# Studentpad

**MHT-CET-XI MATHEMATICS 2022-23**

| Time : 150 Min | Maths : Full Portion Paper | Marks : 100 |
|---|---|---|

## Hints and Solutions

**01)** Ans: **C)** $\frac{2a}{b}$

Sol: $\because \left|\frac{1}{2}\right| = \frac{1}{2} < 1$, $\Rightarrow \left(\frac{1}{2}\right)^n \to 0$ and $\left(\frac{1}{2}\right)^{n+1} \to 0$

$\therefore \frac{1}{2^n} \to 0$ and $\frac{1}{2^{n+1}} \to 0$, $\therefore \frac{\sin(a/2^n)}{\sin(b/2^{n+1})}$

$= \frac{\sin(a/2^n)}{a/2^n} \times \frac{b/2^{n+1}}{\sin(b/2^{n+1})} \times \frac{a/2^n}{b/2^{n+1}}$

$\to (1) \times (1) \times \frac{2a}{b} \to \frac{2a}{b}$

**02)** Ans: **C)** 2

Sol: $y = \frac{x^{2/3} - x^{-1/3}}{x^{2/3} + x^{-1/3}}$, $= \frac{x^{2/3} - \frac{1}{x^{1/3}}}{x^{2/3} + \frac{1}{x^{1/3}}}$

$y = \frac{x^{2/3}.x^{1/3} - 1}{x^{2/3}.x^{1/3} + 1} = \frac{x^{(2/3)+(1/3)} - 1}{x^{(2/3)+(1/3)} + 1} = \frac{x-1}{x+1}$

$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(x+1)}{(x+1)^2}$

$y_1 = \frac{(x+1)1 - (x-1)1}{(x+1)^2} \quad = \frac{(x+1) - (x+1)}{(x+1)^2}$

$(x+1)^2 y_1 \ = 2$

**03)** Ans: **D)** b + ia

Sol: Suppose $\because a^2 + b^2 = 1$

Let a=sinθ, b=cosθ, $\Rightarrow \frac{1+b+ia}{1+b-ia}$

$= \frac{(1+\cos\theta) + i(\sin\theta)}{(1+\cos\theta) - i(\sin\theta)} = \frac{2\cos^2\frac{\theta}{2} + 2i.\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} - 2i.\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}$

$= \frac{\cos\frac{\theta}{2} + i.\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - i.\sin\frac{\theta}{2}} = \frac{\cos\left(\frac{\theta}{2}\right) + i.\sin\left(\frac{\theta}{2}\right)}{\cos\left(-\frac{\theta}{2}\right) + i.\sin\left(-\frac{\theta}{2}\right)}$

$= \cos\left[\frac{\theta}{2} - \left(-\frac{\theta}{2}\right)\right] + i.\sin\left[\frac{\theta}{2} - \left(-\frac{\theta}{2}\right)\right]$

$= (\cos\theta) + i(\sin\theta)$ =b + ia

**04)** Ans: **A)** 512

Sol: $\left(1+x+x^2+x^3\right)^5 \ = (1+x)^5\left(1+x^2\right)^5$

$= \left(1+5x+10x^2+10x^3+5x^4+x^5\right)$

$\left(1+5x^2+10x^4+10x^6+5x^8+x^{10}\right)$

$\Rightarrow$ Coefficient of even power of x

$= (1+10+5) \times 2^5 = 16 \times 32 = 512$

**05)** Ans: **D)** $\pm 1$

Sol: f(x) is continuous at $x = 0$, then

$\Rightarrow f(0) = \lim_{x\to 0} f(x)$

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin^2 ax}{x^2} = \lim_{x\to 0} \frac{\sin^2 ax}{a^2x^2} \times a^2$

$= \left[\lim_{x\to 0} \frac{\sin ax}{ax}\right]^2 \times a^2 = a^2$

$\because f(0) = 1 \Rightarrow a^2 = 1 \therefore a = \pm 1$

**06)** Ans: **A)** x=y

Sol: $\because -1 \le \sin\theta \le 1 \quad \therefore 0 \le \sin^2\theta \le 1 \quad \therefore \sin^2\theta \le 1$

$\therefore \frac{x^2+y^2}{2xy} \le 1 \quad \therefore x^2+y^2 \le 2xy \quad \therefore x^2+y^2-2xy \le 0$

$\therefore (x-y)^2 \le 0 \quad \therefore (x-y)^2 < 0$ or $(x-y)^2 = 0$

But square of a real number cannot be negative

$\therefore (x-y)^2 < 0 \quad \therefore (x-y)^2 = 0 \quad \therefore x = y$

**07)** Ans: **D)** $x + y = 2$

Sol: A line is equally inclined to the axes when its slope is either 1 or -1.

