Studentpad

K-CET MATHEMATICS PAPER 2022-23

Time : 120 Min	Maths : Full Portion Paper	Marks : 60

Hints and Solutions

01) Ans: **D)** coefficient of variation
Sol: By definition

02) Ans: **C)** $\frac{1}{8}$

Sol: Now,

$$\sin 12^\circ \sin 48^\circ \sin 54^\circ = \frac{1}{2}\left(\cos 36^\circ - \cos 60^\circ\right)\cos 36^\circ$$

$$= \frac{1}{2}\left[\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right]\left[\frac{\sqrt{5}+1}{4}\right] = \frac{1}{2}\left[\frac{\sqrt{5}-1}{4}\right]\left[\frac{\sqrt{5}+1}{4}\right]$$

$$= \frac{5-1}{32} = \frac{4}{32} = \frac{1}{8}$$

03) Ans: **D)** Infinite
Sol: For the system of given homogeneous equations,

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & -1 \\ 1 & -3 & 1 \end{vmatrix} = 1(-1-3) - 1(3+1) - 1(-9+1)$$

$$= -4 - 4 + 8 = 0$$

Therefore, there are infinite number of solutions.

04) Ans: **D)** $\alpha / 2$

Sol: $\sin\theta + \sin 3\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow 2\sin 2\theta \cos\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow \sin 2\theta(2\cos\theta + 1) = \sin\alpha$ .....(i)

Here, $\cos\theta + \cos 3\theta + \cos 2\theta = \cos\alpha$

$2\cos 2\theta \cos\theta + \cos 2\theta = \cos\alpha$

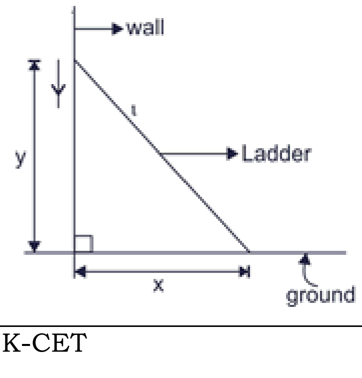
$\cos 2\theta\,(2\cos\theta + 1) = \cos\alpha$ .....(ii)

From (i) and (ii),

$\tan 2\theta = \tan\alpha \Rightarrow 2\theta = \alpha \Rightarrow \theta = \alpha / 2$

05) Ans: **B)** $\frac{25}{\sqrt{3}}$

Sol: A ladder of length l=2m leans against a vertical wall.
Now, the top of ladder begins to slide down the wall at the rate 25 cm/s.
Let the rate at which bottom of the ladder slides away from the wall on the horizontal ground is $\frac{dx}{dt}$ cm / s.



$\because x^2 + y^2 = l^2$ [by Pythagoras theorem]

$\Rightarrow x^2 + y^2 = 4$ $[\because l = 2m]$....(i)

On differentiating both sides of Eq. (i) w.r.t. 't', we get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad \Rightarrow \frac{dx}{dt} = -\left(\frac{y}{x}\right)\frac{dy}{dt} \qquad ...(ii)$$

From Eq. (i), when y=1m, then $x^2 + 1^2 = 4$

$\Rightarrow x^2 = 3 \quad \Rightarrow x = \sqrt{3}\,m \quad [\because x > 0]$

Substituting $x = \sqrt{3}m$ and $y = 1m$ in Eq. (ii), we get

$$\frac{dx}{dt} = -\frac{1}{\sqrt{3}}\left(-\frac{25}{100}\right) m / s \quad \left[\text{given} \frac{dy}{dt} = -25 cm / sec\right]$$

$$= \frac{25}{\sqrt{3}} cm / s$$

06) Ans: **B)** 7, 11, 15, 19

Sol: Let four arithmetic means are A_1, A_2, A_3 and A_4 i.e. $3, A_1, A_2, A_3, A_4, 23$

