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AE 630

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(Q.2) (a) Quad-plane

It is a combined fixed wing and rotary wing aerial vehicle. Additional modes and commands allow it to takeoff and land. It can smoothly transition between plane and copter like modes in both automatic and autopilot assisted modes.

Advantages of quad-plane :-

* Additional rotors that enable it to take off and land vertically
* Significantly greater speed and range of travel
* ability to hover and perform copter-like tasks at any place.

Disadvantages of quad-plane :-

* High cost
* Generally more complex
* requires additional actuators that enable the transition from hover to forward flight. This consequently increases weight & hence efficiency
* decreases.

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(b) Tailsitter

It takes off and land on its tail and hence the name. When it takes off, it is in hovering mode then it shifts to forward moving mode in which the fixed wing component is used. The motors that are needed to produce thrust for forward motion are the same motors that were used for vertical takeoff. In other words we can say that it has a fuselage whose orientation changes after takeoff.

Advantages of tail-sitter

* Easy to store it as a compact design
* Easy to transport and setup.
* Don't need a runaway for takeoff and landing
* Simpler mechanical structures as compared to other VTOL vehicles

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Disadvantages of tail-sitter :-

* When it flies in fix-wing mode, then it undergoes some drag due to dead weight of quadrotor components.
* As compared to others, it has complex control system and transition software.
* While landing it, the platform needs to be stable because the tail-sitter is not much stable ~~because~~ as centre of gravity is not that low and we know that lower the centre of gravity, the more stable the object will be

(C) Quad tilt rotor

It consists of four tilting prop-rotors mounted at the ends of the pair of fixed wings.
In hover and low speed flight, the four rotors are oriented horizontally.
In forward flight, it is required to have some tilt. These vehicles are good options if the task is to lift heavy payloads. These UAVs have the ability to rotate their propellers relative to their body.

## Advantages of quad tilt-rotor:-

(i) Good for lifting heavy payload.

(ii) Being a VTOL, no need of a runaway for takeoff and landing.

(iii) More controllability and stability than non-VTOL vehicles.

(iv) simpler transition mechanism.

## Disadvantages of quad tilt-rotor:-

(i) Rotors are too small for efficient ~~for efficient~~ hover and too large for efficient forward flight.

(ii) Slower forward cruise speed, large fuel consumption

(iii) Complex structure, so there is a risk of system failure

(d) Tilt-wing

In this the wing is horizontal for forward flight and while doing takeoff and landing it rotates up. The design is quite similar to the design of tilt-rotor. These vehicles are fully capable of VTDL operations.

It has some mechanical advantages, like the drag caused by a propeller partially causing air flow over the wing is minimized.

## Advantages of tilt-wing

(i) Having fewer moving parts, reduces the complexity of the physical structure as well as that of the mathematical model.

(ii) Simpler control structures

(iii) Have some advantages of tilt-rotor, like capacity to carry heavy payload, etc.

## Disadvantages of tilt-wing

(i) Likely to be influenced by sudden increase in wind speed in VTOL mode

(ii) Lower hover efficiency

(iii) Large surface area, so crosswinds can push against it.

(e) Bi-plane quadrotor :-

It consists of two pairs of counter rotating propellers arranged in a quadrotor configuration. Vertical takeoff and landing is done in quad-rotor mode. Then through pitching moment provided through RPM variations of propellers, transition is done. Then at high speed, loads are transferred to wings in horizontal flight mode.

Advantages of bi-plane quadrotor :-

(i) Maneuverability of a quadrotor in hover mode

(ii) Simple construction

(iii) Increased compactness with biplane configuration

(iv) No redundant actuators in any of flight modes

Disadvantages of bi-plane quadrotor :

(i) When it is in forward flight then experiences a drag due to propellers.

(2) Stability is defined as the quality or state of being stable where stable means something that is firmly fixed or not likely to change

Static stability

:- An object is said to have static stability if it has initial tendency to return to its equilibrium state whenever it is disturbed from the equilibrium state. It occurs whenever there is no acceleration (linear or angular) of the object.

Dynamic stability

:- An object is said to have dynamic stability if the amplitudes of the response due to disturbance decay in finite time to attain the equilibrium state and not only initial tendency.

