Studentpad

MHT-CET-XII MATHEMATICS 2022-23

Time : 150 Min	Maths : Full Portion Paper	Marks : 100

01) The solution of the differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is

A) $\sin^{-1}\left(\frac{y}{x}\right) = \log x + c$

B) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$

C) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$

D) $\tan^{-1}\left(\frac{y}{x}\right) = -\log x + c$

02) The negation of $p \to (\sim p \vee q)$ is
A) $p \vee (p \vee \sim q)$
B) $p \to \sim(p \vee q)$
C) $p \wedge \sim q$
D) $p \to q$

03) Pair of straight lines perpendicular to each other is represented by

A) $x^2 = 2(x - y)$

B) $2x^2 = y(2x + y)$

C) $2x^2 = 2y(2x + y)$

D) $x^2 + y^2 + 3 = 0$

04) The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$ is

A) 0

B) $\frac{2}{105}$

C) $\frac{22}{7} - \pi$

D) $\frac{71}{15} - \frac{3\pi}{2}$

05) Let for any matrix M, M^{-1} exist, then which of the following is not true?
A) $(M^{-1}) = (M)^{-1}$
B) $(M^2)^{-1} = (M^{-1})^2$
C) $(M^{-1})^{-1} = M$
D) $(M^{-1})^{-1} = (M^{-1})^1$

06) The dual of $(p \wedge t) \vee (c \wedge \sim q)$ where t is a tautology and c is a contradiction, is
A) $(\sim p \wedge c) \wedge (t \vee q)$
B) $(p \vee c) \wedge (t \vee \sim q)$
C) $(\sim p \vee c) \wedge (t \vee q)$
D) $(\sim p \vee t) \wedge (c \vee \sim q)$

07) The maximum value of $f = 4x + 3y$ subject to constraints $x \geq 0, y \geq 2$, $2x + 3y \leq 18$, $x + y \geq 10$ is
A) 34
B) 35
C) 36
D) no optimum value

08) If $P(X = x) = \begin{cases} \frac{x^2}{5}, x = 0, 1, 2 \\ 0, \text{otherwise} \end{cases}$, then

A) $\sum_{x_i = S} P(X = x) = 2$

B) f is not a p.m.f.

C) $\sum_{x_i = S} P(X = x) \neq 1$

D) f is a p.m.f.

09) If p is false and q is true, then
A) $p \wedge q$ is true
B) $p \vee \sim q$ is true
C) $p \to q$ is true
D) $q \to p$ is true

10) If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, then the smallest positive values of A and B are
A) 45°, 45°
B) 45°, 60°
C) 75°, 15°
D) 60°, 30°

11) Find the value of $\frac{\cos 9^0 + \sin 9^0}{\cos 9^0 - \sin 9^0}$.

A) $-\tan 54^0$

B) $\tan 54^0$

C) $-\cot 54^0$

D) $\cot 54^0$

12) The matrix $\begin{pmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{pmatrix}$ is not invertible, if 'a' has the value
A) -1
B) 0
C) 1
D) 2

13) Identify the lines given by

$x^2 + 2xy - 35y^2 - 4x + 44y - 12 = 0$ and the line 5x + 2y - 8 = 0.
A) Concurrent
B) Parallel
C) Coincident
D) None of these

14) A particle is moving in such a way that its displacement 's' at a time 't' given by $s = 2t^2 + 5t + 20$, identify the velocity and acceleration after 2 seconds.
A) 9 units/sec; 4 units $/(\text{sec})^2$
B) 12 units/sec; 2 units$/(\text{sec})^2$
C) 13 units/sec; 4 units$/(\text{sec})^2$
D) 10 units/sec; 4 units$/(\text{sec})^2$

15) Identify the normal unit vector to the plane $x + 2y + 3z - 6 = 0$
A) $\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\P k$
B) $\frac{1}{\sqrt{14}}\hat{i} - \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\P k$
C) $\frac{1}{\sqrt{14}}\hat{i} - \frac{2}{\sqrt{14}}\hat{j} - \frac{3}{\sqrt{14}}\P k$
D) None of these

16) If one of the roots of the equations $x^2 + ax = 0$ and $x^2 + bx + a = 0$ is coincident, then find out the numerical value of (a + b).
A) 2
B) 5
C) 0
D) -1

17) If the area of a circle increases at a uniform rate, then the rate of change of perimeter varies
A) inversely as the diameter
B) directly as the radius
C) directly as the diameter
D) inversely as the radius