$\Rightarrow T_6 = 23 = a + 5d \Rightarrow d = 4$

$\therefore A_1 = 3 + 4 = 7, A_2 = 7 + 4 = 11,$

$A_3 = 11 + 4 = 15, A_4 = 15 + 4 = 19$

07) Ans: **C)** $-2 \le a \le 4$

Sol: $\int_0^a x\,dx \le a + 4 \quad \Rightarrow \frac{a^2}{2} \le a + 4$

$\Rightarrow a^2 \le 2a + 8 \quad \Rightarrow a^2 - 2a - 8 \le 0$

$\Rightarrow (a-4)(a+2) \le 0 \quad \Rightarrow -2 \le a \le 4$

08) Ans: **D)** 5

Sol: S.D. of first n natural numbers $= \sigma = \sqrt{2}$

$$\therefore \sigma^2 = 2 \Rightarrow \frac{\Sigma n^2}{n} - \left(\frac{\Sigma n}{n}\right)^2 = 2$$

$$\Rightarrow \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2 = 2$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = 2$$

$$\Rightarrow \frac{n+1}{2}\left(\frac{2n+1}{3} - \frac{n+1}{2}\right) = 2$$

$$\Rightarrow \frac{n+1}{2}\left(\frac{4n+2-3n-3}{6}\right) = 2$$

$\Rightarrow (n+1)(n-1) = 24 \quad \Rightarrow n^2 - 1 = 24 \quad \Rightarrow n^2 = 25$

$\therefore n = 5$

09) Ans: **A)** $\frac{1}{3x} + \frac{2x^2}{3}$

Sol: Here, $\lim\limits_{t\to x}\dfrac{t^2f(x)-x^2f(t)}{t-x}=1$

$\Rightarrow x^2f'(x)-2xf(x)+1=0$

$\Rightarrow \dfrac{x^2f'(x)-2xf(x)}{\left(x^2\right)^2}+\dfrac{1}{x^4}=0 \;\; \Rightarrow \dfrac{d}{dx}\left(\dfrac{f(x)}{x^2}\right)=-\dfrac{1}{x^4}$

On integrating both sides, we get

$f(x)=cx^2+\dfrac{1}{3x}$

Also, $f(1)=1, \quad c=\dfrac{2}{3}$

Therefore, $f(x)=\dfrac{2}{3}=x^2+\dfrac{1}{3x}$

10) Ans: **C)** $0, \pm\dfrac{1}{2}$

Sol: $\tan^{-1}(x-1)+\tan^{-1}(x)+\tan^{-1}(x+1)=\tan^{-1}3x$

$\Rightarrow \tan^{-1}(x-1)+\tan^{-1}(x)=\tan^{-1}3x-\tan^{-1}(x+1)$

$\Rightarrow \tan^{-1}\left[\dfrac{(x-1)+x}{1-(x-1)(x)}\right]=\tan^{-1}\left[\dfrac{3x-(x+1)}{1+3x(x+1)}\right]$

$\Rightarrow \dfrac{2x-1}{1-x^2+x}=\dfrac{2x-1}{1+3x^2+3x}$

$\Rightarrow (1-x^2+x)(2x-1)=(1+3x^2+3x)(2x-1)$

Simplifying, we get $x=0, \pm\dfrac{1}{2}$

11) Ans: **A)** $A.\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}=2I$

Sol: $A.\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}$

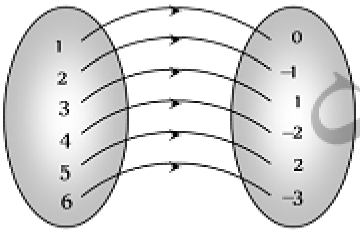
$=\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}=2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=2I$

12) Ans: **C)** one-one and onto both.

Sol: $f: N \to I$

$f(1)=0, f(2)=-1, f(3)=1, f(4)=-2, f(5)=2$ and

$f(6)=-3$ so on.



In such type of function, every element of set A has unique image in set B and there is no element left in set B. Thus, f is one-one and onto function.

