Control flow, continuations, monads

Week 8

- Structured programming
- Procedural abstraction
- Exceptions
- Continuations
- Monads

Turning the clock back

- To understand what structured programming is, let's look at what programs looked like before it:

```
10 IF (X .GT. 0.000001) GO TO 20
11 X = -X
    IF (X .LT. 0.000001) GO TO 50
20 IF (X*Y .LT. 0.00001) GO TO 30
     X = X-Y-Y
30  X = X+Y
     ...
50 CONTINUE
     X = A
     Y = B-A
     GO TO 11
     …
```

- Do you know what this Fortran program does?
  - A: yes, B: no

Go To Statement Considered Harmful

Key Words and Phrases: go to statement, jump instruction, branch instruction, conditional clause, alternative clause, repetitive clause, program intelligibility, program sequencing
CR Categories: 4.22, 5.23, 5.24

EDITOR:

For a number of years I have been familiar with the observation that the quality of programmers is a decreasing function of the density of **go to** statements in the programs they produce. More recently I discovered why the use of the **go to** statement has such disastrous effects, and I became convinced that the **go to** statement should be abolished from all "higher level" programming languages (i.e. everything except, perhaps, plain machine code). At that time I did not attach too much importance to this discovery; I now submit my considerations for publication because in very recent discussions in which the subject turned up, I have been urged to do so.

My first remark is that, although the programmer's activity ends when he has constructed a correct program, the process taking place under control of his program is the true subject matter of his activity, for it is this process that has to accomplish the desired effect; it is this process that in its dynamic behavior has to satisfy the desired specifications. Yet, once the program has been made, the "making" of the corresponding process is delegated to the machine.

My second remark is that our intellectual powers are rather geared to master static relations and that our powers to visualize processes evolving in time are relatively poorly developed. For that reason we should do (as wise programmers aware of our limitations) our utmost to shorten the conceptual gap between the static program and the dynamic process, to make the correspondence between the program (spread out in text space) and the process (spread out in time) as trivial as possible.

Let us now consider how we can characterize the progress of a process. (You may think about this question in a very concrete manner: suppose that a process, considered as a time succession of actions, is stopped after an arbitrary action, what data do we have to fix in order that we can redo the process until the very same point?) If the program text is a pure concatenation of, say, assignment statements (for the purpose of this discussion regarded as the descriptions of single actions) it is sufficient to point in the

M. Broy et al. (eds.), *Pioneers and Their Contributions to Software Engineering*

Go To Statement Considered Harmful

1. Primary way to understand programs: processing code in sequence
2. goto programs: can jump anywhere => spaghetti
3. Eliminate gotos!
4. Need higher level control flow constructs!

Structured Programming with go to Statements

DONALD E. KNUTH

Stanford University, Stanford, California 94305

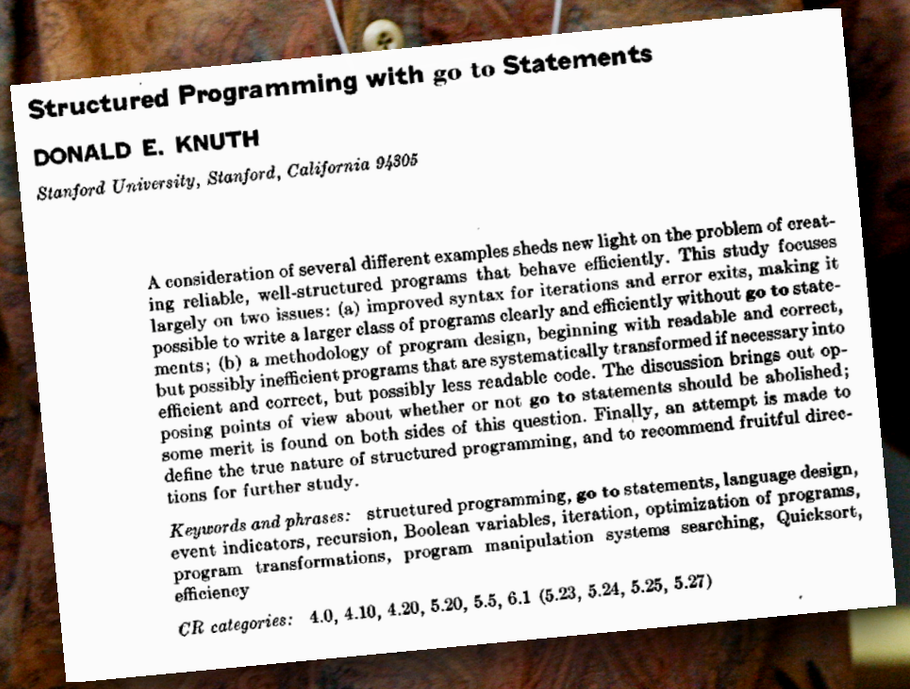
A consideration of several different examples sheds new light on the problem of creating reliable, well-structured programs that behave efficiently. This study focuses largely on two issues: (a) improved syntax for iterations and error exits, making it possible to write a larger class of programs clearly and efficiently without **go to** statements; (b) a methodology of program design, beginning with readable and correct, but possibly inefficient programs that are systematically transformed if necessary into efficient and correct, but possibly less readable code. The discussion brings out opposing points of view about whether or not **go to** statements should be abolished; some merit is found on both sides of this question. Finally, an attempt is made to define the true nature of structured programming, and to recommend fruitful directions for further study.

Keywords and phrases: structured programming, **go to** statements, language design, event indicators, recursion, Boolean variables, iteration, optimization of programs, program transformations, program manipulation systems searching, Quicksort, efficiency

CR categories: 4.0, 4.10, 4.20, 5.20, 5.5, 6.1 (5.23, 5.24, 5.25, 5.27)

1. Eliminate gotos in some cases
2. Introduce gotos in other cases

   Where?



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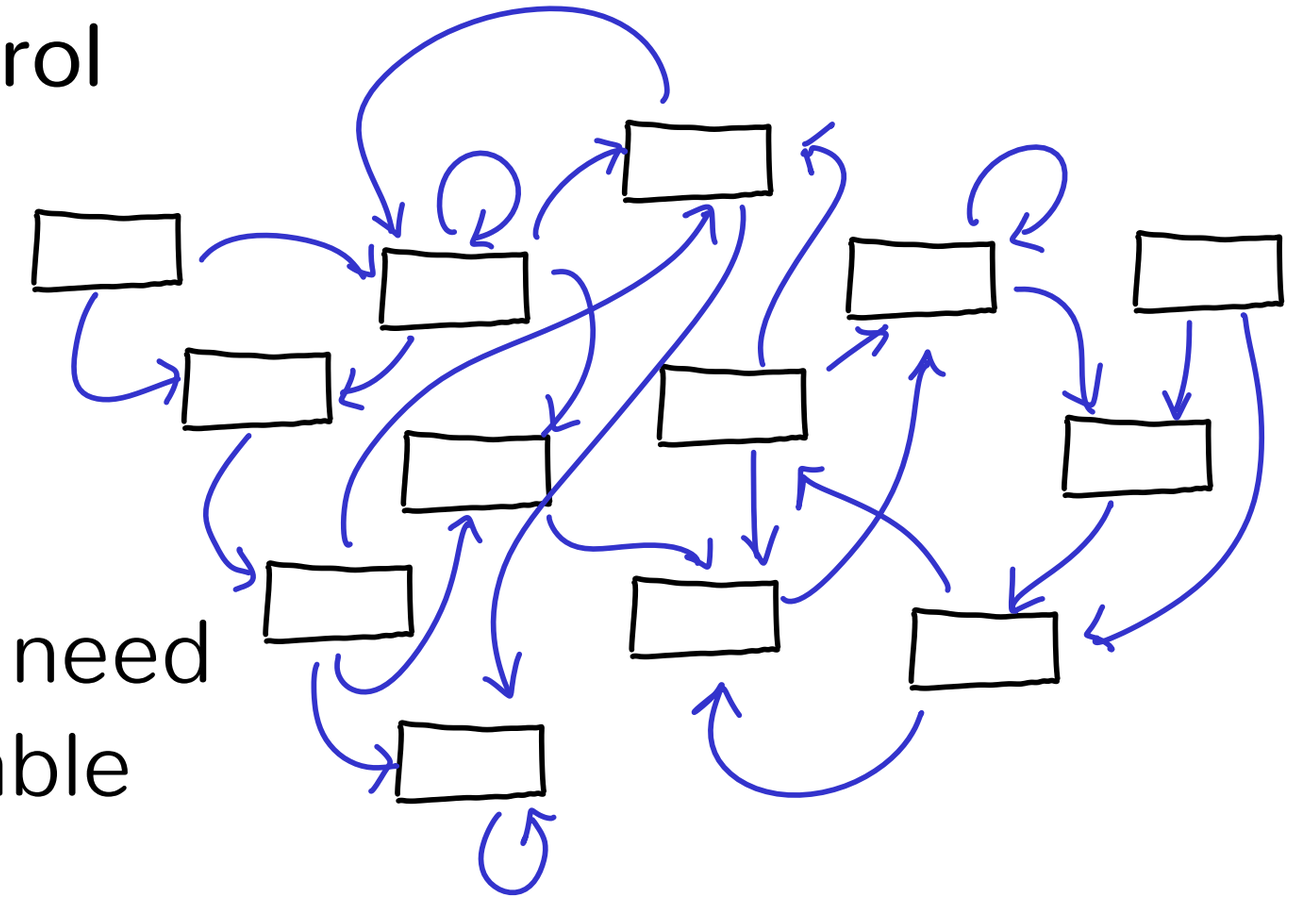
Structured programming

- What is structured programming?
  - Programming paradigm consisting of: control structures, procedures, and blocks
- Why?
  - Largely because gotos make it extremely hard to reason about and prove things about programs

Programming with gotos

- Largely unstructured control flow graphs

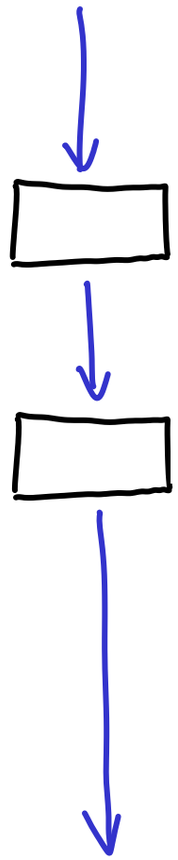


- But: extremely flexible!

