WIMFluence/Response Comparison Scripts User Manual

Developed for the Arkansas Department of Transportation

TRC1701 — Bridge Load Posting Based on Actual Arkansas Truck Traffic

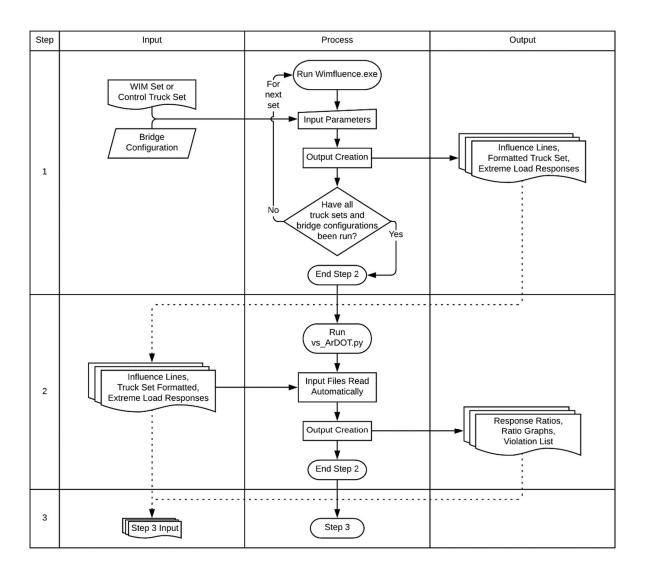
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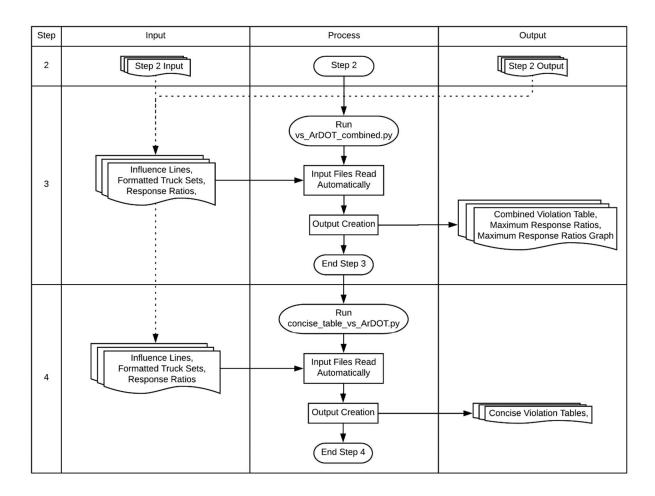
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Table of Contents

Introduction	1
Python Installation and Setup	2
Step 1: WIMFluence.exe	3
Output	3
Input	5
Operation	5
Step 2: vs_ArDOT.py / vs_AASHTO.py	7
Output	7
Input	9
Operation	9
Step 3: vs_ArDOT_combined.py / vs_AASHTO_combined.py / AASHTO_vs_ArDOT_combined.py	9
Output	10
Input	11
Operation	11
Step 4: concise_table_vs_ArDOT.py / concise_table_vs_AASHTO.py /	
concise_table_AASHTO_vs_ArDOT.py	12
Output	12
Input	13
Operation	13
GNU Free Documentation License	15
Appendix	23

This manual aims to comprehensively explain the operation of, inputs for, and outputs of the program and scripts developed for TRC1701. The flowchart below gives an overview of the operation steps involved. The installation of Python and the necessary modules follows this introduction. Each main operation step has its own explanation section afterward. An appendix of code requirements finishes the manual.





Python Installation and Setup

These scripts are created for use with the Anaconda distribution of Python. Navigate to the web address below in an internet browser. From there, download the Anaconda Windows installer for Python 3.X. Run the installer and follow the installation dialog box to install.

https://www.anaconda.com/download/

If these scripts are the only things Python is needed for, Anaconda includes more than is necessary. In that case installing Python and each necessary module independently may be preferential. If inexperienced in such things, the excess overhead of Anaconda can be worthwhile if trying to avoid potential headaches of installing each piece independently. Based on this and the scripts being written for Anaconda, use of the Anaconda distribution is the recommended method for Python installation.