13) Ans: **D)** $\dfrac{1}{2}$

Sol: Let $\alpha=\cos^{-1}\sqrt{p}, \beta=\cos^{-1}\sqrt{1-p}$

and $\gamma=\cos^{-1}\sqrt{1-q}$

$\Rightarrow \cos\alpha=\sqrt{p}, \cos\beta=\sqrt{1-p}$ and $\cos\gamma=\sqrt{1-q}$

Thus, $\sin\alpha=\sqrt{1-p}$, $\sin\beta=\sqrt{p}$ and $\sin\gamma=\sqrt{q}$

The given equation may be written as,

$\alpha+\beta+\gamma=\dfrac{3\pi}{4}$

$\Rightarrow \alpha+\beta=\dfrac{3\pi}{4}-\gamma \;\; \Rightarrow \cos(\alpha+\beta)=\cos\left(\dfrac{3\pi}{4}-\gamma\right)$

$\Rightarrow \cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\cos\left\{\pi-\left(\dfrac{\pi}{4}+\gamma\right)\right\}=-\cos\left(\dfrac{\pi}{4}+\gamma\right)$

$\Rightarrow \sqrt{p}\sqrt{1-p}-\sqrt{1-p}\sqrt{p}=-\left(\dfrac{1}{\sqrt{2}}\sqrt{1-q}-\dfrac{1}{\sqrt{2}}\sqrt{q}\right)$

$\Rightarrow 0=\sqrt{1-q}-\sqrt{q} \Rightarrow 1-q=q \;\; \Rightarrow q=\dfrac{1}{2}$

14) Ans: **B)** $3\pi/4$

Sol: $1-\cos\alpha=\dfrac{1}{2-\sqrt{2}}=1+\dfrac{1}{\sqrt{2}}$

$\Rightarrow \cos\alpha=-\dfrac{1}{\sqrt{2}}=\cos\dfrac{3\pi}{4}$ i.e. $\alpha=\dfrac{3\pi}{4}$

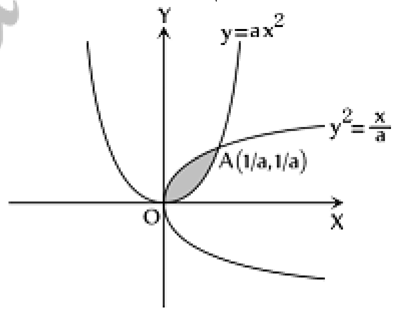
15) Ans: **A)** $\dfrac{1}{\sqrt{3}}$

Sol: The x-coordinate of A is $\dfrac{1}{a}$.

From the given condition,

$1=\int_0^{1/a}\left(\sqrt{\dfrac{x}{a}}-ax^2\right)dx=\dfrac{1}{\sqrt{a}}.\dfrac{2}{3}[x^{3/2}]_0^{1/a}-\dfrac{a}{3}[x^3]_0^{1/a}$

$\therefore a^2=\dfrac{1}{3} \Rightarrow a=\dfrac{1}{\sqrt{3}}$



16) Ans: **D)** $4n^2$

Sol: $\because x=\operatorname{cosec}\theta-\sin\theta$

$\Rightarrow x^2+4$

$=(\operatorname{cosec}\theta-\sin\theta)^2+4=(\operatorname{cosec}\theta+\sin\theta)^2 \quad \ldots(1)$

and $y^2+4=\left(\operatorname{cosec}^n\theta-\sin^n\theta\right)^2+4$

$=\left(\operatorname{cosec}^n\theta+\sin^n\theta\right)^2 \quad \ldots(2)$

Now, $\dfrac{dy}{dx}=\dfrac{\left(\dfrac{dy}{d\theta}\right)}{\left(\dfrac{dx}{d\theta}\right)}$

$=\dfrac{n\left(\operatorname{cosec}^{n-1}\theta\right)(-\operatorname{cosec}\theta\cot\theta)-n\sin^{n-1}\theta\cos\theta}{-\operatorname{cosec}\theta\cot\theta-\cos\theta}$

$=\dfrac{n\left(\operatorname{cosec}^n\theta\cot\theta+\sin^{n-1}\theta\cos\theta\right)}{(\operatorname{cosec}\theta\cot\theta+\cos\theta)}$

$$= \frac{n \cot\theta\left(\operatorname{cosec}^n\theta + \sin^n\theta\right)}{\cot\theta\left(\operatorname{cosec}\theta + \sin\theta\right)}$$

$$= \frac{n\left(\operatorname{cosec}^n\theta + \sin^n\theta\right)}{\left(\operatorname{cosec}\theta + \sin\theta\right)} = \frac{n\sqrt{y^2+4}}{\sqrt{x^2+4}}$$ [From (1) and (2)]

Squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^2 = \frac{n^2\left(y^2+4\right)}{\left(x^2+4\right)} \text{ or } \left(x^2+4\right)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2+4\right)$$

17) Ans: **C)** 256

Sol: f(x) is continuous at x=4

$$\Rightarrow f(4) = \lim_{x\to 4}\frac{x^4-256}{x-4} = \lim_{x\to 4}\frac{x^4-4^4}{x-4} = 4.4^{4-1} = 256$$

18) Ans: **D)** 1

Sol: Let the vertices A,B,C,D of quadrilateral be $(x_1,y_1,z_1),(x_2,y_2,z_2),(x_3,y_3,z_3)$ and (x_4,y_4,z_4) and the equation of the plane PQRS be $u \equiv ax+by+cz+d=0$

Let $u_r = a_r x + b_r y + c_r z + d$,

where r=1,2,3,4 Then, $\frac{AP}{PB}\cdot\frac{BQ}{QC}\cdot\frac{CR}{RD}\cdot\frac{DS}{SA}$

$$= \left(-\frac{U_1}{U_2}\right)\left(-\frac{U_2}{U_3}\right)\left(-\frac{U_3}{U_4}\right)\left(-\frac{U_4}{U_1}\right) = 1$$

19) Ans: **A)** $\frac{10}{3\sqrt{3}}$

Sol: Required distance $= \left|\frac{d - a.n}{|n|}\right|$

$$= \left|\frac{5-(2i-2j+3k).(i+5j+k)}{\sqrt{1+25+1}}\right| = \left|\frac{5-(2-10+3)}{\sqrt{27}}\right|$$

$$= \frac{10}{3\sqrt{3}}.$$

20) Ans: **B)** $x^2+2xy=c$

Sol: $(x+y)dx + xdy = 0 \Rightarrow xdy = -(x+y)dx$

$$\Rightarrow \frac{dy}{dx} = -\frac{x+y}{x}$$

It is homogeneous equation, thus put $y = vx$

i.e. $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = -\frac{x+vx}{x} = -\frac{1+v}{1}$$

$$\Rightarrow x\frac{dv}{dx} = -1-2v \Rightarrow \int\frac{dv}{1+2v} = -\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\log(1+2v) = -\log x + \log c$$

$$\Rightarrow \log\left(1+2\frac{y}{x}\right) = 2\log\frac{c}{x}$$

$$\Rightarrow \frac{x+2y}{x} = \left(\frac{c}{x}\right)^2 \Rightarrow x^2+2xy=c$$

21) Ans: **C)** f(x) is not differentiable at $x=0$.

Sol: As the function is defined for $x \ge 0$ means not defined for $x<0$. Therefore the function neither continuous non- differentiable at $x=0$.

22) Ans: **B)** $f'(a)=0$ and $f''(a)<0$

Sol: The given function $f: R \to R$ is to be maximum, if $f'(a)=0$ and $f''(a)<0$.

23) Ans: **C)** 13500

Sol: At (800, 400), P = 12(800)+6(400) = 12000

At (1050, 150), P = 12(1050)+6(150) = 13500

At (600, 0), P = 12(600)+6(0) = 7200

Hence, maximum value is 13500.

24) Ans: **B)** $4a^2b^2c^2$

Sol: $\Delta = \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$

$$= -2\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}, \text{ ...By } R_1 \to R_1 - (R_2+R_3)$$

$$= -2\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix}, \quad \text{...By } \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$= -2\{-c^2(b^2a^2) + b^2(-c^2a^2)\} = 4a^2b^2c^2$$

25) Ans: **A)** -40

Sol: Three points A, B, C are collinear, if $\overrightarrow{AB} = -20\hat{i} - 11\hat{j}$ and $\overrightarrow{AC} = (a-60)\hat{i} - 55\hat{j}$, then

$$\overrightarrow{AB} \parallel \overrightarrow{AC} \quad \therefore \frac{a-60}{-20} = \frac{-55}{-11} \quad \Rightarrow a = -40$$

