

Studentpad

JEE-MAIN MATHEMATICS - QUADRATIC EQUATIONS 2022-23

Time : 90 Min

Maths : Quadratic Equations

Marks : 120

01) If $x^2 + ax + 10 = 0$ and $x^2 + bx - 10 = 0$ have a common root, then $a^2 - b^2$ is equal to

- A) 40
- B) 30
- C) 20
- D) 10

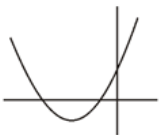
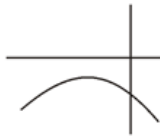
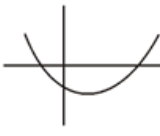

02) If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other root, then the value of $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} =$

- A) - b
- B) b
- C) $b^{\frac{1}{n+1}}$
- D) $-b^{\frac{1}{n+1}}$

03) If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$ have a common root, then find $a : b : c$.

- A) 1 : 2 : 3
- B) 3 : 1 : 2
- C) 1 : 3 : 2
- D) 3 : 2 : 1

04) Identify for which of the following graphs in the quadratic expression $y = ax^2 + bx + c$ the product abc is negative.

- A) 
- B) 
- C) 
- D) 

05) If α , β and γ are the roots of $x^3 + 8 = 0$, then the equation whose roots are α^2, β^2 and γ^2 is

A) $x^3 - 64 = 0$

- B) $x^3 + 64 = 0$
- C) $x^3 - 16 = 0$
- D) $x^3 - 8 = 0$

06) For a, b, c non-zero, real distinct and the equation, $(a^2 + b^2)x^2 - 2b(a + c)x + b^2 + c^2 = 0$ has non-zero real roots. One of these roots is also the root of the equation:

- A) $(b^2 + c^2)x^2 - 2a(b + c)x + a^2 = 0$
- B) $(b^2 - c^2)x^2 + 2a(b - c)x - a^2 = 0$
- C) $a^2x^2 + a(c - b)x - b^2 = 0$
- D) $a^2x^2 - a(b - c)x + b^2 = 0$

07) If a, b, c are real $x^3 - 3b^2 + 2c^3$ is divisible by $x - a$ and $x - b$, then

- A) $a = -b = -c$
- B) $a = b = c, a = -2b = -2c$
- C) $a = 2b = 2c$
- D) None of these

08) Two candidates attempt to solve the equation $x^2 + px + q = 0$. One starts with a wrong value of p and finds the roots to be 2 and 6 and the other starts with a wrong value of q and finds the roots to be 2 and -9. The roots of the original equation are

- A) -3, -4
- B) -2, -3
- C) 3, 4
- D) 2, 3

09) If a, b, c, a_1, b_1, c_1 are rational and equations $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have one only one root in common, then

- A) at least one of $b^2 - ac$ and $b_1^2 - a_1c_1$ must be perfect squares
- B) both $b^2 - ac$ and $b_1^2 - a_1c_1$ must be perfect squares
- C) both $b^2 - ac$ and $b_1^2 - a_1c_1$ may not be perfect squares
- D) cannot say anything

10) If α, β be the roots of $x^2 - px + q = 0$ and α', β' be the roots of $x^2 - p'x + q' = 0$, then the value of $(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (a - \beta')^2 + (\beta - \beta')^2$ is

- A) $2\{p^2 - 2q - p'^2 - 2q' - qq'\}$
- B) $2\{p^2 - 2q - p'^2 - 2q' - pp'\}$
- C) $2\{p^2 - 2q + p'^2 - 2q' - qq'\}$

D) $2\{p^2 - 2q + p'^2 - 2q' - pp'\}$

11) If α, β are the roots of $x^2 - 2x + 4 = 0$, then $\alpha^5 + \beta^5$ is equal to

- A) 64
B) 32
C) 16
D) None of these

12) If x is real and satisfies $x + 2 > \sqrt{x + 4}$, then

- A) $x < -2$
B) $x > 0$
C) $-3 < x < 0$
D) $-3 < x < 4$

13) If α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$, then $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1} =$