To uninstall Anaconda, follow the instructions presented at the website below.

https://docs.anaconda.com/anaconda/install/uninstall

If the Python scripts do not run after double clicking, the .py file extension may need to be associated with Python. Note that the exact wording of the selections in these instructions may vary depending on the Windows version. Right click a script file, select "Open with," then select "Choose another app." From the dialog box that opens, select Python. If multiple Python entries are available, select the one with an icon appearing to represent a program window. The other entry should appear to represent a sheet of paper. Now check the box for "Always use this app to open .py files." If neither entry is available or these steps do not work, additional help may be found on the internet. Alternatively, the python scripts can still be run via command line as described in the "Operation" subsection of each script section.

Step 1:

WIMFluence.exe

Description:

This code reads a file of truck configurations to find the maximum positive moment, negative moment, and shear values at analysis points along a given bridge configuration. The analysis points are spaced at 20ths within each span of the bridge.

Output

Within the following file names, "CLASS" is replaced with the appropriate class name and "BRIDGE" is replaced with a description of the bridge on which the response values are for.

Output files are contained in a folder describing the bridge configuration within the "output" folder.

- IL_BRIDGE.csv: This includes the influence lines for the moment of, shear at the left of, and shear at the right of each analysis point. Analysis points are given as both actual values and ratios to the length of the first span. Influence factors are in terms of the length of the first span.
 - o *spans*: the number of spans in the bridge configuration
 - o span lengths: the lengths (ft) of each span in order from left to right
 - span length ratios: the ratios of span length to first span length in order from left to right
 - o internal support positions: the position (ft) of internal supports in order from left to right
 - internal support position ratios: the ratios of support positions first span length in order from left to right
 - o analysis point: the location at which the response factors are determined
 - o analysis_point_ratio: the ratio of the analysis_point to the length of the first span
 - load_point: the location of the point load causing the responses at analysis_point
 - o moment: the moment response value in terms of point load value (P) at load_point and first span length (L1). Multiply this value by P and L1 to find the actual moment response value.

- o *left_shear*: the shear value at the left side of *analysis_point* in terms of the point load value (*P*) at *load_point*. Multiply this value by *P* to find the actual shear response at the left side
- o right_shear: the shear value at the right side of analysis_point in terms of the point load value (P) at load_point. Multiply this value by P to find the actual shear response at the right side
- **CLASS_formatted.csv**: This is an alternative format list of the trucks given in the truck input file. This format is potentially friendlier for database-like tools. Depending on the size of the input truck file, this file can easily exceed the row limit of a Microsoft Excel spreadsheet.
 - o *truck_index*: the number *n* meaning the *n*th truck in the set
 - o *num axles*: the number of axles on the truck
 - o axle num: the number n meaning the nth axle of the truck
 - o axle_weight: the weight (kips) of the axle defined by axle_num and truck_index
 - o axle_rel_pos: the relative position in feet of the axle defined by axle_num and truck_index to the first axle of the truck defined by truck_index. The negative value indicates the axle being behind the first axle. A value of -5 means the axle is 5 feet behind the first axle.
- **CLASS_extreme_response_BRIDGE.csv**: This gives the maximum and minimum values for moment, left shear, and right shear at each analysis point. The "NaN" values are simply placeholders to allow a script that used this file to run properly. There are six rows per analysis point a minimum and maximum value for each of the three responses.
 - o *truck_index*: the truck number producing the extreme response value within the same row. This corresponds with the truck index given in the alternative format truck file.
 - truck_direction: the direction the truck is facing to produce the extreme response. "f" denotes a
 truck traveling forward along the bridge (left to right) and "b" denotes a truck traveling
 backward (right to left).
 - o first_axle_pos: the position in feet of the front (steering) axle of the truck along the bridge. A forward-facing truck will have other axles to the left of the steering axle while a backward-facing truck will have other axles to the right. A negative value means the first axle is off the bridge to the left of it (before it). This occurs in conjunction with a backwards facing truck resulting in at least the rear axle being on the bridge.
 - o *analysis_point*: the location in feet at which the extreme response value is being found on the bridge.
 - moment: the extreme moment (kip-ft) values at each analysis point. These are the maximum and minimum moment values. In the case of no negative moment values, the minimum value will be zero.
 - o *shear_left*: the extreme shear (kip) values at the left side of the analysis point. These are the maximum and minimum left shear values.
 - o shear_right: the extreme shear (kip) values at the right side of the analysis point. These are the maximum and minimum right shear values.