26) Ans: **B)** $\frac{2}{3}$ and $\frac{1}{2}$

Sol: Probability of solving the problem by $A, P(A) = \frac{1}{2}$

$\Rightarrow$ Probability of solving the problem by B, $P(B) = \frac{1}{3}$

$\Rightarrow$ Probability of not solving the problem by $A = P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$ and probability of not solving the problem by $B = P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

(i) p (the problem is solved)=1-P (none of them solve the problem) $= 1 - P(A' \cap B') = 1 - P(A')P(B')$

($\because$ A and B are independent $\Rightarrow$ A' and B' are independent)

$$= 1 - \left(\frac{1}{2}\times\frac{2}{3}\right) = 1 - \frac{1}{3} = \frac{2}{3}$$

(ii) P (exactly one of them above the problem)

=P(A) P(B')+(A') P(B)

$$= \frac{1}{2}\times\frac{2}{3} + \frac{1}{2}\times\frac{1}{3} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$$

27) Ans: **B)** $e^{1/e}$

Sol: Given that, $y = x^{1/x}$

Taking log, we get, $\log y = \frac{1}{x}\log x$

Differentiating both sides w.r.t. x,

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2} \quad \Rightarrow \frac{dy}{dx} = \frac{1}{x^2}(1 - \log x)x^{1/x}$

For maximum, $\frac{dy}{dx} = 0 \quad \Rightarrow x = e$;

$\therefore y_{max} = e^{1/e}$

28) Ans: **C)** $\frac{7}{2}$

Sol: The given planes are $2x + y + 2z - 8 = 0$

i.e. $4x + 2y + 4z - 16 = 0$ .....(i)

and $4x + 2y + 4z + 5 = 0$ .....(ii)

The distance between two parallel planes

$= \left|\frac{-16-5}{\sqrt{4^2+2^2+4^2}}\right| = \frac{21}{6} = \frac{7}{2}$

29) Ans: **A)**

$\frac{1}{2}\left[2^n \cos\left(2x + n\frac{\pi}{2}\right) - 4^n \cos\left(4x + n\frac{\pi}{2}\right)\right]$

Sol: We know, $\sin x \sin 3x = \frac{1}{2}[\cos 2x - \cos 4x]$

30) Ans: **C)** $\sin(3x+4) + 3(x-1)\cos(3x+4)$

Sol: Given $\int_1^b f(x)dx = (b-1)\sin(3b+4)$

By differentiating both sides w.r.t.b,

we get $\Rightarrow f(b) = 3(b-1)\cos(3b+4) + \sin(3b+4)$

$\Rightarrow f(x) = \sin(3x+4) + 3(x-1)\cos(3x+4)$.

31) Ans: **A)** 4

Sol: $\lim_{n\to\infty}(3^n + 4^n)^{\frac{1}{n}} = \lim_{n\to\infty}(4^n)^{\frac{1}{n}}\left[\frac{3^n}{4^n} + 1\right]^{\frac{1}{n}}$

$= \lim_{n\to\infty} 4\left[1 + \frac{1}{\left(\frac{4}{3}\right)^n}\right]^{1/n} = 4\lim_{n\to\infty}\left[1 + \frac{1}{\left(\frac{4}{3}\right)^n}\right]^{1/n}$

$= 4\left[1 + \frac{1}{\infty}\right]^0 = 4\times(1)^0 = 4\times 1 = 4$.

32) Ans: **B)** $7A + 1$

Sol: Given, $A^2 = A$,

We have $(I+A)^3 = (I)^3 + (A)^3 + 3I.A(I+A)$

$= I + A^2.A + 3A(I+A) \quad (\because I^3 = I, I.A = A)$

$= I + A.A + 3A(I+A) \quad (\because A^2 = A)$

$= I + A^2 + 3(A.I + A^2) \quad = I + A + 3(A+A) \quad (\because A^2 = A)$

$= I + A + 3(2A) = I + A + 6A = 7A + I$

33) Ans: **C)** -4

Sol: Given, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

Expanding along C_1

$\Rightarrow (2a^2 + 4) - 2(-4a - 20) = 86$

$\Rightarrow 2a^2 + 4 + 8a + 40 = 86 \quad \Rightarrow 2a^2 + 8a - 42 = 0$

$\Rightarrow a^2 + 4a - 21 = 0 \Rightarrow a^2 + 7a - 3a - 21 = 0$

$\Rightarrow a(a+7) - 3(a+7) = 0 \quad \Rightarrow (a+7)(a-3) = 0$

$\therefore a = -7, 3$ therefore, Sum of these number $= 3 - 7 = -4$

34) Ans: **D)** $(2,-4,1)$

Sol: Equation of planes bisecting the angles between the planes

$a_1x + b_1y + c_1z + d_1 = 0$ and

$a_2x + b_2y + c_2z + d_2 = 0$, are

$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \pm\frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$