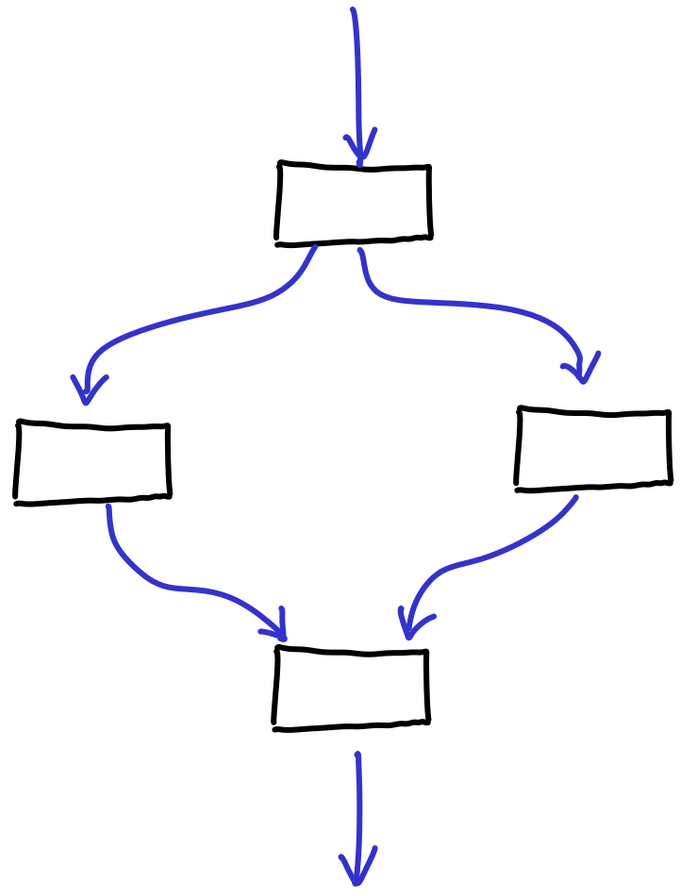
Q: What constructs do we need to express any computable function?

A: Sequence, selection, iteration

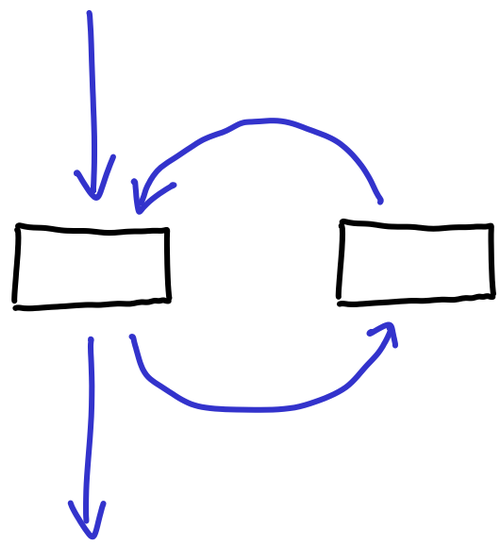
CFG of sequencing



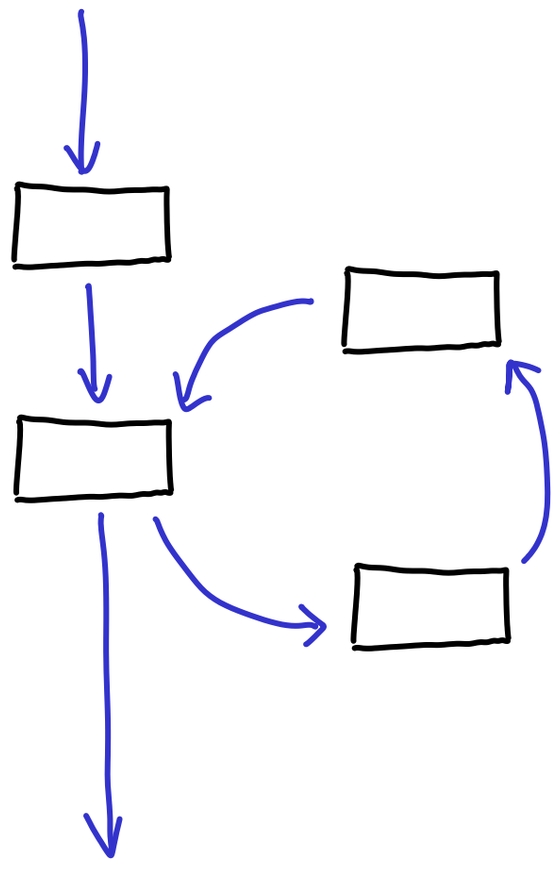
CFG of if-else statement



CFG of while loop



CFG of for loop



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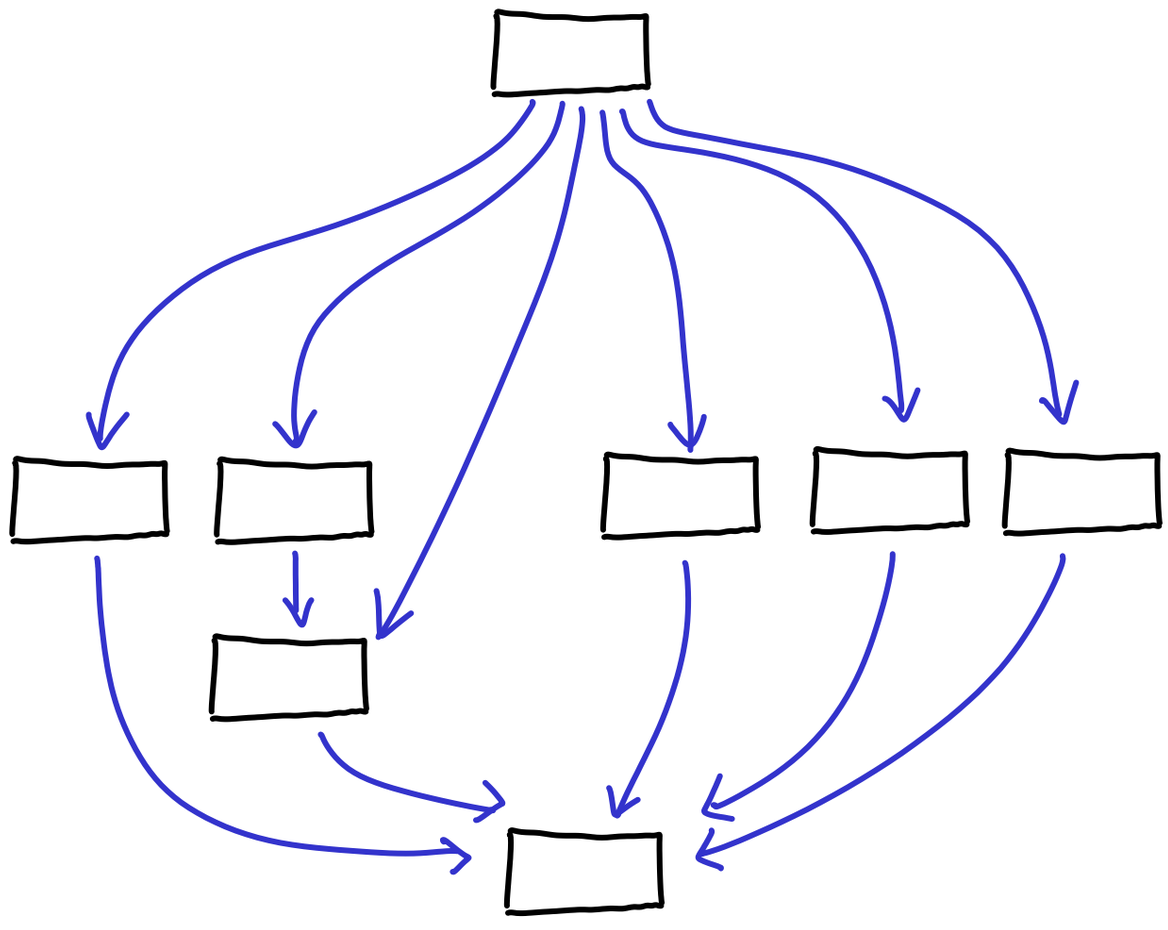

