Studentpad

JEE - Main Full Portion 2020-21		
Time : 120 Min	**Maths : Full Portion Paper**	**Marks : 150**

Hints and Solutions

01) Ans: **1)** $1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{x^3}{16}$

Sol: $(1-x)^{3/2}$

$=[1+\frac{3}{2}(-x)+\frac{3}{2}.\frac{1}{2}.\frac{1}{2!}(-x)^2+\frac{3}{2}.\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{3!}(-x)^3+...]$

$=1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{x^3}{16}$ (only four terms).

02) Ans: **4)** 2

Sol: The equation of circle passing through $(0,0),(2,0)$ and $(0,-2)$ is $x^2+y^2-2x+2y=0$.

If it passes through $(k,-2)$, then

$k^2+4-2k-4=0 \Rightarrow k=0,2$

Since, $(0,-2)$ is already a point on circle $\therefore$ k = 2.

03) Ans: **1)** Only (1)

Sol: It is obvious.

04) Ans: **1)** Always passes through a fixed point

Sol: Solving equation of parabola with x -axis (y=0),

we get $(a-b)x^2+(b-c)x+(c-a)=0$,which should have two equal values of x, as x-axis touches the parabola.

$\therefore (b-c)^2-4(a-b)(c-a)=0$

$\Rightarrow (b+c-2a)^2=0$

$\Rightarrow (b+c-2a)^2=0 \Rightarrow -2a+b+c=0$

Therefore, $ax+by+c=0$ always passes through (-2, 1).

05) Ans: **2)** $\sqrt{2}\log\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)+c$

Sol: $\int\frac{1}{\sqrt{1+\sin x}}dx=\int\frac{1}{\sqrt{2}\,\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}dx$

$=\frac{1}{\sqrt{2}}\int \text{cosec}\left(\frac{x}{2}+\frac{\pi}{4}\right)dx=\sqrt{2}\log\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)+c$

06) Ans: **3)** $\left(\frac{3}{5}\right)^7$

Sol: On trial, $n=15$ as any of the 15 numbers can be on the selected coin and $m=9$ as the largest number is 9 and hence it can be 1 or 2 or 3 ... or 9.

$\therefore$ Required probability $=\left(\frac{9}{15}\right)^7=\left(\frac{3}{5}\right)^7$

07) Ans: **2)** $\sqrt{\frac{5}{2}}$

Sol: The distance between the pair of straight lines given by the equation

$ax^2+2hxy+by^2+2gx+2fy+c=0$ is $2\sqrt{\frac{g^2-ac}{a(a+b)}}$.

on comparing, $a=1$, $b=9$, $c=-4$, $g=3/2$

$=2\times\sqrt{\frac{9/4-(-4)}{1(1+9)}}=2\times\sqrt{\frac{25/4}{10}}=\sqrt{5/2}$

08) Ans: **4)** Both (1) and (3)

Sol: $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$

Let, $\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}=\theta \Rightarrow \cos 2\theta=\frac{\sqrt{5}}{3}$

But $\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \frac{\sqrt{5}}{3}=\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow \sqrt{5}+\sqrt{5}\tan^2\theta=3-3\tan^2\theta$

$\Rightarrow (\sqrt{5}+3)\tan^2\theta=3-\sqrt{5} \Rightarrow \tan^2\theta=\frac{3-\sqrt{5}}{3+\sqrt{5}}$

$\Rightarrow \tan^2\theta=\frac{(3-\sqrt{5})^2}{4} \Rightarrow \tan\theta=\frac{3-\sqrt{5}}{2}$

Rationalizing, we get

$\Rightarrow \tan\theta=\frac{3-\sqrt{5}}{2}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{2}{3+\sqrt{5}}$

09) Ans: **4)** ${}^{n+1}C_{r+1}$

Sol: ${}^{r}C_r+{}^{r+1}C_r+{}^{r+2}C_r......+{}^{n-1}C_r+{}^{n}C_r$

$={}^{r+1}C_{r+1}+{}^{r+1}C_r+{}^{r+2}C_r+.....+{}^{n-1}C_r+{}^{n}C_r$

$={}^{r+2}C_{r+1}+{}^{r+2}C_r+.....+{}^{n-1}C_r+{}^{n}C_r$

$={}^{r+3}C_{r+1}+......+{}^{n-1}C_r+{}^{n}C_r$.