Equation of given planes are

$2x - y + 2z - 4 = 0 \quad ...(i)$

and $x + 2y + 2z - 2 = 0 \quad ...(ii)$

Now, equation of planes bisecting the angles between the planes (i) and (ii) are

$\frac{2x - y + 2z - 4}{\sqrt{4+1+4}} = \pm\frac{x + 2y + 2z - 2}{\sqrt{1+4+4}}$

$\Rightarrow 2x - y + 2z - 4 = \pm(x + 2y + 2z - 2)$

On taking (+ve) sign, we get a plane

$x - 3y = 2 \quad ...(iii)$

On taking (-ve) sign, we get a plane

$3x + y + 4z = 6 \quad ...(iv)$

So, from the given options, the point $(2,-4,1)$ satisfy the plane of angle bisector $3x + y + 4z = 6$

35) Ans: **D)** $1 - \frac{\pi}{4}$

Sol: $\int_0^{\pi/4}\tan^2 x dx = \int_0^{\pi/4}(\sec^2 x - 1)dx$

$= [\tan x]_0^{\pi/4} - [x]_0^{\pi/4} = 1 - \frac{\pi}{4}$

36) Ans: **A)** $2Tx - a^2y + 2aT = 0$

Sol: If line cuts off the axes at A and B,

area of triangle is $\frac{1}{2}\times OA \times OB = T$

$\Rightarrow \frac{1}{2}.a.OB = T \quad \Rightarrow OB = \frac{2T}{a}$

The equation of line is $\frac{x}{-a} + \frac{y}{2T/a} = 1$

$\Rightarrow 2Tx - a^2y + 2aT = 0$

37) Ans: **B)** $3b - \frac{a}{3}$

Sol: As $\overline{OA} = a, \overline{OB} = b$ and $2AC = CO$

By section formula, $\overrightarrow{OC} = \frac{2}{3}a$.

$\therefore |\overrightarrow{CD}| = 3|\overrightarrow{OB}| \Rightarrow \overrightarrow{CD} = 3b$

$\Rightarrow \overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \frac{2}{3}a + 3b$

$\therefore \overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \frac{2}{3}a + 3b - a = 3b - \frac{1}{3}a$.

38) Ans: **D)** $(-1)^k (k-1)\pi$

Sol: $f'(k-0) = \lim\limits_{h\to 0} \frac{[k-h]\sin\pi(k-h) - [k]\sin\pi k}{-h}$

$= \lim\limits_{h\to 0} \frac{(-1)^{k-1}(k-1)\sin\pi h - k\times 0}{-h}$

$= \lim\limits_{h\to 0} \frac{(-1)^{k-1}(k-1)\sin\pi h}{-h} = (-1)^k.(k-1)\pi$

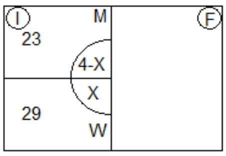
39) Ans: **C)** 5

Sol: $f'(1) = \lim\limits_{h\to 0} \frac{f(1+h) - f(1)}{h}$; Since, function is differentiable, hence it is continuous as it is given that $\lim\limits_{h\to 0} \frac{f(1+h)}{h} = 5$ and thus $f(1) = 0$.

$\therefore f'(1) = \lim\limits_{h\to 0} \frac{f(1+h)}{h} = 5$

40) Ans: **A)** 48,1

Sol: See the following Venn diagram.



$n(I) = 29 + 23 = 52$

$n(F) = 100 - 52 = 48$,

$n(M \cup D) = n(M) + n(D) - n(M \cap D)$

$\Rightarrow 24 = 23 + 4 - n(M \cap D)$

$\therefore n(M \cap D) = 3 \quad \Rightarrow n(W \cap D) = 4 - 3 = 1$

41) Ans: **A)** 1/2

Sol: Total number of ways to arrange 3 boys and 2 girls are 5!.
Considering given condition, following cases may arise.

