Studentpad

JEE-MAIN MATHEMATICS PERMUTATIONS AND COMBINATIONS 2022-23

Time : 90 Min	**Maths : Permutations and Combinations**	**Marks : 120**

Hints and Solutions

01) Ans: **B)** 14
Sol:
$99\times97\times95\times\ldots\times51=\dfrac{100!}{(100\times98\times96\times\ldots\times52)}\times\dfrac{1}{50!}$

$=\dfrac{100!\times25!}{2^{25}\times50!\times50!}$

maximum power of 3 in 100!
$=\left[\dfrac{100}{3}\right]+\left[\dfrac{100}{9}\right]+\left[\dfrac{100}{27}\right]+\left[\dfrac{100}{81}\right]$
=33 + 11 + 3 + 1= 48.
maximum power of 3 in 50!
$=\left[\dfrac{50}{3}\right]+\left[\dfrac{50}{9}\right]+\left[\dfrac{50}{27}\right]$ =16 + 5 + 1 = 22
maximum power of 3 in 25!
$=\left[\dfrac{25}{3}\right]+\left[\dfrac{25}{9}\right]$ = 8 + 2 =
$\therefore$ Exponent of $3=48+10-(22\times2)=14$

02) Ans: **A)** $\begin{pmatrix} n+2 \\ r \end{pmatrix}$

Sol: Given expression $={}^nC_r+2.{}^nC_{r-1}+{}^nC_{r-2}$
$=({}^nC_r+{}^nC_{r-1})+({}^nC_{r-1}+{}^nC_{r-2})={}^{n+1}C_r+{}^{n+1}C_{r-1}={}^{n+2}C_r$

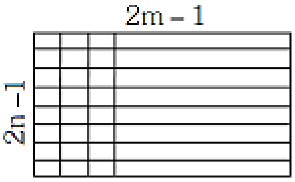
03) Ans: **A)** ${}^{56}C_4$

Sol: ${}^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3=$

${}^{50}C_4+\left({}^{50}C_3+{}^{51}C_3+{}^{52}C_3+\ldots{}^{55}C_3\right)$

By taking first two terms together and adding them and following the same pattern, we get
${}^{56}C_4$, $[\because {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r]$.

04) Ans: **A)** m^2n^2
Sol: Along horizontal side one unit can be taken in $(2m-1)$ ways and 3 unit side can be taken in $2m-3$ ways.
$\therefore$ Number of ways of selecting a side horizontally is $(2m-1+2m-3+2m-5+\ldots.+3+1)$
Similarly, the number of ways along vertical side is $(2n-1+2n-3+\ldots.+5+3+1)$.



$\therefore$ Total number of rectangles
$=[1+3+5+\ldots..+(2m-1)]\times[1+3+5+\ldots.+(2n-1)]$

$=\dfrac{m(1+2m-1)}{2}\times\dfrac{n(1+2n-1)}{2}=m^2n^2$.

05) Ans: **D)** None of these
Sol: IUAENSRNC
$\therefore$ Required number of words are $\dfrac{6!}{2!}\times4!=8640$

06) Ans: **A)** ${}^7C_3\,.\,9+{}^7C_5\,.\,44+{}^7C_1\,.\,265$
Sol: No. of ways getting one correct
$={}^7C_1 6!\left(1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}+\ldots+\dfrac{1}{6!}\right)={}^7C_1\,.\,265$
No. of ways getting two correct
$={}^7C_2\,.\,5!\left(1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}+\ldots-\dfrac{1}{5!}\right)={}^7C_2\,.\,4$
No. of ways getting three correct
$={}^7C_3\,.\,4!\left(1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}+\dfrac{1}{4!}\right)={}^7C_3\,.\,9$
Require no. of ways
$={}^7C_3\,.\,9+{}^7C_2\,.\,44+{}^7C_1\,.\,265$

