Studentpad

MHT-CET-XII MATHEMATICS 2022-23

Time : 150 Min	Maths : Full Portion Paper	Marks : 100

Hints and Solutions

01) Ans: **C)** $\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$

Sol: Putting y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ in the given equation,

$$v + x\frac{dv}{dx} = \frac{x^2 + vx^2 + v^2x^2}{x^2} = 1 + v + v^2$$

$$\Rightarrow x\frac{dv}{dx} = 1 + v^2 \quad \Rightarrow \frac{dv}{1+v^2} = \frac{dx}{x}$$

Integrating both sides, we get, $\tan^{-1} v = \log x + c$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log x + c \quad \ldots \left[\because v = \frac{y}{x}\right]$$

02) Ans: **C)** $p \wedge \sim q$

Sol: $\sim[p \to (\sim p \vee q)] \equiv p \wedge \sim(\sim p \vee q)$

$\equiv p \wedge (p \wedge \sim q) \equiv p \wedge \sim q$

03) Ans: **C)** $2x^2 = 2y(2x + y)$

Sol: As $2x^2 = 2y(2x+y) \Rightarrow x^2 - 2xy - y^2 = 0$

$\therefore$ Coefficient of x^2 + coefficient of $y^2 = 1 - 1 = 0$.

Thus, the lines are perpendicular.

04) Ans: **C)** $\frac{22}{7} - \pi$

Sol: $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$

$$= \int_0^1 \left(x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2}\right)dx$$

$$= \left[\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4}{3}x^3 + 4x - 4\tan^{-1}x\right]_0^1$$

$$= \frac{1}{7} - \frac{2}{3} + 1 - \frac{4}{3} + 4 - 4\left(\frac{\pi}{4}\right) = \frac{22}{7} - \pi$$

05) Ans: **D)** $(M^{-1})^{-1} = (M^{-1})^1$

Sol: $(M^{-1})^{-1} \neq (M^{-1})^1$

$\therefore (M^{-1})^{-1} = (M^{-1})^1$ is not true.

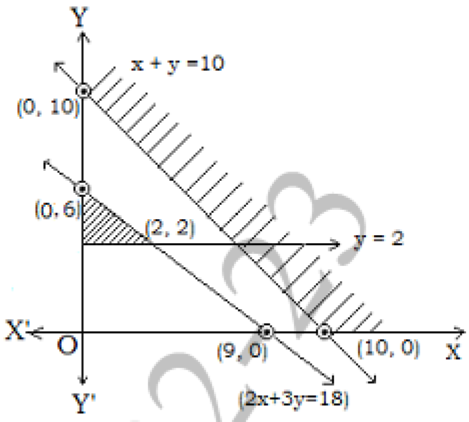
06) Ans: **B)** $(p \vee c) \wedge (t \vee \sim q)$

Sol: Dual of '$\vee$' is '$\wedge$' and of 't' is 'c'.

07) Ans: **D)** no optimum value

Sol: Here, feasible region is the empty set and there is no point in common.

Hence, there is no optimum value of objective function.



08) Ans: **D)** f is a p.m.f.

Sol: $P(X=0) = \frac{0^2}{5} = 0; P(X=1) = \frac{1^2}{5} = \frac{1}{5};$

$P(X=2) = \frac{2^2}{5} = \frac{4}{5}$

$$\sum_{x_i = S} P(X = x) = 0 + \frac{1}{5} + \frac{4}{5} = 1$$

Function is a p.m.f.

09) Ans: **C)** $p \to q$ is true

Sol: When p is false and q is true, then $p \wedge q$ is false, $p \vee \sim q$ is false, ($\because$ both p and ~q are false) and $q \to p$ is also false, only $p \to q$ is true.