- Truck set (WIM)1 file within folder "input" with file extension of ".txt" or ".csv"
- The name of the file before the period is used as the name of the truck set in subsequent files.
 - The first line is a header line that is ignored by the code. This can contain anything the user desires, such as column names or truck set descriptions
 - O Subsequent lines detail individual trucks. The first value in the line shall be the number of axles on the truck. The following value is the weight (kips) of the first axle. Following those are the alternating axle spaces (ft) and axle weights (kips) in order from front to back. Axle spaces shall be integer values. Anything on the line after the last axle weight is ignored by the code, so truck labels can be placed there if desired. Labels can be left off if so desired. Spaces shall separate values. Any other delimiter will cause the code to fail. The following example lines detail the AASHTO SU5 truck and ArDOT Code 4.

```
5 12 10 8 4 8 4 17 4 17 AASHTO SU5 3 11 8 20 4 14 ArDOT Code 4
```

Operation

With the input files in the "input" folder, run the executable (run via double click, command line, etc.). Follow the prompt that follows.

- Input the name of the truck set file to be ran. (Ex. "ArDOT.txt", "Class_7.csv")
- Tell whether the truck file is in metric ("y") or not ("n"). If the file is in metric units, then the axle spaces and weights describe in the Inputs section shall be in decimeters and 100 kilograms (0.1 tonne). Metric units will be converted to kips and nearest foot values.
- Input the number of spans
- Input the length (ft) of each span. Span lengths shall be in integer feet values.

The program then creates the "output" folder and the subfolder describing the bridge configuration.

The folder location is given in the command line interface and the output file names are shown. A description of the bridge configuration is given followed by the run time of the code.

Repeat this process for each input file and bridge configuration desired.

The program can be run quickly at the command line by echoing the input parameters in order then piping that into the program execution. Example:

```
(echo ArDOT.txt n 2 30 20) | WIMFluence.exe
```

The PowerShell equivalent is given below.

```
"ArDOT.txt n 2 30 40" | ./ WIMFluence.exe
```

In the two examples above, the truck input file is given followed by the yes or no for metric units, the number of spans, and the lengths of each span. That information is then piped into the input text stream of the program execution.

```
Select Windows PowerShell
                                                                                                               X
PS C:\Users\User\Desktop\Research-Bridge\Research_Code>
Input the name of the WIM input file in 'input' folder:
Class_7.csv
Are the truck parameters in metric (decimeters and 100 kilograms)? y/n
Input the number of spans:
spans: 2
Input the length of span 1:
20
Input the length of span 2:
IL_2spans_20_40.csv
Class_7_formatted.csv
Class_7_extreme_response_2spans_20_40.csv
Number of Spans:
Length of Span 1:
                                                20
3
1
2
0
Total Length Ratio:
Length Ratio of Span 1:
Length Ratio of Span 2:
Position Ratio of Support 1:
Position Ratio of Support 2:
Position Ratio of Support 3:
Time taken: 21.43s
```

Steps 2-4: Data Analysis Python Scripts

The Python scripts are written for the Anaconda distribution of Python 3. The use of Anaconda is recommended, but the modules can be installed individually if so desired.

