Selecting Inequalities for Sharp Identification in Models with Set-Valued Predictions

(work in progress)

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Outline

Motivation

- Many partially identified models have the following structure:
  - Y — outcome, X — covariates, U — latent variables.
  - The model gives a set-valued prediction such that, by assumption,

$$Y \in G(U|X;\theta) \text{ a.s.}$$

  - The sharp identified set contains all θ such that the observed $P_{Y|X}$ corresponds to the distribution of some $Y \in G(U|X;\theta)$.
- The sharp identified set is characterized by Artstein's inequalities:
  - Using all of the inequalities is often infeasible in practice.
  - But: many of them are actually not informative.

Question: Which inequalities should we choose for identification?

Relevant Applications

- Entry games with multiple equilibria
  - Tamer 2003; Ciliberto and Tamer 2009;
- Network formation models
  - de Paula, Richards-Shubik, Tamer 2018; Sheng 2020;
- Discrete choice with heterogeneous or counterfactual choice sets
  - Barseghyan, Coughlin, Molinari, and Teitelbaum 2021;
  - Manski 2007;
- Discrete choice with endogenous explanatory variables and IVs
  - Chesher and Rosen 2017;
- Auctions
  - Haile and Tamer 2003;

Related Literature

- Theory of random sets for identification
  - Beresteanu and Molinari 2008;
  - Beresteanu, Molchanov, and Molinari 2011;
- Core-determining classes
  - Galichon and Henry 2011;
  - Chesher and Rosen 2017;
  - Luo and Wang 2018;
- Review articles
  - Ho and Rosen 2015;
  - Molinari 2019;
  - Chesher and Rosen 2019;

Example 1: Static Entry Game

- N firms choose $y_j \in \{0, 1\}$ and receive payoffs:

$$\pi_j(\boldsymbol{y}, \varepsilon_j, \theta) = y_j \cdot (\alpha_j + \delta_j(N(\boldsymbol{y}) - 1) + \varepsilon_j),$$

  where $N(\boldsymbol{y})$ is the number of firms on the market.

- Assume $\delta_j < 0$, and $U = (\varepsilon_1, \dots, \varepsilon_N) \sim F(\,\cdot\,; \gamma)$ on $\mathbb{R}^N$, observed by the players but not the researcher. Denote $\theta = (\alpha, \beta, \gamma)$.

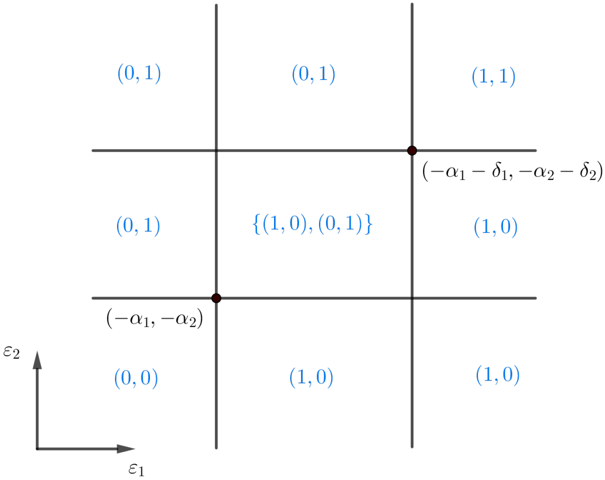
- The set-valued prediction $G(U; \theta)$ corresponds to the set of pure strategy Nash Equilibria, i.e., $\boldsymbol{y} = (y_1, \dots, y_N)$ satisfying

$$y_j = \mathbb{1}\{\alpha_j + \delta_j(N(\boldsymbol{y}) - 1) + \varepsilon_j \geqslant 0\}$$

  for all $j = 1, \dots, N$.