10) Ans: **D)** 60°, 30°

Sol: $\sin(A+B) = 1$ and $\cos(A-B) = \frac{\sqrt{3}}{2}$

$\Rightarrow A + B = \frac{\pi}{2}$ and $A - B = \frac{\pi}{6}$ i.e. $A = \frac{\pi}{3}, B = \frac{\pi}{6}$

11) Ans: **B)** $\tan 54^0$

Sol: Dividing num. and denom, by $\cos 9^0$,

we get $\frac{1+\tan 9^0}{1-\tan 9^0} = \frac{\tan 45^0 + \tan 9^0}{1 - \tan 45^0 \tan 9^0}$

$= \tan\left(45^0 + 9^0\right) = \tan 54^0$

12) Ans: **C)** 1

Sol: Matrix is not invertible, if $\begin{vmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow 1(2-5) - a(1-10) + 2(1-4) = 0$

$\Rightarrow -3 + 9a - 6 = 0 \Rightarrow a = 1$

13) Ans: **A)** Concurrent

Sol: As $h^2 \neq ab$ they are neither parallel nor coincident.

$x^2 + 2xy - 35y^2 - 4x + 44y - 12 = 0$
a = 1, h = 1, b = - 35, g = - 2, f = 22, c = -12
Point of intersection is $\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$

$\equiv \left(\frac{(1)(22)+35(-2)}{-35-1}, \frac{(-2)(1)-1(22)}{-35-1}\right)$

$\equiv \left(\frac{22-70}{-36}, \frac{-24}{-36}\right) \equiv \left(\frac{4}{3}, \frac{2}{3}\right)$.

The line 5x + 2y - 8 = 0 passes through $\left(\frac{4}{3}, \frac{2}{3}\right)$

Thus, the given lines are concurrent.

14) Ans: **C)** 13 units/sec; 4 units/$(\text{sec})^2$

Sol: We have $s = 2t^2 + 5t + 20$

Velocity = $v = \frac{ds}{dt} = 4t + 5 \Rightarrow \left(\frac{ds}{dt}\right)_{t=2} = 4(2) + 5 = 13$

the velocity is 13 units per second.

Acceleration = $a = \frac{dv}{dt} = 4 \quad \Rightarrow \left(\frac{dv}{dt}\right)_{t=2} = 4$

Therefore, the acceleration is 4 units/ $(\text{sec})^2$.

15) Ans: **A)** $\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

Sol: Given plane is $x + 2y + 3z - 6 = 0$, which can be written as $\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 6$

Therefore, a vector normal to the plane is $\vec{n} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and a unit vector normal to the plane is $\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{1+4+9}} \quad = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$

16) Ans: **D)** -1

Sol: Let is the coincident (common) root of the given equations, then

$1\alpha^2 + a\alpha + b = 0$, $1\alpha^2 + b\alpha + a = 0$

$\therefore \frac{\alpha^2}{\begin{vmatrix} a & b \\ b & a \end{vmatrix}} = -\frac{\alpha}{\begin{vmatrix} 1 & b \\ 1 & a \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & a \\ 1 & b \end{vmatrix}}$

$\therefore \frac{a^2}{a^2-b^2} = \frac{\alpha}{b-a} = \frac{1}{b-a}$

$\therefore \frac{a^2}{a^2-b^2} = \frac{1}{b-a}$ and $\frac{\alpha}{b-a} = \frac{1}{b-a}$

$\therefore \alpha^2 = -(a+b)$ and $\alpha = 1$

$\therefore 1^2 = -(a+b) \quad \therefore a + b = -1$

17) Ans: **D)** inversely as the radius

Sol: Let A be the area of the circle, P be the perimeter when the radius is r at any time t, then $A = \pi r^2$, $P = 2\pi r$.