Out of given lines the line $x + y = 2$ is at a distance of $\sqrt{2}$ units from the origin.

**08)** Ans: **A)** $29x + 4y + 5 = 0$

Sol: Given, $A(-1, 6), B(-3, -9)$ and $C(5, -8)$.

Let O be the midpoint of BC.

Therefore, $D = \left(\frac{-3+5}{2}, \frac{-9+(-8)}{2}\right) \Rightarrow D\left(1, -\frac{17}{2}\right)$

Now, equation of median AD,

$\frac{y-6}{x+1} = \frac{\frac{-17}{2} - 6}{1+1} \quad \Rightarrow \frac{y-6}{x+1} = \frac{-17 \ -12}{2 \times 2}$

$\Rightarrow \frac{y-6}{x+1} = \frac{-29}{4} \quad \Rightarrow 4y - 24 = -29x -$

$\Rightarrow 29x + 4y + 5 = 0$

**09)** Ans: **A)** 90

$\boxed{2}\ \boxed{3.4.5}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ } > 23000$

$1 \times 3 \times 3 \times 2 \times 1 = 18$

OR

$\boxed{3.4.5}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ } > 23000$

Sol: $3 \times 4 \times 3 \times 2 \times 1 = 72$

Therefore, total number of required numbers=18 + 72 = 90

**10)** Ans: **B)** 120

Sol: Here, $2\log_b a^4.\log_c b^3.\log_a c^5$

$= 8\log_b a.3\log_c b.5\log_a c$

$= \frac{8\log a}{\log b}.\frac{3\log b}{\log c}.\frac{5\log c}{\log a} = 120$

**11)** Ans: **B)** $f : R \to [0, \infty), f(x) = x^2$

Sol: The function $f(x) = 3x + 1, x \in R$ is one - one and onto

$\therefore f(x)$ is invertible.

The function $f(x) = x^2, x \in R$ is not one -one due to $f(-4) = f(4) = 16$

$\therefore f(x)$ is not invertible.

We have, $f(x) = \frac{1}{x^3}, x \in R^+$

Let $f(x_1) = f(x_2)$ for $x_1, x_2 \in R^+ \quad \Rightarrow \frac{1}{x_1^3} = \frac{1}{x_2^3}$

$\Rightarrow x_1^3 - x_2^3 = 0 \quad \Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) \neq 0$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one- one.

For onto : Let f(x)=k

$\Rightarrow \frac{1}{x^3} = k \quad \Rightarrow x^3 = \frac{1}{k} \quad \Rightarrow x = \frac{1}{k^{1/3}}$

For $k \in R^+, x = \frac{1}{k^{1/3}} \in R^+ \quad \Rightarrow f$ is onto

$\therefore f(x)$ is invertible.

**12)** Ans: **D)** $\frac{3}{5}$

Sol: Given, $P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}, P(A \cup B) = \frac{4}{5}$

$\therefore P(A \cap B) = P(A/B)P(B) = \frac{1}{2}.\frac{3}{5} = \frac{3}{10}$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10} \Rightarrow P(A) = \frac{1}{5} + \frac{3}{10} = \frac{1}{2}$

$\therefore P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

We know, $P(B \cap A') + P(B \cap A) = P(B)$

$[\because A' \cap B \text{ and } A \cap B \text{ are mutually exclusive events}]$

$\therefore P(B \cap A') = P(B) - P(B \cap A) = \frac{3}{5} - \frac{3}{10} = \frac{3}{10}$

$\therefore P(B/A') = \frac{P(B \cap A')}{P(A')} = \frac{3/10}{1/2} = \frac{3}{5}$

**13)** Ans: **B)** 8

Sol: Consider ${}^{2n}P_3 = 2({}^nP_4)$

$\therefore (2n)(2n - 1)(2n - 2) = 2.n(n - 1)(n - 2)(n - 3)$

$\therefore (2n - 1).2(n - 1) = (n - 1)(n - 2)(n - 3)$

$\therefore 2(2n - 1) = (n - 2)(n - 3)$

$\therefore 4n - 2 = n^2 - 5n + 6 \quad \therefore n^2 - 9n + 8 = 0$

$\therefore (n - 1)(n - 8) = 8 \quad \therefore n = 1$ or $n = 8$

Suppose n = 1, then ${}^{2n}P_3 = {}^2P_3$ and ${}^nP_4 = {}^1P_4$ both of which are not defined.