18) Let $a = \log_3(3x)$ and $b = \log_{x^3}$, then find out the limiting value of a^b as $x \to 1$.
A) e
B) 1
C) -1
D) 1/e

19) If a force of magnitude 3 units in the direction 2i+3j+6k acts at the point (1,1,1), then what is its moment about the point (-1,2,3) is?
A) $\frac{24}{7}(-2k + i)$
B) $\frac{24}{7}(-2i + j)$
C) $\frac{24}{7}(-2j + k)$
D) none of these

20) Natural numbers 1, 2, 3, ...,2n are written down on 2n cards. One card is selected at random from these cards. Let the probability of drawing a number I is proportional to I, then find the probability of drawing an odd number in one draw.
A) $\frac{n-1}{n}$
B) $\frac{1}{2}$
C) $\frac{n+2}{n+3}$
D) $\frac{n}{2n+1}$

21) Considering only the principal values, if $\tan(\cos^{-1} x) = \sin\left[\cot^{-1}\left(\frac{1}{2}\right)\right]$, then x is equal to
A) $\frac{1}{\sqrt{5}}$
B) $\frac{2}{\sqrt{5}}$
C) $\frac{\sqrt{5}}{3}$
D) $\frac{3}{\sqrt{5}}$

22) Suppose ${}^{2n}P_3 = 2({}^{n}P_4)$, then evaluate n.
A) 12
B) 8
C) 6
D) 4

23) If f(x) = |x - 2|, then
A) $\lim_{x \to 2^+} f(x) \neq 0$
B) f(x) is continuous at x = 2
C) $\lim_{x \to 2^-} f(x) \neq 0$
D) $\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$

24) $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx =$
A) $\tan x + \cot x + c$
B) $\text{cosec } x - \cot x + c$
C) $\tan x - \cot x + c$
D) $\sec x - \text{cosec } x + c$

25) If the foot of the perpendicular from $(0,0,0)$ to a plane is $(1,2,3)$, then what is the equation of the plane?
A) $2x + y + 3z = 14$
B) $x + 2y + 3z = 14$
C) $x + 2y + 3z + 14 = 0$
D) $x + 2y - 3z = 14$

26) Which of the following equation is linear?

A) $x\frac{dy}{dx}+y^2=\sin x$

B) $\frac{dy}{dx}+3y=xy^2$

C) $x^2\frac{dy}{dx}+y=e^x$

D) $\frac{dy}{dx}+xy^2=1$

27) The solution of the differential equation $x\frac{d^2y}{dx^2}=1$, given that $y=1,\frac{dy}{dx}=0$ when $x=1$, is

A) $y=x\log x-x$

B) $y=x\log x+x$

C) $y=x\log x-x+2$

D) $y=x\log x+x+2$

28) If a fair coin is tossed 8 times, then the probability that it shows heads at least once is

A) $\frac{3}{256}$

B) $\frac{1}{256}$

C) $\frac{85}{256}$

D) $\frac{255}{256}$

29) If $y=\frac{\sqrt{x}(2x+3)^2}{\sqrt{x+1}}$, then $\frac{dy}{dx}=$

A) $y\left[\frac{1}{3x}+\frac{4}{2x+3}+\frac{1}{2(x+1)}\right]$

B) $y\left[\frac{1}{2x}+\frac{4}{2x+3}-\frac{1}{2(x+1)}\right]$

C) $y\left[\frac{1}{3x}+\frac{4}{2x+3}+\frac{1}{x+1}\right]$

D) None of these

30) Identify the equation of the plane passing through the point $(-1,3,2)$ and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$.

A) $7x+8y+3z+25=0$

B) $7x-8y+3z-25=0$

C) $7x-8y+3z+25=0$

D) $7x-8y-3z+25=0$

31) The probability that a certain kind of component will survive a check test is 0.6. The probability that exactly two of the next four components tested survive is

A) 0.03456

B) 0.03465

C) 0.3465

D) 0.3456

32) Differentiate w.r.t. x $\frac{\sec x+\tan x}{\sec x-\tan x}$

A) $\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)$

B) $-\tan\left(\frac{\pi}{4}+\frac{x}{2}\right).\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)$

C) $\sec\left(\frac{\pi}{4}+\frac{x}{2}\right).\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

D) $\tan\left(\frac{\pi}{4}+\frac{x}{2}\right).\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)$

33) If AB=h is the hypotenuse of a right angled triangle ABC, then find out $\overline{AB}.\overline{AC}+\overline{BC}.\overline{BA}+\overline{CA}.\overline{CB}=$