We are given that $\frac{dA}{dt} = A$ (Constant)

$\Rightarrow \frac{d}{dt}\left(\pi r^2\right) = k$

$\Rightarrow (\pi 2r)\frac{dr}{dt} = k \Rightarrow \frac{dr}{dt} = \frac{k}{2\pi r}$ ....(i)

Therefore, $\frac{dP}{dt} = \frac{d}{dt}(2\pi r) = 2\pi\frac{dr}{dt} \quad = 2\pi\left(\frac{k}{2\pi r}\right) = \frac{k}{r}$

$\Rightarrow \frac{dP}{dt} \propto \frac{1}{r}$. (using (1))

18) Ans: **A)** e

Sol: $a = \log_3(3x) = \log_3 3 + \log_3 x = 1 + \log_3 x = 1 + h$,

Say $b = \log_x 3 = \frac{1}{\log_3 x} = \frac{1}{h}$,

Here $\because x \to 1$, $\Rightarrow h = \log_3 x \to 0$, $\Rightarrow a^b \to e$

19) Ans: **C)** $\frac{24}{7}(-2j + k)$

Sol: $\frac{24}{7}(-2j + k)$

20) Ans: **D)** $\frac{n}{2n+1}$

Sol: Suppose P(i)-ki, where k is a constant of proportionality.
Thus P(1) + P(2) + ...+ P(2n)=1

$\Rightarrow k(1+2+...+2n)=1 \Rightarrow k = \frac{1}{n(2n+1)}$

$\Rightarrow P(i) = \frac{i}{n(2n+1)}$

$\therefore$ Probability of drawing an odd number is

$P(1)+P(2)+...+P(2n-1) = \frac{1}{n(2n+1)}\left[1 + 3 + ... + (2n-1)\right]$

$= \frac{1}{n(2n+1)} \times n^2 = \frac{n}{2n+1}$.

21) Ans: **C)** $\frac{\sqrt{5}}{3}$

Sol: put $\cot^{-1}\left(\frac{1}{2}\right) = \theta \quad \Rightarrow \cot\theta = \frac{1}{2}$

$\therefore \sin\theta = \frac{2}{\sqrt{5}}$. Put $\cos^{-1} x = \phi$,

$\therefore x = \cos\ \phi$

Also, $\tan\phi = \frac{2}{\sqrt{5}}$, $\quad \therefore x = \cos\phi = \frac{\sqrt{5}}{3}$

22) Ans: **B)** 8

Sol: Consider ${}^{2n}P_3 = 2({}^{n}P_4)$
$\therefore (2n)(2n - 1)(2n - 2) = 2.n(n - 1)(n - 2)(n - 3)$
$\therefore (2n - 1).2(n - 1) = (n - 1)(n - 2)(n - 3)$
$\therefore 2(2n - 1) = (n - 2)(n - 3)$
$\therefore 4n - 2 = n^2 - 5n + 6 \ \therefore n^2 - 9n + 8 = 0$
$\therefore (n - 1)(n - 8) = 8 \quad \therefore n = 1$ or $n = 8$
Suppose n = 1, then ${}^{2n}P_3 = {}^{2}P_3$ and ${}^{n}P_4 = {}^{1}P_4$ both of which are not defined.

So, there is only one possible solution : n = 8

23) Ans: **B)** f(x) is continuous at x = 2

Sol: Here, f(2) = 0

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} |2-h-2| = 0$$

$$\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0} |2+h-2| = 0$$

Hence, it is continuous at x = 2

24) Ans: **D)** $\sec x - \operatorname{cosec} x + c$

Sol: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \right) dx$

$= \int (\sec x \tan x + \operatorname{cosec} x \cot x) dx$

$= \sec x - \operatorname{cosec} x + c$

25) Ans: **B)** $x + 2y + 3z = 14$

Sol: Foot of perpendicular from $(0,0,0)$ to the plane is $(1,2,3)$ then $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Normal to the plane is $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

Therefore, equation of plane is $\vec{r}.\vec{n} = \vec{a}.\vec{n}$

$\Rightarrow \left(x\hat{i} + y\hat{j} + z\hat{k} \right).\left(\hat{i} + 2\hat{j} + 3\hat{k} \right) = 1 + 4 + 9 = 14$

$\Rightarrow x + 2y + 3z - 14 = 0$

26) Ans: **C)** $x^2 \frac{dy}{dx} + y = e^x$

Sol: $x^2 \frac{dy}{dx} + y = e^x \Rightarrow \frac{dy}{dx} + \frac{y}{x^2} = \frac{e^x}{x^2}$, which is a linear equation.