B	G	G	B	B
G	G	B	B	B
G	B	G	B	B
G	B	B	G	B
B	G	B	G	B

So, number of favourable ways $= 5 \times 3! \times 2! = 60$

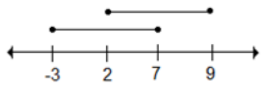
Therefore, required probability $= \frac{60}{120} = \frac{1}{2}$

42) Ans: **B)** $(1, 7/3]$

Sol: From given, $f(x) = 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$

$\Rightarrow \text{Range} = (1, 7/3]$

43) Ans: **C)** $A - B = [-3, 2)$

Sol: 

From diagram $A \cap B = [2, 7]$,

$(A \cup B)' = (-\infty, -3) \cup (9, \infty) \quad \Rightarrow A - B = [-3, 2)$

$\Rightarrow A - B' = [2, 7]$ where $B' = (-\infty, 2) \cup (9, \infty)$

44) Ans: **B)** n^2

Sol: Here, $\alpha + \beta = 1 + n^2$; $\alpha\beta = \frac{1}{2}(1 + n^2 + n^4)$

$\therefore \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (1 + n^2)^2 - 2.\frac{1}{2}(1 + n^2 + n^4)$

$= 1 + n^4 + 2n^2 - 1 - n^2 - n^4 \quad \Rightarrow \alpha^2 + \beta^2 = n^2$

45) Ans: **B)** (-6, 11)

Sol: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$

$cA = \begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix}$; $dI = \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}$

From given, $A^{-1} = \frac{1}{6}[A^2 + cA + dI]$

$\Rightarrow 6 = 1 + c + d$ ...(By equality of matrices)

Here, (-6, 11) satisfies the relation.

46) Ans: **A)** isosceles

Sol: The points of intersection of three lines are $A(1,1)$, $B(2,-2)$, $C(-2,2)$.

Now, $|AB| = \sqrt{1+9} = \sqrt{10}$, $|BC| = \sqrt{16+16} = \sqrt{32}$

and $|CA| = \sqrt{9+1} = \sqrt{10}$

So, triangle is an isosceles.

47) Ans: **D)** 300

Sol: Number of girls in the class = 5 and number of boys in the class = 7

Now, total ways of forming a team of 3 boys and 2 girls $= {}^7C_3 . {}^5C_2 = 350$

But, if two specific boys are in team, then number of ways $= {}^5C_1 . {}^5C_2 = 50$

Required ways, i.e. the ways in which two specific boys are not in the same team $= 350 - 50 = 300$.

Alternate Method

Number of ways when A is selected and B is not $= {}^5C_2 \cdot {}^5C_2 = 100$

Number of ways when B is selected and A is not $= {}^5C_2 \cdot {}^5C_2 = 100$

Number of ways when both A and B are not selected $= {}^5C_3 \cdot {}^5C_2 = 100$

Therefore, required ways $= 100 + 100 + 100 = 300$.

48) Ans: **C)** $\frac{e^{a^2}-1}{2}$ sq. unit

Sol: Required area = $\int_0^a y \; dx = \int_0^a x e^{x^2} dx$

Now, putting $x^2 = t \Rightarrow dx = \frac{dt}{2x}$, as $x = 0 \Rightarrow t = 0$

and $x = a \Rightarrow t = a^2$,

$\therefore \frac{1}{2}\int_0^{a^2} e^t dt = \frac{1}{2}[e^t]_0^{a^2} = \frac{e^{a^2}-1}{2}$ sq. unit.

49) Ans: **B)** $x = \cot y + c$

Sol: $\frac{dy}{dx} + \sin^2 y = 0 \Rightarrow -\frac{dy}{\sin^2 y} = dx$

Integrating, we get $x = \cot y + c$.