Illustration: Static Entry Game with Two Players

Values of $G(U;\theta)$



Example 2: English Auction

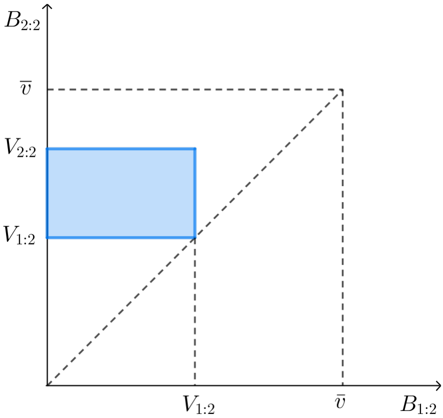
- A symmetric ascending auction with N bidders:
  - $U = (V_{1:N}, \dots, V_{N:N}) \sim F$ on $[0, \overline{v}]^N$ — ordered valuations.
  - $(B_{1:N}, \dots, B_{N:N})$ — ordered final bids.
  - No reserve price, no minimal bid increment.
- Assume that bidders:
  1. Do not bid above their valuation: $B_{j:N} \leqslant V_{j:N}$.
  2. Do not let the others win at an acceptable price: $V_{N-1:N} \leqslant B_{N:N}$.
- The set-valued prediction corresponds to the set of bids satisfying assumptions 1 and 2:

$$G(U; F) = S \cap \prod_{j=1}^{N-1} [\underline{v}, V_{j:N}] \times [V_{N-1:N}, V_N],$$

where $S = \{x \in [0, \overline{v}]^N : x_1 \leqslant \dots \leqslant x_N\}$.

Illustration: English Auction with Two Players

Example realization of $G(U; F)$



Outline

Formal Setup

- I abstract away from (i.e. condition on) the covariates.
- Basics:
  - $(\Omega, \mathcal{F}, P)$ a probability space; Θ a parameter space;
  - $Y : \Omega \to (\mathcal{Y}, \mathcal{A}) \subseteq \mathbb{R}^{d_Y}$ an observed outcome with $Y \sim P_Y$;
  - $U : \Omega \to \mathcal{U} \subseteq \mathbb{R}^{d_U}$ latent variables with $U \sim F_\theta$;
- Set-valued predictions:
  - $G : \mathcal{U} \times \Theta \to \mathcal{Y}$ is a measurable correspondence;
  - $Y \in G(U;\theta)$ a.s. by the model's assumptions;
  - The distribution of G is given by:

$$C_{G(U;\theta)}(A) \equiv P(G(U;\theta) \subseteq A) = \int \mathbf{1}(G(u;\theta) \subseteq A) dF_\theta(u)$$

- I will suppress the dependence of G and C_G from θ.

Some Terminology

- Let G be a random set defined on $(\Omega, \mathcal{F}, P)$. Its distribution can be caracterized by the containment functional:

$$C_G(A) = P(G \subseteq A).$$

- A selection of G is a random variable Y with $Y(w) \in G(w)$ P-a.s. The set of distributions of all selections of G is called the core.

- A fundamental result due to Artstein (1983):

$$\mu \in \text{Core}(G) \iff \mu(A) \geqslant C_G(A) \text{ for all } A \in \mathcal{A},$$

where μ denotes a probability distribution on $(\mathcal{Y}, \mathcal{A})$.

Identification Via Artstein's Inequalities

- The model produces a set-valued prediction G, while the researcher observes a single outcome $Y \in (\mathcal{Y}, \mathcal{A})$.
- The sharp identified set contains all θ such that the observed P_Y corresponds to the distribution of a selection of G:

$$\Theta_I = \{\theta \in \Theta : P_Y(A) \geqslant C_G(A), \text{ for all } A \in \mathcal{A}\}$$

- Note that:
  - Continuous $Y \implies$ infinite number of inequalities;
  - Discrete $Y \implies$ finite number of inequalities;
  - In both cases, many of the inequalities may be not informative.

Core-Determining Classes

- A class $\mathcal{C} \subset \mathcal{A}$ of subsets of $\mathcal{Y}$ is core-determining if

$$\mu(A) \geqslant C_G(A) \quad \forall A \in \mathcal{C} \iff \mu(A) \geqslant C_G(A) \quad \forall A \in \mathcal{A}$$

  for any probability distribution μ on $(\mathcal{Y}, \mathcal{A})$.

- In words: given $\mathcal{C}$, all other subsets of $\mathcal{Y}$ are not informative.

- Such $\mathcal{C}$ may have a much smaller cardinality then $\mathcal{A}$. To obtain such class, we need to identify redundant inequalities.

