DISCONTINUOUS SDES

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1. Symmetric Stable Processes

Consider the stochastic process,

$$dX(t) = \begin{cases} dW(t) & |X(t)| < m, \\ 0 & |X(t)| = m, \end{cases} \tag{1}$$

where $m > 0$, $X(0) = 0$ and the process $dW(t)$ satisfies

$$dW(t) = \xi(t)\sqrt{dt},$$

where $\xi(t) \sim \mathcal{N}(0, 1)$. Typical samples from this distribution exhibit Brownian motion from their initial state, and converge to either m or $-m$. We make intuitive observations for specific cases of m, which serve also as boundary conditions:

(1) When $m = 0$, the stochastic process $X(t) = 0$ is deterministic. This is a trivial case.
(2) When $m = \infty$, the stochastic process $X(t) = W(t)$ is a Brownian motion.
(3) The conditional probability that X converges to m given prior that $X = x$ satisfies the relations,

$$\mathcal{P}(X(t) = m | X(\tau) = x) = \begin{cases} 1 & x = m, \\ 1/2 & x = 0. \\ 0 & x = -m. \end{cases}$$