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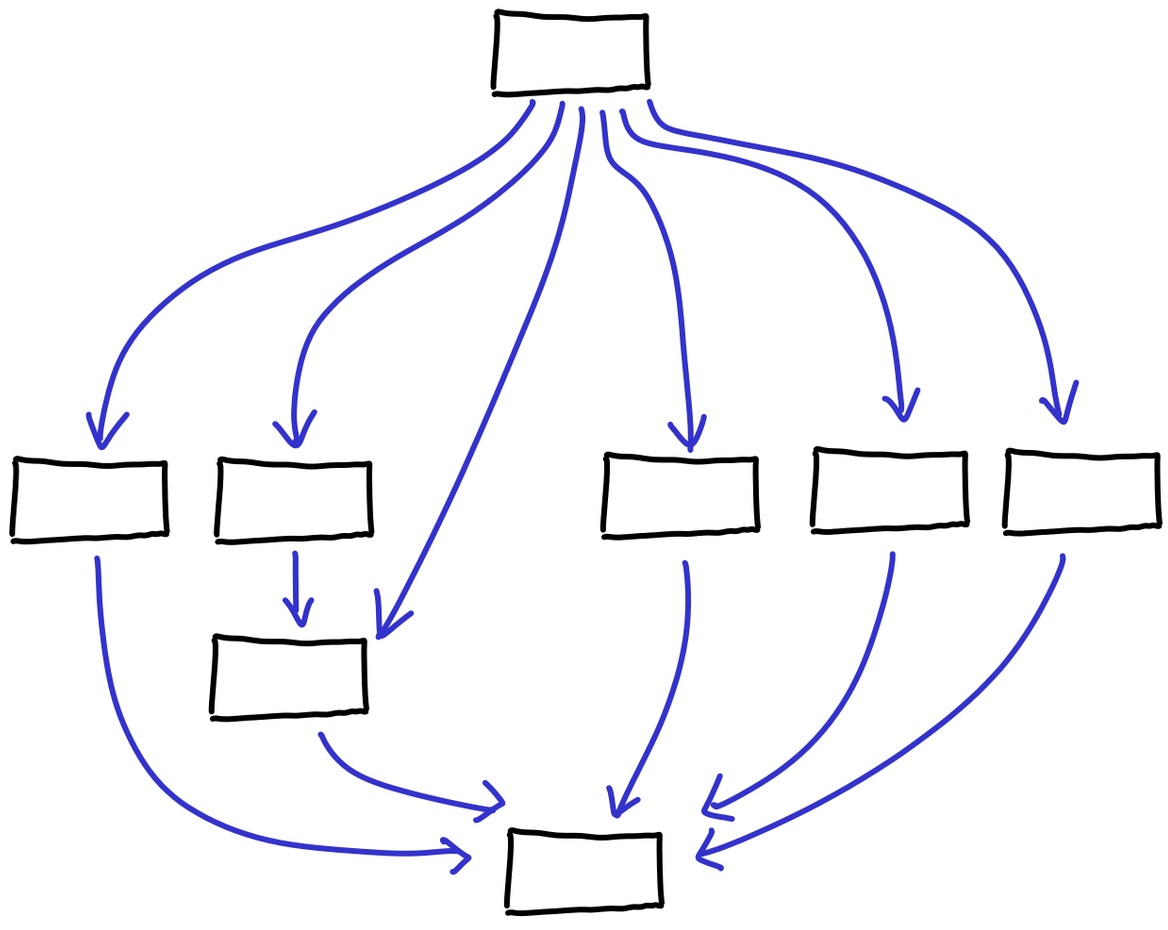

switch statement!

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Procedural abstraction

- We can chain blocks using the previous constructs to program any computable function
  - Downside:
- Procedural abstraction
  - Organize code into subroutines that can be called multiple times from different blocks
  - Upside: code reuse and better organization
  - Downside: complicates CFGs

Procedural abstraction

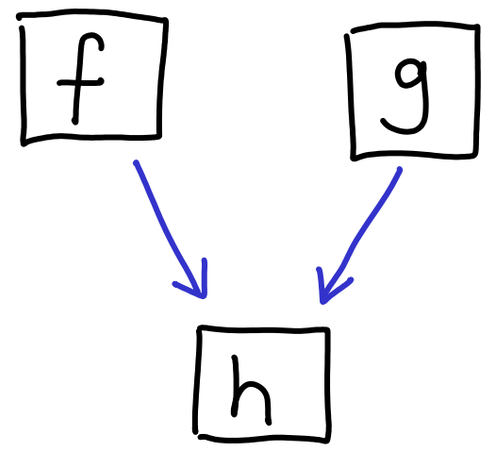
- We can chain blocks using the previous constructs to program any computable function
  - Downside: programs are giant blobs of code
- Procedural abstraction
  - Organize code into subroutines that can be called multiple times from different blocks
  - Upside: code reuse and better organization
  - Downside: complicates CFGs

CFG for function calls

```
function f(x) {
  return h(x) + 1;
}

function g(x) {
  return h(x) - 1;
}

function h(x) {
  return x * 2;
}
```



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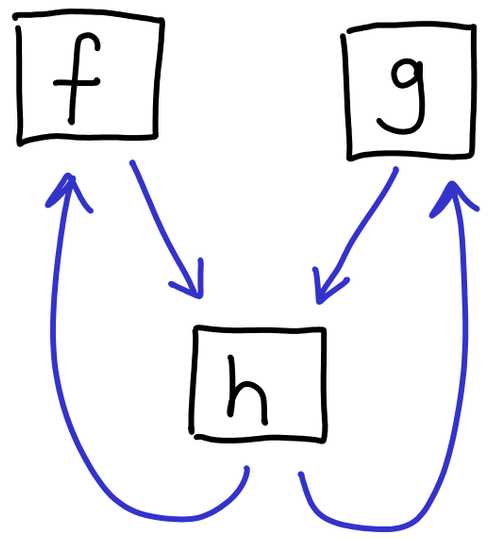

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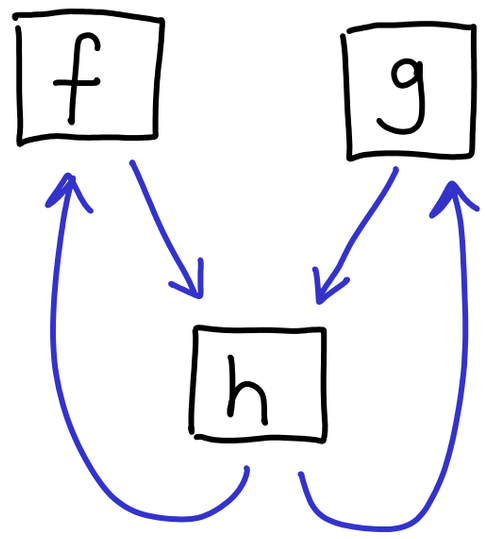

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Don’t know where h was called from until runtime!

What about return?

- There may be a lot of places where we could return to from a function call

- In general: determining the interprocedural CFG of a program is hard and super important

  - Modern attacks hijack control flow to execute arbitrary code (control-flow integrity)

- At runtime: how do we know where to go?

  - We keep track of return pointer on the stack!

Dynamic control flow

- The return pointer on the stack dictates where control goes
- Do we need to keep track of where control goes for if-statements, while loops, etc. on the stack as well?
  - No! We know exactly where execution will go if condition is true or false!

Dynamic control flow

- The return pointer on the stack dictates where control goes
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  - A: yes, B: no

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Exceptions

- Two main language constructs
  - raise/throw
  - handler/catch
- Used to terminate part of a computation
  - Jump out a construct
  - Pass data as part of the jmp
  - Return to the most recent site set up to handle ex

Exceptions

- Can we determine statically where to return to when we throw?

  - A: yes, B: no

Exceptions

- Can we determine statically where to return to when we throw?

➤ A: yes, B: no

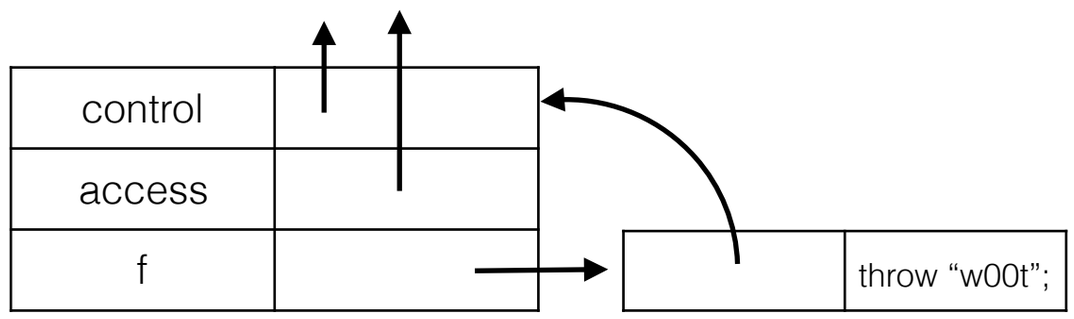
Exceptions

- How do we know where to return?
  - keep track of handler information on the stack
  - throw returns to the handler frame that is found on the stack
- How is exception handling scoped?
  - User knows how to handle error
  - Author of library function does not

Simple example

```
function f(y) {
  throw “w00t”;
}

try {
  f(1);
} catch (e) {
  console.log(e);
}
```



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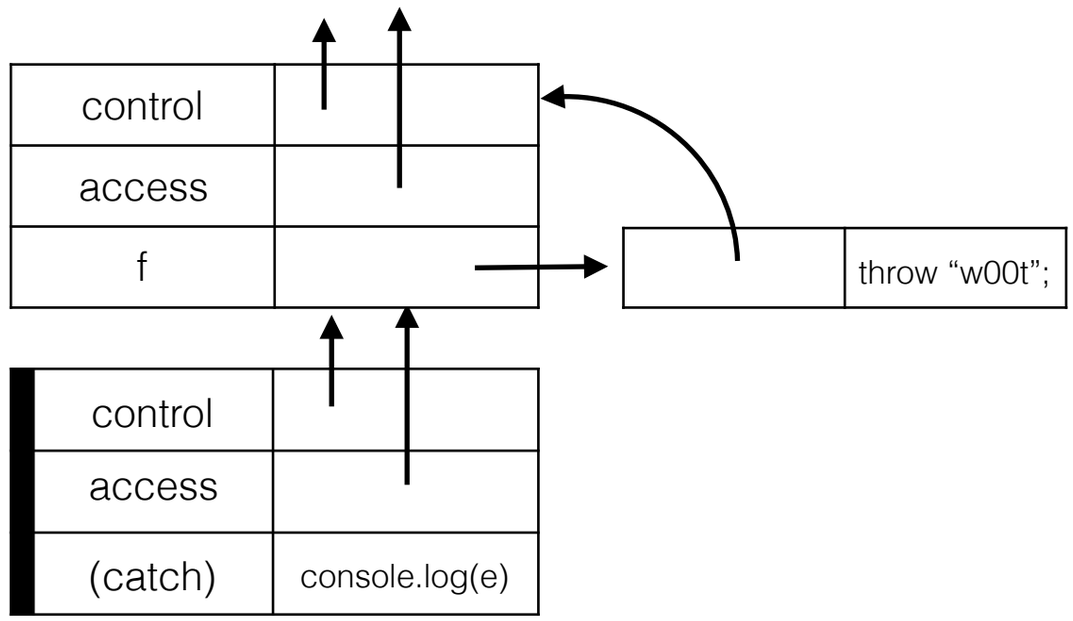

