K-Means and EM - HW2

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1 Algorithms

1.1 K-Means

Following functions are written for the K-Means Algorithm

 $\mathbf{getrandomcentrioids}(list points, k)$: Selects k random centroids from the available list of data points

stopcheck(oldcentroids, updated centroids, iterations): Termination condition for k-means. Algorithm terminates when centroid in n-1th step is same as the nth step centroid

pointlabel(listpoints, centroids): Assigns data points to the closest centroid

 $\mathbf{makearray}(listvariable): \mathbf{Helper}$ function to convert from dictionary to list

calcentroid(*listvar*): Calculates the centroid for the list of datapoints

getnewcentroid(object): Recalculates the centroid for a cluster

printclusters(): main function to call the functions

main(listpoints, centroids): Prints and plots the clusters in the desired manner

The Output obtained for K-Means is shown in Figure-1. The centroids obtained are mentioned below.

Centroids for K-means:

```
{'x': -1.0393701035692309, 'y': -1.2380392655538461}
{'x': 5.1729039155918359, 'y': 4.1359136770612261}
{'x': 0.49711036355555549, 'y': 1.266963754611111}
```

1.2 EM for clustering using a GMM

Following functions are written for the EM Algorithm

stopcheck(clusters, oldclusters): Terminating condition for the algorithm

 $\mathbf{weightedmean}(list points)$: Calculates the weighted mean of each cluster by taking into consideration soft-membership of each point

weightedcovar(listpoints, riclist): Calculates the co-variance matrix using the array of soft-membership of each point with respect to the cluster

 $\mathbf{newclusters}(list points)$: Updates standard deviation, mean and amplitude of each cluster after each iteration

covariance (listpoints): Calculates the initial co-variance matrix of each cluster without weight

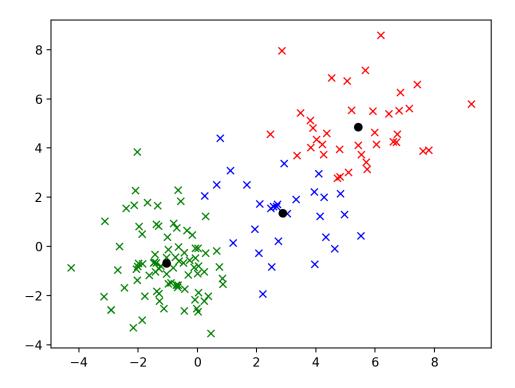


Figure 1: Cluster Obtained for K-Means

meancal(listpoints): Calculates the initial mean of each cluster without weight

 ${\bf ricinitialization}(list points):$ Randomly initializes soft membership of each point with respect to each cluster to begin the algorithm

splitcontent(listpoints): Helper method to calculate the initial parameters of the clusters

ric(clusters, listpoints): Calculates the soft membership of each point with respect to clusters

main(listpoints, centroids): Main function to call other functions and coordinate the data flow

The Outputs obtained for EM using GMM are mentioned below.

Mean: [4.5494457059982665, 3.2285096745683166] Covariance: [[3.39797759 2.57105296

[2.57105296 4.99869074]] Amplitude: 0.403147739541

Mean: [0.22042129662400683, 2.9338991440633113] Covariance: [[6.49188751 5.11468536]

[5.11468536 6.98699934]] Amplitude: 0.0749724735652

2 Software Familiarization

2.1 Scikit-learn

K-Means and GMM can be implemented using Scikit as follows



Figure 2: Original Image 96,615

GaussianMixture(n_components=1, covariance_type='full', tol=0.001, reg_covar=1e-06, max_iter=100, n_init=1, init_params='kmeans', weights_init=None, means_init=None, precisions_init=None, random_state=None, warm_start=False, verbose=0, verbose_interval=10)[source]

2.2 Improvements for Implemented Algorithm

- The Algorithm written for the purpose of this assignment has a restriction of creating 3 clusters. This should be made flexible to increased to desirable number of clusters
- A predict function needs to be written where test data can be predicted without manual-tracing which is currently employed
- This algorithm is restricted to data points in 2 dimensions. This should be expanded over to multiple dimensions.

3 Applications

3.1 K-Means for Color Quantization

K-Means can be used to perform a pixel-wise Vector Quantization (VQ) of an image, reducing the number of colors required to show the image drastically, while preserving the overall appearance quality. Figure 2, 3 and 4 obtained from Scikit Learn clearly elaborate this application.

3.2 Response Theory Models

In psychometrics $^{[NCME]}$ a field of study concerned with the theory and technique of psychological measurement, EM used for item parameters and latent abilities of item response theory models $^{[NCME]}$ a paradigm for the design, analysis, and scoring of tests, questionnaires, and similar instruments measuring abilities, attitudes, or other variables. It is the preferred method for developing scales in the in the Graduate Record Examination (GRE) and Graduate Management Admission Test (GMAT).



Figure 3: Quantized Image K-Means 64 colors



Figure 4: Quantized Image Random 64 Colors

4 Responsibility division between Teammates

We, Karthik and Praneet have brain-stormed and implemented the assignment with equal contribution for every line of code.

5 References

http://scikit-learn.org/stable/modules/generated/sklearn.mixture.GaussianMixture.html http://scikit-learn.org/stable/modules/generated/sklearn.cluster.KMeans.html http://stanford.edu/~cpiech/cs221/handouts/kmeans.html http://scikit-learn.org/stable/auto_examples/cluster/plot_color_quantization.html# sphx-glr-auto-examples-cluster-plot-color-quantization-py https://en.wikipedia.org/wiki/Expectation%E2%80%93maximization_algorithm#Applications https://en.wikipedia.org/wiki/Item_response_theory http://www.ncme.org/ncme/NCME/Resource_Center/Glossary/NCME/Resource_Center/Glossary1.aspx?hkey=4bb87415-44dc-4088-9ed9-e8515326a061#anchorP