Identifying Redundant Sets (1)

- Let $\mathsf{supp}(G)$ denote the support of G, and U_G denote the set of all unions of elements in $\mathsf{supp}(G)$.
- For an arbitrary $A \subseteq \mathcal{Y}$, consider $\tilde{A} \in \mathsf{U}_G$ defined as

$$\tilde{A} = \bigcup \{B \mid B \subseteq A, B \in \mathsf{supp}(G)\}.$$

- Then, A is redundant given $\tilde{A}$, because:

$$\mu(A) \geqslant \mu(\tilde{A}) \geqslant C_G(\tilde{A}) = C_G(A).$$

- Therefore, U_G is core-determining.

Identifying Redundant Sets (2)

- Denote $G^{-}(A) = \{u : G(u) \subseteq A\}$.
- Suppose that, for some $A \in \mathsf{U}_G$, there are $A_1, A_2 \in \mathsf{U}_G$ such that:
  1. $A_1 \cap A_2 = \varnothing$;
  2. $A_1 \cup A_2 = A$;
  3. $G^{-}(A_1 \cup A_2) = G^{-}(A_1) \cup G^{-}(A_2)$;
- Then, A is redundant given A_1 and A_2, because

$$\mu(A) = \mu(A_1) + \mu(A_2) \geqslant C_G(A_1) + C_G(A_2) = C_G(A)$$

- Therefore, a class $\mathcal{C}$ such that for every $A \in \mathsf{U}_G \backslash \mathcal{C}$ there exist $A_1, A_2 \in \mathcal{C}$ satisfying 1, 2, 3 above is core-determining.

A Core-Determining Class

Theorem 1

Let U_G denote the set of all unions of elements of the support of a random set G. Let $\mathcal{C} \subseteq \mathsf{U}_G$ be a class of sets such that for every $A \in \mathsf{U}_G \backslash \mathcal{C}$ there exist $A_1, A_2 \in \mathcal{C}$:

1. $A_1 \cap A_2 = \varnothing$;
2. $A_1 \cup A_2 = A$;
3. $G^-(A) = G^-(A_1) \cup G^-(A_2)$;

Then, $\mathcal{C}$ is core-determining.

Comments:

- This is a simplified version of Theorem 3 in Chesher/Rosen 2017.
- In words: the unions of "small" sets are redundant.
- Very helpful, but we can do even better.

Identifying Redundant Sets (3)

- Suppose that, for some $A \in \mathsf{U}_G$, there are $A_1, A_2 \in \mathsf{U}_G$ such that:
  1. $A_1 \cap A_2 = A$;
  2. $A_1 \cup A_2 = \mathcal{Y}$;
  3. $G^-(A) = G^-(A_1) \cap G^-(A_2)$;
- Then, A is redundant given A_1 and A_2, because:

$$1 + \mu(A) = \mu(A_1) + \mu(A_2) \geqslant C_G(A_1) + C_G(A_2) = 1 + C_G(A).$$

- **Comment:** Denote $G^{-1}(A) = \{u : G(u) \cap A \neq \varnothing\}$. Then:
  - Condition 3 is equivalent to $G^{-1}(A^c) = G^{-1}(A_1^c) \cup G^{-1}(A_2^c)$.
  - If $\mathcal{Y} = \mathcal{Y}_1 \cup \mathcal{Y}_2$: $\mathcal{Y}_1 \cap \mathcal{Y}_2 = \varnothing$, and $G^-(\mathcal{Y}) = G^-(\mathcal{Y}_1) \cup G^-(\mathcal{Y}_2)$, the above argument can be applied within $\mathcal{Y}_1$ and $\mathcal{Y}_2$ (or further partitions) separately.

A New Core-Determining Class

First Main Result

Theorem 2

Suppose that there are no $\mathcal{Y}_1, \mathcal{Y}_2$ with $\mathcal{Y}_1 \cup \mathcal{Y}_2 = \mathcal{Y}$, $\mathcal{Y}_1 \cap \mathcal{Y}_2 = \varnothing$, $G^-(\mathcal{Y}) = G^-(\mathcal{Y}_1) \cup G^-(\mathcal{Y}_2)$. Let $\mathcal{C} \subseteq \mathsf{U}_G$ be a class of sets such that for each $A \in \mathsf{U}_G \backslash \mathcal{C}$ there exist $A_1, A_2 \in \mathcal{C}$ for which at least one of the following sets of conditions holds:

1. $A_1 \cap A_2 = \varnothing$; $A_1 \cup A_2 = A$; $G^-(A) = G^-(A_1) \cup G^-(A_2)$.
2. $A_1 \cap A_2 = A$; $A_1 \cup A_2 = \mathcal{Y}$; $G^-(A) = G^-(A_1) \cap G^-(A_2)$.