Solving similar way, we get

${}^{n-1}C_{r+1}+{}^{n}C_r+{}^{n}C_r={}^{n}C_{r+1}+{}^{n}C_r={}^{n+1}C_{r+1}$

10) Ans: **2)** -54

Sol: From the given conditions

$f(x)=-a(x+1)(x-5)$, $a>0$ y- intercept is 10

$\therefore 10=-a(1)(-5) \quad \Rightarrow a=2$

$\therefore f(x)=-2(x+1)(x-5)$

Put x = 8, $\Rightarrow f(8)=-2(8+1)(8-5)=-54$

11) Ans: **4)** 1

Sol: Let the vertices A,B,C,D of quadrilateral be

$(x_1,y_1,z_1),(x_2,y_2,z_2),(x_3,y_3,z_3)$ and (x_4,y_4,z_4) and the equation of the plane PQRS be
$u \equiv ax+by+cz+d=0$
Let $u_r = a_r x + b_r y + c_r z + d$,
where r=1,2,3,4 Then, $\frac{AP}{PB}\cdot\frac{BQ}{QC}\cdot\frac{CR}{RD}\cdot\frac{DS}{SA}$
$$=\left(-\frac{U_1}{U_2}\right)\left(-\frac{U_2}{U_3}\right)\left(-\frac{U_3}{U_4}\right)\left(-\frac{U_4}{U_1}\right)=1$$

12) Ans: **2)** $(x-y)(y-z)(z-x)(xy+yz+zx)$

Sol: $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = \frac{1}{xyz}\begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}$

(using $R_1 \to xR_1$ and $R_2 \to yR_2$ and $R_3 \to zR_3$)

$=\frac{xyz}{xyz}\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$ (take out xyz common from C_3)

$=\begin{vmatrix} x^2 & x^3 & 1 \\ y^2-x^2 & y^3-x^3 & 0 \\ z^2-x^2 & z^3-x^3 & 0 \end{vmatrix}$ (using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$)

Expanding corresponding to C_3, we get

$=1\begin{vmatrix} y^2-x^2 & y^3-x^3 \\ z^2-x^2 & z^3-x^3 \end{vmatrix}$

$=\left[(y^2-x^2)(z^3-x^3)-(z^2-x^2)(y^3-x^3)\right]$

$=(y+x)(y-x)(z-x)(z^2+x^2+xz)$
$\quad -(z+x)(z-x)(y-x)(y^2+x^2+xy)$

$=(y-x)(z-x)\begin{bmatrix}(y+x)(z^2+x^2+xz) \\ -(z+x)(y^2+x^2+xy)\end{bmatrix}$

$=(y-x)(z-x)\begin{bmatrix} yz^2+yx^2+xyz+xz^2+x^3+x^2z \\ -zy^2-zx^2-xyz-xy^2-x^3-x^2y\end{bmatrix}$

$=(y-x)(z-x)\left[yz(z-y)+x(z^2-y^2)\right]$

$=(y-x)(z-x)\left[yz(z-y)+x(z-y)(y+z)\right]$

$=(y-x)(z-x)\left[(z-y)(xy+yz+zx)\right]$

$=(x-y)(y-z)(z-x)(xy+yz+zx)$

13) Ans: **3)** $x+3y=0$ and $y-3x=0$

Sol: Let the equation of the chord OA of the circle
$x^2+y^2-2x+4y=0$ ...(i)
by y=mx ...(ii)
On solving Eqs.(i) and (ii), we get
$\Rightarrow x^2+m^2x^2-2x+4mx=0$
$\Rightarrow (1+m^2)x^2-(2-4m)x=0$
$\Rightarrow x=0$ and $x=\frac{2-4m}{1+m^2}$
Hence, the points of intersection
are $(0,0)$ and $A\left(\frac{2-4m}{1+m^2},\frac{m(2-4m)}{1+m^2}\right)$.

$\Rightarrow OA^2=\left(\frac{2-4m^2}{1+m^2}\right)^2(1+m^2) = \frac{(2-4m)^2}{1+m^2}$

Since, OAB is an isosceles right-angled triangle
$OA^2=\frac{1}{2}AB^2$, where AB is a diameter of the given circle $OA^2=10$

$\Rightarrow \frac{(2-4m)^2}{1+m^2}=10 \Rightarrow 4-16m+16m^2=10(1+m)^2$

$\Rightarrow 3m^2-8m-3=0 \Rightarrow m=3$ or $-\frac{1}{3}$

Therefore, the required equation are
$y=3x$ or $x+3y=0$.

