CS583 Assignment 1 Solutions

Question 1

a. No, A and B are not independent.

b. No, A and C are not independent. (If decimals rounded, the Yes is accepted)

c. Yes, A and C independent given B

d. Yes, A and D independent

e. No, A and D are not independent given C. (If decimals rounded, the Yes is accepted)

Question 2

a. $2^{n-1}2^{m-1} - 1 = 2^{m+n-2} - 1$

b. $3^{n-1}3^{m-1} - 1 = 3^{m+n-2} - 1$

c. Every X_i has i values,

$$(2 \times 3 \times \ldots \times n \quad \times \quad 2 \times 3 \times \ldots \times m) - 1 = n!m! - 1$$

d. $2^{n-1} tables \ \times \ (2^m - 1) \ values \ per \ table = 2^{n-1}(2^m - 1)$

e. $3^{n-1} tables \ \times \ (3^m - 1) \ values \ per \ table = 3^{n-1}(3^m - 1)$

f. $n! \ tables \times \ (m! - 1)$values per table = $(m! - 1)n!$

Question 3

Naïve Bayes' representation can be shown as:

$P(Y) \prod_{i=1}^{n-1} P(X_i|Y)$

where $P(Y)$ is the target variable and $P(X_i)$ are the feature variables.

For each $P(X_i|Y)$, for binary variables, number of parameters $2 \ tables \times 1 \ value \ per \ table$

Hence, for $n - 1$ product terms, $2(n - 1)$ independent parameters.

$P(Y)$ needs 1 independent variable, so total number =

$$2(n-1) + 1$$
$$\implies 2n - 1$$

Question 4

a. $P(A) \ P(B) \ P(C|A, B) \ P(D|C) \ P(F|C) \ P(E) \ P(G|D, E, F) \ P(H|G)$

b. We need to add-up the number of independent parameters for each of the above probabilities

$(n-1) + (n-1) + (n^2 \times n - 1) + (n - 1 \times n) + (n - 1 \times n) + (n-1) + (n^3 \times n - 1) + (n - 1 \times n)$

c.

