CHAPTER 4

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Constraint equations: Consider the (1-dimensional) constraint equation,

$$g(q(t)) = 0. \tag{1}$$

Taking a time derivative and substituting $\dot{q} = p/m$ yields

$$\frac{d}{dt} g(q(t)) = g'(q)p/m = 0,$$

this is the cotangency condition. Taking a second time derivative yields

$$\begin{aligned}\frac{d}{dt}(g'(q)p/m) &= g''(q)\dot{q}p/m + g'(q)\dot{p}/m \\ &= g''(q)p^2/m^2 + g'(q)\dot{p}/m \\ &= 0.\end{aligned}$$

A generalization of this gives equation (4.10),

$$\boldsymbol{g}'(\boldsymbol{q}(t))\boldsymbol{M}^{-1}\dot{\boldsymbol{p}}(t) + \boldsymbol{\phi} = \boldsymbol{0}, \tag{2}$$

where $\phi_k = \boldsymbol{p}^T \boldsymbol{M}^{-1} \nabla^2 \boldsymbol{g} \boldsymbol{M}^{-1} \boldsymbol{p}$.