Then $\mathcal{C}$ is core-determining.

Comments:

- In words: the intersections of “large” sets are also redundant.
- Useful in practice: may not observe data in small subsets of $\mathcal{Y}$.
- If $\mathcal{Y}$ does not satisfy the stated assumption, can work with $\mathcal{Y}_i$.

Example: English Auction

- Recall: F is the joint distribution of $U = (V_{1:N}, \dots, V_{N:N})$.
- The model delivers a set-valued prediction:

$$G(U;F) = S \cap \prod_{j=1}^{N-1} [\underline{v}, V_{j:N}] \times [V_{N-1:N}, V_N].$$

  By assumption, $B = (B_{1:N}, \dots, B_{N:N}) \in G(U;F)$.

- Let $\mathcal{F}$ be a set of distributions of U satisfying some assumptions on the information structure (e.g., IPV, affiliated values, etc.)
- The sharp identified set for F is:

$$\mathcal{F}_I = \{F \in \mathcal{F} : P(B \in A) \geqslant P(G(U;F) \subseteq A), \text{ for all } A \subseteq S\}$$

  where $S = \{x \in [0, \overline{v}]^N : x_1 \leqslant \cdots \leqslant x_N\}$.

Illustration: English Auction with 2 Players

Examples of sets in U_G

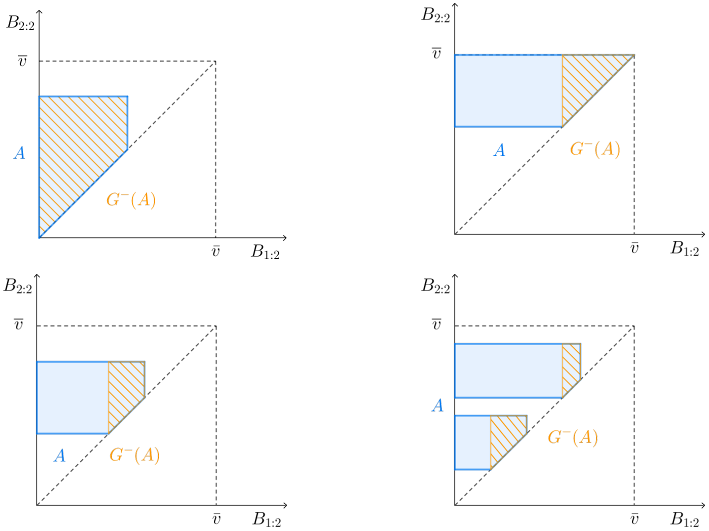


Illustration: English Auction with 2 Players

Unions of "Small" Sets: A is redundant given A_1 and A_2.