07) Ans: **B)** n = m = 9
Sol: Problem is same as dividing 17 identical things in two groups $\Rightarrow n=\dfrac{17+1}{2}=9$
No effect if two diamonds are different as necklace can be flipped over $\Rightarrow n=m=9$

08) Ans: **B)** 20
Sol: Point of intersection of diagonal ${}^nC_4=70$
$\Rightarrow n(n-1)(n-2)(n-3)=70\times24$ n = 8
Total no. of diagonals = ${}^nC_2-n={}^8C_2-8=20$

09) Ans: **B)** 6
Sol: ${}^nP_r=720.\,{}^nC_r$
$\Rightarrow {}^nP_r\div{}^nC_r=720\quad\Rightarrow r!=720=6!$ i.e. r = 6

10) Ans: **B)** 300
Sol: Numbers between 999 and 10000 are of four digit numbers.
The four digit numbers formed by digits 0,2,3,6,7,8 are ${}^6P_4=360$.
But, here those numbers are also involved which begin from 0. Hence, we take those numbers as three digit numbers.
On taking initial digit 0, the number of ways to fill remaining 3 places from five digits 2, 3, 6, 7, 8 are ${}^5P_3=60$

$\therefore$ Required numbers = 360 - 60 = 300.

11) Ans: **A)** 64
Sol: A selection of 3 balls so as to include at least one black ball, can be made in the following 3 mutually exclusive ways as
(i) 1 black ball and 2 others
$= {}^3C_1 \times {}^6C_2 = 3 \times 15 = 45$
(ii) 2 black balls and one other
$= {}^3C_2 \times {}^6C_1 = 3 \times 6 = 18$
(iii) 3 black balls and no other $= {}^3C_3 = 1$
$\therefore$ Total no. of ways = 45 + 18 + 1 = 64

12) Ans: **B)** $r!$
Sol: Obviously, product of any r consecutive natural numbers is always divisible by $r!$.

13) Ans: **A)** 480
Sol: After fixing 1 at one position out of 4, places 3 places can be filled by ${}^7P_3$ ways. But some numbers whose fourth digit is zero, thus such type of ways $= {}^6P_2$
$\therefore$ Total ways $= {}^7P_3 - {}^6P_2 = 480$

14) Ans: **C)** 535
Sol: Number of points of intersection of 37 straight lines is ${}^{37}C_2$. But 13 of them pass through the point A. Thus, instead of getting ${}^{13}C_2$ points we get merely one point.
Similarly, 11 straight lines out of the given 37 straight lines intersect at B. Hence instead of getting ${}^{11}C_2$ points, we get only one point.
$\therefore$ No. of intersection points of the lines is
${}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2 = 535.$

15) Ans: **A)** 10.2^7
Sol: No. of selections = coefficient of x^8 in
$(1+x+x^2+....+x^8)(1+x+x^2+....+x^8).(1+x)^8$
= coefficient of x^8 in $\dfrac{(1-x^9)^2}{(1-x)^2}(1+x)^8$
= coefficient of x^8 in $(1+x)^8(1-x)^{-2}$
= coefficient of x^8 in
$({}^8C_0 + {}^8C_1x + {}^8C_2x^2 + ... + {}^8C_8x^8)$
$\times(1+2x+3x^2+4x^3+....+9x^8+...)$
$= 9.{}^8C_0 + 8.{}^8C_1 + 7.{}^8C_2 + ... + 1.{}^8C_8$
$= C_0 + 2C_1 + 3C_2 + ... + 9C_8 \qquad [C_r = {}^8C_r]$
Now, $C_0x + C_1x^2 + ... + C_8x^9 = x(1+x)^8$
On differentiating with respect to x, we get
$C_0 + 2C_1x + 3C_2x^2 + ... + 9C_8x^8 = (1+x)^8 + 8x(1+x)^7$
By putting $x = 1$, we get
$C_0 + 2C_1 + 3C_2 + ... + 9C_8$
$= 2^8 + 8.2^7 = 2^7.(2+8) = 10.2^7$