27) Ans: **C)** $y = x \log x - x + 2$

Sol: $x \frac{d^2y}{dx^2} = 1 \Rightarrow \frac{d^2y}{dx^2} = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \log x + c_1$

On integrating twice, $\Rightarrow y = x \log x - x + c_1 x + c_2$

Given $y = 1$ and $\frac{dy}{dx} = 0$ at $x = 1 \Rightarrow c_1 = 0$ and $c_2 = 2$

$\therefore$ The required solution is $y = x \log x - x + 2$.

28) Ans: **D)** $\frac{255}{256}$

Sol: Let p denotes the probability of getting head in a single toss of a coin, then

$p = \frac{1}{2}$ $\quad q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Here n = 8

$P(X = x) = {}^8C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x} = {}^8C_x \left(\frac{1}{2}\right)^8$

The probability of getting exactly 5 heads is

$P(X = 5) = {}^8C_5 \left(\frac{1}{2}\right)^8 = 56 \left(\frac{1}{2}\right)^8 = \frac{7}{32}$

The probability of getting larger number of heads than tails is

$P(X \ge 5) = P(X = 5) + P(x = 6) + P(X = 7) + P(X = 8)$

$P(X \ge 5) = {}^8C_5 \left(\frac{1}{2}\right)^8 + {}^8C_6 \left(\frac{1}{2}\right)^8 + {}^8C_7 \left(\frac{1}{2}\right)^8 + {}^8C_8 \left(\frac{1}{2}\right)^8$

$P(X \ge 5) = (56 + 28 + 8 + 1)\left(\frac{1}{2}\right)^8 = (93)\left(\frac{1}{2}\right)^8$

$P(X \ge 5) = \frac{93}{256}$

The probability of getting at least one head is

$P(X \ge 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$P(X \ge 1) = 1 - {}^8C_0 \left(\frac{1}{2}\right)^8 = 1 - (1)\left(\frac{1}{256}\right)$

$P(X \ge 1) = \frac{255}{256}$

29) Ans: **B)** $y\left[\frac{1}{2x} + \frac{4}{2x+3} - \frac{1}{2(x+1)}\right]$

Sol: $y = \frac{\sqrt{x}(2x+3)^2}{\sqrt{x+1}}$

$\Rightarrow \log y = \frac{1}{2} \log x + 2 \log (2x+3) - \frac{1}{2} \log (x+1)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + \frac{2.2}{(2x+3)} - \frac{1}{2(x+1)}$

i.e. $\frac{dy}{dx} = y\left[\frac{1}{2x} + \frac{4}{2x+3} - \frac{1}{2(x+1)}\right]$

30) Ans: **C)** $7x - 8y + 3z + 25 = 0$

Sol: The required planes passes through the point having position vector $\vec{a} = -\hat{i} + 3\hat{j} + 2\hat{k}$.

Normal vector $\vec{n}$ is perpendicular to the vectors

$\vec{n}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{n}_2 = 3\hat{i} + 3\hat{j} + \hat{k}$

$\therefore \vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{vmatrix} = -7\hat{i} + 8\hat{j} - 3\hat{k}$

Therefore, the equation of the required plane is $\vec{r}.\vec{n} = \vec{a}.\vec{n}$

$\Rightarrow \vec{r}.(-7\hat{i} + 8\hat{j} - 3\hat{k}) = (-\hat{i} + 3\hat{j} + 2\hat{k}).(-7\hat{i} + 8\hat{j} - 3\hat{k})$