These require truck set files have been named "Class_#.csv" or "Class_#.txt" when run through the above program. Either control truck set (ArDOT or AASHTO) must have also been already run.

These scripts should be placed within the "output" folder produced by the above code. They are listed in the order to be run.

Python Modules: os, winsound, numpy, pandas, matplotlib, timeit

Description: This script walks through the folders within "output" to find response ratios for each truck class compared to the control truck set (ArDOT or AASHTO). Files are created detailing the response ratios for shear, positive moment, and negative moment; the corresponding response values; and the trucks creating those response values.

Output

Within the following file names, "#" is replaced with the appropriate class number, "CONTROL" is replaced with either "ArDOT" or "AASHTO" for whichever is used as a control, and "BRIDGE" is replaced with a description of the bridge on which the response values are for.

The following four files are within the bridge-specific folder within the "output" folder.

- ratios_moment_neg_Class_#_vs_CONTROL_BRIDGE.csv: gives the negative moment ratios for each analysis point as well as the moment and trucks creating them.
- ratios_moment_pos_Class_#_vs_CONTROL_BRIDGE.csv: gives the positive moment ratios for each analysis point as well as the moment and trucks creating them.
- ratios_shear_Class_#_vs_CONTROL_BRIDGE.csv: gives the shear ratios for each analysis point as well as the shear and trucks creating them.
- The following columns are the same for the three previously described files. "RESPONSE" should be replaced with the corresponding response type (moment neg, moment pos, shear)
 - o analysis_point: the location (ft) along the bridge at which the response ratio occurs
 - o span_position: the location of the analysis point within its bridge span. The integer part tells which support is left of the analysis point. The fractional part tells how far within the span the analysis point occurs. In "0.35", the "0" indicates the external support is to the left of the analysis point. The ".35" indicates the analysis point is 35% of the way through the span. "2.85" indicates 85% through the third bridge span. "2.00" indicates the end of the second span and the beginning of the third.
 - o ratio_moment_neg_max: the ratio of the negative moment values of the truck class to the control set. "max" refers to the response values being the extreme ones produced by the truck set when run through the previous program.
 - Class # RESPONSE max: the response value in class # producing the corresponding ratio
 - CONTROL_RESPONSE_max: the response value in the control set producing the corresponding ratio
 - o Class # truck index: the truck number of class # producing the corresponding response value
 - Class_#_truck_direction: the direction the class # truck is facing to produce the corresponding response value. "f" denotes a truck traveling forward along the bridge (left to right) and "b" denotes a truck traveling backward (right to left).

- Class_#_first_axle_pos: the location (ft) of the first axle of the class # truck to produce corresponding response value. A forward-facing truck will have other axles to the left of the steering axle while a backward-facing truck will have other axles to the right. A negative value means the first axle is off the bridge to the left of it (before it). This occurs in conjunction with a backwards facing truck resulting in at least the rear axle being on the bridge.
- CONTROL_truck_index: the truck number of the control set producing the corresponding response value
- CONTROL_truck_direction: the direction the control set truck is facing to produce the
 corresponding response value. "f" denotes a truck traveling forward along the bridge (left to
 right) and "b" denotes a truck traveling backward (right to left).
- o CONTROL_first_axle_pos: the location (ft) of the first axle of the control set truck to produce corresponding response value. A forward-facing truck will have other axles to the left of the steering axle while a backward-facing truck will have other axles to the right. A negative value means the first axle is off the bridge to the left of it (before it). This occurs in conjunction with a backwards facing truck resulting in at least the rear axle being on the bridge.
- Class_#_vs_CONTROL_BRIDGE.png: a graph of the three response ratios for each analysis point along the bridge. The positive moment is not graphed near the supports due to small moment values creating large ratios (e.g. 0.1 kips / 0.01kips = 10).