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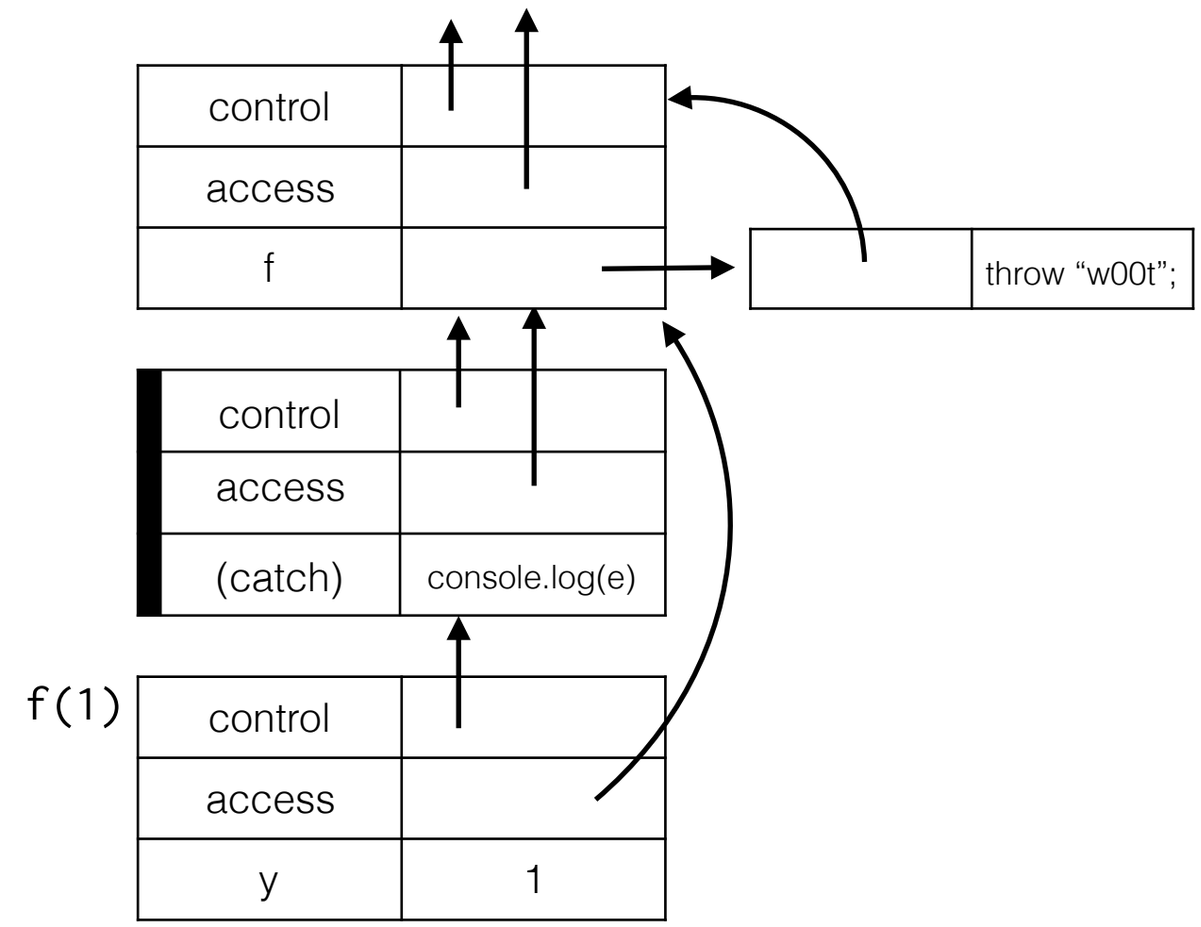

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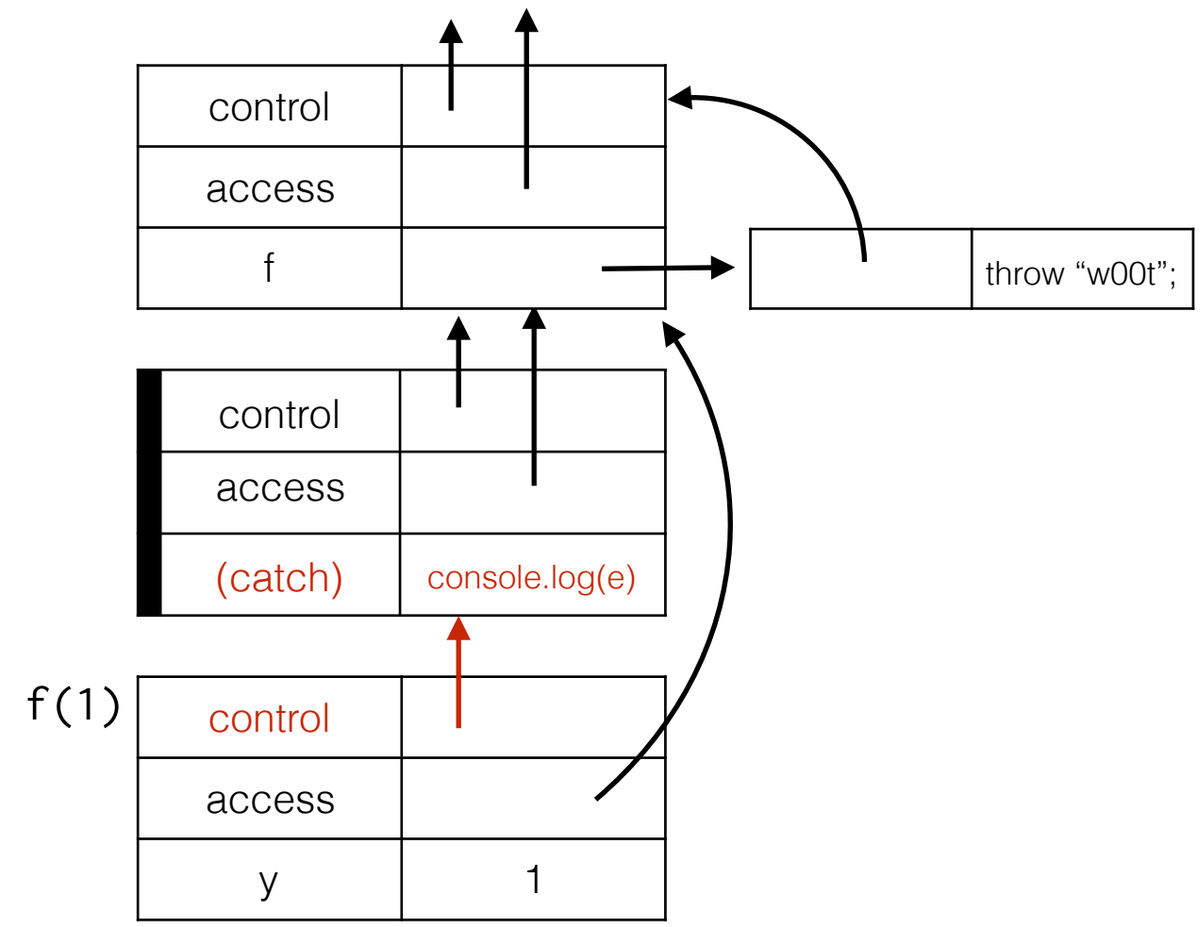

A more complicated example

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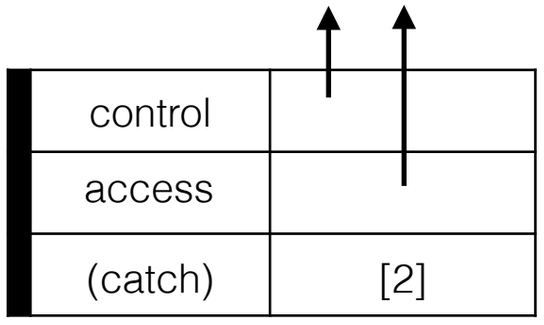