50) Ans: **A)** $2^{n/2} \cos \frac{n\pi}{4}$

Sol: As $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots.. + C_n x^n$

Put $x = i$, on both the sides, we get

$(1+i)^n = (C_0 - C_2 + C_4 - \ldots) + i(C_1 - C_3 + C_5 - \ldots)$ ..(i)

Also, $1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$ in amplitude modulus form

$\Rightarrow (1+i)^n = 2^{n/2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^n$

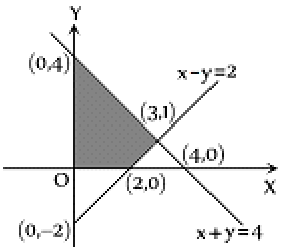
$= 2^{n/2}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right)$ ....(ii)

By equating the real parts in (i) and (ii),

$C_0 - C_2 + C_4 - C_6 + \ldots.. = 2^{n/2} \cos\frac{n\pi}{4}$

51) Ans: **D)** $x = 0, y = 4, z = 8$

Sol: Given that, $z = x + 2y$



$\therefore$ Max $z = 0 + 4(2) = 8$

52) Ans: **A)** continuous but not differentiable at $x = 2$.

Sol: $\lim_{h\to 0^-} 1 + (2-h) = 3$, $\lim_{h\to 0^+} 5 - (2+h) = 3$, $f(2) = 3$

$\therefore$ f is continuous at $x = 2$

Now, $Rf'(x) = \lim_{h\to 0} \frac{5-(2+h)-3}{h} = -1$

$Lf'(x) = \lim_{h\to 0} \frac{1+(2-h)-3}{-h} = 1$

$\because Rf'(x) \neq Lf'(x)$, $\therefore$ f is not differentiable at $x = 2$.

53) Ans: **A)** - 5

Sol: Given $f(-x) = -f(x)$

We know, $\int_{-a}^{a} f(x)dx = 0 = \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx$

$\Rightarrow \int_{-1}^{0} f(x)\,dx + \int_0^1 f(x)\,dx = 0 \quad \Rightarrow \int_{-1}^{0} f(x)\,dx = -$

$\Rightarrow \int_{-1}^{0} f(t)\,dt = -5$

54) Ans: **B)** [-1, 3]

Sol: $-\sqrt{1+(-\sqrt{3})^2} \le (\sin x - \sqrt{3}\cos x) \le \sqrt{1+(-\sqrt{3})^2}$

$-2 \le (\sin x - \sqrt{3}\cos x) \le 2$

$-2 + 1 \le (\sin x - \sqrt{3}\cos x + 1) \le 2 + 1$

$-1 \le (\sin x - \sqrt{3}\cos x + 1) \le 3$ i.e. range $= [-1, 3]$

$\therefore$ For f to be onto $S = [-1, 3]$

55) Ans: **C)** $\pi / 2$

Sol: $I = \left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2}$

56) Ans: **A)** 0

Sol: $I = \int_{-\pi/2}^{\pi/2} (3\sin x + \sin^3 x)\,dx = 0$,

(As Function $(3\sin x + \sin^3 x)$ is an odd function).

57) Ans: **A)** > 7

Sol: Given that, ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3 \quad \Rightarrow {}^{n}C_4 > {}^{n}C_3$

$\frac{{}^{n}C_4}{{}^{n}C_3} > 1 \quad \Rightarrow \frac{n-3}{4} > 1 \quad \Rightarrow n > 7$

58) Ans: **B)** $y = \log(\sec x) + (x-2)e^x + c_1 x + c_2$

Sol: $\frac{d^2y}{dx^2} = \sec^2 x + xe^x$

By integrating, $\frac{dy}{dx} = \tan x + xe^x - e^x + c_1$

Again integrating,

$y = \log(\sec x) + xe^x - e^x - e^x + c_1 x + c_2$

$\therefore$ Required solution is

$y = \log(\sec x) + (x-2)e^x + c_1 x + c_2$

59) Ans: **C)** $\sin x^2$

Sol: $f: R \to R, f(x) = \sin x$ and $g: R \to R, g(x) = x^2$

$\Rightarrow fog(x) = f(g(x)) = f(x^2) = \sin x^2$

60) Ans: **A)** increasing.

Sol: $f(x) = e^x \quad \Rightarrow f'(x) = e^x > 0, \forall x$

$\therefore f'(x)$ is increasing for all x.