$\Rightarrow -7x + 8y - 3z = 25 \quad \Rightarrow 7x - 8y + 3z + 25 = 0$

31) Ans: **D)** 0.3456

Sol: Let p denotes the probability that the component will survive a check test, then

p = 0.6 $\quad$ q = 1 - p = 1 - 0.6 = 0.4

Here n = 4

$p(X = x) = {}^4C_x (0.6)^x (0.4)^{4-x}$

The probability that exactly 2 components tested survive is

$P(X = 2) = {}^4C_2 (0.6)^2 (0.4)^2$

$P(X = 2) = (6)(0.36)(0.16) = 0.3456$

32) Ans: **D)** $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right).\sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right)$

Sol: Let $y = \dfrac{\sec x + \tan x}{\sec x - \tan x} = \dfrac{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}{\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}} = \dfrac{1 + \sin x}{1 - \sin x}$

$= \dfrac{1 - \cos\left(\dfrac{\pi}{2} + x\right)}{1 + \cos\left(\dfrac{\pi}{2} + x\right)} \quad \left[\because \cos\left(\dfrac{\pi}{2} + x\right) = -\sin x\right]$

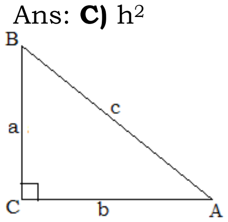
$= \dfrac{2\sin^2\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)}{2\cos^2\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)} = \tan^2\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$

$\therefore \dfrac{dy}{dx} = \dfrac{d}{dx}\left[\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)\right]^2$

$= 2\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right) . \dfrac{d}{dx} \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$

$= \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right) . \sec^2\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$

33) Ans: **C)** h^2

Sol: 

$\overline{AB}.\overline{AC} + \overline{BC}.\overline{BA} + \overline{CA}.\overline{CB}$

$= bc \cos A + ac \cos B + 0 = c(b \cos A + a \cos B)$

$= c(c) = c^2 = h^2$

34) Ans: **D)** None of these

Sol: Here, $I = \int \dfrac{dx}{x(x^5 + 1)} = \int \dfrac{dx}{x^6\left(1 + \dfrac{1}{x^5}\right)}$

Put $1 + \dfrac{1}{x^5} = t \Rightarrow \dfrac{-5}{x^6} dx = dt$

$\Rightarrow I = -\dfrac{1}{5}\int \dfrac{dt}{t} = -\dfrac{1}{5}\log t + c$

$I = -\dfrac{1}{5}\log\left(1 + \dfrac{1}{x^5}\right) + c = -\dfrac{1}{5}\log\left(\dfrac{x^5 + 1}{x^5}\right) + c$

$\therefore I = \dfrac{1}{5}\log\left(\dfrac{x^5}{x^5 + 1}\right) + c$

35) Ans: **B)** a

Sol: As we know, $\int_{-a}^{a} \sqrt{\dfrac{a - x}{a + x}} dx = a\pi$

$\therefore$ On comparing it with the given expression, we get k = a.

36) Ans: **D)** $4x^2 + 4xy - 3y^2 = 0$

Sol: $x + y = 3 \quad \Rightarrow \dfrac{x}{3} + \dfrac{y}{3} = 1$ .....(i)

Also, $y^2 = 4x$ .....(ii)

$\therefore y^2 = 4x\left[\dfrac{x}{3} + \dfrac{y}{3}\right]$ ....[from (i) and (ii)]

$\Rightarrow 3y^2 = 4x^2 + 4xy \qquad \Rightarrow 4x^2 + 4xy - 3y^2 = 0$

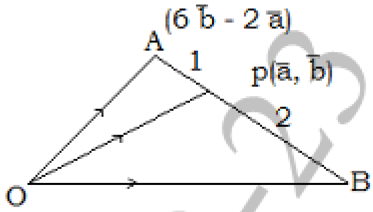
37) Ans: **D)** 3

Sol: Given, $\tan^{-1} x = \sin^{-1}\left[\dfrac{3}{\sqrt{10}}\right]$

$\Rightarrow x = \tan\left\{\sin^{-1}\left[\dfrac{3}{\sqrt{10}}\right]\right\} = \tan\{\tan^{-1} 3\} = 3$