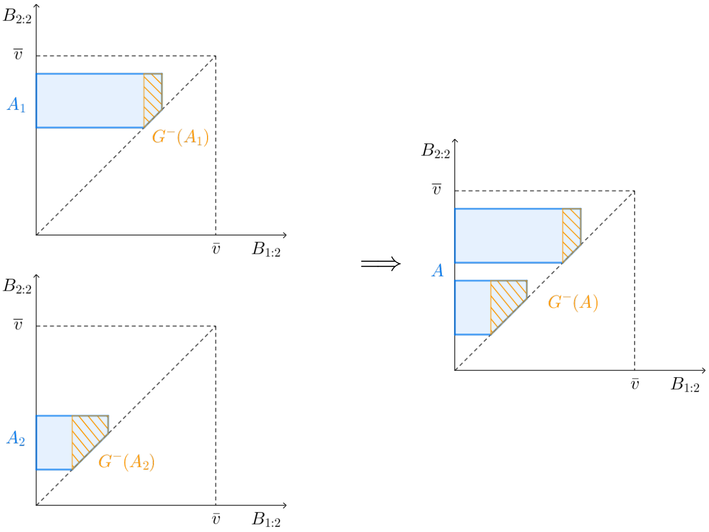
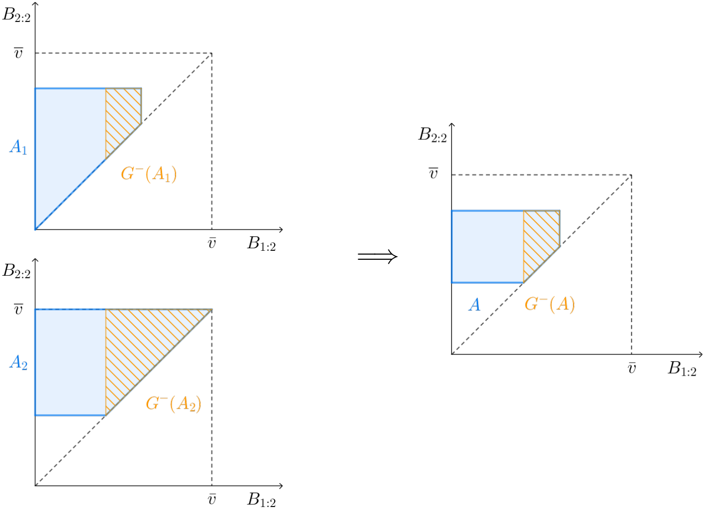


Illustration: English Auction with 2 Players

Intersections of “Large” Sets: A is redundant given A_1 and A_2.



Outline

The Smallest Core Determining Class

- If $\mathcal{Y}$ is finite, we can find the smallest core-determining class using linear programming.
- For every subset $A \subseteq \mathcal{Y}$, define the quantity:

$$\lambda(A) = \min_{\mu \in \Delta(\mathcal{Y})} \left\{ \mu(A) \,\middle|\, \mu(\tilde{A}) \geqslant C_G(\tilde{A}) \text{ for all } \tilde{A} \neq A \right\}$$

  where $\Delta(\mathcal{Y})$ is the set of probability distributions on $\mathcal{Y}$.

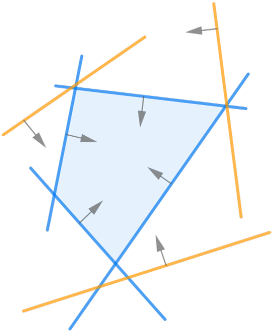
- If $\lambda(A) < C_G(A)$, then A must belong to any core-determining class. In fact, the class of all such sets:

$$\mathcal{C}^* = \{A \subseteq \mathcal{Y} : \lambda(A) < C_G(A)\}$$

  is the smallest core-determining class.[1]

[1] Follows from the literature on redundancy in LP, e.g., Telgen 1983.

Illustration: Redundant Constraints



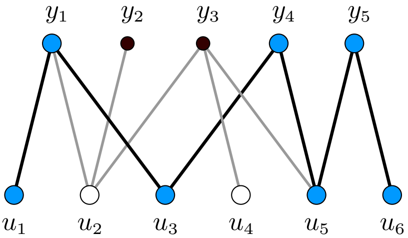
- Intuitively, if $\lambda(A) \geqslant C_G(A)$, we can remove the corresponding constraint without changing the feasible set. Blue constraints are critical, orange constraints are redundant.

Discrete Random Set as a Bipartite Graph

- The above characterization of $\mathcal{C}^*$ is nice, but:
  - It seems to depend on θ via $C_G(A) = P(G(U, \theta) \subseteq A)$. Therefore, we might need to compute $\mathcal{C}^*$ for each θ, which is impractical.
  - How is this related to Theorem 2?
- If $\mathcal{Y}$ is finite, so is $\text{supp}(G) = \{G_1, \ldots, G_K\}$, and we can partition the space of latent variables as $u_k \equiv \{u \in \mathcal{U} : G(u) = G_k\}$.
- G can be represented as an undirected bipartite graph $\mathbf{B}_G$ with two groups of vertices $\mathcal{U} = \{u_1, \ldots, u_K\}$ and $\mathcal{Y} = \{y_1, \ldots, y_S\}$ and edges (u_k, y_s) if $y_s \in G(u_k)$.
- Then, the conditions of Theorem 2 can be translated into the properties of this graph.