A) h

B) $\frac{1}{2}h^2$

C) h^2

D) $2h^2$

34) $\int\frac{dx}{x(x^5+1)}=$

A) $\frac{1}{5}\log x^5\left(\frac{x^5}{x^5+1}\right)+c$

B) $\frac{1}{5}\log x^5(x^5+1)+c$

C) $\frac{1}{5}\log x^5\left(\frac{1+x^5}{x^5}\right)+c$

D) None of these

35) If $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx=k\pi$, then $k=$

A) -a

B) a

C) -2a

D) 2a

36) Equations of pair of straight lines through origin and intersection points of line $x+y=3$ and parabola $y^2=4x$, is

A) $3x^2-4xy+4y^2=0$

B) $3x^2+4xy-4y^2=0$

C) $3x^2+4xy+4y^2=0$

D) $4x^2+4xy-3y^2=0$

37) The value of x which satisfies the equation $\tan^{-1}x=\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ is

A) -1/3

B) -3

C) 1/3

D) 3

38) A and B are two points. The position vector of A

is $6\bar{b}-2\bar{a}$. A point P divides the line AB in the ratio 1:2. If $\bar{a}-\bar{b}$ is the position vector of P, then the position vector of B is given by

A) $7\bar{a}-15\bar{b}$

B) $15\bar{a}+7\bar{b}$

C) $15\bar{a}-7\bar{b}$

D) $7\bar{a}+15\bar{b}$

39) The first group has 100 items with mean 45 and variance 49. If the combined group has 250 items with mean 51 and varisnce 130, then find the standard deviation of second group.

A) 12

B) 13

C) 10

D) 9

40) The probability distribution of a r.v. X is

X=x	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

Then Var(X)=

A) 3.6

B) 2.16

C) 3.44

D) 2

41) Write the equation of the tangent to the circle $x^2+y^2-2y=0$ at the point (-1, 1).

A) y + 1 = 0

B) x + 1 = 0

C) y - 1 = 0

D) x - 1 = 0

42) The area of the region bounded by the curve y = x|x|, X-axis and the ordinates x=1, x= -1 is given by

A) zero

B) $\frac{1}{3}$

C) $\frac{2}{3}$

D) 1

43) If $f(x)=\frac{x+3}{x-2}, g(x)=\frac{2x+3}{x-1}$, then which of the following is correct?

A) f is not inverse of g but g is inverse of f

B) f and g are inverse of each other

C) f is inverse of g but g is not inverse of f

D) f and g are not inverse of each other

44) Let A be the invertible matrix, which of the following is not true?

A) $\left(A^T\right)^{-1}=\left(A^{-1}\right)^T$

B) $(A^2)^{-1}=(A^{-1})^2$

C) $A^{-1}=|A|^{-1}$

D) $(A^3)^{-1}=(A^{-1})^3$

45) The rate of change of $\sqrt{(x^2+16)}$ with respect to $\frac{x}{x-1}$ at $x=3$ is

A) -3

B) $-\frac{12}{5}$

C) 2

D) $\frac{11}{5}$

46) Variables of the objective function of the linear programming problem are

A) zero

B) negative

C) zero or negative

D) zero or positive

47) If $\int f(x)\sin x\cos x\,dx=\frac{1}{2(b^2-a^2)}\log(f(x))+c$, then $f(x)=$

A) $\frac{1}{a^2\cos^2 x-b^2\sin^2 x}$

B) $\frac{1}{a^2\cos^2 x+b^2\sin^2 x}$

C) $\frac{1}{a^2\sin^2 x-b^2\cos^2 x}$

D) $\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$

48) Let p and q be the statements $p: 4x+5y\le 20$, $q: 3x^2+2y^2\le 6$

A) p but not q is a constraint of L.P.P.

B) q but not p is a constraint of L.P.P.

C) both p and q can be constraint of L.P.P.

D) neither p nor q is a constraint of L.P.P.

49) What is the equation of median passing through A of ΔABC, where $A\cong(-1,6)$, $B\cong(-3,-9)$ and $C\cong(5,-8)$?

A) $29x+4y+5=0$

B) $29x-4y-5=0$

C) $13x-14y-47=0$

D) $13x+14y+47=0$

50) The area bounded by the curve $y^2=8x$ and the line x=2 is

A) $\frac{16}{3}$ sq.units

B) $\frac{32}{3}$ sq.units

C) $\frac{23}{3}$ sq.units

D) $\frac{13}{2}$ sq.units