16) Ans: **D)** $3 \times {}^{m+1}C_4$
Sol: $\alpha = {}^mC_2 \Rightarrow \alpha = \dfrac{m(m-1)}{2}$
$\therefore {}^{\alpha}C_2 = \dfrac{\alpha(\alpha-1)}{2} = \dfrac{1}{2}.\dfrac{m(m-1)}{2}\left\{\dfrac{m(m-1)}{2} - 1\right\}$
$= \dfrac{1}{8}m(m-1)(m-2)(m+1)$
$= \dfrac{1}{8}(m+1)m(m-1)(m-2) = 3.{}^{m+1}C_4$

17) Ans: **C)** 66
Sol: No. of ways = coefficient of x^{15} in the expansion
$(1+x+x^2+x^3+x^4+x^5)\,(1+x+x^2+.......+x^{10})$
$(1+x+x^2+......+x^{15})$
$\therefore (1+x+x^2+x^3+x^4+x^5)(1+x+x^2+.....+x^{10})$
$(1+x+x^2+...+x^{15}) = (1-x^6-x^{11})(1+{}^3C_1x+{}^4C_2x^2$
$+......+{}^6C_4x^4 + {}^{11}C_9x^9 + {}^{17}C_{15}x^{15} + ...)$
$= .... + ... + x^{15}(-{}^{11}C_9 - {}^6C_4 + {}^{17}C_{15})$
$= .... + ... + x^{15}(-55-15+136) = x^{15} \times 66$
$\therefore$ The coefficient of $x^{15} = 66$

18) Ans: **B)** 505
Sol: The number of numbers when repetition is allowed $= 5^4$.
The number of numbers when digits cannot be repeated $= {}^5P_5$
Thus, the required number of numbers $= 5^4 - 5!$

19) Ans: **B)** 576
Sol: $P(n,r) = 1680 \quad \Rightarrow \dfrac{n!}{(n-r)!} = 1680$ .....(i)
$C(n,r) = 70 \quad \Rightarrow \dfrac{n!}{r!\,(n-r)!} = 70$ .....(ii)
i.e. $\dfrac{1680}{r!} = 70$, [From (i) and (ii)]
$r! = \dfrac{1680}{70} = 24 \quad \Rightarrow r = 4$
$\therefore P(n,4) = 1680$
$\Rightarrow n(n-1)(n-2)(n-3) = 1680 \quad \Rightarrow n = 8$
$\Rightarrow 8 \times 7 \times 6 \times 5 = 1680$
Now, $69n + r! = 69 \times 8 + 4! = 552 + 24 = 576$

20) Ans: **B)** 36
Sol: The word MOBILE has three even places and three odd places. It contains 3 consonants and 3 vowels. In three odd places we have to fix up 3 consonants which can be done in ${}^3P_3$ ways.
Now, remaining three places, remaining three letters can be fixed up in ${}^3P_3$ ways.
$\therefore$ Total number of ways $= {}^3P_3 \times {}^3P_3 = 36$.

21) Ans: **D)** 480

Sol: After fixing 1 at one position out of 4 places, 3 places can be filled by ${}^{7}P_{3}$ ways. But some numbers whose fourth digit is zero, so such type of ways $= {}^{6}P_{2}$

Thus, total ways $= {}^{7}P_{3} - {}^{6}P_{2} = 480$

22) Ans: **A)** $\left({}^{m+2}C_{2}\right)^{2}$

Sol: Each set is consisting of m + 2 parallel lines and each parallelogram is formed by choosing two straight lines from the first set and two straight lines from the second set. Two straight lines from the first set can be chosen in ${}^{m+2}C_{2}$ ways and two straight lines from the second set can be chosen in ${}^{m+2}C_{2}$ ways.