Discrete Random Set as a Bipartite Graph

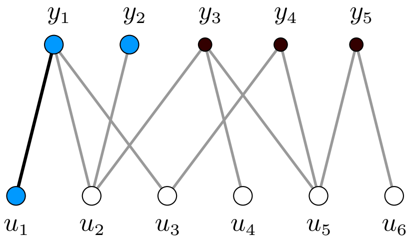
Example to Fix Ideas



- Here, $\text{supp}(G) = \{\{y_1\}, \{y_1, y_2, y_3\}, \{y_1, y_4\}, \{y_3\}, \{y_3, y_4, y_5\}, \{y_5\}\}$.
- For example, $G(u_1) = \{y_1\}$, and $G(u_5) = \{y_4, y_5\}$.
- If $A = \{y_1, y_4, y_5\}$, $G^-(A) = \{u_1, u_3, u_4, u_5\}$, $G^{-1}(A) = G^-(A) \cup \{u_2\}$.
- If $U \sim F$, then $C_G(A) = P(G^-(A)) = F(\{u_1, u_3, u_4, u_5\})$.

Redundant Sets and Bipartite Graphs

Condition 1: Only Elements of U_G Matter.



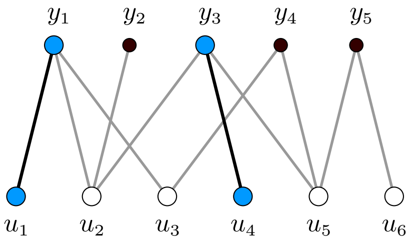
- Consider $A = \{y_1, y_2\}$, $\tilde{A} = \{y_1\}$. Then A is redundant given $\tilde{A}$:

$$\mu(A) \geqslant \mu(\tilde{A}) \geqslant C_G(\tilde{A}) = C_G(A)$$

- Note: the subgraph induced by $(A, G^{-}(A))$ is disconnected.

Redundant Sets and Bipartite Graphs

Condition 2: Unions of "Small" Sets Are Redundant.



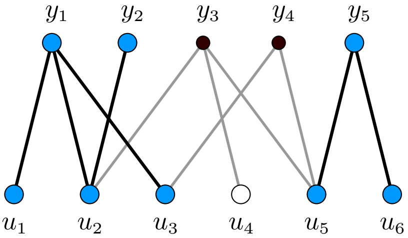
- Consider $A = \{y_1, y_3\}$, $A_1 = \{y_1\}$, and $A_2 = \{y_3\}$. Then, A is redundant given A_1, A_2:

$$\mu(A) = \mu(A_1) + \mu(A_2) \geqslant C_G(A_1) + C_G(A_2) = C_G(A)$$

- Note: the subgraph induced by $(A, G^{-}(A))$ is disconnected.

Redundant Sets and Bipartite Graphs

Condition 3: Intersections of Large Sets Are Redundant.



- Consider $A = \{y_3, y_4\}$, $A_1 = \{y_1, y_2, y_3, y_4\}$ and $A_2 = \{y_3, y_4, y_5\}$. Note that $A = A_1 \cap A_2$, $A_1^c = \{y_5\}$ and $A_2^c = \{y_1, y_2\}$.
- Condition 2 in Theorem 2 equivalently: $G^{-1}(A_1^c) \cap G^{-1}(A_2^c) = \varnothing$. Therefore, A is redundant given A_1 and A_2.
- Note: the subgraph induced by $(A^c, G^{-1}(A^c))$ is disconnected.

The Smallest Core-Determining Class

Second Main Result

Theorem 3

Let G be a discrete random set with the bipartite graph $\mathbf{B}_G$ on $(\mathcal{U}, \mathcal{Y})$. Suppose that the distribution F on $\mathcal{U}$ satisfies $F(\{u\}) > 0$ for all $u \in \mathcal{U}$. Then the class $\mathcal{C}$ of subsets $A \subseteq \mathcal{Y}$ such that:

1. *The subgraph induced by $(A, G^{-}(A))$ is connected.*
2. *The subgraph induced by $(A^c, G^{-1}(A^c))$ is connected.*

is the smallest core-determining class, i.e. $\mathcal{C} = \mathcal{C}^*$.