The following file is within the "output" folder.

- violation_list_vs_CONTROL.csv: a list of bridge configuration and class combinations and whether a ratio exceeds 1 or not
 - o span configuration: description of the bridge span for which the response ratios are found
 - o class: the truck class for which the response ratios are found
 - mom_pos_max_ratio: the maximum positive moment ratio produced by class in span configuration
 - mom_pos_position: the location (ft) within the bridge at which the positive moment ratio occurs
 - mom_pos_span_position: the position within the span at which the positive moment ratio occurs. The integer indicates the support to the left of the position (0 being the left external support). The decimal indicates how far into the span the point is (.35 being 35% into the span).
 - mom_neg_max_ratio: the maximum negative moment ratio produced by class in span configuration
 - mom_neg_position: the location (ft) within the bridge at which the negative moment ratio occurs
 - mom_neg_span_position: the position within the span at which the negative moment ratio occurs. The integer indicates the support to the left of the position (0 being the left external support). The decimal indicates how far into the span the point is (.35 being 35% into the span).
 - o shear_max_ratio: the maximum shear ratio produced by class in span_configuration
 - shear_position: the location (ft) within the bridge at which the shear ratio occurs
 - o shear_span_position: the position within the span at which the shear ratio occurs. The integer indicates the support to the left of the position (0 being the left external support). The decimal indicates how far into the span the point is (.35 being 35% into the span).
 - o violation: boolean value telling whether a response ratio greater than 1 occurs or not

The inputs for this script are the outputs of the previous program. Files are already in their proper locations.

Operation

Ensure Python and the necessary modules are installed first. See the Python installation section for more information.

Double click the script file to run it in the current directory. The script is open in a new command prompt window. This window will close after script execution.

If preferred, the script can be run via Command Prompt or PowerShell. Right click in the folder window the script is in then click "Open command window here" or "Open PowerShell window here." Type "python vs_ArDOT.py" or "python vs_AASHTO.py" then press Enter. This method will run the script within the current window and keep the window open after execution.

During execution, the window will display the current folder it is walking through. With knowledge of the bridge configurations present, this output can be used to measure the script's progress. At the end, the duration of the run is reported in seconds.

Step 3: vs_ArDOT_combined.py / vs_AASHTO_combined.py / AASHTO_vs_ArDOT_combined.py

Python Modules: os, winsound, numpy, pandas, matplotlib, timeit, sys

Description: This script takes the response ratios produced by the previous script and combines them across all truck classes for each bridge configuration. The maximum response ratio among all truck classes is found for each analysis point. This allows one to focus on the response ratio behavior of specific bridge configurations regardless of which truck class produces the response ratios. The truck

class producing each maximum response ratio is reported in the output files to track which classes consistently produce the greatest responses.

AASHTO_vs_ArDOT_combined.py does not combine the response ratios of different truck classes like the other two do. It still produces each of the output files produced by the other two, so its inclusion in this section is appropriate.

Output

Within the following names, "CONTROL" is replaced with either "ArDOT" or "AASHTO" for whichever is used as a control, "BRIDGE" is replaced with a description of the bridge on which the response values are for, and "RESPONSE" is replaced with the corresponding response type (moment_pos, moment_neg, shear or moment_positive, moment_negative, shear).

For AASHTO_vs_ArDOT_Combined.py, "All" is replaced with "AASHTO" to reflect the script's nature.

