ONGOING QUESTIONS

DAVID LI

1. Symplectic structure with holonomic constraints

(1) Define a flow map $\mathcal{F}_t(\boldsymbol{z}) = \boldsymbol{Z}(t, \boldsymbol{z})$ for which $\boldsymbol{Z} = (Z_1, \cdots, Z_{2N_c})$ satisfies the symplectic condition

$$d\boldsymbol{Z} \wedge \boldsymbol{J} d\boldsymbol{Z} = d\boldsymbol{z} \wedge \boldsymbol{J} d\boldsymbol{z},$$

where $\boldsymbol{J}$ is the symplectic structure matrix. Using product rule,

$$\begin{aligned}\frac{d}{dt}(d\boldsymbol{Z} \wedge \boldsymbol{J} d\boldsymbol{Z}) &= \frac{d}{dt}(d\boldsymbol{Z}) \wedge \boldsymbol{J} d\boldsymbol{Z} + d\boldsymbol{Z} \wedge \frac{d}{dt}(\boldsymbol{J} d\boldsymbol{Z}) \\ &= \frac{d}{dt}(d\boldsymbol{Z}) \wedge \boldsymbol{J} d\boldsymbol{Z} + d\boldsymbol{Z} \wedge \boldsymbol{J} \frac{d}{dt}(d\boldsymbol{Z}) \\ &= 0,\end{aligned}$$

Since $\boldsymbol{z}$ does not depend on time. Assume that $\frac{d}{dt}(d\boldsymbol{Z}) = d\dot{\boldsymbol{Z}}$. Then

$$\frac{d}{dt}(d\boldsymbol{Z} \wedge \boldsymbol{J} d\boldsymbol{Z}) = d\dot{\boldsymbol{Z}} \wedge \boldsymbol{J} d\boldsymbol{Z} + d\boldsymbol{Z} \wedge \boldsymbol{J} d\dot{\boldsymbol{Z}} = 0.$$

Interpretation. The vector $\boldsymbol{z}$ represents a preset configuration of entries q_i and p_i, and is independent of time.