Bootstrapping and Simulating Quantities of Interest

Advanced Quantitative Methods 2019

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- Non-parametric method

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4. Conduct the analysis on the new, bootstrapped, dataset, and calculate all quantities of interest
5. Repeat 2-4 J times
6. Use the J bootstrapped estimates of your quantities of interest to calculate the estimated values and confidence bounds for your quantities of interest

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- Allows for a 'brute-force' approach to the distribution of quantities of interest
- Bootstrap aggregation (bagging) is useful in creating better predictions
- Can take multiple forms, including asymmetric bootstrapping, boosting, etc (useful when trying to maximize predictive power on certain classes)

Bootstrapping - Assumption

The sample is large enough to be a representation of the underlying distribution of values

Bootstrapping - R-Time

Example in R

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- Aims to mimic 'repeated sampling' by making draws from the sampling distribution (typically a multivariate normal distribution)
- Similar to bootstrapping, but based on parametric assumptions

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- Expected vs. predicted value (Carsey & Harden):
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  - Predicted value: 1,000 draws from the sampling distribution for the estimates, calculate $\hat{Y}$, draw from the residuals of the model, present the distribution of $\hat{Y} + \varepsilon$

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  - Example 2: Estimating the different effects of a variable dependent on the value of another variable (interaction)
  - Exampel 3: Visualizing the marginal effect of a variable in a non-linear model

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  7. Take the mean of 6 as your point estimate, and use appropriate quantiles for the confidence bounds

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Translating a table using simulated QI

Table I. Multinomial Logit: Civil War Terminations, 1944–97

	Outcomes			
Variable	*Government*	*Rebel*	*Truce*	*Treaty*
Bureaucracy	–.38	–7.09**	1.36	–.40
	.84	2.81	2.21	.65
Democracy	–.21	.13	–.17	.08
	.18	.27	.34	.13
Army	.26**	.31**	.57***	.29***
	.11	.14	.139	.10
Duration	–.12*	–.03	.06	–.14**
	.06	.11	.09	.06
Duration^2	.0002	–.0003	–.00008	.0004**
	.0002	.0006	.00028	.0002
Exports	53.75*	80.59	427.00***	59.31**
	31.04	35.90	118.65	30.33
Gini	.12	.25	–2.12***	.06
	.20	.22	.66	.18
Borders	1.17**	3.20***	3.86***	.93*
	.53	1.03	1.11	.48
Ethnicity	[illegible]	[illegible]	[illegible]	[illegible]

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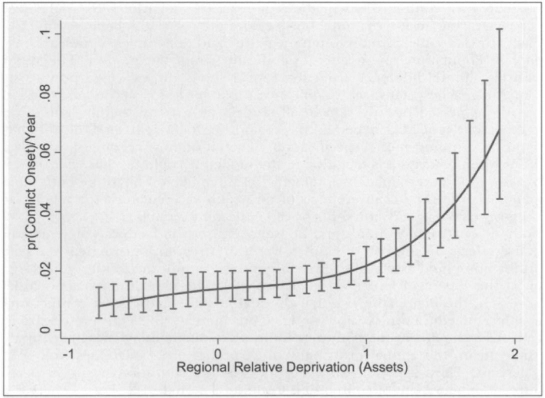
Table II. Probabilities and Expected Durations Based on Reduced Model

Effect	*Ongoing*	*Government*	*Rebel*	*Truce*	*Treaty*	*Duratio*
Baseline	.003(.00,.03)	.64(.30,.89)	.007(.00,.06)	.05(.00,.30)	.31(.07,.65)	65
Bureaucracy 25th	.003(.00,.02)	.69(.27,.92)	.04(0,.31)	.03(.00,.29)	.24(.05,.60)	56
Bureaucracy 75th	.004(.00,.04)	.56(.21,.86)	.003(0,.02)	.08(.00,.36)	.35(.07,.74)	73
Army 25th	.02(.00,.15)	.63(.29,.88)	.007(.00,.04)	.04(.00,.28)	.30(.07,.65)	63
Army 75th	.002(.00,.02)	.64(.29,.89)	.007(.00,.07)	.05(.00,.31)	.30(.07,.64)	65
Dur./dur.^{2} 25th	.002(.00,.01)	.64(.27,.89)	.01(.00,.09)	.04(.00,.38)	.31(.07,.67)	–
Dur./dur.^{2} 75th	.02(.00,.08)	.50(.16,.80)	.01(.00,.06)	.06(.00,43)	.42(.14,.73)	–
Exports 25th	.015(.00,.10)	.68(.30,.92)	.01(.00,.05)	.02(.00,.16)	.28(.05,.66)	71
Exports 75th	.003(.00,.03)	.61(.27,.88)	.007(.00,.06)	.06(.00,.37)	.31(.08,.65)	64
Borders 25th	.008(.00,.06)	.52(.16,.85)	.006(.00,.04)	.05(.00,32)	.41(.08,.82)	77
Borders 75th	.002(.00,.02)	.70(.33,.92)	.01(.00,.10)	.04(.00,.37)	.25(.06,.57)	61
Ethnicity 25th	.003(.00,.02)	.65(.25,.92)	.01(.00,.12)	.10(.00,.45)	.23(.02,.64)	59
Ethnicity 75th	.004(.00,.03)	.60(.26,.86)	.01(.00,.04)	.03(.00,.32)	.36(.11,.68)	69

Visualizing Marginal Effects and Interactions using simulated QI



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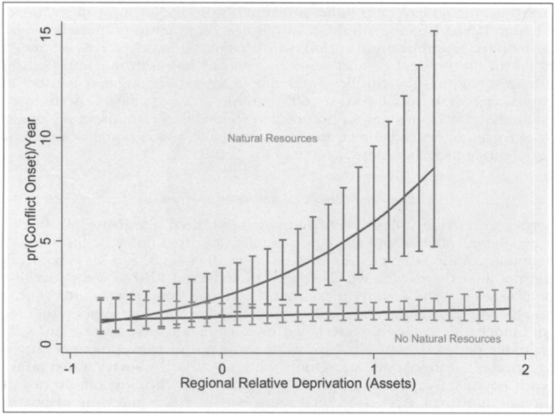

Bootstrapping - R-Time

Example in R

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- Bootstrapping - a non-parametric solution