Neural Network Application to Stochastic Overlapping Generations Economies

Kevin Mott

January 28, 2022

Economic Model

Notation: z_i^t indicates value of economic variable z at time t for the i^{th} youngest agent.

Economic Variable:	**Notation**:
Endowment income	ω_i^t
Consumption	c_i^t
Cash-on-hand	x_i^t
Equity holdings	e_i^t
Equity price	p^t
Equity dividend	δ^t
Total resources	$w^t = \delta^t + \sum_{i=1}^{L} \omega_i^t$
Shock	s^t

Intertemporal budget constraints:

$$c_1^t = \omega_1^t - p^t e_1^t - q^t b_1^t$$

$$c_i^t = \omega_i^t + \left(p^{t+1} + \delta^{t+1}\right) e_{i-1}^{t-1} + b_{i-1}^{t-1} - p^t e_i^t - q^t b_i^t \quad \text{for } i \in \{2, \ldots, L-1\}$$

$$c_L^t = \omega_L^t + \left(p^{t+1} + \delta^{t+1}\right) e_{L-1}^{t-1} + b_{L-1}^{t-1}$$

Leading to the consumer's problem:

$$\max_{\left(e_i^{t+\tau}\right)_{\tau=0}^{L-1}, \left(b_i^{t+\tau}\right)_{\tau=0}^{L-1}} \left\{ u\left(c_i^t\right) + \sum_{\tau=1}^{L-1} \beta^\tau \mathbb{E}_t \left[u\left(c_i^{t+\tau}\right)\right] \right\} \quad \text{s.t. Budget Constraints}$$

Market clearing conditions:

$$0 = \sum_{i=1}^{L-1} b_i^t \qquad\qquad 1 = \sum_{i=1}^{L-1} e_i^t$$

Theory tells us the relevant state variable at time t is

$$\sigma^t := \left((e_i^t)_{i=1}^{L-1}, (b_i^t)_{i=1}^{L-1}, s^t\right)$$

Neural Network Computation

Forward time iteration to create Ergodic grid. Starting at the solution to the equity-only deterministic steady state: $\sigma^0 = \left(\bar{e}, \bar{b}, s^0\right)$.

Network Architecture

At time t:

- Input: $\sigma^t = \left((e_i^t)_{i=1}^{L-2}, (b_i^t)_{i=2}^{L-1}, s^t\right) \in \mathbb{R}^{2L-3}$
- Output: $y^t(\sigma^t) := \left((e_i^t)_{i=1}^{L-2}, (b_i^t)_{i=1}^{L-2}, p^t, q^t\right) \in \mathbb{R}^{2L-2}$

So we see that each period, the neural network takes as input the output of last period. Run this forward for T periods.

Use market clearing for e_{L-1}^t and b_{L-1}^t in each period t.

Loss Function

I use Euler equation errors as loss function. At time t:

$$\mathcal{E}^t := \sum_{i=1}^{L-1} \left| \frac{(u')^{-1}\left[\frac{\beta \mathbb{E}_t\left[u'(c_{i+1}^{t+1})\right]}{q^t}\right]}{c_i^t} - 1 \right| + \sum_{i=1}^{L-1} \left| \frac{(u')^{-1}\left[\frac{\beta \mathbb{E}_t\left[(p^{t+1}+\delta^{t+1})u'(c_{i+1}^{t+1})\right]}{p^t}\right]}{c_i^t} - 1 \right|$$

Training

If the parameters of the neural network are Γ, the optimization scheme is:

$$\min_{\Gamma} \sum_{t=0}^{T} \mathcal{E}^t$$