(a) $\dot{x} = ax$

$$\frac{dx}{dt} = ax$$

$$\frac{dx}{x} = a\,dt \Rightarrow \int \frac{dx}{x} = \int a\,dt$$

$$\Rightarrow \ln x = at + c_1 \Rightarrow c_2 e^{at} = x$$

$\Rightarrow x = ke^{at}$

if $a < 0 \Rightarrow$ decaying $\Rightarrow$ dynamic stability

Since the amplitude is decaying hence there must be a restoring force $\Rightarrow$ static stability

if $a > 0 \Rightarrow$ system will become unbounded $x$ will keep on increasing

(b) $\dot{X} = AX$

The general solution of such equations are

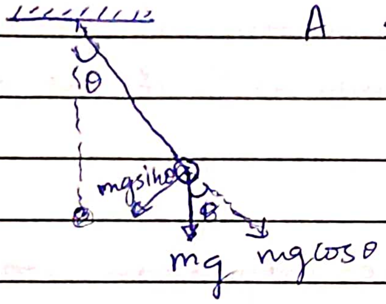
$$X = K e^{At}$$

where $e^{At}$ = Laplace inverse of $[(sI-A)^{-1}]$

$I$ = Identity matrix.

If the eigen values of A are negative, then it will have dynamic stability and hence static stability.

(c) A simple pendulum




In this diagram we can say that $mg\sin\theta$ is acting as a restoring force, hence it is statically stable.

Now there are two cases.

(I) if there is damping ⇒ dynamic stability

(II) if there is no damping ⇒ amplitude won't decay ⇒ No dynamic stability.

(d) Pitch dynamics of a fixed wing UAV:-

In longitudinal stability (pitching), we say that the vehicle is stable about its lateral axis.

It depends on the following:

* Location of wing with respect to centre of gravity
* Location of horizontal tail surfaces with respect to the centre of gravity
* area or size of tail surfaces.

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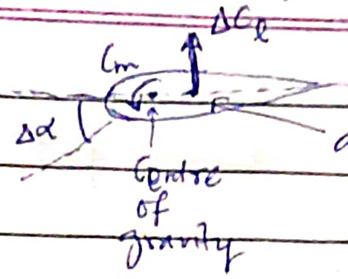


[assumed that aerodynamic ~~centre lies behind centre of~~ gravity]

aerodynamic centre (the point at which pitching moment coefficient for the airfoil does not vary ~~with lift coefficient~~)

If ~~[illegible]~~ there is a disturbance $\Delta\alpha$

$\Rightarrow$ ~~$\Delta C_\ell$ acts~~ will act on the aerodynamic centre due to which a moment $\Delta C_m$ will be generated about centre of gravity.

Here $C_m$ (i.e pitching moment) can be understood as tendency to return to its original (equilibrium) ~~position~~ orientation, so it has static stability.

$$\Rightarrow \frac{\partial C_m}{\partial C_\alpha} < 0$$

$$\Rightarrow C_{m_\alpha} < 0$$

(e) Attitude dynamics of quadcopter

$$\text{error} \quad e = \begin{bmatrix} \phi - \phi_d \\ \theta - \theta_d \\ \psi - \psi_d \end{bmatrix}$$

$\phi$ = roll
$\theta \Rightarrow$ pitch
$\psi \rightarrow$ yaw

$$\Rightarrow \dot{e} = \begin{bmatrix} \dot{\phi} - \dot{\phi}_d \\ \dot{\theta} - \dot{\theta}_d \\ \dot{\psi} - \dot{\psi}_d \end{bmatrix}$$

Assuming the disturbances to be very small,

$$\dot{\phi} \approx p, \quad \dot{\theta} \approx q, \quad \dot{\psi} \approx r$$

$$\Rightarrow \ddot{\theta} = \dot{q} = \frac{m}{I_{yy}}$$

$$\Rightarrow I_{yy}\ddot{\theta} = m$$

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Calculating the transfer function,

$$s^2\theta(s) = \frac{m(s)}{I_{yy}} \qquad \left(\text{assuming } \theta(s)\big|_{s=0} = 0\right)$$

$$\frac{\theta(s)}{m(s)} = \frac{1}{s^2 I_{yy}} = \text{Transfer function}$$

Here, we can see that two roots (repeated ones) are at origin so it is unstable. (In fact if there is any repeated roots at imaginary axis, then system will be unstable).
So we need proper controllers for its stability.

