(1)

(a) $\forall P(x) \Rightarrow Q(x)$

Drop implication

$\forall x \quad \neg P(x) \vee Q(x)$

Drop universal quantifiers

$\neg P(x) \vee Q(x)$

(b) $\forall x \forall y \; P(x,y) \Rightarrow Q(x)$

$\forall x \forall y \; \neg P(x,y) \vee Q(x)$ (Eliminate Implication)

$\neg P(x,y) \vee Q(x)$ (Drop universal quantifiers)

(c) $\exists x \; P(x) \wedge Q(x)$

$P(A) \wedge Q(A)$ (Skolemize)

(d) $\exists x \exists y \; P(x,y) \wedge Q(x,y)$

$\exists x \exists y \; P(x,y) \wedge Q(x,y)$

~~$\exists x \; P(x, F(x)) \wedge Q(F(x), x)$~~ (~~Drop $\exists$~~)

~~$P(A, F(A))$~~ $\wedge$ ~~Q~~$(F(A), A)$ (~~skolemize~~)

$\exists x \; P(x, A) \wedge Q(x, A)$ (Skolemize)

$P(B, A) \wedge Q(B, A)$ (Skolemize)

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(e) $\exists x \forall y \; P(x,y) \Rightarrow Q(x)$

$\forall y \; P(A,y)$ (Skolemize)

A is a constant

$P(A,y)$ (Drop universal quantifiers)

(f) $\forall x \exists y \; P(x,y)$

$\forall x \; P(x, F(x))$ (Skolemize)

$P(x, F(x))$ (Drop universal quantifiers)

(g) $\forall x \forall y \exists z \; P(x,y,z)$

$\forall x \forall y \; P(x, y, F(x,y))$ (Skolemize)

$P(x, y, F(x,y))$ (Drop universal quantifiers)

(h) $\exists x \forall y \forall z \; P(x,y,z)$

$\forall y \forall z \; P(A,y,z)$ (Skolemize)

$P(A,y,z)$ (Drop universal quantifiers)

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i) $\forall x [\exists y \; p(x,y) \wedge Q(y)] \Rightarrow R(x)$

$\forall x \neg (\exists y \; p(x,y) \wedge Q(y)) \vee R(x)$ (Eliminate implication)

$\forall x [\forall y \neg p(x,y) \vee \neg Q(y)] \vee R(x)$ (Move $\neg$ inwards)

$\neg p(x,y) \vee \neg Q(y) \vee R(x)$ (Drop universal quantifiers)

~~j) $\forall x (\forall y \; p(y,x)$~~

j) $\forall x (\forall y \; p(x,y) \Rightarrow Q(y)) \Rightarrow R(x)$

$\forall x [\forall y \neg p(x,y) \vee Q(y)] \Rightarrow R(x)$

$\forall x \neg [\forall y \neg p(x,y) \vee Q(y)] \vee R(x)$ } → (Eliminate implication)

$\forall x [\exists y \; p(x,y) \wedge \neg Q(y)] \vee R(x)$ (move $\neg$ inwards)

$\forall x [p(x, F(x)) \wedge \neg Q(F(x))] \vee R(x)$ (Skolemize)

$[p(x, F(x)) \wedge \neg Q(F(x))] \vee R(x)$ (Drop universal quantifiers)

$[p(x, F(x)) \vee R(x)] \wedge [\neg Q(F(x)) \vee R(x)]$ [Distribute $\wedge$ $\vee$ over $\wedge$]

