Studentpad

Vector 2022-23

Time : 90 Min	**Phy : Vector**	**Marks : 120**

Hints and Solutions

01) Ans: **D)** $2\sqrt{10}$

Sol: $\vec{r} = \vec{r}_2 - \vec{r}_1 = (-2\hat{i} - 2\hat{j} + 0\hat{k}) - (4\hat{i} - 4\hat{j} + 0\hat{k})$

$\vec{r} = -6\hat{i} + 2\hat{j} + 0\hat{k}$

$\therefore \ |\vec{r}| = \sqrt{(-6)^2 + (2)^2 + 0^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

02) Ans: **C)** $\dfrac{\pi\sqrt{2}}{30}$ cm / s

Sol: Change in velocity,

$$\Delta v = 2v\sin\left(\frac{90^\circ}{2}\right) = 2v\sin 45^\circ = 2v \times \frac{1}{\sqrt{2}} = \sqrt{2}v$$

$$= \sqrt{2} \times r\omega = \sqrt{2} \times 1 \times \frac{2\pi}{60} = \frac{\sqrt{2}\pi}{30}\text{cm / s}$$

03) Ans: **C)** $\sqrt{3}$

Sol: Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is $\hat{n}_s = \hat{n}_1 + \hat{n}_2$ or

$n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta \ \Rightarrow \hat{n}_s = 1 + 1 + 2\cos\theta$

As it is given that n_s is also an unit vector,

therefore, $1 = 1 + 1 + 2\cos\theta \Rightarrow \cos\theta = -\dfrac{1}{2}$

$\therefore \theta = 120^\circ$

Now the difference vector is $\hat{n}_d = \hat{n}_1 - \hat{n}_2$ or

$n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1 + 1 - 2\cos(120^\circ)$

$\therefore \ n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \ \Rightarrow n_d = \sqrt{3}$

04) Ans: **B)** $(A^2 + B^2 + AB)^{1/2}$

Sol: $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A}.\vec{B})$

$AB\sin\theta = \sqrt{3}AB\cos\theta \Rightarrow \tan\theta = \sqrt{3} \ \therefore \theta = 60^\circ$

Let, $|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$= \sqrt{A^2 + B^2 + 2AB\left(\frac{1}{2}\right)} = (A^2 + B^2 + AB)^{1/2}$$

05) Ans: **C)** 150°

Sol: $\dfrac{B}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ....(i)

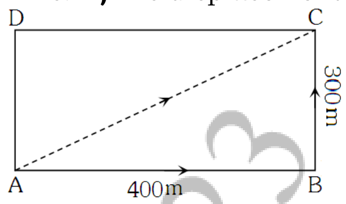
$\therefore \ \tan 90^\circ = \dfrac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$

$\therefore \ \cos\theta = -\dfrac{A}{B}$

$\therefore$ from (i) $\dfrac{B^2}{4} = A^2 + B^2 - 2A^2 \Rightarrow A = \sqrt{3}\dfrac{B}{2}$

$\cos\theta = -\dfrac{A}{B} = -\dfrac{\sqrt{3}}{2} \quad \therefore \ \theta = 150^\circ$

06) Ans: **B)** His displacement is 700 m.



Sol:

Displacement $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\Rightarrow AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(400)^2 + (300)^2} = 500$ m

Distance = AB + BC = 400 + 300 = 700 m

07) Ans: **A)** 24

Sol: Using, $\hat{k}.\hat{k} = 1, \hat{i}.\hat{j} = 0$ Power

$P = \vec{F}.\vec{v} = (7\hat{i} + 6\hat{k}).(3\hat{j} + 4\hat{k}) = 24$ watt

08) Ans: **B)** 4

09) Ans: **C)** $4\hat{i} - 8\hat{k}$

Sol: Radius vector is calculated as,

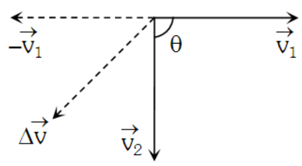
$\vec{r} = \vec{r}_2 - \vec{r}_1 = (2\hat{i} - 3\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} + \hat{k}) \ \therefore \vec{r} = -4\hat{j}$

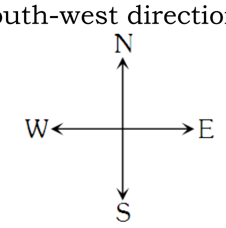
Linear momentum $\vec{p} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{L} = \vec{r} \times \vec{p} = (-4\hat{j}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & 0 \\ 2 & 3 & -1 \end{vmatrix} = 4\hat{i} - 8\hat{k}$$

10) Ans: **C)** 14.14 ms^{-1} in south-west direction.





