

①

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$$a) P(X_2, X_3, X_4, X_5, \dots, X_9)$$

$$= P(X_2) P(X_3) P(X_4) P(X_5 | X_2, X_3) P(X_6 | X_3, X_4) P(X_7 | X_5) \\ P(X_8 | X_3, X_5, X_6) P(X_9 | X_5, X_7, X_8)$$

b) i) No. of Independent parameters

$$= 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 - 1$$

$$= 362880 - 1$$

$$= 362879$$

ii) X_2

$$X_{21} = 1$$

X_{22} 1-leaf value

X_3

$$X_{31} = 2$$

$$X_{32} = 1$$

$$X_{33} = 1$$

X_4

X_{41}

$$X_{42} = 3$$

X_{43}

X_{43}

		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{43}
X_{21}	X_{31}	-	-	-	-	-	24
X_{21}	X_{32}	-	-	-	-	-	
X_{21}	X_{33}	-	-	-	-	-	
X_{22}	X_{31}	-	-	-	-	-	
X_{22}	X_{32}	-	-	-	-	-	
X_{22}	X_{33}	-	-	-	-	-	

In similar way,

$$X_6 = 30 ; X_7 = 60 ; X_8 = 630 ; X_9 = 2240$$

For BN,

$$P(X_1, X_2, \dots, X_9)$$

$$= P(X_2) P(X_3) P(X_4) P(X_5 | X_2, X_3) P(X_6 | X_3, X_4) P(X_7 | X_5) \\ P(X_8 | X_3, X_5, X_6) P(X_9 | X_5, X_7, X_8)$$

$$= 1 + 2 + 3 + 24 + 30 + 60 + 630 + 2240$$

Independent params:
 $= 2990$

(c)

$$i) X_2 \perp X_3 = \text{True}$$

$$ii) X_2 \perp X_3 \mid X_8 = \text{False}$$

$$iii) X_2 \perp X_3 \mid X_6 = \text{True}$$

$$iv) X_2 \perp X_4 \mid X_9 = \text{False}$$

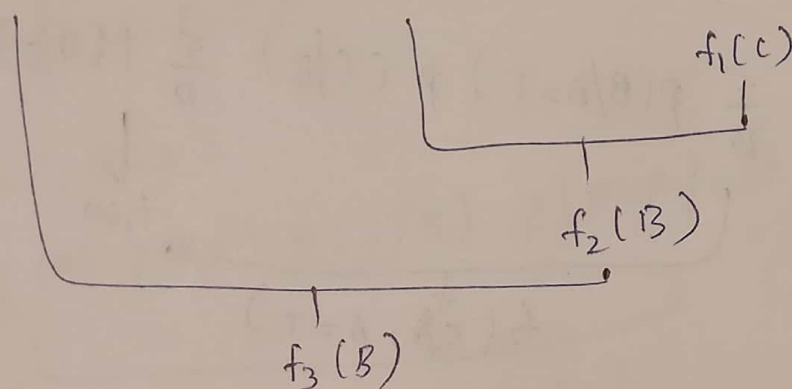
$$v) X_7 \perp X_6 = \text{True}$$

② a) $p(B)$

$$p(A, B, C, D) = p(A) p(B|A) p(C|B) p(D|C)$$

Query $\rightarrow B$; order $A - C - D$

$$= \sum_A p(A) p(B|A) \sum_C p(C|B) \sum_D p(D|C)$$



C	$f_1(C)$
T	$0.82 + 0.18 = 1$
F	$0.37 + 0.63 = 1$

$$p(D=T | C=T) + p(D=F | C=T)$$

$$p(D=T | C=F) + p(D=F | C=F)$$

B	$f_2(B)$
T	$0.4 \times 0.1 \times 1 + 0.6 \times 0.8 \times 0.1 = 0.52$
F	$0.4 \times 0.9 \times 1 + 0.6 \times 0.2 \times 1 = 0.48$

②⑥ $p(C/A=T)$

BN for ABCD is with evidence $A=T$

$$p(A,B,C,D) = p(A=T) p(B/A=T) p(C/B) p(D/C)$$

Query C, Evidence $A=T$, order B,D variables.

$$p(A=T) \sum_B p(B/A=T) p(C/B) \sum_D p(D/C)$$

\downarrow
 $f_1(C)$

\downarrow
 $f_2(C, A=T)$

\downarrow
 $f_3(C, A=T)$

C	$f(C)$
T	$0.82 + 0.18 = 1$
F	$0.37 + 0.63 = 1$

C	A=T	$f_2(C, A=T)$
T	T	$0.1 \times 0.7 \times 1 + 0.9 \times 0.4 \times 1 = 0.43$
F	T	$0.1 \times 0.3 \times 1 + 0.9 \times 0.6 \times 1 = 0.57$