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(2) a) $P(x)$

b) $P(A)$

$\{x/A\}$

c) $P(x) \vee Q(x, A)$

d) $P(B) \vee Q(x, A)$

$\{x/B\}$

e) $P(x) \vee Q(A, x)$

f) $P(x) \vee Q(A, B) \longrightarrow$

$\{x/B\}$

g) $P(x, A) \vee Q(A, x)$

h) $P(B, y) \vee Q(y, B) \longrightarrow \{x/B, y/A\}$

i) $P(x) \vee Q(f(x))$

j) $P(A) \vee Q(f(A)) \longrightarrow \{x/A\}$

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k) $P(x, A) \vee Q(F(x), x)$

l) $P(B, y) \vee Q(F(B), B)$

$\{x/B, y/A\}$

m) $P(x, A) \vee Q(F(x), x)$

n) $P(B, y) \checkmark Q(F(A), A)$

Fail

o) $P(x, y) \vee Q(F(A), B)$ $\rightarrow$ $\{x/F(A), y/B\}$

p) $P(x, y) \vee Q(x, y)$

q) $P(x, y) \vee Q(F(A), A)$ $\rightarrow \{x/F(A), y/A\}$

r) $P(x, y) \vee Q(x, y)$

s) $P(x, y) \vee Q(F(x), y)$

t) $P(z, y) \vee Q(z, y)$

Fail

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(3)

a) $P(A, C) = P[A=T, C=T \mid B)$

$= P(A=T, C=T \mid B=T) + P(A=T, C=T \mid B=F)$

$= 0.014 + 0.012$

$= 0.014 + 0.012$

$= 0.026$

A	B	C	$P(A, B, C)$
T	T	T	0.014
T	F	T	0.012

=

(b) $P(C) = \sum_A \sum_B P(A, B, C=T)$

C	A	B	$P(A, B, C=T)$
T	T	T	0.014
T	F	T	0.392
T	T	F	0.012
T	F	F	0.144

$P(C) = 0.014 + 0.392 + 0.012 + 0.144$

$= 0.562$

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(c) $P(A|C) = \frac{P(A,C)}{P(C)}$

$= \frac{0.026}{0.562}$

$= \frac{0.026}{0.562}$

$= 0.0462$

(d) $P(A,B|C)$

$P(A,B|C) = \frac{P(A=T, B=T, C=T)}{P(C)}$

$= \frac{0.014}{0.562}$

$= 0.0249$

A	B	C	
T	T	T	0.014

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(e) $P(B/A, C)$

$$= \frac{P(B, A, C)}{P(A, C)}$$

$$= \frac{P(B=T, A=T, C=T)}{P(A=T, C=T)}$$

$$= \frac{0.014}{0.026}$$

$$= 0.0538$$

$P(A, C)$

$$= P(A=T, C=T / B=T) + P(A=T, C=T / B=F)$$

$$= 0.014 + 0.012$$

$$= 0.026$$

A	C	B	$P(A, C)$
T	T	T	0.014
T	T	F	0.012
			0.026

=

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(4) (a)

i) $P(X_2)$ = 1 Independent parameter

ii) $P(X_n)$ = 1 Independent parameter

iii) $P(X_2, X_3, \cdots X_n)$

$= 2^{(n-1)} - 1$ independent parameters.

iv) $P(X_2 / X_3 \cdots X_n)$

$= (2-1)(2^{n-2})$

$= 2^{(n-2)}$ independent parameters

(v) $P(X_2 \cdots X_{n-1} / X_n)$

$= 2^{n-2} - 1$ independent parameters.

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(b)

i) $P(X_2) = 1$ Independent parameter

ii) $P(X_n) = (n-1)$ Independent parameters

iii) $P(X_2, X_3 \cdots X_n)$

$= (2-1)(3-1)(4-1) \cdots (n-2)(n-1)$

$= (1)(2)(3) \cdots (n-2)(n-1)$

$= (n-1)!$ Independent parameters

iv) $P(X_2 / X_3 \cdots X_n)$

$= (2-1)(3 \times 4 \times \cdots \times n)$

$= (3 \times 4 \times \cdots \times n) \cdot (2/2)$

$= \frac{n!}{2}$ Independent parameters

(v) $P(X_2, X_3 \cdots X_{n-1} / X_n)$

$= [1, 2, \cdots (n-2)][n]$

$= \frac{(n-1)}{(n-1)} (1, 2, \cdots (n-2))(n)$

$= \frac{n!}{(n-1)}$

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