Sol:

The magnitude of vector remains the same, only direction changes by θ therefore

$\overrightarrow{\Delta\upsilon} = \overrightarrow{\upsilon_2} - \overrightarrow{\upsilon_1}, \ \overrightarrow{\Delta\upsilon} = \overrightarrow{\upsilon_2} + (\overrightarrow{-\upsilon_1})$. Magnitude of change in vector $|\overrightarrow{\Delta\upsilon}| = 2\upsilon\sin(\dfrac{\theta}{2})$

$|\overrightarrow{\Delta\upsilon}| = 2 \times 10 \times \sin(\dfrac{90^\circ}{2}) = 10\sqrt{2} = 14.14$ m/s and

Direction is south-west as shown in figure.

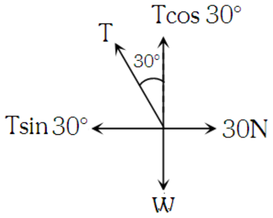
11) Ans: **D)** 1 hr

Sol: Distance between the train $s_{rel.} = 90$ km,

and Relative velocity $v_{rel.} = 60 - (-30) = 90$ km / hr.

Hence, time when they collide $= \frac{s_{rel.}}{v_{rel.}} = \frac{90}{90} = 1$ hr.

12) Ans: **B)** $30\sqrt{3}, 60$



Sol:
From the figure,

$T\sin 30^o = 30$ ....(i)

$T\cos 30^o = W$ ....(ii)

By solving above equations, we get

$W = 30\sqrt{3}N$ and $T = 60N$

13) Ans: **A)** 1

Sol: $(\vec{A}+\vec{B})$ is perpendicular to $(\vec{A}-\vec{B})$.

$\Rightarrow (\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = 0$ or $A^2 + \vec{B}.\vec{A} - \vec{A}.\vec{B} - B^2 = 0$

As per commutative property of dot product

$\vec{A}.\vec{B} = \vec{B}.\vec{A}$

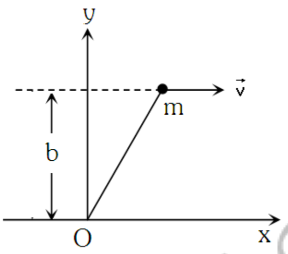
$\therefore A^2 - B^2 = 0$ or $A = B$

Hence, the ratio of magnitudes A/B = 1

14) Ans: **C)** $-mvb\hat{k}$

Sol: As we know, Angular momentum

$\vec{L} = \vec{r} \times \vec{p}$ in terms of component becomes



$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

As motion is in x-y plane (z = 0 and $P_z = 0$),

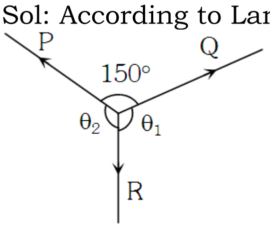
so $\vec{L} = \vec{k}(xp_y - yp_x)$.

Here, x = v t, y = b, $p_x = mv$ and $p_y = 0$

$\therefore \vec{L} = \vec{k}\left[vt \times 0 - bmv\right] = -mvb\hat{k}$

15) Ans: **C)** 1

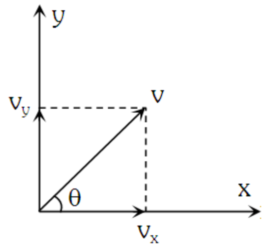
Sol: According to Lami's theorem,



$$\frac{P}{\sin\theta_1} = \frac{Q}{\sin\theta_2} = \frac{R}{\sin 150^\circ} \Rightarrow \frac{1.93}{\sin\theta_1} = \frac{R}{\sin 150^\circ}$$

$$R = \frac{1.93 \times \sin 150^\circ}{\sin\theta_1} = \frac{1.93 \times 0.5}{0.9659} = 1$$

16) Ans: **D)** $\tan^{-1}(2)$



Sol:
Given that, $v_y = 20$ and $v_x = 10$

$\therefore$ velocity $\vec{v} = 10\hat{i} + 20\hat{j}$

Direction of velocity with x axis is

$\tan\theta = \frac{v_y}{v_x} = \frac{20}{10} = 2$

$\therefore \theta = \tan^{-1}(2)$

17) Ans: **B)** Three

18) Ans: **D)** 100 seconds

Sol: Relative speed of police with respect to thief $= 10 - 9 = 1m/s$

Instantaneous separation = 100 m

We know, Time $= \frac{\text{distance}}{\text{veclotiy}} = \frac{100}{1} = 100$ s.

19) Ans: **C)** $\sqrt{274}$

Sol: Sum of the vectors is given

as $\vec{R} = 5\hat{i} + 8\hat{j} + 2\hat{i} + 7\hat{j} = 7\hat{i} + 15\hat{j}$

Then, magnitude of $\vec{R} = |\vec{R}| = \sqrt{49 + 225} = \sqrt{274}$

20) Ans: **D)** 45 km/hr

Sol: Two cars (A and B) are moving with same velocity, the relative velocity of one car (B) with respect to the other car (A),

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = v - v = 0$ and the relative separation between them (5 km) always remains the same.

Now if the velocity of car (C) moving in opposite direction to (A) and (B), is $\vec{v}_C$ relative to ground then the velocity of car (C) relative to (A) and (B) will be $\vec{v}_{rel.} = \vec{v}_C - \vec{v}$. But as $\vec{v}$ is opposite to v_C.