38) Ans: **A)** $7\bar{a} - 15\bar{b}$

Sol: $\overline{OP} = \dfrac{1(\overline{OB}) + 2(6\bar{b} - 2\bar{a})}{1 + 2}$



$\Rightarrow 3(\bar{a} - \bar{b}) = \overline{OB} + 12\bar{b} - 4\bar{a} \qquad \Rightarrow \overline{OB} = 7\bar{a} - 15\bar{b}$

39) Ans: **A)** 12

Sol: Here $n_1 = 100, \bar{x}_1 = 45, \sigma_1^2 = 49, n = 250, \bar{x}_c = 51,$

$\sigma_c^2 = 130$

$\therefore n_2 = n - n_1 = 250 - 100 = 150$

Combined mean is $\bar{x}_c = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$\therefore 51 = \dfrac{100 \times 45 + 150 \times \bar{x}_2}{100 + 150} \Rightarrow 51 = \dfrac{4500 + 150\bar{x}_2}{250}$

$12750 = 4500 + 150\bar{x}_2 \Rightarrow 8250 = 150\bar{x}_2 \Rightarrow \bar{x}_2 = 55$

$d_1 = \bar{x} - \bar{x}_c$ and $d_2 = \bar{x}_2 - \bar{x}_c$

$\therefore d_1 = 45 - 51 = -6$ and $d_2 = 55 - 51 = 4$

Now $\sigma_c^2 = \dfrac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}$

$\Rightarrow 130 = \dfrac{100(49 + 36) + 150\left(\sigma_2^2 + 16\right)}{250}$

$\Rightarrow 32500 = 100(85) + 150\sigma_2^2 + 2400$

$\Rightarrow 30100 = 8500 + 150\sigma_2^2 \Rightarrow 21600 = 150\sigma_2^2$

$\Rightarrow 144 = \sigma_2^2$

$\therefore \sigma_2 = 12$

40) Ans: **B)** 2.16

Sol: $\sum_{x_i = S} P(X = x) = 1$

$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$4k = 0.4 \Rightarrow k = 0.1$

x_i	p_i	$p_i x_i$	x_i^2	$p_i x_i^2$
-2	0.1	-0.2	4	0.4
-1	0.1	-0.1	1	0.1
0	0.2	0	0	0
1	0.2	0.2	1	0.2
2	0.3	0.6	4	1.2
3	0.1	0.3	9	0.9
		$\Sigma = 0.8$		$\Sigma = 2.8$

$E(X)=\mu=\sum p_i x_i = 0.8$

$Var(X)=\sum p_i\ x_i^2-\mu^2 = 2.8-(0.8)^2$

$Var(X)=2.8-0.64=2.16$

41) Ans: **B)** x + 1 = 0
Sol: Here circle is $x^2+y^2-2y=0$
Here 2g=0,=-2, c = 0, $P(x_1,y_1)\equiv P(-1,1)$
$\therefore g=0, f=-1, c=0$
The point P lies on the circle.
The equation of tangent to a circle
is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$
$\therefore x(-1)+y(1)+(0)(x-1)+(-1)(y+1)+0=0$
$\therefore -x+y+-y-1=0 \Rightarrow -x-1=0 \Rightarrow x+1=0$

42) Ans: **C)** $\frac{2}{3}$

Sol: Here, required area = $\int_{-1}^{1} x|x|\,dx$

$= \int_{-1}^{0} -x^2dx + \int_{0}^{1} x^2\,dx = \left[\frac{-x^3}{3}\right]_1^0 + \left[\frac{x^3}{3}\right]_0^1 = \left|\frac{-1}{3}\right| + \left|\frac{1}{3}\right| = \frac{2}{3}$

43) Ans: **B)** f and g are inverse of each other
Sol: Here, $f(x)=\frac{x+3}{x-2}, g(x)=\frac{2x+3}{x-1}$

$\therefore (fog)(x)=f(g(x))=f\left(\frac{2x+3}{x-1}\right)=\frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}$

$=\frac{2x+3+3x-3}{2x+3-2x+2}=\frac{5x}{5}=x$

$\therefore (gof)(x)=g(f(x))=g\left(\frac{x+3}{x-2}\right)=\frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}$

$=\frac{2x+6+3x-6}{x+3-x+2}=\frac{5x}{5}=x$
Therefore, f and g are inverse of each other

44) Ans: **C)** $A^{-1} = |A|^{-1}$
Sol: $A^{-1}=|A|^{-1}$ is not true.
L.H.S. is a matrix and R.H.S. is a number.