So, there is only one possible solution : n = 8

**14)** Ans: **A)** G.P.

Sol: $\because x + y = 2y$

$\Rightarrow e^{-x}.e^{-z} = e^{-x-z} = e^{-2y} = (e^{-y})^2$

Therefore $e^{-x}, e^{-y}, e^{-z}$ are in G.P.

**15)** Ans: **A)** 0.6

Sol: Let E and F be the events that students study Mathematics and Biology respectively.

$\therefore P(E) = \frac{40}{100} = 0.4$ and $P(F) = \frac{30}{100} = 0.3$

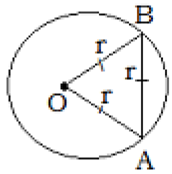
Also, $P(E \cap F) = \frac{10}{100} = 0.1$

We have to find the probability that a student studies Mathematics or Biology, i.e., $P(E \cup F)$

Now, $P(E \cup F) = 0.4 + 0.3 - 0.1 = 0.6$

**16)** Ans: **B)** $\frac{20\pi}{3}$ cm

Sol: Let r be the radius of circle with centre at O.
Diameter of circle = 40 cm
Radius of circle = r = 20 cm




Let AB be the chord of the circle, then Minor arc of circle is arc AB
Here OA = OB = AB = r = 20 cm
So $\Delta OAB$ is an equilateral triangle, then

$m\angle AOB = \theta = 60^0 = \left(\frac{\pi}{3}\right)^c$

Thus, length of arc of circle is $s = r\theta^c$

$= (20)\left(\frac{\pi}{3}\right) = \frac{20\pi}{3}$ cm

**17)** Ans: **D)** $\frac{2}{a_1 + a_n}\left(\frac{1}{a_1} + \frac{1}{a_2} + ...... + \frac{1}{a_n}\right)$

Sol: $\frac{1}{a_1} + \frac{1}{a_n} = \frac{a_1 + a_n}{a_1.a_n}$ ....(1)

$\frac{1}{a_1}+\frac{1}{a_{n-1}}=\frac{a_2+a_{n-1}}{a_2.a_{n-1}}=\frac{a_1+a_n}{a_2.a_{n-1}}$

$=\frac{(a_1+d)+(a_n-d)}{a_2.a_{n-1}}=\frac{a_1+a_n}{a_2.a_{n-1}}$

$\therefore \frac{1}{a^2}+\frac{1}{a_{n-1}}=\frac{a_1+a_n}{a_2.a_{n-1}}$ ...(2)

$\frac{1}{a_n}+\frac{1}{a_1}=\frac{a_1+a_n}{a_n.a_1}$ ...(n)

Adding equation (1) to (n): $2\left(\frac{1}{a_1}+\frac{1}{a_2}+.....+\frac{1}{a_n}\right)$

$=(a_1+a_n)\left(\frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+....+\frac{1}{a_na_1}\right)$

$\therefore \frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+......+\frac{1}{a_na_1}$

$=\frac{2}{a_1+a_n}\left(\frac{1}{a_1}+\frac{1}{a_2}+....+\frac{1}{a_n}\right)$

**18)** Ans: **B)** 14

Sol: $1, a_1, a_2, .......a_m, 31$ are in A.P.

The common difference d is given by

$31=1+(m+1)d \Rightarrow d=\frac{30}{m+1}$

$\frac{a_7}{a_{m-1}}=\frac{5}{9}\Rightarrow\frac{1+7d}{1+(m-1)d}=\frac{5}{9}$

$\Rightarrow\frac{5}{9}=\frac{1+\frac{210}{m+1}}{1+\frac{(m-1)30}{m+1}}=\frac{m+211}{31m-29}$

$\Rightarrow 5(31m-29)=9(m+211)$

$\Rightarrow 146\text{ m}=2044\Rightarrow m=14$

**19)** Ans: **A)** 2

Sol: Let $A\equiv(3,-2), B\equiv(1,0), C\equiv(-1,-2), D\equiv(1,-4)$

Points A, B C and D are concyclic.