$\therefore v_{rel} = v_c - (-30) = (v_C + 30)$ km/hr.

$\therefore$ The time taken by it to cross the cars A and B.

$$t = \frac{d}{v_{rel}} \Rightarrow \frac{4}{60} = \frac{5}{v_C + 30} \Rightarrow v_C = 45 \text{ km/hr.}$$

21) Ans: **C)** 6i +8j

Sol: Relative velocity = (3i + 4j) - (- 3i - 4j) = 6i + 8j

22) Ans: **D)** are equal to each other in magnitude.

Sol: When two vectors are perpendicular, then their dot product must be equal to zero.

According to the problem,

$(\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = 0$

$\vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$

$\Rightarrow A^2 - B^2 = 0 \Rightarrow A^2 = B^2 \quad \therefore A = B$

It means two vectors are equal to each other in magnitude.

23) Ans: **A)** arbitrary vectors which have the original vector as their resultant.

24) Ans: **B)** $\tan\frac{\alpha}{2} = \left(\frac{A-B}{A+B}\right)\tan\frac{\theta}{2}$

Sol: As, $A = A\hat{A} = B\hat{B}$. Let, θ be the angle between A and B. As per question $\cos\alpha = \frac{(A\hat{A}+B\hat{B})(A\hat{B}+B\hat{A})}{|A\hat{A}+B\hat{B}|\,|A\hat{B}+B\hat{A}|}$

or $\cos\alpha = \frac{2AB+(A^2+B^2)\cos\theta}{\sqrt{(A^2+B^2+2AB\cos\theta)^2}}$

or $2AB+(A^2+B^2)\cos\theta = (A^2+B^2)\cos\alpha + 2AB\cos\theta\cos\alpha$

or $2AB(1-\cos\alpha\cos\theta) = (A^2+B^2)(\cos\alpha-\cos\theta)$

or $\frac{2AB}{A^2+B^2} = \frac{\cos\alpha-\cos\theta}{1-\cos\alpha\cos\theta}$

or $\frac{2AB}{(A^2+B^2)} = \frac{\cos\alpha-\cos\theta}{1-\cos\alpha\cos\theta}$

or $\frac{2AB+(A^2+B^2)}{(A^2+B^2)-2AB} = \frac{(\cos\alpha-\cos\theta)+(1-\cos\alpha\cos\theta)}{(1-\cos\alpha\cos\theta)+(\cos\alpha-\cos\theta)}$

or $\frac{(A+B)^2}{(A-B)^2} = \frac{(1+\cos\alpha)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\alpha)} = \frac{\tan^2\theta/2}{\tan^2\alpha/2}$

or $\tan\frac{\alpha}{2} = (\frac{A-B}{A+B})\tan\frac{\theta}{2}$

25) Ans: **D)** Work
Sol: Acceleration, displacement and electric field are vector quantities.

26) Ans: **C)** 20 km/hr
Sol: A man standing on a road means is at rest w.r.t. the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground.

Let, $\vec{v}_{rg} =$ velocity of rain with respect to the ground.

$\vec{v}_{mg} =$ velocity of the man with respect to the ground.

and $\vec{v}_{rm} =$ velocity of the rain with respect to the man,

We have equation, $\vec{v}_{rg} = \vec{v}_{rm} + \vec{v}_{mg}$ ...(i)

Taking horizontal components equation (i) gives

$v_{rg}\sin 30^\circ = v_{mg} = 10$ km / hr

$v_{rg} = \frac{10}{\sin 30^\circ} = 20$ km / hr

27) Ans: **20** Sol: Let n be the number of storeys above the 15th storey. Then height fallen is $h-4n$ metres.

Using $v^2 - u^2 = 2gh,$

we have $(20)^2 - 0 = 2\times10\times4n$ which gives

$n = \frac{400}{80} = 5.$

So, the total number of storeys $= 15+5 = 20.$

28) Ans: **45** Sol: Here, $u = 30\,ms^{-1}$. From the figure the acceleration a = slope of line

$AB = \frac{30ms^{-1}}{-3s} = 10\,ms-2.$

The maximum height is reached when final $V = 0.$
Using the values of u, v and a in relation $v^2 - u^2 = 2as$, we get $0 - 30\times30 = 2\times-10\times s$

or $s = \frac{30\times 30}{20} = 45\,m$

29) Ans: **87** Sol: $A_x = 50, \theta = 60^0$

Then $\tan\theta = A_y / A_x$ or $A_y = A_x\tan\theta$

or $A_y = 50\tan 60^0 = 50\times\sqrt{3} = 87N$

30) Ans: **120** Sol: $F^2 = F^2 + F^2 + 2F^2\cos\theta$

or $F^2 = 2F^2(1+\cos\theta)$ or $1+\cos\theta = \frac{1}{2}$

or $\cos\theta = -\frac{1}{2}$ or $\theta = 120^0$

$\therefore \cos 120^0 = -\frac{1}{2} = -0.5$