- All_vs_CONTROL_RESPONSE_violation_table.csv: This table shows problem response ratios and their corresponding truck class in a format somewhat reminiscent of the bridge configurations represented. It can be viewed as a text-based alternative to the response ratio figures also produced for each bridge configuration. This table has the benefit of showing all bridge configurations in a single file and reporting only the response ratios exceeding 1.
 - The first row is the span position of the response ratio reported. Span position refers to the fractional location of a position within its span of the bridge. The integer portion represents the support index (0 being the initial external support or leftmost support). The decimal portion is the percentage within the span at which the location occurs (ex. .35 indicates 35% within the span).
 - The first column gives the bridge configuration represented in each row. The numbers after "spans" gives the span lengths (ft) from left to right.
 - The rest of the table is the response ratios occurring at each span position for each bridge configuration. Ratios not greater than 1 are represented by "—" instead of being reported explicitly. This allows one to quickly pick out the issue response ratios across all bridge configurations and span positions. "N/A" is reported near each bridge support in the positive moment table. Ratios near the supports are neglected due to extreme ratios being produced by small moment values.

The following files are bridge-specific.

- ratios_RESPONSE_All_vs_CONTROL_BRIDGE.csv: the greatest response ratios at each analysis point across all truck classes
 - analysis_point: the location (ft) into the bridge at which the response ratio occurs
 - o *span_position*: the position within the span at which the response ratio occurs. The integer indicates the support to the left of the position (0 being the left external support). The decimal indicates how far into the span the point is (.35 being 35% into the span).

- ratio_RESPONSE_max: the greatest response ratio occurring at the analysis point across all truck classes
- o class_RESPONSE_max: the extreme response value of a class truck producing the response ratio
- CONTROL_RESPONSE_max: the extreme response value of the control set truck producing the response ratio
- o class_truck_index: the truck number of the class truck producing the response ratio
- class_truck_direction: the direction the class truck is facing to produce the response value. "f"
 denotes a truck facing forward (or to the right) along the bridge. "b" denotes a truck facing
 backward (or to the left) along the bridge.
- class_first_axle_pos: the position (ft) onto the bridge of the first axle of the class truck to
 produce the response value. A negative value indicates the first axle is off the bridge and before
 (or to the left) of it.
- o CONTROL_truck_index: the truck number of the control set truck producing the response ratio
- CONTROL_truck_direction: the direction the control set truck is facing to produce the response value. "f" denotes a truck facing forward (or to the right) along the bridge. "b" denotes a truck facing backward (or to the left) along the bridge.
- CONTROL_first_axle_pos: the position (ft) onto the bridge of the first axle of the control set truck to produce the response value. A negative value indicates the first axle is off the bridge and before (or to the left) of it.
- o truck_class: the truck class of the truck producing the response value
- ratios_RESPONSE_All_vs_CONTROL_BRIDGE.png: A graphical representation of the contents of the previous file. The truck class producing each response ratio is reported toward the top of the figure.

Input

The inputs for this script are the outputs of the previous program and script. Files are already in their proper locations.

Operation

Ensure Python and the necessary modules are installed first. See the Python installation section for more information.

Double click the script file to run it in the current directory. The script is open in a new command prompt window. This window will close after script execution.

If preferred, the script can be run via Command Prompt or PowerShell. Right click in the folder window the script is in then click "Open command window here" or "Open PowerShell window here." Type "python" and the script name separated with a space. Press Enter. This method will run the script within the current window and keep the window open after execution.

During execution, the window will display the base name of the current folder it is walking through. The period in the first line of output represents the parent directory of the script. As this is the first folder the script walks through, it is reported along with all the bridge-specific folders. The use of a period to represent the current folder path is a shorthand convention. With knowledge of the bridge configurations present, this output can be used to measure the script's progress.

```
Windows PowerShell

PS C:\Users\User\Desktop\Research-Bridge\Research_Code\output> python vs_ArDOT_combined.py
C:\Users\User\Desktop\Research-Bridge\Research_Code\output

2spans_20_40
3spans_100_150_100
6spans_50_50_50_50_50
PS C:\Users\User\Desktop\Research-Bridge\Research_Code\output>

✓
```

Step 4: concise_table_vs_ArDOT.py / concise_table_vs_AASHTO.py / concise_table_AASHTO_vs_ArDOT.py

Python Modules: os, winsound, pandas

Description: This script takes the response ratios produced by the first script and combines them across all truck classes for each bridge configuration. The maximum response ratio among all truck classes is found for each analysis point. This allows one to focus on the response ratio behavior of specific bridge configurations regardless of which truck class produces the response ratios. The truck class producing each maximum response ratio is reported in the output files to track which classes consistently produce the greatest responses. This script differs from the previous script in the format of the response ratio table.

concise_table_AASHTO_vs_ArDOT.py does not combine the response ratios across classes. It produces the same output table for only AASHTO compared to ArDOT.