Let P(h, k) be the centre of the circle passing through the points A, B and C

$\therefore PA=PB=PC \quad PA=PB\Rightarrow PA^2=PB^2$

$\therefore (h-3)^2+(k+2)^2=(h-1)^2+(k-0)^2$

$\therefore h^2-6h+9+k^2+k^2+4k+4=h^2-2h+1+k^2$

$\therefore -4h+4k=-12$

$\therefore -h+k=-3$ ...(i)

$PA=PC\Rightarrow PA^2=PC^2$

$\therefore (h-3)^2+(k+2)^2=(h+1)^2+(k+2)$

$\therefore h^2-6h+9=h^2+2h+1\Rightarrow -8h=-8$

$\therefore h=1$ .. (ii)

From (i) and (ii), we get

$k=1-3=-2$

Thus, centre is $P\equiv(1,-2)$ and radius

$=AP=\sqrt{(3-1)^2+(-2+2)^2}=\sqrt{(2)^2+0}=\sqrt{4}=2$

**20)** Ans: **C)** It is not necessary that either A = O or B = O

**21)** Ans: **A)** $\frac{1}{32}$

Sol: $1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}.\cos\frac{x^2}{4}$

$=\left(1-\cos\frac{x^2}{2}\right)-\cos\frac{x^2}{4}\left(1-\cos\frac{x^2}{2}\right)$

$=\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)$

**22)** Ans: **B)** 64

Sol: $\because \alpha,\beta$ satisfy : $2x^2$-35x+2=0

$\therefore \alpha\beta=2/2=1$

$2\alpha^2-35\alpha+2=0,\quad 2\beta^2-35\beta+2=0$

$\therefore 2\alpha-35=2,\quad \beta(2\beta-35)=-2$

$\therefore 2\alpha-35=-\frac{2}{\alpha},\quad 2\beta-35=-\frac{2}{\beta}$

$\therefore (2\alpha-35)^3.(2\beta-35)^3$

$=\left(-\frac{2}{\alpha}\right)^3.\left(-\frac{2}{\beta}\right)^3=\left(-\frac{8}{\alpha^3}\right)\left(-\frac{8}{\beta}\right)$

$=\frac{64}{(\alpha\beta)^3}=\frac{64}{(1)^3}=64$

**23)** Ans: **A)** f(3)

Sol: Taking 2 out from $C_2$, (x-3) from $R_1$ and (x-5) from $R_2$

$$f(x)=2(x-3)(x-5)\begin{vmatrix}1 & x+3 & 3(x^2+3x+9)\\ 1 & x+5 & 4(x^2+5x+25)\\ 1 & 1 & 3\end{vmatrix}$$

Applying $R_2-R_1$ and $R_1-R_3$

$$f(x)=2(x-3)(x-5)\begin{vmatrix}0 & x+2 & 3(x^2+3x+8)\\ 0 & 2 & x^2+11x+73\\ 1 & 1 & 3\end{vmatrix}$$

$\Rightarrow f(3)=0,\quad f(5)=0$

$$f(1)=2(1-3)(1-5)\begin{vmatrix}0 & 3 & 36\\ 0 & 2 & 85\\ 1 & 1 & 3\end{vmatrix}=......=2928$$

$\therefore$ f(1).f(3)+f(3).f(5)+f(5).f(1) =0+0+0=0 ; i.e. f(3)

**24)** Ans: **D)** 3

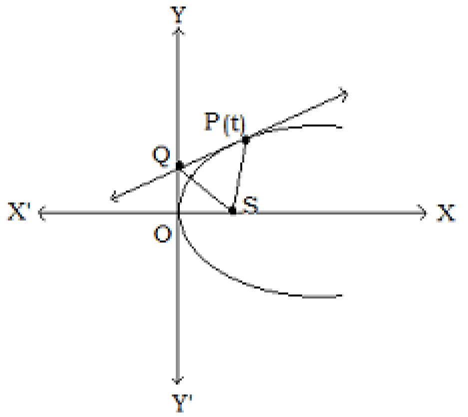
Sol: $\because s+s^2=1 \quad \therefore s=1-s^2=c^2 \quad \therefore c^2=s$

$\therefore c^2+c^4=s+s^2=1$ (Given)

**25)** Ans: **C)** $90^0$

Sol: Given parabola is $y^2=4ax$

Focus is S(a, 0).