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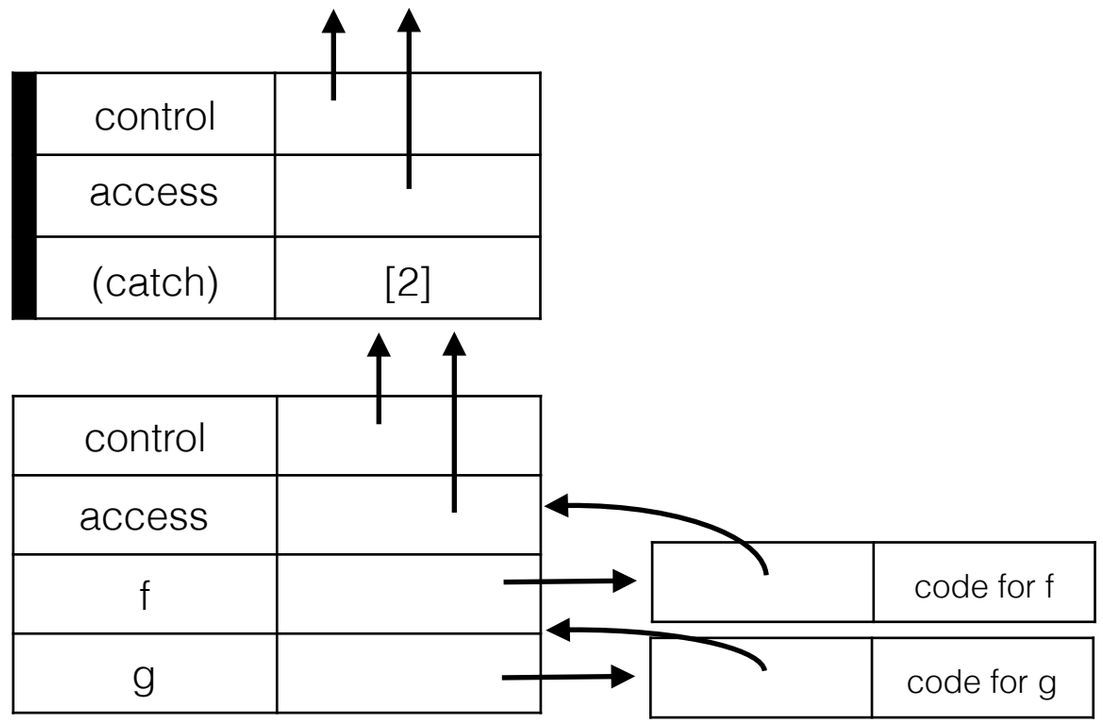

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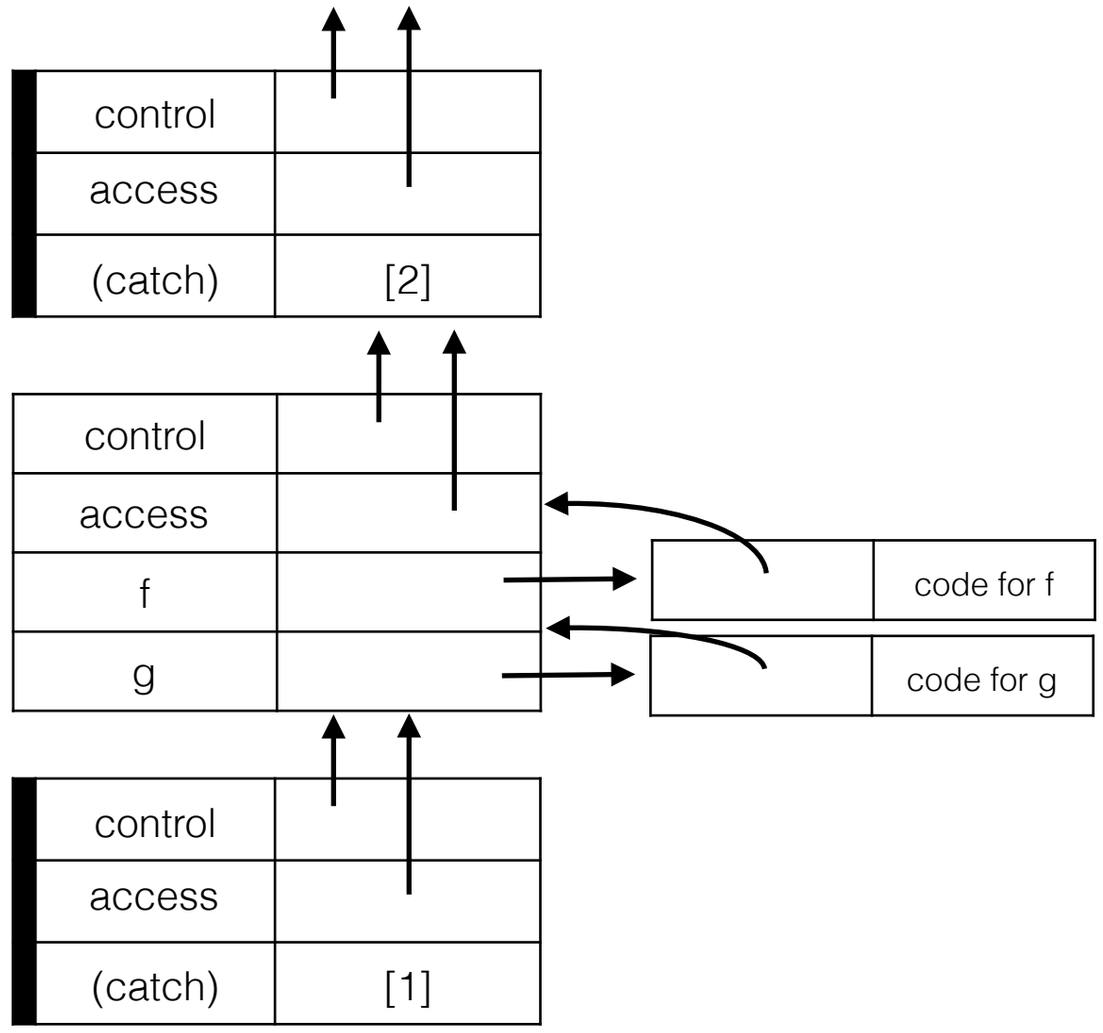

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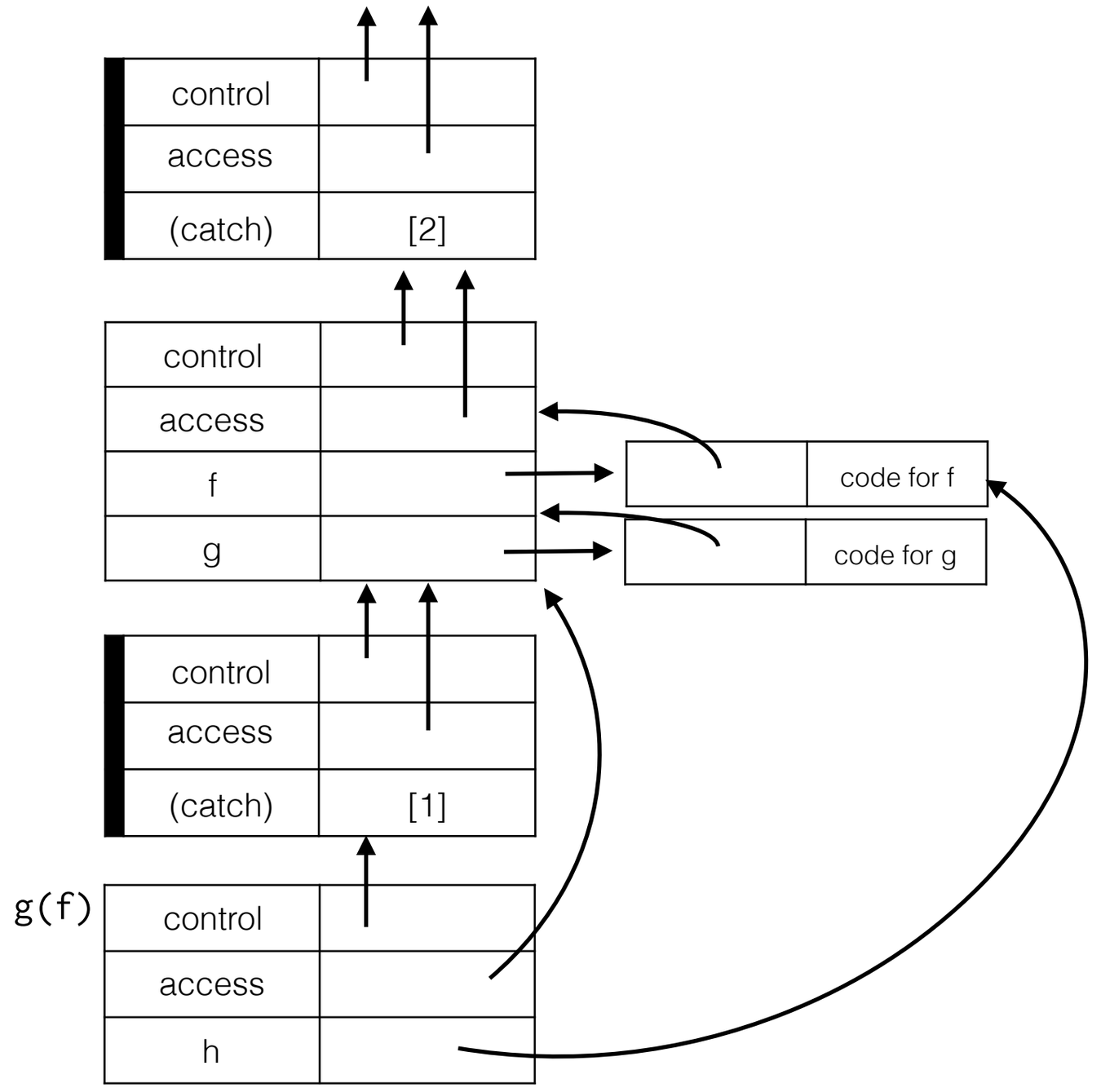

