Studentpad

Set theory, relations and functions 2022-23

Time : 90 Min	Maths : Set Theory, Relations and Functions	Marks : 120

01) Consider $f(\theta)=\sin\theta(\sin\theta+\sin 3\theta)$. Then, $f(\theta)$

A) ≥ 0, only when $\theta \geq 0$

B) ≤ 0, only when $\theta \leq 0$

C) ≥ 0, for all real θ

D) ≤ 0, for all real θ

02) The domain of the function $f(x)=\sqrt{x-x^2}+\sqrt{4+x}+\sqrt{4-x}$ is

A) [0, 1]
B) [0, 4]
C) [-4, 4]
D) $[-4,\infty)$

03) The function $f(x)=\sin\left(\log(x+\sqrt{x^2+1})\right)$ is

A) odd function.
B) even function.
C) periodic function.
D) neither even nor odd.

04) A survey shows that 63% of the Indians like mangoes while 76% like apples. x% of the Indians like both mangoes and apples. Then find the value of x.

A) $x=63$

B) $39\leq x\leq 63$

C) $x=39$

D) None of these

05) The function $f:[0,3]\to[1,29]$, defined by $f(x)=2x^3-15x^2+36x+1$, is which of the following?

A) One-one but not onto
B) Onto but not one-one
C) One-one and onto
D) Neither one-one nor onto

06) If $f(x)=\sin x+\cos x, g(x)=x^2-1$, then $g\{f(x)\}$ is invertible in which domain?

A) $\left[0,\frac{\pi}{2}\right]$

B) $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$

C) $[0,\pi]$

D) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

07) What is the domain of R if $R=\{(x,y)\mid x,y\in Z, x^2+y^2\leq 4\}$ is a relation in Z?

A) {0, 1, 2}
B) {- 2, - 1, 0, 1, 2}
C) {0, - 1, - 2}
D) None of these

08) Consider $f(x)=x^2$ and $g(x)=\sin x$ for all $x\in R$. Then, determine the set of all x satisfying $(fogogof)(x)=(gogof)(x)$, where $(fog)(x)=f(g(x))$.

A) $\pm\sqrt{n\pi}, n\in\{0,1,2,....\}$

B) $\pm\sqrt{n\pi}, n\in\{1,2,...\}$

C) $2n\pi, n\in\{...,-2,-1,0,1,2,...\}$

D) $\pi/2+2n\pi, n\in\{...,-2,-1,0,1,2,...\}$

09) Which of the following function is even function?

A) $f(x)=x\left(\frac{a^x-1}{a^x+1}\right)$

B) $f(x)=\frac{a^x+1}{a^x-1}$

C) $f(x)=\frac{a^x-a^{-x}}{a^x+a^{-x}}$

D) $f(x)=\sin x$

10) Let $n(A)=3$, $n(B)=6$ and $A\subseteq B$. Then how many elements are there in $A\cap B$?

A) 9
B) 6
C) 3
D) None of these

11) Two finite sets have m and n elements. The total number of subjects of the first set is 48 more than the total number of subjects of the second set. What are the respective values of m and n?

A) 7, 6
B) 7, 4
C) 6, 4
D) 6, 3

12) If $f(x)=\cos(\log x)$, then the value of $f(x).f(4)-\frac{1}{2}\left[f\left(\frac{x}{4}\right)+f(4x)\right]$

A) ± 1
B) -1
C) 0
D) 1

13) Suppose R and S be two equivalence relations on a set A, then _____.

A) $R\cup S$ is an equivalence relation on A
B) $R-S$ is an equivalence relation on A
C) $R\cap S$ is an equivalence relation on A
D) None of these

14) If co-ordination is R, then find out which of the following functions is not a surjective (onto) function?

A) $f(x) = x + \cos(\pi[x]), (\text{where } [.] \text{denotes the G.I.F})$

B) $f(x) = \log_e |\log_e(-[x])|, (\text{where } [.] \text{denotes the G.I.F})$

C) $f(x) = \tan^{-1} x - \frac{x}{\sqrt{1+x^2}}$

D) $f(x) = \frac{x}{x^2-1}$

15) Suppose $f(x) = (x+1)^2$ for $x \geq -1$. If g(x) is the function whose graph is reflection of the graph of f(x) with respect to the line $y = x$, then what is g(x).

A) $\sqrt{x+1}, x \geq -1$

B) $\frac{1}{(x+1)^2}, x > -1$

C) $-\sqrt{x} - 1, x \geq 0$

D) $\sqrt{x} - 1, x \geq 0$

16) If $f(x) = \sin x + \cos x, g(x) = x^2 - 1$, then in the domain, $g(f(x))$ is invertible?

A) $[0, \pi]$

B) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D) $\left[0, \frac{\pi}{2}\right]$

17) Which of the following is the void relation on set A?

A) Reflexive
B) Reflexive and symmetric
C) Symmetric and transitive
D) Reflexive and transitive

18) Let n be a fixed +ve integer. Define a relation R on set Z of integers by, $aRb \Leftrightarrow n \mid a - b \mid$. Then R is ____.

A) transitive
B) reflexive
C) symmetric
D) all of above

19) If A, B and C are any three sets, then A - (B ∪ C) = _____.

A) (A - B) ∩ (A - C)
B) (A - B) ∪ (A - C)
C) (A - B) ∪ C
D) (A - B) ∩ C

20) If $f(x) = \log_a x$ and $F(x) = a^x$, then F[f(x)] is

A) f[F(2x)]

B) f[F(x)]

C) F | f(2x) |

D) F[(x)]

21) If A = {1, 2, 3}, B = {1, 3, 5}. A relation $R: A \to B$ is defined by R = {(1, 3), (1, 5), (2, 1)}.

Then R^{-1} is represented as:

A) {(1, 2), (5, 1), (3, 1)}
B) {(1,2), (3,1), (1,3), (1,5)}
C) {(1, 2), (3, 1), (2, 1)}
D) None of these

22) Let A is set of even natural numbers less than 8 and B is set of prime numbers less than 7. Then what is the number of relations from A to B?

A) 2^{9-1}

B) 3^2

C) 9^2

D) 2^9

23) If $N_a = \{an : n \in N\}$, then $N_3 \cap N_4 =$

A) N_{12}

B) N_7

C) N_4

D) N_3

24) If A={a, b, c}, B={b, c, d}, C={a, b, d, e}, then find $A \cap (B \cup C)$.

A) {b, c, d}
B) {a, b, c}
C) {a, b, d, e}
D) {e}

25) If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, then $X \cup Y$ = _____.

A) N
B) X
C) Y
D) None of these

26) In a group of 1000 people there are 750 who can speak Hindi and 400 who can speack Bengali. Estimate the number of people those can speack Bengali only.

27) The set A has 3 elements and B has 6 elements. The minimum number of elements of $A \cup B$ can be ____.

28) Given the relation $R = \{(1,2),(2,3)\}$ on the set $\{1,2,3\}$, what is the minimum number of ordered pairs which when added to R make it an equivalence relation?

29) Let $S = \{1,2,3,4\}$. What is the total number of unordered pairs of disjoint subsets of S?

30) In a survey of 200 students of a higher secondary school it was found that 120 studied Mathematics (M), 90 studied Physics (P) and 70 studies Chemistry (C), 40 studied M and P, 30 studied P and C, 50 studied C and M, and 20 studied none of these subjects. Estimate the number of students who studied all three subjects.