Consider $P(t) \equiv P(at^2, 2at)$ be any point on the parabola. Equation of tangent to the parabola at P(t) is $yt = x + at^2$ ... (i)

Tangent at P(t) meets the y-axis in Q.
Put x = 0 in (i), we get

$yt = 0 + at^2 \Rightarrow yt = at^2 \Rightarrow y = at$

$\therefore Q \equiv (0, at)$

Slope of PQ is $m_1 = \frac{2at - at}{at^2 - 0} = \frac{at}{at^2} = \frac{1}{t}$

Slope of SQ is $m_2 = \frac{0 - at}{a - 0} = -\frac{at}{a} = -t$

Now $m_1 m_2 = \frac{1}{t} - t = -1$

Therefore SP subtends a right angle at Q.

**26)** Ans: **D)** $x^2 + y^2 - 6x - 3y + 8 = 0$

Sol: Consider C (h, k) be the centre of the circle.
Consider the circle passes through point A(2, 3).
Consider the line 2x + 3y = 4 touches the circle at the point B (2, 0)

$\therefore CA = CB \Rightarrow CA^2 = CB^2$

$(h-2)^2 + (k-3)^2 = (h-2)^2 + (k-0)^2$

$\therefore k^2 - 6k + 9 = k^2 \Rightarrow -6k = -9 \Rightarrow k = \frac{3}{2}$

CB is perpendicular to the line 2x + 3y - 4 = 0.
⇒ (Slope of CB) (Slope of line 2x + 3y - 4 = 0) = - 1

$\therefore \left(\frac{k-0}{h-2}\right)\left(-\frac{2}{3}\right) = -1 \Rightarrow 2k = 3h - 6$

$\Rightarrow 2\left(\frac{3}{2}\right) = 3h - 6$

$\therefore 3 + 6 = 3h \Rightarrow 9 = 3h \rightarrow h = 3$

$\therefore C(h,k) \equiv C\left(3, \frac{3}{2}\right)$ and

$r = CB = \sqrt{(3-2)^2 + \left(\frac{3}{2} - 0\right)^2} = \sqrt{1^2 + \left(\frac{3}{2}\right)^2} = \sqrt{1 + \frac{9}{4}}$

$= \sqrt{\frac{13}{4}}$ Equation of circle is $(x-h)^2 + (y-k)^2 = r^2$

$\therefore (x-3)^2 + \left(y - \frac{3}{2}\right)^2 = \left(\sqrt{\frac{13}{4}}\right)^2$

$\therefore x^2 - 6x + 9 + y^2 - 3y + \frac{9}{4} = \frac{13}{4}$

$\therefore x^2 + y^2 - 6x - 3y + 8 = 0$

**27)** Ans: **D)** $-(21x^{-4} - 8x^{-5} + 20x^{-6})$

Sol: $y = \frac{7x^2 - 2x + 4}{x^5} = \frac{7x^2}{x^5} - \frac{2x}{x^5} + \frac{4}{x^5}$

$= \frac{7}{x^3} - \frac{2}{x^4} + \frac{4}{x^5} = 7(x^{-3}) - 2(x^{-4}) + 4(x^{-5})$

**28)** Ans: **C)** $2^{n-1}A$

Sol: If n = 1, then $A^n = A^1 = A$

Thus possible options are (b) and (c)

$A^n = A^2 = AA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$= 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A, \therefore A^2 = 2A$

This also happens in both (b) and (c).
If n=3, then

$A^n = A^3 = A^2.A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

$= 4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 4A, \therefore A^3 = 4A$

This happens only in (c).
Therefore correct option is (c).