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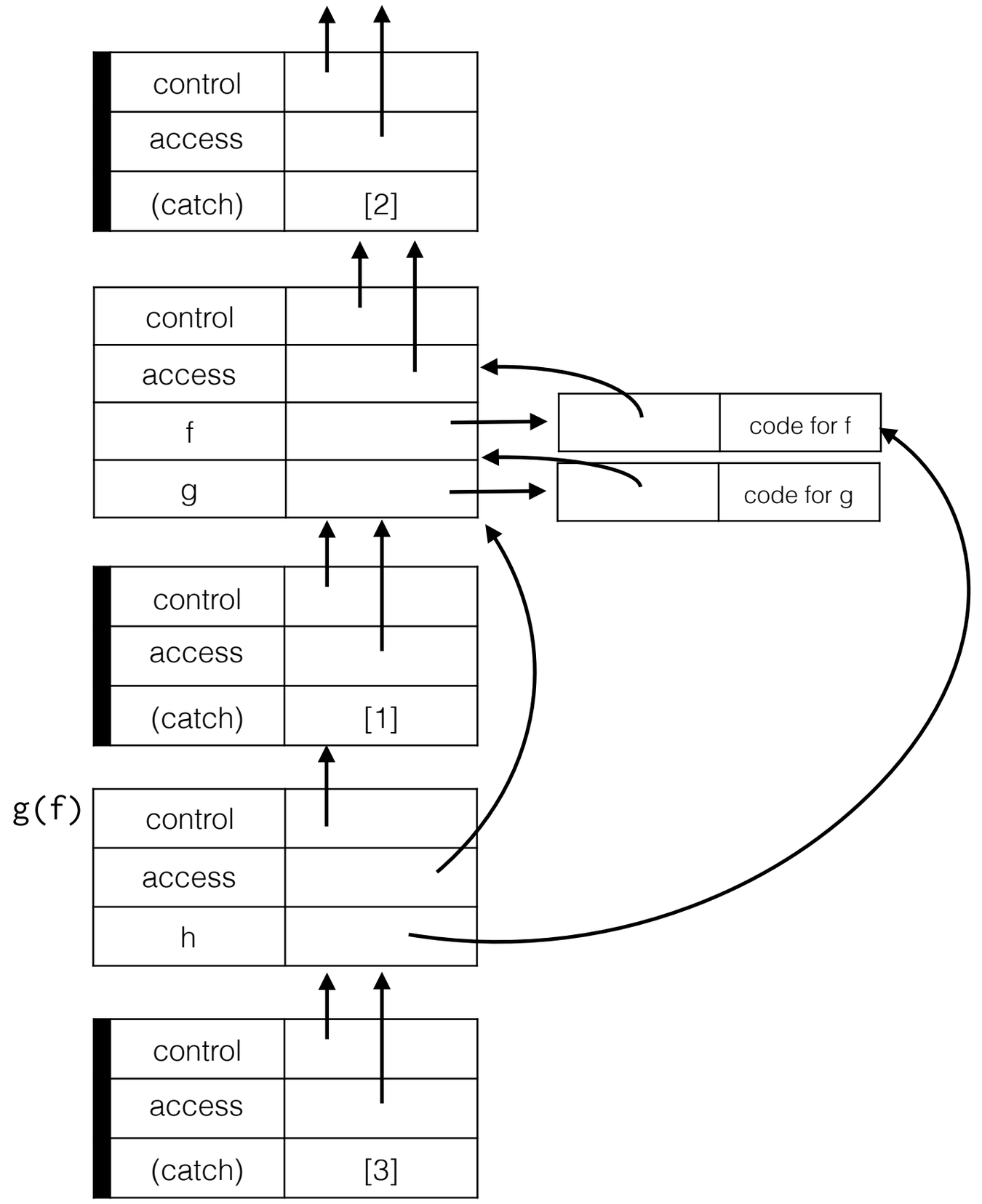

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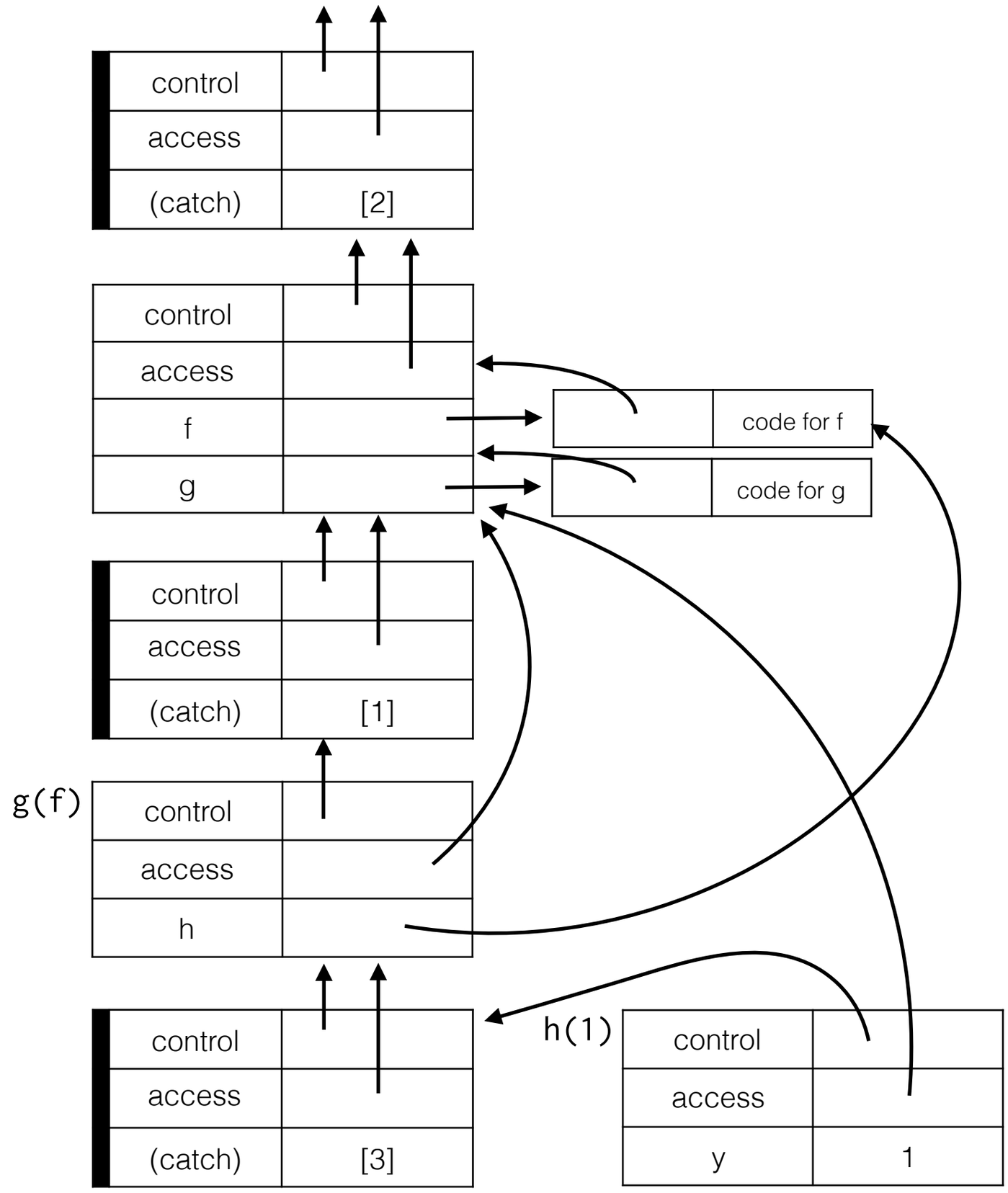

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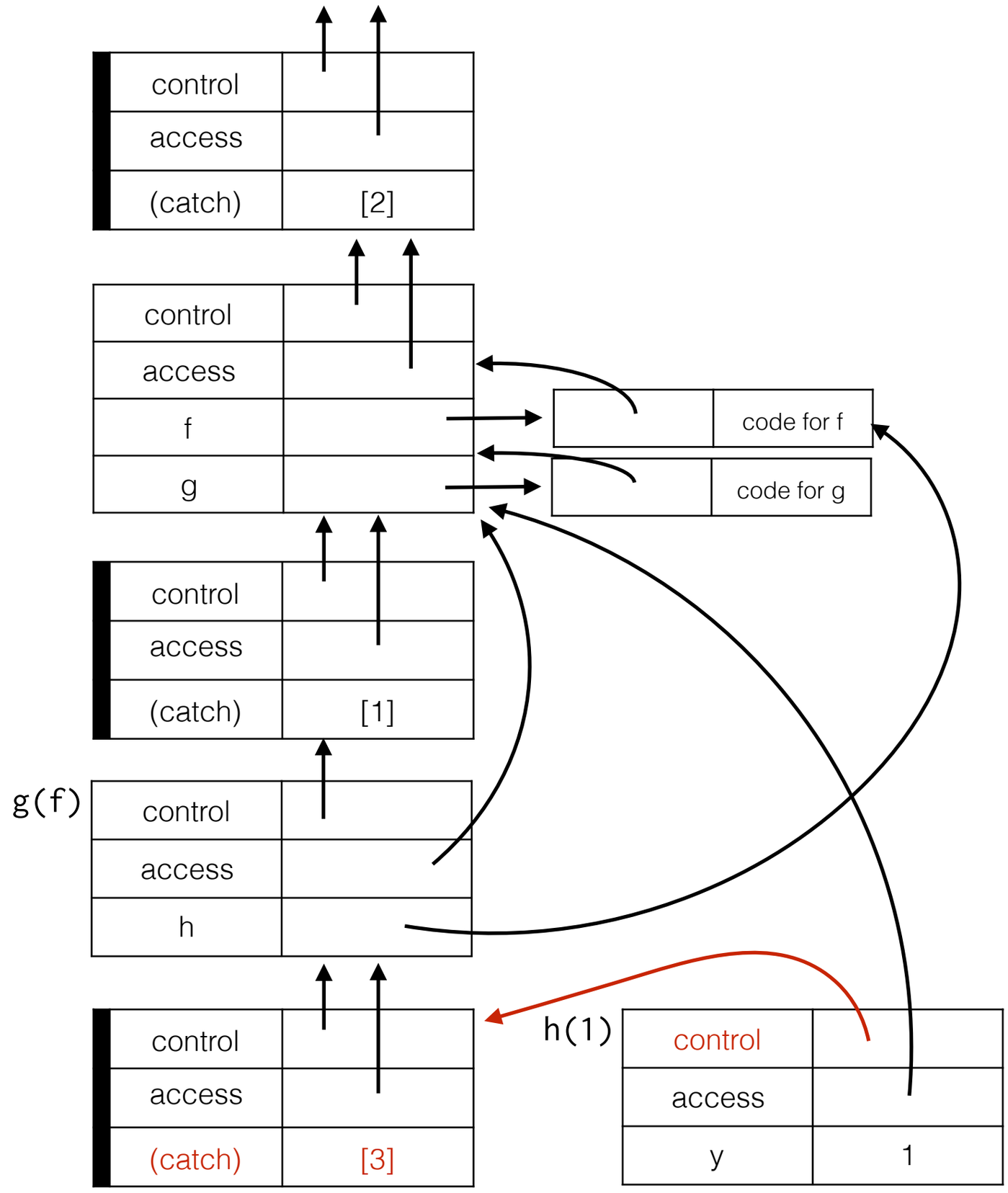