⑤

A \ C	$f_3(C, A=T)$		Normalize	
T	$0.4 * 0.43 = 0.172$		$\frac{0.172}{0.172 + 0.228}$	≈ 0.43
F	$0.4 * 0.57 = 0.228$		$\frac{0.228}{0.172 + 0.228}$	≈ 0.57

⑥ $P(A, B | C=T, D=F)$

$$P(A, B, C=T, D=F) = P(A) P(B|A) P(C=T|B) P(D=F|C=T)$$

$f(A, B, C=T, D=F)$

A	B	$f(A, B, C=T, D=F)$	Normalize
T	T	$0.4 * 0.1 * 0.7 * 0.18 = 0.005$	$\frac{0.005}{0.0999} \approx 0.05$
T	F	$0.4 * 0.9 * 0.4 * 0.18 = 0.0259$	$\frac{0.0259}{0.0999} \approx 0.26$
F	T	$0.6 * 0.8 * 0.7 * 0.18 = 0.0604$	$\frac{0.0604}{0.0999} \approx 0.65$
F	F	$0.6 * 0.2 * 0.4 * 0.18 = 0.0086$	$\frac{0.0086}{0.0999} \approx 0.086$

(3)

a) P(B) Action?

Bayesian Network is

$$p(x, y, z) = p(x) p(y|x) p(z|y)$$

Eliminate z since it is not relative and $f(1,1)$

y	$p(x) p(y x)$
T	$0.4 * 0.2 + 0.6 * 0.7 = 0.50$
F	$0.4 * 0.8 + 0.6 * 0.3 = 0.32 + 0.18$ $= 0.50$

Action a :

$$a \quad y=T + a \quad y=F$$

MEU(a)

$$= 0.5 * 800 + 0.5 * 200$$

$$= 400 + 100$$

$$= 500$$

Action $\sim a$:

$$\sim a \quad y=T + \sim a \quad y=F$$

Action $\sim a$ MEU:

$$0.5 * 400 + 0.5 * 1000$$

$$= 200 + 500$$

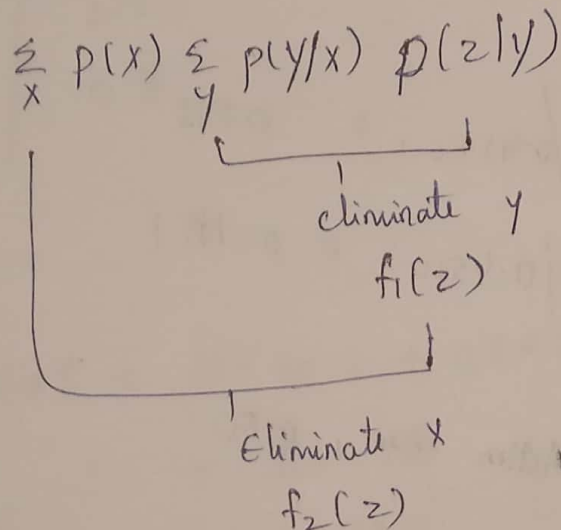
$$= 700$$

Action to take $\sim a$ with MEU 700

③ b) VOI (z) = ?

⑦

Bayesian network



x	z	$p(y x) p(z y)$
T	T	$0.2 * 0.9 + 0.8 * 0.2 = 0.34$
T	F	$0.2 * 0.1 + 0.8 * 0.8 = 0.66$
F	T	$0.7 * 0.9 + 0.3 * 0.2 = 0.69$
F	F	$0.7 * 0.1 + 0.3 * 0.8 = 0.31$

z	$p(x) f_1(x, z) = f_2(z)$
T	$0.4 * 0.34 + 0.6 * 0.69 = 0.55$
F	$0.4 * 0.66 + 0.6 * 0.31 = 0.45$

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MEU when $p(y/z=T)$

$$= p(y) p(z=T/y)$$

y	$p(y) p(z=T/y)$
T	$0.5 \times 0.9 = 0.45 / 0.45 + 0.1 = 0.818$
F	$0.5 \times 0.2 = 0.1 / 0.45 + 0.1 = 0.181$

Action a, MEU:

$$a \quad y=T + a \quad y=F$$

$$\begin{aligned} \text{MEU} &= 800 \times 0.818 + 0.181 \times 200 \\ &= 654.4 + 36.2 \\ &= 690.6 \end{aligned}$$

Action na, MEU:

$$na \quad y=T + na \quad y=F$$

$$\begin{aligned} \text{MEU} &= 400 \times 0.818 + 1600 \times 0.181 \\ &= 327.2 + 181 \\ &= 508.2 \end{aligned}$$