**29)** Ans: **C)** 2

Sol: We know that, $\sin 0 = 0, \sin\left(\frac{\pi}{6}\right)^c = \frac{1}{2}$,

$\sin\left(\frac{\pi}{3}\right)^c = \frac{\sqrt{3}}{2}$ and $\sin\left(\frac{\pi}{2}\right)^c = 1$

$\Rightarrow \sin^2 0 + \sin^2\left(\frac{\pi}{6}\right)^c + \sin^2\left(\frac{\pi}{3}\right)^c + \sin^2\left(\frac{\pi}{2}\right)^c$

$= 0 + \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 = 0 + \frac{1}{4} + \frac{3}{4} + 1 = 2$

**30)** Ans: **A)** $6, \frac{14}{3}$

Sol: From given data, we make the following table

| x | $x^2$ |
|---|---|
| 2 | 4 |
| 3 | 9 |
| 11 | 121 |
| x | $x^2$ |
| $\sum x = 16 + x$ | $\sum x^2 = 134 + x^2$ |

But we know that, variance $= \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$\Rightarrow \frac{134 + x^2}{4} - \left(\frac{16 + x}{4}\right)^2 = \frac{49}{4}$ (given)

$\Rightarrow \frac{134+x^2}{4} - \frac{\left(256+x^2+32x\right)}{16} = \frac{49}{4}$

$\Rightarrow \frac{3x^2 32x+280}{18} = \frac{49}{4}$

$\Rightarrow 280+3x^2-32x = \frac{49}{4}\times 16$

$\Rightarrow 280+3x^2-32x = 196$

$\Rightarrow 3x^2-32x+84=0$

$\Rightarrow (x-6)(3x-14)=0$

$\Rightarrow x=6, x=\frac{14}{3}$

Hence, the values of x are 6 and $\frac{14}{3}$.

**31)** Ans: **C)** $\frac{3\pi}{10}$

Sol: Since, f(x) is continuous at x = 0

$\therefore f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow 2k = \lim_{x\to 0}\frac{3\sin\pi x}{5x} = \lim_{x\to 0}\frac{3\sin\pi x}{5(\pi x)}\times\pi = \frac{3\pi}{5} \quad \Rightarrow k = \frac{3\pi}{10}$

**32)** Ans: **A)** 0

Sol: $(x, y)\to(1, 0)$ means $x\to 1$ and $y \to 0$

Also, y=x - 1, $\Rightarrow$ D= $x^3$-$y^2$-1=($x^3$-1)-$y^2$

$= (x-1)(x^2+x+1)-(x-1)^2$

$= (x-1)(x^2+x+1-x+1) = (x-1)(x^2+2)$

$\therefore \lim_{(x,y)\to(1,0)}\frac{y^3}{x^3-y^3-1} = \lim_{x\to 1}\frac{(x-1)^3}{(x-1)(x^2+2)}$

$= \lim_{x\to 1}\frac{(x-1)^2}{x^2+2} = \frac{(1-1)^2}{1^2+2} = \frac{0}{3} = 0$

**33)** Ans: **D)** $\frac{\sqrt{3}}{2}$

Sol: $\sin 70^0 \cos 10^0 - \cos 70^0 \sin 10^0$

$= \sin\left(70^0-10^0\right)$

$= \sin 60^0 = \frac{\sqrt{3}}{2}$.

**34)** Ans: **D)** $\frac{462a^5}{b^6}$

Sol: Suppose $x^{-7}$ occurs in $(r+1)^{th}$ term.

We have $T_{r+1} = {}^nC_r x^{n-r}a^r$ in $(x+a)^n$

In the given question, $n=1, x=ax, a=\frac{-1}{bx^2}$

$\therefore T_{r+1} = {}^{11}C_r (ax)^{11-r}\left(\frac{-1}{bx^2}\right)^r$

$= {}^{11}C_r a^{11-r}b^{-r}x^{11-3r}(-1)^r$

This term contains $x^{-7}$ if $11-3r=-7 \Rightarrow r=6$

Therefore, coefficient of $x^{-7}$ is

${}^{11}C_6 (a)^5\left(\frac{-1}{b}\right)^6 = \frac{462}{b^6}a^5$

**35)** Ans: **A)** $\frac{[1.3.5...(2n-1)]^2}{1.3.5...(4n-1)}$

Sol: ${}^{4n}C_2 : {}^{2n}C_n = \frac{(4n)!}{(2n)!.(2n)!}\times\frac{n!.n!}{(2n)!}$ .....(1)

