applies to any situation exhibiting a finite number of individuals, a finite number of “bins” that serve to classify individuals, and a preference function that allows us rate or rank population distributions among those bins.

2.1 # 1

Let’s say we have a population of people that can be sorted into income classes. Perhaps there exists some distribution of people among these income classes that maximizes their nation’s GDP or overall welfare. How can we arrive at a such a distribution?

2.2 # 2

Let’s say we have a set of employees that work for a profit-maximizing firm and that there is some distribution of employees among wage brackets that maximizes the firm’s profit. How many employees should we promote/demote per timestep to converge to this optimum distribution?

2.3 # 3

Let’s say we have a population of people hailing from various countries and we can alter the likelihood that they immigrate to our nation via our laws or incentives. Oded Galor (a Brown Professor!) *et al.* showed that, empirically, there exists an ideal heterozygosity (genetic diversity) that maximizes a nation’s income per capita. Furthermore, we know the heterozygosity of all other nations and how immigration from these other nations can effect our own heterozygosity, also courtesy of Galor, *et al.*’s work. How may we adjust our immigration rates to ensure we converge to this empirically optimum heterozygosity?[1]

[1] `http://media.virbcdn.com/files/8a/78fedbd35de3e73a-Galor-Lecture4-2016-H.pdf`