Discussion

- Note that $\mathcal{C}$ is the same class of sets as in Theorem 2:
  - Much simpler characterization than that of Luo and Wang 2018 for discrete random sets.
  - Often dramatically smaller than $2^{\mathcal{Y}}$ and the class in Theorem 1.
- Provides a simple and practical characterization:
  - Recall that $F = F_\theta$, so that the values of $C_G(A)$ depend on θ.
  - However, $\mathcal{C}$ does not depend on θ, provided that F_θ is non-degenerate. By definition, $\mathcal{C}$ also does not depend on P_Y.
  - Therefore: the smallest core-determining class is the same in population and in finite samples.
- $\mathcal{C}$ is fully determined by simple properties of the bipartite graph:
  - It can be efficiently computed by parsing $\mathbf{B}_G$.
  - Note that we only need to compute it once.

Implications (1)

Corollary 3.1

Let G be a discrete random set with the bipartite graph $\mathbf{B}_G$ on $(\mathcal{U}, \mathcal{Y})$. Suppose that the distribution F on $\mathcal{U}$ is non-degenerate, and the outcome space can be partitioned as $\mathcal{Y} = \bigcup_{k=1}^{K} \mathcal{Y}_k$ with (i) $\mathcal{Y}_i \cap \mathcal{Y}_j = \varnothing$ for $i \neq j$, and (ii) $G^{-1}(\mathcal{Y}_j) \cap G^{-1}(\mathcal{Y}_i) = \varnothing$. Let $\mathcal{C}_i$ be the class of all subsets $A \subseteq \mathcal{Y}_i$ such that:

1. *The subgraph induced by $(A, G^{-}(A))$ is connected.*
2. *The subgraph induced by $(\mathcal{Y}_i \backslash A, G^{-1}(\mathcal{Y}_i \backslash A))$ is connected.*

Then, $\mathcal{C} = \cup_{k=1}^{K} \mathcal{C}_i$ is the smallest core-determining class.

Example: Entry Game with Substitutes

- N firms choosing $y_j \in \{0,1\}$, so that $\mathcal{Y} = \{0,1\}^N$, with payoffs:

$$\pi_j(\boldsymbol{y}, \varepsilon_j, \theta) = y_j \cdot (\alpha_j + (N(\boldsymbol{y}) - 1)\delta_j + \varepsilon_j),$$

  where $N(\boldsymbol{y})$ is the number of entrants. Assume that $\delta_j < 0$, $\varepsilon \sim F_\varepsilon(\cdot;\gamma)$ with full support.

- The set of Nash Equilibria can not contain two equilibria with different numbers of entrants $\implies$ partition $\mathcal{Y}$ accordingly.

- Characterizing sharp identified sets (same as in CR17):
  - $N = 3$: 254 inequalities in total / 15 in the new class.
  - $N = 4$: 65534 inequalities in total / 94 in the new class.
  - $N = 5$ is infeasible;

Example: Entry Game with Complementarities

- N firms choose $y_j \in \{0, 1\}$ and receive payoffs:

$$\pi_j(\boldsymbol{y}, \varepsilon_j, \theta) = y_j \cdot (\alpha_j + \delta_j(N(\boldsymbol{y}) - 1) + \varepsilon_j),$$

  where $N(\boldsymbol{y})$ is the number of entrants. Assume that $\delta_j > 0$, $\varepsilon \sim F_\varepsilon(\cdot; \gamma)$ with full support.

- The set of Nash Equilibria only contains equilibria with different number of entrants.