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(1)(a) Rigid body dynamics

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

I am transforming it to inertial frame from body frame to get force acting on body.

$$\vec{v} = R\vec{v}_b \quad , \quad \hat{\vec{v}}_b = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\vec{F} = m\frac{d}{dt}(R\vec{v}_b)$$

where $\hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$

$$\vec{F} = m(R\hat{\omega}\vec{v}_b + R\dot{\vec{v}}_b)$$

$$\frac{\vec{F}}{m} = R(\vec{\omega}\times\vec{v}_b + \dot{\vec{v}}_b)$$

$\omega = [\omega_1 \ \omega_2 \ \omega_3]^T = [p \ q \ r]^T$

$$\frac{R^T\vec{F}}{m} = R^TR(\vec{\omega}\times\vec{v}_b + \dot{\vec{v}}_b)$$

[as $R^TR = I$]

$$\Rightarrow \frac{\vec{F}_b}{m} = \vec{\omega}\times\vec{v}_b + \dot{\vec{v}}_b$$

Rotational dynamics

$$\vec{M} = \frac{d}{dt}(\vec{H}_I)$$

$$J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}$$

In a similar way as I did earlier

$$\vec{H}_I = R\vec{H}_b$$

$(\vec{H}_b = J\omega)$

$$\vec{M}_I = \frac{d}{dt}(R\vec{H}_b) = \dot{R}J\omega + RJ\dot{\omega}$$

$$\vec{M}_I = R\hat{\omega}J\omega + RJ\dot{\omega}$$

$$\vec{M}_I = R(\hat{\omega}J\omega + J\dot{\omega})$$

$$R^T\vec{M}_I = R^TR(\hat{\omega}J\omega + J\dot{\omega})$$

(as $R^TR = I$)

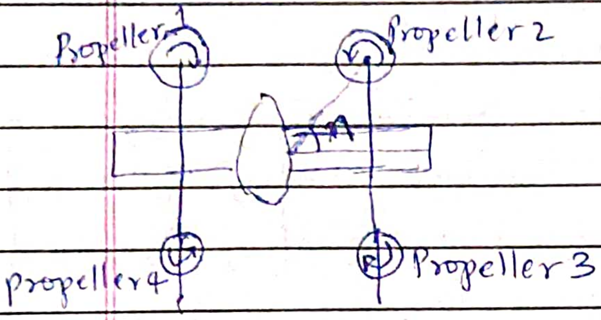
$$\vec{M}_b = \hat{\omega}J\omega + J\dot{\omega}$$

$$\Rightarrow \vec{M}_b = \omega\times J\omega + J\dot{\omega}$$

## (b) Rotor aerodynamics

$$F_i = C_{Tq}\,\omega_i^2 \qquad M_i = C_{Mq}\,\omega_i^2$$

where $F_i$ = Force
$M_i$ = Moment
$\omega_i$ = angular speed
$C_{T_q}, C_{M,q} \rightarrow$ propeller constants.




$\eta$ = angle between the line out to each propeller & $i^b$-axis

Diagonally opposite propellers are moving in the same sense.
The total thrust produced by the propellers are given by sum of all force generated by propellers.
Roll & pitching moments are given by differences between forces applied by propellers across respective axes.
Yaw moment is given by the difference in reaction torques generated by opposite rotating propellers.

$$\begin{bmatrix} F^b_{Tq} \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix} = \begin{bmatrix} C_{T_q}(\omega_1^2+\omega_2^2+\omega_3^2+\omega_4^2) \\ C_{T_q}\cos(\eta)(\omega_1^2-\omega_2^2-\omega_3^2+\omega_4^2) \\ C_{T_q}\sin(\eta)(\omega_1^2+\omega_2^2-\omega_3^2-\omega_4^2) \\ C_{M_q}(\omega_1^2-\omega_2^2+\omega_3^2-\omega_4^2) \end{bmatrix}$$

## (c) Wing aerodynamics

Lift generated by the wing will be the forces and moments along with the drag force and pitching moment

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$F^b_{lift} = \frac{1}{2} \rho S C_L V_a^2$ where, $S$ = area of wing
$C_L$ = Lift coefficient
$V_a$ = wind speed in vehicle frame

$F^b_{drag} = \frac{1}{2} \rho S C_D V_a^2$ where $C_D$ = drag coefficient
(rest symbols same as before)

$M = \frac{1}{2} \rho S C_m C V_a^2$ where $C_m$ = coefficient for pitch moment
(Pitching moment)
$C$ = chord length.
(rest symbols same as before)

(d) Forces and moments in body frame:

$$F^b = R\begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix} + \frac{1}{2} \rho S V_a^2 \begin{bmatrix} -C_D \\ 0 \\ -C_L \end{bmatrix} + \begin{bmatrix} F_{TH} \\ 0 \\ 0 \end{bmatrix}$$

(also $T = \rho A C_T R^2 \left( \Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2 \right)$)

In case there is a propeller at front

$$M^b = \begin{bmatrix} \tau^b_{roll} \\ \tau^b_{pitch} \\ \tau^b_{yaw} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2} \rho S C_m C V_a^2 \\ 0 \end{bmatrix} + \begin{bmatrix} \tau^b_{roll,F} \\ 0 \\ 0 \end{bmatrix}$$