1. True
2. False
3. False
4. False
5. True
6. False
7. False
8. False
9. True
10. False
11. True
12. True
13. False

Question 5

Part a

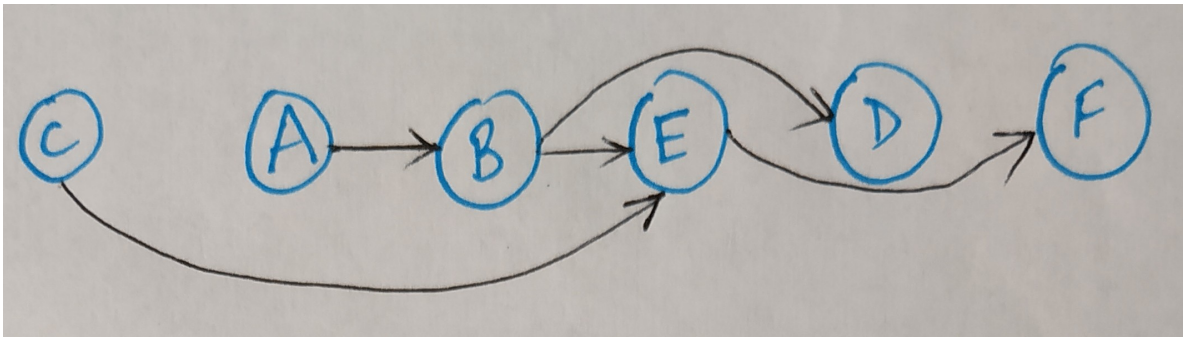
$A \perp C|\phi$

$B \perp C|A$

$E \perp A|B, C$

$D \perp A, C, E|B$

$F \perp A, B, C, D|E$



Part b

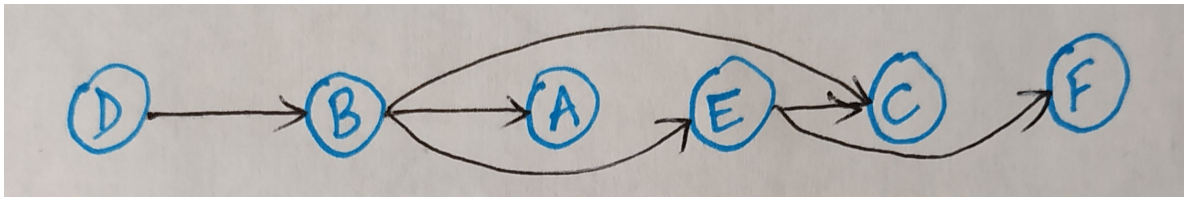
$B \not\perp D$

$A \perp D|B$

$E \perp A, D|B$

$C \perp A, D|B, E$

$F \perp D, B, A, C|E$



Part c

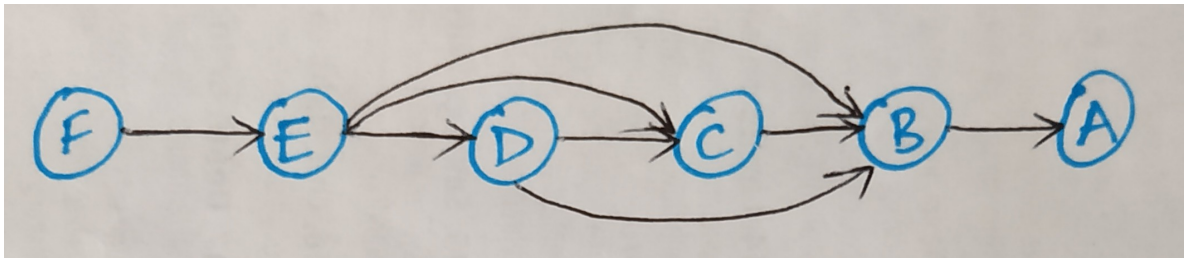
$F \not\perp E$

$D \perp F|E$

$C \perp F|E, D$

$B \perp F|D, E, C$

$A \perp C, D, E, F|B$



Part d

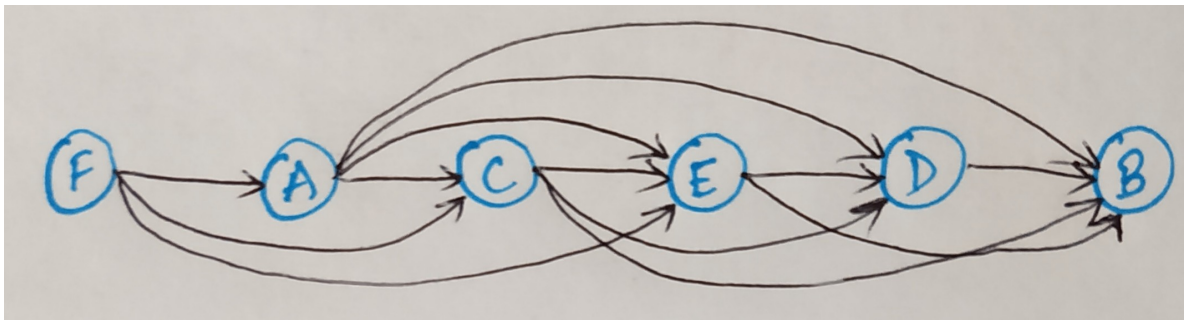
$F \not\perp A$

$C \not\perp A, F$

$E \not\perp F, A, C$

$D \perp F|A, C, E$

$B \perp F|A, C, D, E$

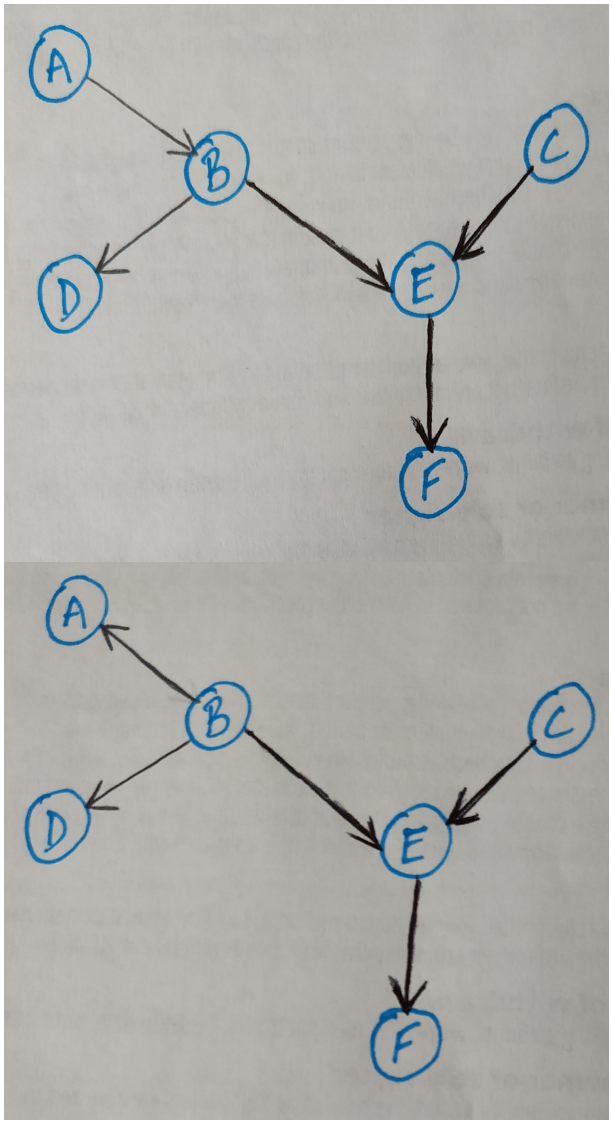


Question 6

There is 1 *immorality* in $B \rightarrow E \leftarrow C$. This should remain unchanged in the I-Equivalent Structures

The $E \rightarrow F$ cannot be changed as it will introduce a new immorality.

So on the relationship between A,B and B,D can be changed in a way where no new immoralities are introduced.



Question 7

a. 0.725

b. 0.725

c. 0.4901

d. 0.3419

e. 0.95

f. 0.95