Dynamic vs. static scoping

- Again: exceptions follow dynamic scoping rules!
- Which handler would have been called if we had used static/ lexical scoping rules?
  - A: [1]
  - B: [2]
  - C: [3]

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Continuations

- Historical accident: to perform “asynchronous” computations many languages forced users to write their code in continuation passing style
  - Algol 60, Landin’s SECD machine, Scheme
  - See Reynolds’ The Discoveries of Continuations
- History repeats itself: JavaScript!

Example: async programming

```
fs.readFile('myfile.txt', (err, data) => {
  console.log(data);
  processData(data);
});
```

- When you write an explicit callback:
  - you are implementing cooperative multithreading!
  - callback is a way for thread of execution to "save its current sate" and let other code to run while it waits

Example: debugger

- Debugger is tool that builds on continuations
  - execution pauses at set breakpoint:
  - can inspect memory
  - can continue running the program (have continuation to rest of the program!)

Plan

- What is a continuation?
- Continuation-passing style
- Short summary of how to use continuations to implement control flow

Continuations are implicit in your code

- Code you write implicitly manages the future (continuation) of its computation

- Consider: (2*x + 1/y) * 2

A. Multiply 2 and x

B. Divide 1 by y

C. Add A and B

D. Multiply C and 2

Continuations are implicit in your code

- Code you write implicitly manages the future (continuation) of its computation

- Consider: (2*x + 1/y) * 2

A. Multiply 2 and x

B. Divide 1 by y — current computation

C. Add A and B

D. Multiply C and 2

rest of the program, current continuation

Continuations are implicit in your code

- Code you write implicitly manages the future (continuation) of its computation

- Consider: (2*x + 1/y) * 2

A. Multiply 2 and x

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```
let before = 2*x;
let cont = curResult =>
  (before + curResult) * 2;
cont(1/y)
```

Node.js example

- Implicit continuation:

```
const data = fs.readFileSync(‘myfile.txt’)
console.log(data);
processData(data);
```

- Explicit continuation

```
fs.readFile(‘myfile.txt', callback)
function callback (err, data)  {
  console.log(data);
  processData(data);
};
```

Continuation passing style (CPS)

- Some languages let you get your hands on the current continuation
  - call/cc (call with current continuation) is used to call a function and give it the current continuation
  - Why is this powerful?

- Most languages don’t let you get your hands on the current continuation: transform your code to CPS!

Continuation passing style (CPS)

- Some languages let you get your hands on the current continuation
  - call/cc (call with current continuation) is used to call a function and give it the current continuation
  - Why is this powerful?
    A: let's some inner function bail out and continue program by calling continuation
- Most languages don't let you get your hands on the current continuation: transform your code to CPS!

Continuation passing style

- Why do we want to do this?
  - Makes control flow explicit: no return!
  -
- So? Why should you care about this?
  -
  -

Continuation passing style

- Why do we want to do this?
  - Makes control flow explicit: no return!
  - Makes evaluation order explicit
- So? Why should you care about this?
  -
  -

Continuation passing style

- Why do we want to do this?
  - Makes control flow explicit: no return!
  - Makes evaluation order explicit
- So? Why should you care about this?
  - IR of a number of (research) languages
  - Turns function returns, exceptions, etc.: single jmp instruction! Can get rid of runtime stack!

To CPS, by example

```
function zero() {
    return 0;
}
```

To CPS, by example

```
function zero() {
   return 0;
}
```

```
function zero(cc) {
   cc(0);
}
```

cc is a function
(the current continuation)

continue execution by calling cc

To CPS, by example

```
function fact(n) {
  if (n == 0) {
    return 1;
  } else {
    return n* fact (n-1);
  }
}
```

To CPS, by example

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function fact(n) {
  if (n == 0) {
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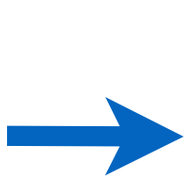
    return n* fact (n-1);
  }
}
```



```
function fact(n, cc) {
  if (n == 0) {
    cc(1);
  } else {
    fact(n-1, r => cc(n*r));
  }
}
```

To CPS, by example

```
function fact(n, cc) {
  if (n == 0) {
    cc(1);
  } else {
    fact(n-1, r => cc(n*r));
  }
}
```

```
fact(3, id) ->
fact(2, rA => id(3*rA)) ->
fact(1, rb => (rA => id(3*rA))(2*rb)) ->
fact(0, rc => (rb => (rA => id(3*rA))(2*rb))(1*rc)) ->
(rc => (rb => (rA => id(3*rA))(2*rb))(1*rc))(0) ->
(rb => (rA => id(3*rA))(2*rb))(1*0) ->
(rA => id(3*rA))(2*1*0) ->
id(3*2*1*0)
```

To CPS, by example

```
function twice(f, x) {
 return f(f(x));
}
```

```
function cmp(f, g, x) {
 return f(g(x));
}
```

To CPS, by example

```
function twice(f, x) {
 return f(f(x));
}
```

```
function twice(f, x, cc) {
 f(x, r => f(r, cc));
}
```

```
function cmp(f, g, x) {
 return f(g(x));
}
```

```
function cmp(f, g, x, cc) {
 g(x, r => f(r, cc));
}
```

To CPS, by example

```
function twice(f, x) {
 let r = f(x);
 return f(r);
}
```

To CPS, by example

```
function twice(f, x) {
 let r = f(x);
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}
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```
function twice(f, x, cc) {
 f(x, r => f(r, cc));
}
```

To CPS, the rules

- Function decls take extra argument: the continuation
  - `function (x) {` ➡ `function (x, cc) {`
- There are no more returns! Call continuation instead
  - `return x;` ➡ `cc(x);`
- Lift nested function calls out of subexpressions
  - `let r = g(x);`
`stmt1`
`stmt2` ➡ `g(x, r => {`
`  stmt1 ; stmt2`
`})`

Why is this useful?

- Makes control flow explicit
  - Compilers like this form since they can optimize code
- Multithreaded programming
- Event based programming such as GUIs

Continuations are extremely powerful

- Generalization of goto!
- Can implement control flow constructs using continuations
- How do we do if statements?
- How do we do exceptions?

Exceptions w/ continuations

```
1. function f() { throw “w00t”; }
2.
3. try {
4.   f();
5.   console.log(“no way!”);
6. } catch (e) {
7.   console.log(e);
8. }
9. console.log(“cse130 is lit”);
```

Exceptions w/ continuations

current cont = line 9

```
1. function f() { throw “w00t”; }
2.
3. try {
4.   f();
5.   console.log(“no way!”);
6. } catch (e) {
7.   console.log(e);
8. }
9. console.log(“cse130 is lit”);
```