45) Ans: **B)** $-\frac{12}{5}$

Sol: Suppose, $y=\sqrt{x^2+16}$ and $z=\frac{x}{x-1}$

$\frac{dy}{dx}=\frac{1}{2}(x^2+16)^{-1/2}(2x)$ and $\frac{dz}{dx}=\frac{x-1-x}{(x-1)^2}=\frac{-1}{(x-1)^2}$

$\therefore \frac{dy}{dz}=\frac{-x}{\sqrt{x^2+16}}\ \frac{1}{1/(x-1)^2} \Rightarrow \left(\frac{dy}{dz}\right)_{x=3}=\frac{-3(2)^2}{5}=\frac{-12}{5}$

46) Ans: **D)** zero or positive
Sol: This is a fundamental concept.

47) Ans: **D)** $\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$

Sol: As $\int f(x)\sin x\cos x\,dx=\frac{1}{2(b^2-a^2)}\log(f(x))+c$

$\therefore\ f(x)\sin x\cos x=\frac{1}{2(b^2-a^2)}\cdot\frac{1}{f(x)}f'(x)$
Differentiating both sides w.r.t. x

$\Rightarrow 2(b^2-a^2)\sin x\cos x=\frac{f'(x)}{f(x)^2}$

$\Rightarrow \int(2b^2\sin x\cos x-2a^2\sin x\cos x)dx=\int\frac{f'(x)}{\{f(x)\}^2}dx$

$\Rightarrow \pm(-b^2\cos^2 x-a^2\sin^2 x)=-\frac{1}{f(x)}$

$\Rightarrow f(x)=\pm\frac{1}{(a^2\sin^2 x+b^2\cos^2 x)}$

48) Ans: **A)** p but not q is a constraint of L.P.P.
Sol: ‘p’ is linear while ‘q’ is non-linear.

49) Ans: **A)** $29x+4y+5=0$
Sol: Given, $A(-1,6), B(-3,-9)$ and $C(5,-8)$.
Let O be the midpoint of BC.

Therefore, $D=\left(\frac{-3+5}{2},\frac{-9+(-8)}{2}\right) \Rightarrow D\left(1,-\frac{17}{2}\right)$

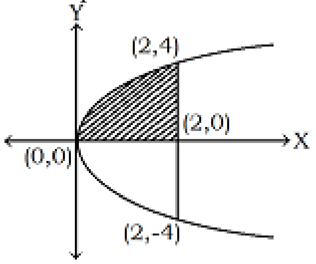
Now, equation of median AD,

$\frac{y-6}{x+1}=\frac{\frac{-17}{2}-6}{1+1} \quad \Rightarrow \frac{y-6}{x+1}=\frac{-17-12}{2\times 2}$

$\Rightarrow \frac{y-6}{x+1}=\frac{-29}{4} \quad \Rightarrow 4y-24=-29x-$
$\Rightarrow 29x+4y+5=0$

50) Ans: **B)** $\frac{32}{3}$ sq.units

Sol: x=2 meets $y^2=8x \Rightarrow y^2=16 \Rightarrow y=\pm 4$
$\therefore$ the points of intersection are (2,4) and (2,-4).



$\therefore$ Required area $=2\int_0^2\sqrt{8x}dx=4\sqrt{2}\int_0^2 x^{\frac{1}{2}}dx$

$=4\sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2=\frac{8\sqrt{2}}{3}\left[2\sqrt{2}-0\right]=\frac{32}{3}$ sq.units