Maximum Expected Utility

$$\text{MEU} (y/z=T)$$

$$= 690.6$$

when $(y|z=F)$, MEU?

y	$P(y) P(z=F y)$
T	$0.5 * 0.1 = 0.05 / 0.45 = 0.111$
F	$0.5 * 0.8 = 0.4 / 0.45 = 0.889$

MEU, when action a

$$\begin{aligned} \text{MEU} &= 0.111 * 800 + 200 * 0.889 \\ &= 266.6 \end{aligned}$$

MEU, when action a

$$\begin{aligned} \text{MEU} &= 7a \ y=T + 7a \ y=F \\ &= 0.111 * 400 + 0.889 * 1000 \\ &= 933.4 \end{aligned}$$

MEU, when $y|z=F = 933.4$

$$\begin{aligned} \text{VOI}(z) &= 0.55 * 690.6 + 0.45 * 933.4 - 700 \\ &\approx 99.86 \end{aligned}$$

(3) (c) $VOI(x)$

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y	$(y/x=T)$
T	0.2
F	0.8

when $y=T$:

$$\begin{aligned}MEU(a) &= 800 * 0.2 + 0.8 * 200 \\&= 160 + 160 \\&= 320\end{aligned}$$

$$\begin{aligned}MEU(\sim a) &= 400 * 0.2 + 1000 * 0.8 \\&= 80 + 800 \\&= 880\end{aligned}$$

MEU , when $y=T$ is 880

y	$(y/x=F)$
T	0.7
F	0.3

MEU(a) :

$$y=T \quad a + y=F \quad a$$

$$= 800 * 0.529 + 200 * 0.471$$

$$= 423.2 + 94.2$$

$$= 517.4$$

MEU($\sim a$) :

$$y=T \quad \sim a + y=F \quad \sim a$$

$$= 400 * 0.529 + 1000 * 0.471$$

$$= 211.6 + 471$$

$$= 682.6$$

$$MEU(a, x=T, z=T) = 682.6$$

MEU when $x=F, z=T$

y	$P(x=F)$	$P(y/x=F)$	$P(z=T/y)$
T	0.6	0.7	0.9
F	0.6	0.3	0.2

$0.6 * 0.7 * 0.9 = 0.378 \rightarrow \frac{0.378}{0.414} = 0.913$

$0.6 * 0.3 * 0.2 = 0.036 \rightarrow \frac{0.036}{0.414} = 0.087$

$$\begin{aligned}
 MEU(a, y/x \neq F) &= 0.7 * 800 + 0.3 * 200 \\
 &= 560 + 60 \\
 &= 620
 \end{aligned}$$

$$\begin{aligned}
 MEU(\bar{a}, y/x \neq F) &= 400 * 0.7 + 1000 * 0.3 \\
 &= 280 + 300 \\
 &= 580.
 \end{aligned}$$

$$\begin{aligned}
 VOI(X) &= 0.4 * 880 + 0.6 * 620 - 700 \\
 &= 352 + 372 - 700 \\
 &= 24
 \end{aligned}$$

(d) Given $z \neq T$, $VOI(X) = ?$

MEU when $z \neq T$; 690%

MEU when $x \neq T$; $z \neq T$:

$$p(y/x \neq T, z \neq T) = p(x \neq T) p(y/x \neq T) p(z \neq T/y)$$

y			
T	$0.4 * 0.2 * 0.9 = 0.072$	$\frac{0.072}{0.136} = 0.529$	
F	$0.4 * 0.8 * 0.2 = 0.064$	$\frac{0.064}{0.136} = 0.471$	

Action a

$$y = T \cdot a + y = F \cdot a$$

$$= 800 * 0.913 + 200 * 0.087$$

$$= 730.4 + 17.4$$

$$= 747.8$$

Action na :

$$y = T \cdot na + y = F \cdot na$$

$$MEU = 400 * 0.913 + 1000 * 0.087$$

$$= 365.2 + 87$$

$$= 452.2$$

x	$f_i(x, z=T)$
T	$0.4 * 0.2 * 0.9 + 0.4 * 0.8 * 0.2 = 0.072 + 0.064$ $= 0.0136 \xrightarrow[\text{ize}]{\text{Normal}} \frac{0.0136}{0.55} = 0.25$
F	$0.6 * 0.7 * 0.9 + 0.6 * 0.3 * 0.2 = 0.378 + 0.036$ $= 0.414 \xrightarrow[\text{ize}]{\text{Normal}} \frac{0.414}{0.55} \approx 0.75$

An

$$VOI(x, z=T) = 0.25 * 682.6 + 0.75 * 747.8 - 690.6$$

$$= 170.65 + 560.85 - 690.6$$

$$= 40.9$$