Here, (4n)!=1.2.3.4.5.6...(4n-1).(4n)

=[1.3.5(4n-1)][2.4.6....(4n)]

=[1.3.5....(4n-1)][2(1).2(2).2(3)...2(2n)]

=[1.3.5...(4n-1)] [1.2.3...(2n)]

$[2\times 2\times 2\times$.......(2n)times]

=[1.3.5......(4n-1)].(2n)!.$2^{2n}$

$(2n)! = [1.3.5...(2n-1)].n!.2!$

$\therefore$ From (1), Given expression

$= \frac{[1.3.5...(4n-1).(2n)!.2^{2n}}{[1.3.5...(2n-1)].n!.2^n\times(2n)!}\times\frac{n!.n!}{(2n)!}$

$= \frac{[1.3.5.....(4n-1)].2^n}{[1.3.5....(2n-1)]}\times\frac{n!}{[1.3.5....(2n-1).n!.2^n}$

$= \frac{[1.3.5....(4n-1)]}{[1.3.5....(2n-1)]^2}$

**36)** Ans: **A)** 4

Sol: $(12)(\log 4)^3 = \lim_{x\to 0}\frac{(4^x-1)^3}{\sin\left(\frac{x}{p}\right)\log\left(1+\frac{x^2}{3}\right)}$

$= \lim_{x\to 0}\frac{\left(\frac{4^x-1}{x}\right)^3}{\frac{\sin\left(\frac{x}{p}\right)}{x}\frac{\log\left(1+\frac{x^2}{3}\right)}{x^2}}$

$\therefore 12(\log 4)^3 = p\,(3)(\log 4)^3 \qquad \Rightarrow p = 4$

**37)** Ans: **B)** $\frac{462a^5}{b^6}$

Sol: $\frac{462a^5}{b^6}$

**38)** Ans: **A)** 1

Sol: Consider,

$\frac{\log a}{x+y-2z} = \frac{\log b}{y+z-2x} = \frac{\log c}{z+x-2y} = k$

$\therefore \frac{\log a}{x+y-2z} = k, \frac{\log b}{y+z-2x} = k, \frac{\log c}{z+x-2y} = k$

$\therefore \log a = k(x+y-2z), \log b = k(y+z-2x)$, log c = k (z + x - 2y)

$\therefore \log(abc) = k(x+y-2z+y+z-2x+z+x-2y)$

$\therefore \log(abc) = k(x+y-2z+y+z-2x+z+x-2y)$

$\therefore \log(abc) = 0 = \log 1 \quad \therefore abc = 1$

**39)** Ans: **C)** A =B

Sol: $A \times B = B \times A$ iff $A = B$.

**40)** Ans: **A)** $\{(c,a),(a,b),(b,c)\}$

Sol: We are given that

$f(a) = c, f(b) = a$ and $f(c) = b$

$\therefore a = f^{-1}(c), b = f^{-1}(a)$ and $c = f^{-1}(b)$.

Therefore $f^{-1} = \{(c,a),(a,b),(b,c)\}$.

Alternatively, $f^{-1} = \{(y,x):(x,y) \in f\}$

$= \{(c,a),(a,b),(b,c)\}$.

**41)** Ans: **A)** 7

Sol: $(\sin+\operatorname{cosec})^2 + (\cos+\sec)^2$

$=\sin^2+\operatorname{cosec}^2 + 2(\sin)(\csc) + \cos^2 + \sec^2 + 2(\cos)(\sec)$

$=(\sin^2+\cos^2) + 2(1) + 2(1) + \sec^2 + \operatorname{cosec}^2$

$=(1) + 2 + 2 + (1+\tan^2) + (1+\cot^2)=\tan^2+\cot^2+7$

**42)** Ans: **C)** A.P.

Sol: Given determinant can be written as sum of determinants as

$$\begin{vmatrix} x & x & x \\ x & x & x \\ x & x & x \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ a & b & c \end{vmatrix} = 0$$

$$\Rightarrow 0 + \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ a & b & c \end{vmatrix} = 0$$

This determinant will be 0 if $R_1 \equiv R_3$ or $R_2 \equiv R_3$

In both the cases, a,b,c will be in A.P.