14) Ans: **3)** $\frac{\sqrt{3}}{2}$

Sol: $\cos^2\left(\frac{\pi}{3}-x\right)-\cos^2\left(\frac{\pi}{3}+x\right)$

$=\left[\cos\left(\frac{\pi}{3}-x\right)+\cos\left(\frac{\pi}{3}+x\right)\right]$
$\left[\cos\left(\frac{\pi}{3}-x\right)-\cos\left(\frac{\pi}{3}+x\right)\right]$

$=\left(2\cos\frac{\pi}{3}\cos x\right)\left(2\sin\frac{\pi}{3}\sin x\right)$

$=\sin\frac{2\pi}{3}\sin 2x=\frac{\sqrt{3}}{2}\sin 2x$

Therefore, maximum value of given expression is $\frac{\sqrt{3}}{2}$.

15) Ans: **3)** 252
Sol: Given $2^n=1024$, $\therefore n=10$
$\therefore$ Greatest coefficient is ${}^{10}C_5=252$

16) Ans: **3)** A.P.
Sol: Suppose
$a^{1/x}=b^{1/y}=c^{1/z}=k \Rightarrow a=k^x,\ b=k^y, c=k^z$
Now, a, b, c are in G.P.
$\Rightarrow b^2=ac \Rightarrow k^{2y}=k^x.k^z=k^{x+z} \Rightarrow 2y=x+z$
i.e. x, y, z are in A.P.

17) Ans: **4)** $4xy=x^4+c$

Sol: $\frac{dy}{dx}+\frac{y}{x}=x^2$ is of the form $\frac{dy}{dx}+Py=Q$.

$\therefore$ I.F.= $e^{\int\frac{1}{x}dx}=e^{\log x}=x$

$\therefore$ Required solution $xy=\int x.x^2dx+c$

$\Rightarrow\ xy = \frac{x^4}{4} + c \Rightarrow\ 4xy = x^4 + c$

18) Ans: **4)** $\pi/12$

Sol: $\int_1^{\sqrt{3}} \frac{1}{1+x^2}dx = [\tan^{-1}x]_1^{\sqrt{3}} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

19) Ans: **3)** R^+

Sol: R^+ {because y is always positive $\forall\ x \in R$ }

20) Ans: **1)** one maximum and one minimum.

Sol: $f(x) = 2x^3 - 3x^2 - 12x + 4$

$f'(x) = 6x^2 - 6x - 12$

Now $f'(x) = 0 \quad \Rightarrow x^2 - x - 2 = 0 \quad \Rightarrow x = 2, -1$

Now $f''(x) = 12x - 6 \quad \Rightarrow f''(2)=+ve,\ f''(-1)=-ve$

$\therefore$ Given function contains one maximum and one minimum.

21) Ans: **2)** $3\cos(\alpha+\beta+\gamma)$

Sol: $\cos\alpha + \cos\beta + \cos\gamma = 0$ and

$\sin\alpha + \sin\beta + \sin\gamma = 0$

Let $a = \cos\alpha + i\sin\alpha$; $b = \cos\beta + i\sin\beta$ and

$c = \cos\gamma + i\sin\gamma$.

$\therefore a+b+c = (\cos\alpha + \cos\beta + \cos\gamma) + i(\sin\alpha + \sin\beta + \sin\gamma)$

$= 0 + i0 = 0$

If $a+b+c=0$, then $a^3+b^3+c^3 = 3abc$ or

$(\cos\alpha + i\sin a)^3 + (\cos\beta + i\sin\beta)^3 + (\cos\gamma + i\sin\gamma)^3$

$= 3(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)(\cos\gamma + i\sin\gamma)$

$= (\cos3\alpha + i\sin3\alpha) + (\cos3\beta + i\sin3\beta) + (\cos3\gamma + i\sin3\gamma)$

$= 3[\cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)]$

i.e. $\cos3\alpha + \cos3\beta + \cos3\gamma = 3\cos(\alpha+\beta+\gamma)$.

22) Ans: **4)** $f(x_1, x_2) = x_1 : x_2,\ x_1, x_2 \in \{0,1\}$

Sol: $f(x_1, x_2) = x_1 : x_2,\ x_1, x_2 \in \{0,1\}$

23) Ans: **4)** does not exist because left hand limit is not equal to right hand limit.