This script will crash if it finds that there are no instances of response ratios greater than 1.

Output

- RESPONSE_violation_vs_CONTROL_ordered_table.csv: This gives the response ratios greater than
 1 for each bridge configuration and reports other information for each ratio. It is sorted from
 greatest response ratio to least.
 - ratio_RESPONSE: the ratio of the greatest response produced by a truck class to the control truck set
 - o bridge_span: a description of the bridge configuration on which the response ratio occurs

- truck_class: the truck class producing the response ratio. This column is nonexistent for the output of AASHTO_vs_ArDOT_combined.py as the column contents would always be "AASHTO."
- o analysis point: the location (ft) into the bridge at which the response ratio occurs
- o *span_position*: the position within the span at which the response ratio occurs. The integer indicates the support to the left of the position (0 being the left external support). The decimal indicates how far into the span the point is (.35 being 35% into the span).
- class_truck: a description of the class truck producing the response ratio. Values preceding the
 dash are the lengths (ft) of the axle spaces from front to back. Values after the dash are the axle
 weights (kips) from front to back. For the output of AASHTO_vs_ArDOT_combined.py, this
 column is named AASHTO_truck and gives the name of the truck instead of a description of it.
- o *gross_weight*: the gross weight (kips) of the class truck producing the response ratio. This column is omitted in the output of **AASHTO_vs_ArDOT_combined.py**.
- o *total_length*: the total length (ft) of the class truck producing the response ratio. This column is omitted in the output of **AASHTO_vs_ArDOT_combined.py**.
- o CONTROL_truck: the name of the control set truck producing the response ratio.
- class_RESPONSE: the actual response value producing the response ratio. Moment is in kip-ft
 and shear is in kips. This column is renamed AASHTO_RESPONSE in the output of
 AASHTO vs ArDOT combined.py.
- CONTROL_RESPONSE: the actual response value from the control set producing the response ratio

The inputs for this script are the outputs of the previous program and scripts. Files are already in their proper locations provided the previous program and scripts have been run.

Operation

Ensure Python and the necessary modules are installed first. See the Python installation section for more information.

Double click the script file to run it in the current directory. The script is open in a new command prompt window. This window will close after script execution.

If preferred, the script can be run via Command Prompt or PowerShell. Right click in the folder window the script is in then click "Open command window here" or "Open PowerShell window here." Type "python" and the script name separated with a space. Press Enter. This method will run the script within the current window and keep the window open after execution.

During execution, the window will display the base name of the current folder it is walking through and the class truck set it is importing. The period in the first line of output represents the parent directory of

the script. As this is the first folder the script walks through, it is reported along with all the bridge-specific folders. The use of a period to represent the current folder path is a shorthand convention. Class truck sets are imported once then kept in memory for the duration of the script run. Class truck sets are thus only reported after the first bridge configuration in which they are encountered. With knowledge of the bridge configurations and truck classes present, this output can be used to measure the script's progress.

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- The research program requires C++11. It may work with newer C++ standards, but that is untested.
- The research program is compiled using the GNU GCC compiler for C++11 with the following compilation flags
 - o -std=c++11
 - o -fexpensive-optimizations
 - o -O3
- The Python scripts are written for Python 3.6.2 using Anaconda 4.3.29
- Python modules and versions:
 - o os
 - o matplotlib 2.0.2
 - o numpy 1.13.1
 - o pandas 0.20.3
 - o sys
 - o timeit
 - o winsound