**43)** Ans: **A)** GP

Sol: Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ be any two points on the parabola $y^2 = 4ax$, then point of intersection of tangents at P and Q will be

$T = [at_1t_2, a(t_1+t_2)]$

Now, $SP = a(t_1^2+1)$

$SQ = a(t_2^2+1)$

$ST = a\sqrt{(t_1^2+1)(t_2^2+1)}$

$ST^2 = SP.SQ$

$\therefore$ SP, ST and SQ are in G.P.

**44)** Ans: **D)** Brand I shows more variability

Sol: Given, $\overline{x}_1 = 36, \overline{x}_2 = 48, \sigma_1 = 8, \sigma_2 = 10$

$\therefore$ C.V. of Brand $I = 100 \times \frac{\sigma_1}{\overline{x}_1} = 100 \times \frac{8}{36} = 22.22$

and

C.V. of Brand $II = 100 \times \frac{\sigma_2}{\overline{x}_2} = 100 \times \frac{10}{48} = 20.83$

As C.V. of Brand I> C.V. of Brand II, Brand I shows more variability

**45)** Ans: **A)** $\frac{\sqrt{6}}{3}$

Sol: As x lies in Quadrant III, we have

$\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \quad \Rightarrow \frac{x}{2}$ lies in Quadrant II

$\Rightarrow \sin\frac{x}{2} > 0$

Now, $2\sin^2\frac{x}{2} = (1-\cos x) = \left(1+\frac{1}{3}\right) = \frac{4}{3}$

$\Rightarrow \sin^2\frac{x}{2} = \frac{2}{3} \qquad \Rightarrow \sin\frac{x}{2} = +\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

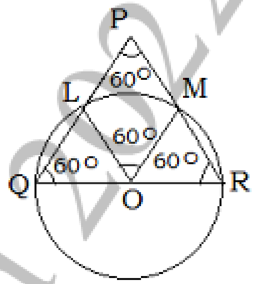
$\left[\because \sin\frac{x}{2} > 0\right]$

**46)** Ans: **C)** $\frac{7}{8}$

Sol: $\frac{7}{8}$

**47)** Ans: **A)** $3\pi\,cm$

Sol: $\Delta PQR$ is an equilateral triangle.




Consider O be the centre of the circle drawn on side QR as diameter.

Let the circle intersect side PQ at L and side PR at M, then arc LM is intercepted within the triangle

But l(OQ) = l(OL), then $m\angle OQL = m\angle OLQ = 60^0$

$\Rightarrow m\angle LOQ = 60^0$

Similarly $m\angle MOR = 60^0 \Rightarrow m\angle LOM = 60^0$

Here $r = 9\,cm, \theta = 60^0 = \left(\frac{\pi}{3}\right)^c$

Thus, length of arc $LM = r\theta^c = (9)\left(\frac{\pi}{3}\right) = 3\pi cm$

**48)** Ans: **B)** e

Sol: Given that, $y=e^{1+\log x}$

$\frac{dy}{dx} = \frac{d}{dy}e^{(1+\log x)} = e^{(1+\log x)}.(0+\frac{1}{x})$

$= \frac{1}{x}e^{1+\log x} = \frac{1}{x}(e^1.e^{\log x}) = \frac{1}{x}.e^1.x = e$

**49)** Ans: **A)** $\pm$ 10

Sol: Root of $x^2+kx+24=0$ are in the the ration 2:3,

Suppose the roots are $2\alpha$ and $3\alpha$.

$\therefore 2\alpha + 3\alpha = -\frac{k}{1}$ and $2\alpha.3\alpha = \frac{24}{1}$

$\therefore 5\alpha = -k$ ..(1) and $\alpha^2 = 4$ ..(2)

From (1), $\alpha = -\frac{k}{5}$

From (2), $(-\frac{k}{5})^2 = 4 \quad \therefore \frac{k^2}{25} = 4$

$\therefore k^2 = 100, \Rightarrow k = \pm 10$

**50)** Ans: **B)** $\frac{-3}{10}$

Sol: The middle term in the expansion of

$(1+\alpha x)^4 = T_3 = {}^4C_2 (\alpha x)^2 = 6\alpha^2 x^2$

The middle term in the expansion of

$(1-\alpha x)^6 = T_4 = {}^6C_3 (-\alpha x)^3 = -20\alpha^3 x^3$

According to the question

$6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\frac{3}{10}$