Studentpad

JEE-MAIN MATHEMATICS - COMPLEX NUMBERS 2022-23

Time : 90 Min	Maths : Complex Numbers	Marks : 120

01) If $\omega(\neq 1)$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$, then find the value of A and B.

A) $1, 0$
B) $1, 1$
C) $0, 1$
D) $-1, 1$

02) If ω is the cube root of unity, then $(3+5\omega+3\omega^2)^2 + (3+3\omega+5\omega^2)^2 =$

A) - 4
B) 0
C) 4
D) None of these

03) If for complex numbers z_1 and z_2, $\arg(z_1/z_2) = 0$, then $|z_1 - z_2|$ is equal to

A) $||z_1| - |z_2||$
B) $|z_1| + |z_2|$
C) $|z_1| - |z_2|$
D) 0

04) $i\log\left(\frac{x-i}{x+i}\right)$ is equal to

A) $-\pi - 2\tan^{-1} x$
B) $-\pi + 2\tan^{-1} x$
C) $\pi - 2\tan^{-1} x$
D) $\pi + 2\tan^{-1} x$

05) If $x + \frac{1}{x} = \sqrt{3}$, then x=

A) $\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$
B) $\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}$
C) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$
D) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

06) $R(z^2) = 1$ is represented by

A) the hyperbola $x^2 - y^2 = 1$.
B) parabola or a circle.
C) the parabola $x^2 + y^2 = 1$.
D) all the above.

07) The value of $\frac{4(\cos 75^\circ + i\sin 75^\circ)}{0.4(\cos 30^\circ + i\sin 30^\circ)}$ is

A) $\frac{10}{\sqrt{2}}(1+i)$
B) $\frac{10}{\sqrt{2}}(1-i)$
C) $\frac{\sqrt{2}}{10}(1-i)$
D) $\frac{\sqrt{2}}{10}(1+i)$

08) The equation $z\bar{z} + a\bar{z} + \bar{a}z + b = 0, b \in R$ represents a circle if

A) $|a|^2 > b$
B) $|a|^2 = b$
C) $|a|^2 < b$
D) None of these

09) If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

A) x = 2n+1, where n is any positive integer.
B) x = 4n+1, where n is any positive integer.
C) x = 2n, where n is any positive integer.
D) x = 4n, where n is any positive integer.

10) If $i^2 = -1$, then sum $i + i^2 + i^3 + \ldots$ to 1000 terms is equal to

A) 0
B) i
C) - 1
D) 1

11) One of the cube roots of unity is

A) $\frac{1+i\sqrt{3}}{2}$
B) $\frac{-1+i\sqrt{3}}{2}$
C) $\frac{1-i\sqrt{3}}{2}$
D) $\frac{\sqrt{3}-i}{2}$

12) If $a = \cos\theta + i\sin\theta$, then $\frac{1+a}{1-a} =$

A) $i\cot\frac{\theta}{2}$
B) $\cot\theta$
C) $\cot\frac{\theta}{2}$
D) $i\tan\frac{\theta}{2}$

13) The amplitude of $\frac{1+\sqrt{3}\,i}{\sqrt{3}-i}$ is

A) $\pi/2$
B) $\pi/3$
C) $\pi/6$
D) π

14) If $\frac{1}{x}+x=2\cos\theta$, then $x^n+\frac{1}{x^n}$ is equal to
A) $2\sin n\theta$
B) $2\cos n\theta$
C) $\sin n\theta$
D) $\cos n\theta$

15) If $1, \omega, \omega^2$ are the roots of unity, then $(1-2\omega+\omega^2)^6$ is equal to
A) 81
B) 243
C) 246
D) 729

16) If $z=x-iy$ and $z^{\frac{1}{3}}=p+iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)/(p^2+q^2)$ is equal to
A) 2
B) 1
C) - 1
D) - 2

17) If $x+iy=\frac{3}{2+\cos\theta+i\sin\theta}$, then x^2+y^2 is equal to
A) $4x-3$
B) $3x-4$
C) $4x+3$
D) None of these

18) The amplitude of $e^{e^{-i\theta}}$ is equal to
A) $-\sin\theta$
B) $\sin\theta$
C) $e^{\sin\theta}$
D) $e^{\cos\theta}$

19) In the argand diagram, if O, P and Q represents the origin, the complex numbers z and $z+iz$ respectively, then the angle $\angle OPQ$ is
A) $\frac{2\pi}{3}$
B) $\frac{\pi}{2}$
C) $\frac{\pi}{3}$
D) $\frac{\pi}{4}$

20) Let z_1 and z_2 be two complex numbers with α and β as their principal arguments such that $\alpha+\beta>\pi$, then principal arg (z_1z_2) is
A) $\alpha+\beta+2\pi$
B) $\alpha+\beta-\pi$
C) $\alpha+\beta+\pi$
D) $\alpha+\beta$

21) If α and β are imaginary cube roots of unity, then $\alpha^4+\beta^4+\frac{1}{\alpha\beta}=$
A) 0
B) 1
C) 2
D) 3

22) If $z_1=1+i, z_2=-2+3i$ and z_3=ai/3, where $i^2=-1$, are collinear then the value of a is
A) 5
B) 4
C) 3
D) - 1

23) Conjugate of $1+i$ is
A) 1 + i
B) 1 - i
C) 1
D) i

24) If $(1+i\sqrt{3})^9=a+ib$, then b is equal to
A) 0
B) 1
C) 9^3
D) 256

25) If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$, then $\cos3\alpha+\cos3\beta+\cos3\gamma$ equals to
A) $3\sin(\alpha+\beta+\gamma)$
B) $3\cos(\alpha+\beta+\gamma)$
C) $\cos(\alpha+\beta+\gamma)$
D) 0

26) If z_1 and z_2 are two non-zero complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$, then $\arg(z_1)-\arg(z_2)$ is equal to
A) 0
B) $-\pi$
C) $-\frac{\pi}{2}$
D) $\frac{\pi}{2}$

27) If $\left|z-\frac{4}{z}\right|=2$, then what is the maximum value $|z|$?

28) Let $z=a-\frac{i}{2}; a\in IR$. Then evaluate $|i+z|^2-|i-z|^2$.

29) $\left|\frac{1}{(2+i)^2}-\frac{1}{(2-i)^2}\right|=?$

30) What is the radius of the circle, represented by the equation $z\bar{z}+(2-3i)z+(2+3i)\bar{z}+4=0$?