$\therefore$ Total number of parallelograms formed

$= {}^{m+2}C_{2} . {}^{m+2}C_{2} = \left({}^{m+2}C_{2}\right)^{2}$.

23) Ans: **A)** 2454

Sol: The word 'MATHEMATICS' contains 2M,2T,2A,H,E,I,C,S. Hence, 4 letters can be chosen in the following ways.

Case I: 2 alike of one kind and 2 alike of second kind i.e. ${}^{3}C_{2}$

$\therefore$ No. of words $= {}^{3}C_{2} \dfrac{4!}{2!\,2!} = 18$

Case II: 2 alike of one kind and 2 different

i.e. ${}^{3}C_{1} \times {}^{7}C_{2} \Rightarrow$ No. of words $= {}^{3}C_{1} \times {}^{7}C_{2} \times \dfrac{4!}{2!} = 756$

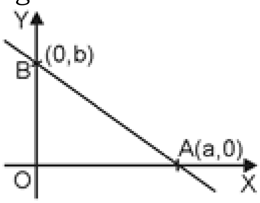
Case III: All are different

i.e. ${}^{8}C_{4} \Rightarrow$ No. of words $= {}^{8}C_{4} \times 4! = 1680$

$\therefore$ Total number of words are 2454.

24) Ans: **A)** 36

Sol: As per given information, we have the following figure.



(Note that as a and b are integers so they can be negative also). Here 0(0, 0), A (a, 0) and B (0, b) are the three vertices of the triangle.

Clearly, $OA = |a|$ and $OB = |b|$.

$\therefore$ Area of $\Delta OAB = \dfrac{1}{2}|a||b|$.

But area of such triangles is given as 50 sq units.

$\therefore \dfrac{1}{2}|a||b| = 50 \quad \Rightarrow |a||b| = 100 = 2^{2}.5^{2}$

Number of ways of distributing two 2's in $|a|$ and $|b| = 3$

|a|	|b|
0	2
1	1
2	0

$\Rightarrow 3$ ways

Similarly, number of ways of distributing two 5's in $|a|$ and $|b| = 3$ways.

$\therefore$ Total number of ways of distributing 2's and 5's $= 3 \times 3 = 9$ ways

Note that for one value of $|a|$, there are 2 possible values of a and for one value of $|b|$, there are 2 possible values of b.

Number of such triangles possible $= 2 \times 2 \times 9 = 36$.

So, number of elements in S is 36.

25) Ans: **B)** 196

Sol: Consider square of 2×2 in which we have 4 triples of squares (In shape 'L') which have common vertex. We have such 7×7 squares of size 2×2

Thus number of ways of choosing 3 squares from a chess board so that they have exactly one common vertex $= 7 \times 7 \times 4 = 196$

26) Ans: **D)** None of these

Sol: As, the first 2 women select the chairs amongst 1 to 4 in ${}^{4}P_{3}$ ways.

Now, from the remaining 6 chairs, three men could be arranged in ${}^{6}P_{3}$.

So, total number of arrangements $= {}^{4}P_{2} \times {}^{6}P_{3}$.

27) Ans: **18** Sol: The number of numbers in which the odd digits occupy the odd places

$= \dfrac{4!3!}{2!2!2!} = \dfrac{24 \times 6}{8} = 18$

28) Ans: **375** Sol: Required numbers $= 3 \times 5 \times 5 \times 5 = 375$.

29) Ans: **625** Sol: A number is divisible by 4 if last 2 digit number is divisible by 4.

Last two digit numbers divisible by 4 from (1, 2, 3, 4, 5) are 12, 24, 32, 44, 52

Therefore, the number of 5 digit number which are divisible by 4, from the digit (1, 2, 3, 4, 5) and digit is repeated is $5 \times 5 \times 5 \times (5 \times 1) = 625$

30) Ans: **360** Sol: The required number of arrangements $= \dfrac{6!}{2} = \dfrac{720}{2} = 360$.