- Characterizing sharp identified sets:
  - $N = 3$: 254 in total / 85 in CR17 / 36 in the new class;
  - $N = 4$: 65534 in total / 18667 in CR17 / 553 in the new class;
  - $N = 5$ is infeasible;

Implications (2)

Corollary 3.2

Let G be a discrete random set with the bipartite graph $\mathbf{B}_G$ on $(\mathcal{U}, \mathcal{Y})$, and U_G denote the set of all unions of elements of supp(G). Suppose that there are linear orders $\succcurlyeq_{\mathcal{Y}}$ and $\succcurlyeq_{\mathcal{U}}$ such that:

$$u_j \succcurlyeq_{\mathcal{U}} u_i \quad \Longrightarrow \quad \begin{cases} \min\{y \mid y \in G(u_j)\} \succcurlyeq_{\mathcal{Y}} \min\{y \mid y \in G(u_i)\} \\ \max\{y \mid y \in G(u_j)\} \succcurlyeq_{\mathcal{Y}} \max\{y \mid y \in G(u_j)\} \end{cases},$$

Then:

$$\mathcal{C} = \mathsf{U}_G \cap \{\{y_1, \ldots, y_k\}, \{y_k, \ldots, y_K\} : k = 1, \ldots, K\}$$

is the smallest core-determining class.

- **Note:** This extends Theorem 4 in Galichon and Henry (2011) who also assume that that $G(u)$ is a connected segment.

Outline

Main Idea

- Using the above results is helpful for many finite games, but sometimes even the smallest core-determining class is infeasible.
- Therefore, In practice, we will have to switch to some outer set. Which one?
- Approach: impose assumptions on the underlying selections in a way that if they are wrong, we still get a sensible outer set.
- Below, I focus on finite games.

Dimensionality Reduction

Local Games

- Suppose that a game Γ can be split into local games $\Gamma_1, \ldots, \Gamma_S$ according to the rule that "what happens in Γ_s stays in Γ_s:"
  - Payoffs from Γ_s depend only on the outcome of Γ_s.
  - Equilibrium in $\Gamma \iff$ Equilibria in all Γ_s.
- Example: Gualdani (2019)
  - Directed network formation / N players / Nash Equilibrium;
  - Γ_s is a "market entry" game with complementarities, in which $(N-1)$ players decide whether to link to player s;
  - Payoffs are appropriately separable;
- In a general network formation game: $\approx 2^{2^{N^2}}$ inequalities.

Dimensionality Reduction

Local Games

- Assuming that the local games are statistically independent, including equilibrium selection rules, simplifies the analysis:
  - Gualdani 2019 shows that: $|\mathcal{C}(\Gamma)| = \sum_{s=1}^{S} |\mathcal{C}(\Gamma_s)|$ but eventually selects the inequalities by hand.
  - This paper: can identify and use the smallest $\mathcal{C}(\Gamma_s)$.
- Characterizing sharp identified sets:
  - $N = 3$: 254 inequalities in total / 15 in the smallest class.
  - $N = 4$: $\approx 2^{64}$ inequalities in total / 144 in the smallest class.
  - $N = 5$: $\approx 2^{1024}$ inequalities in total / 2765 in the smallest class.
- Side note: for inference with many inequalities, one can use the procedure from Chernozhukov/Chetverikov/Kato 2018.

Dimensionality Reduction

Some Thoughts on Exchangeability

- When N is moderate or large, it is common to label players with several known/observable types (e.g., $\delta = \delta(x_j)$ below).
- Consider an entry game with symmetric firms:

$$\pi_j(\boldsymbol{y}, \varepsilon_j, \theta) = y_j \cdot (\alpha + (N(\boldsymbol{y}) - 1)\delta + \varepsilon_j)$$

  Assume $\delta < 0$, $\varepsilon \sim F(\,\cdot\,;\gamma)$ with full support, and $\theta = (\alpha, \delta, \gamma)$.

- Is observing $\boldsymbol{y} = (1, \ldots, 0)$ more informative than $N(\boldsymbol{y}) = 1$?
  - If $F(\cdot)$ and the equilibrium selection mechanism are appropriately exchangeable, it should not be.
  - Aggregating outcomes this way dramatically simplifies the analysis.
  - Exchangeability is testable; If it fails, we get an outer set for Θ_I.

Outline

Conclusion / Further Research

- This paper:
  - Provided a simpler characterization for sharp identified set in a class of models with set-valued predictions.
  - Proposed a new and simpler core-determining class;
  - Derived the smallest possible core-determining class for discrete random sets, which can be efficiently computed.
  - Discussed other tools for dimensionality reduction.
- Further Research:
  - An `R` package to compute the smallest core-determining class;
  - The smallest core-determining class for continuous random sets?
  - Can we aggregate outcomes by types without losing information?