Exceptions w/ continuations

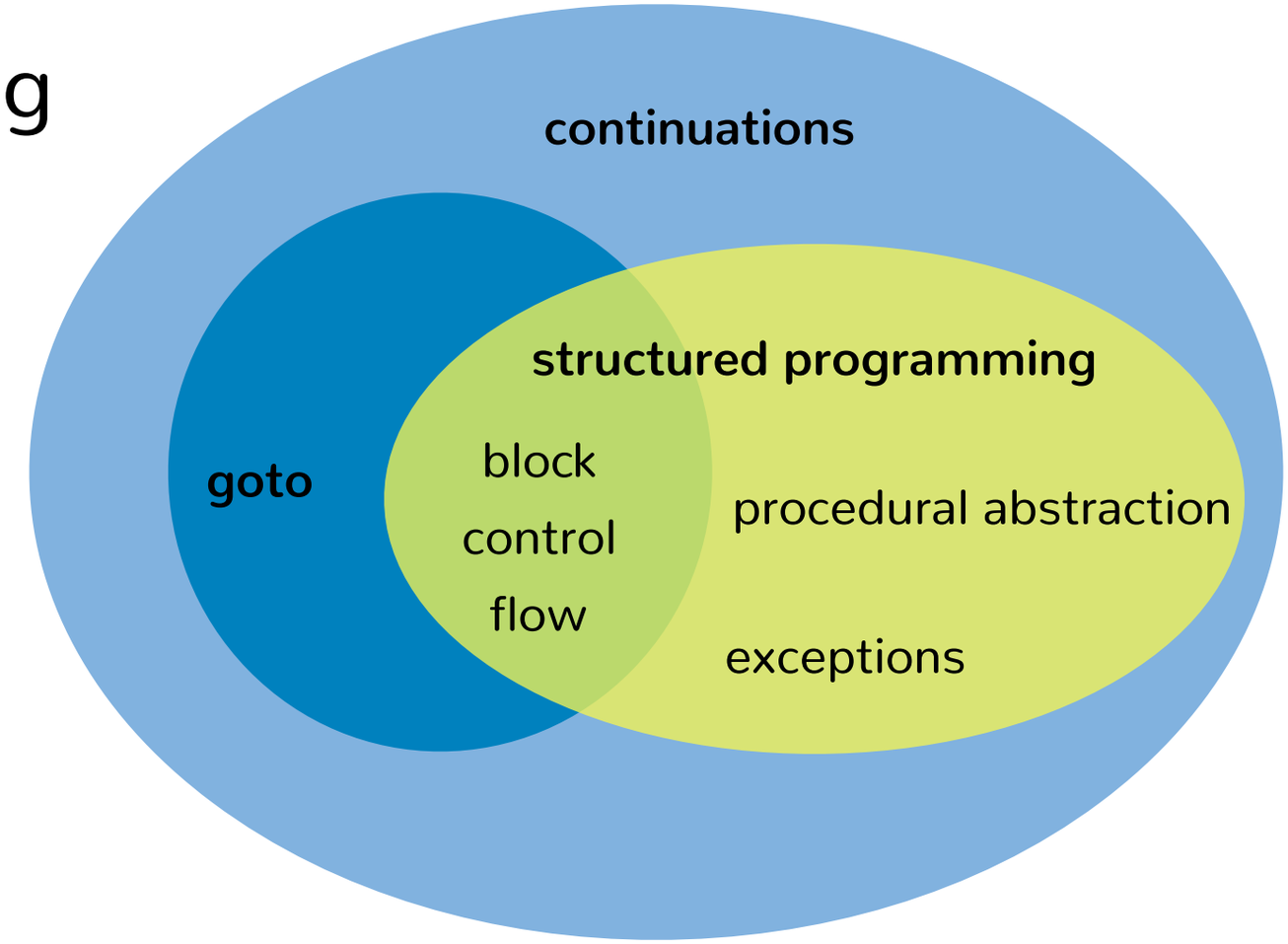
success cont =
line 5;previous cc = lines 5;9

fail cont =
lines 6-8;previous cc = lines 6-9

```
1. function f() { throw “w00t”; }
2.
3. try {
4.   f();
5.   console.log(“no way!”);
6. } catch (e) {
7.   console.log(e);
8. }
9. console.log(“cse130 is lit”);
```

Putting continuations in context

- Structured programming
- Procedural abstraction
- Exceptions
- Continuations



Week 8

- Structured programming
- Procedural abstraction
- Exceptions
- Continuations
- Monads

Can we do IO as usual?

```
ls :: [(), ()]
ls = [putChar ‘x’, putChar ‘y’]
```

Is this okay? A: yes, B: no

Laziness gets in the way?

- Depending on evaluation order order of effects may vary or may not even be observed
  - E.g., `length ls` vs. `head ls`
- Without laziness, are we okay? A: yes, B: no
  -

Laziness gets in the way?

- Depending on evaluation order order of effects may vary or may not even be observed
  - E.g., `length ls` vs. `head ls`
- Without laziness, are we okay? A: yes, B: no
  - Laziness forces us to take a more principled approach!

Monad IO

- Extend category of values with actions
- A value of type (IO a) is an action
- When performed, the action of type IO a may perform some I/O before it delivers a result of type a
- How to think about actions:
  - ➤

Monad IO

- Extend category of values with actions
- A value of type (IO a) is an action
- When performed, the action of type IO a may perform some I/O before it delivers a result of type a
- How to think about actions:
  - `type IO a = World -> (a, World)`

```
getChar :: IO Char
```

IO actions are first-class

- What does this mean? (Recall: first-class functions)
  -
  -
  -

IO actions are first-class

- What does this mean? (Recall: first-class functions)
  - Can return actions from function
  - Can pass actions as arguments
  - Can create actions in functions

```
putChar :: Char -> IO ()
```

How do we create actions?

- The return function:
  - Worst name ever: has nothing to do with terminating early
  - Given value produce IO action that doesn't perform any IO and only delivers the value
  - Type:

How do we create actions?

- The return function:
  - Worst name ever: has nothing to do with terminating early
  - Given value produce IO action that doesn't perform any IO and only delivers the value
  - Type: `return :: a -> IO a`

Example: return

- `return 42`

- ```
  f x =  if x
            then return “what”
            else return “no way!”
  ```
  ```

How do we create actions?

- The compose function (>>)
  - Given an IO action act_1 and action act_2 produce a bigger action, which when executed:
    - executes act_1
    - execute act_2 and deliver the value produced by act_2
  - Type:

How do we create actions?

- The compose function (>>)
  - Given an IO action act_1 and action act_2 produce a bigger action, which when executed:
    - executes act_1
    - execute act_2 and deliver the value produced by act_2
- Type: `(>>) :: IO a -> IO b -> IO b`

Example: >>

- `return 42 >> putChar ‘A’ >> putChar ‘B’`

- ```
  f x =  putStrLn “hello world” >>
          if x == “hello”
            then return x
            else return “bye bye!”
  ```
  ```

How do we create actions?

- The bind function (>>=)
  - Like (>>), but doesn't drop the result of first action: it chains the result to the next action (which may use it)
  - Type:
- Can we define (>>) in terms of (>>=)? A: yes, B: no

How do we create actions?

- The bind function (>>=)
  - Like (>>), but doesn't drop the result of first action: it chains the result to the next action (which may use it)
  - Type: `(>>=) :: IO a -> (a -> IO b) -> IO b`
- Can we define (>>) in terms of (>>=)? A: yes, B: no

(>>) via (>>=)

- Recall:
  - `(>>=) :: IO a -> (a -> IO b) -> IO b`
  - `(>>)  :: IO a ->      IO b    -> IO b`
- From this:
  - `(>>) act1 act2 =`

(>>) via (>>=)

- Recall:
  - `(>>=) :: IO a -> (a -> IO b) -> IO b`
  - `(>>)  :: IO a ->      IO b    -> IO b`
- From this:
  - `(>>) act1 act2 = act1 >>= \_ -> act2`

Example: >>=

- `return 42 >>= (\i -> putChar (chr i))`
- `echo :: IO ()`
  `echo =`

Example: >>=

- `return 42 >>= (\i -> putChar (chr i))`
- ```
  echo :: IO ()
  echo = getChar >>= (\c -> putChar c)
  ```
  ```

Example: >>=

- ```
  echoTwice :: IO ()
  echoTwice =
  ```

- ```
  getTwoChars :: IO (Char, Char)
  getTwoChars =
  ```

Example: >>=

- ```
  echoTwice :: IO ()
  echoTwice =  getChar >>= \c ->
               putChar c >>= _ ->
               putChar c
  ```

- ```
  getTwoChars :: IO (Char, Char)
  getTwoChars =
  ```

Example: >>=

- ```
  echoTwice :: IO ()
  echoTwice =  getChar >>= \c ->
               putChar c >>= _ ->
               putChar c
  ```

- ```
  getTwoChars :: IO (Char, Char)
  getTwoChars =  getChar >>= \c1 ->
                 getChar >>= \c2 ->
                 return (c1, c2)
  ```

Do notation

- Syntactic sugar to make it easier create big actions from small actions

- ```
  getTwoChars :: IO (Char, Char)
  getTwoChars = do
      c1 <- getChar
      c2 <- getChar
      return (c1, c2)
  ```
  ```

Do notation: de-sugaring

- do x <- e
  s ➡

- do e
  s ➡

- do e ➡

Do notation: de-sugaring

- `do x <- e`
`   s` ➡ `e >>= \x -> do s`
- `do e`
`   s` ➡ `e >> do s`
- `do e` ➡ `e`

How do we execute actions?

- Haskell program has to define main function
  - ```
    main :: IO ()
    ```
- To execute an action it has to be bound!
    ```

wc -l in Haskell

Mutable references in IO monad!

- data IORef a = ...
  - readIORef ::
  - writeIORef ::
  - atomicModifyIORef ::

Can we escape IO monad?

- Is it okay to define a function of type: IO a -> a

No! unsafePerformIO can be used to violate type safety

Monads are cool!

- Principled way to expose imperative programming in FP languages

- Evaluation order is explicit

- Idea goes beyond IO: you can define your own monad
  - Monad is a type class (with return and >>=)
  - E.g., LIO monad does security checks before performing, say, a readFile to prevent data leaks

Monads are a type class?

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b
```

hasmap.hs

Functor type class

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

- Laws
  -
  -
- What does this mean?

Functor type class

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

- Laws
  - `fmap id = id`
  - `fmap (f . g) = fmap f . fmap g`
- What does this mean?

Monad type class

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b
```

- Laws
  - ➤
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- What does this mean?

Monad type class

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b
```

- Laws
  - `return a >>= k = k a`
  - `m >>= return = m`
  - `m >>= (\x -> k x >>= h) = (m >>=k) >>= h`
- What does this mean?

Why do these matter?

- Theorem: `putStr r >> putStr s = putStr (r ++ s)`

- Proof (base case):

Why do these matter?

- Theorem: `putStr r >> putStr s = putStr (r ++ s)`

- Proof (inductive case):

Example instance that's not IO?

```
instance Monad Maybe where
  return :: a -> Maybe a

  (>>=)  :: Maybe a -> (a -> Maybe b) -> Maybe b
```