- A) 5
B) 4
C) 3
D) 2

14) If the roots of the given equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ are real, then

- A) $p \in (0, 2\pi)$
B) $p \in (0, \pi)$
C) $p \in (-\pi, 0)$
D) $p \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

15) The expression $x^2 + 2bx + c$ has the positive value, if

- A) $b^2 < c$
B) $c^2 < b$
C) $b^2 - 4c < 0$
D) $b^2 - 4c > 0$

16) If $b > a$, then the equation $(x - a)(x - b) = 1$ has

- A) one root in $(-\infty, a)$ and the other in $(b, +\infty)$.
B) both roots in $(b, +\infty)$.
C) both roots in $(-\infty, a)$.
D) both roots in $[a, b]$.

17) What is the number of real solutions of $\sqrt{2x - 4} - \sqrt{x + 5} = 1$?

- A) 2
B) 1
C) 0
D) Infinite

18) The roots of the equation $x^4 - 2x^3 + x = 380$ are

- A) $-5, -4, \frac{1 \pm 5\sqrt{-3}}{2}$

B) $5, 4, \frac{-1 \pm 5\sqrt{-3}}{2}$

C) $-5, 4, -\frac{1 \pm 5\sqrt{-3}}{2}$

D) $5, -4, \frac{1 \pm 5\sqrt{-3}}{2}$

19) If α_1, α_2 and β_1, β_2 are the roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively and system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-zero solution, then

- A) $b^2 pr = q^2 ac$
B) $a^2 qc = p^2 br$
C) $c^2 ar = r^2 pb$
D) None of these

20) The solution of the equation $2x^2 + 3x - 9 \leq 0$ is given by

- A) $\frac{3}{2} \leq x \leq 3$
B) $-3 \leq x \leq \frac{3}{2}$
C) $-3 \leq x \leq 3$
D) $\frac{3}{2} \leq x \leq 2$

21) The roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are

- A) 1, 8
B) 1, -8
C) 1, 4
D) 1, -4

22) The equation formed by decreasing each root of $ax^2 + bx + c = 0$ by 1 is $2x^2 + 8x + 2 = 0$, then

- A) $b = a + c$
B) $b = -c$
C) $c = -a$
D) $a = -b$

23) What is the possible value of p for which graph of the function $f(x) = 2p^2 - 3p \tan x + \tan^2 x + 1$

does not lie below x -axis for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$?

- A) 3
B) 8
C) 4
D) 0

24) Let α, β be the roots of $x^2 + bx + 1 = 0$. Then

find the equation whose roots are $-\left(\alpha + \frac{1}{\beta}\right)$ and $-\left(\beta + \frac{1}{\alpha}\right)$.

- A) $x^2 - 2bx + 4 = 0$
B) $x^2 + 2bx + 4 = 0$

C) $x^2 = 0$

D) $x^2 - bx + 1 = 0$

25) If a and b ($\neq 0$) are roots of the equations

$$x^2 + ax + b = 0 \text{ then what is the least value of}$$

$$x^2 + ax + b \ (x \in \mathbb{R}) ?$$

A) $\frac{1}{4}$

B) $-\frac{1}{4}$

C) $\frac{9}{4}$

D) $-\frac{9}{4}$

26) Identify the number having two digits such that it is 4 times the sum and three times the product of its two digits.

27) If a, b, c are positive, then what is the least value of $(a + b + c)(1/a + 1/b + 1/c)$?

28) Some points on the plane are marked and they are connected pair wise by line segments. If the total number of line segments formed is 10, then what is the number of marked points on the plane?

29) What are the number of real solutions of the equation $|x^2 + 4x + 3| + 2x + 5 = 0$?

30) Let α and β be the roots of equation

$$x^2 - 6x - 2 = 0. \text{ If } a_n = \alpha^n - \beta^n, \text{ for } n \geq 1, \text{ then find}$$

the value of $\frac{a_{10} - 2a_8}{2a_9}$.