Sol: $f(1+) = \lim\limits_{h\to0} f(1+h) = \lim\limits_{h\to0} \frac{\sqrt{1-\cos 2h}}{h}$

$= \lim\limits_{h\to0} \sqrt{2}\frac{\sin h}{h} = \sqrt{2}$

$f(1-) = \lim\limits_{h\to0} f(1-h) = \lim\limits_{h\to0} \frac{\sqrt{1-\cos(-2h)}}{-h}$

$= \lim\limits_{h\to0} \sqrt{2}\frac{\sin h}{-h} = -\sqrt{2}$.

Hence, limit does not exist as left hand limit is not equal to right hand limit.

24) Ans: **4)** $\frac{\pi}{3}, \pi - \cos^{-1}\frac{3}{5}$

Sol: $5\cos2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0$

$\Rightarrow\ 5(2\cos^2\theta - 1) + (1+\cos\theta) + 1 = 0$

$\Rightarrow\ 10\cos^2\theta + \cos\theta - 3 = 0$

$\Rightarrow\ (5\cos\theta + 3)(2\cos\theta - 1) = 0$

$\Rightarrow\ \cos\theta = \frac{1}{2}, \cos\theta = -\frac{3}{5} \Rightarrow \theta = \frac{\pi}{3}, \pi - \cos^{-1}\left(\frac{3}{5}\right)$

25) Ans: **3)** $\frac{\pi}{3}$

Sol: $a+b+c=0 \Rightarrow a+b=-c \Rightarrow (a+b).(a+b) = |c|^2$

Thus, $|a|^2 + |b|^2 + 2|a||b|\cos\theta = |c|^2$ where, θ is the angle between a and b.

Thus, $\cos\theta = \frac{49-9-25}{2.3.5} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

26) Ans: **1)** 8

Sol: $\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix} = \begin{vmatrix} y+z & x-z & x-y \\ 2y & 2x & 0 \\ 2z & 0 & 2x \end{vmatrix}$

... By $R_2 \to R_2 + R_1$ and $R_3 \to R_3 + R_1$

$= 4\begin{vmatrix} y+z & x-z & x-y \\ y & x & 0 \\ z & 0 & x \end{vmatrix}$

$= 4[(y+z)(x^2) - (x-z)(xy) + (x-y)(-zx)]$

$= 4[x^2y + zx^2 - x^2y + xyz - zx^2 + xyz] \ = 8xyz$

$\therefore k = 8$

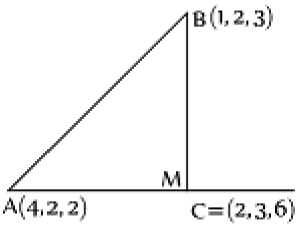
27) Ans: **2)** $2y = 1$

Sol: The equation of diameter of parabola is $y = \frac{2a}{m}$. Here $a = \frac{1}{4}, m = 1 \Rightarrow y = \frac{2\times1/4}{1} \Rightarrow 2y = 1$

28) Ans: **1)** $\sqrt{10}$

Sol: From the figure, $BM^2 = AB^2 - AM^2$ .....(i)

Now, $\overrightarrow{AB} = -3i + 0j + k \Rightarrow AB^2 = \overrightarrow{AB}^2 = 9 + 1 = 10$



AM = Projection of $\overrightarrow{AB}$ in direction of $\vec{C} = 2i + 3j + 6k$

$\therefore\ AM = \frac{\overrightarrow{AB}.\vec{C}}{|\vec{C}|} = \frac{(-3i+0j+k).(2i+3j+6k)}{7} = 0$

$\therefore\ BM^2 = 10 - 0 = 10 \Rightarrow BM = \sqrt{(10)}$, {from(i)}.

29) Ans: **2)** $\left(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}\right)$

Sol: Clearly orthocentre 'H' lies on the line $x - y = 0$.

Now distance of $O(0,0)$ from the line $x + y - 1 = 0$ is $\frac{1}{\sqrt{2}}$.

$\therefore \text{OH} = \frac{2}{3} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{3}$ (since triangle is equilateral, centroid coincides with orthocentre)

$\therefore \text{orthocentre} \equiv \left(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}\right)$

30) Ans: **2)** 1 - i

Sol: Suppose, $z = 1 + i \Rightarrow \bar{z} = 1 - i$