

Assignment 4 Solutions

Question 1

a

$$P(C) \propto \sum_B P(C|B) \sum_A P(A)P(B|A)$$

B	A	$\psi(B,A)$	$\tau(B)$
T	T	$0.3 * 0.4$	0.6
T	F	$0.8 * 0.6$	
F	T	$0.7 * 0.4$	0.4
F	F	$0.2 * 0.6$	

$$P(C) \propto \sum_B P(C|B)\tau(B)$$

C	B	$\psi(C,B)$	$\tau(C)$
T	T	$0.9 * 0.6$	0.7
T	F	$0.4 * 0.4$	
F	T	$0.1 * 0.6$	0.3
F	F	$0.6 * 0.4$	

$$P(C) = \langle 0.7, 0.3 \rangle$$

b

$$P(C|A=t) \propto P(A=t) \sum_B P(B|A=t) \cdot P(C|B)$$

C	B	$\psi(C,B)$	$\tau(C)$
T	T	$0.3 * 0.9$	0.55
T	F	$0.7 * 0.4$	
F	T	$0.3 * 0.1$	0.45
F	F	$0.7 * 0.6$	

$$P(C|A=t) \propto P(A=t) \cdot \tau(C)$$

$$P(C|A=t) = \langle 0.55, 0.45 \rangle$$

c

$$P(C|A=t, B=t) \propto P(A=t) \cdot P(B=t|A=t) \cdot P(C|B=t)$$

C	$P(A=t).P(B=t A=t).P(C B=t)$
T	$0.7 * 0.3 * 0.9 = 0.108$
F	$0.7 * 0.3 * 0.1 = 0.012$
	$\Sigma=0.12$

$$\text{Normalize}$$

$$P(C|A = t, B = t) = < 0.9, 0.1 >$$

Question 2

a

Variable	All Factors	Participates	New Factor After *	# *s	New Factor After +	# +s	# Ops
B	P(A), P(B A), P(D), P(C B,D), P(E C)	P(B A), P(C B,D)	$\psi_1(B,A,C,D)$	$1*2*2*2 = 16$	$\tau_1(A,C,D)$	$1*2*2*2 = 8$	24
C	P(A), P(D), P(E C), $\tau_1(A,C,D)$	P(E C), $\tau_1(A,C,D)$	$\psi_2(E,C,A,D)$	$1*2*2*2 = 16$	$\tau_2(E,A,D)$	$1*2*2*2 = 8$	24
A	P(A), P(D), $\tau_2(E,A,D)$	P(A), $\tau_2(E,A,D)$	$\psi_3(A,D,E)$	$1*2*2*2 = 8$	$\tau_3(D,E)$	$1*2*2 = 4$	12
D	P(D), $\tau_3(D,E)$	P(D), $\tau_3(D,E)$	$\psi_4(D, E)$	$1*2*2 = 4$	$\tau_4(E)$	$1*2 = 2$	6
Normalize	$\tau_4(E)$					1	$1 + 2(\text{divs}) = 3$
Total							69

b

Variable	All Factors	Participates	New Factor After *	# *s	New Factor After +	# +s	# Ops
A	P(A), P(B A), P(D), P(C B,D), P(E C)	P(A), P(B A)	$\psi_1(A,B)$	$1*2*2 = 4$	$\tau_1(B)$	$1*2 = 2$	6
D	P(D), P(C B,D), P(E C), $\tau_1(B)$	P(D), P(C B,D)	$\psi_2(D,C,B)$	$1*2*2*2 = 8$	$\tau_2(C,B)$	$1*2*2 = 4$	12
B	P(E C), $\tau_1(B)$, $\tau_2(C,B)$	$\tau_1(B)$, $\tau_2(C,B)$	$\psi_3(B,C)$	$1*2*2 = 4$	$\tau_3(C)$	$1*2 = 2$	6
C	P(E C), $\tau_3(C)$	P(E C), $\tau_3(C)$	$\psi_4(E,C)$	$1*2*2 = 4$	$\tau_4(E)$	$1*2 = 2$	6
Normalize	$\tau_4(E)$					1	$1 + 2(\text{divs}) = 3$
Total							33

Question 3

a

$$P(A) = \phi(A) \cdot \phi(B) \cdot \phi(C) \cdot \phi(A, B) \cdot \phi(A, C) \cdot \phi(B, C)$$

$$P(A) \propto \phi(A) \cdot \sum_B \phi(B) \cdot \phi(A, B) \sum_C \phi(C) \cdot \phi(A, C) \cdot \phi(B, C)$$

A	B	C	$\psi(A, B, C)$	$\tau(A, B)$
T	T	T	$1 * 6 * 1 = 6$	86
T	T	F	$8 * 1 * 10 = 80$	
T	F	T	$1 * 6 * 10 = 60$	68
T	F	F	$8 * 1 * 1 = 8$	
F	T	T	$1 * 1 * 1 = 1$	481
F	T	F	$8 * 6 * 10 = 480$	
F	F	T	$1 * 1 * 10 = 10$	58
F	F	F	$8 * 6 * 1 = 48$	

$$P(A) \propto \phi(A) \cdot \sum_B \phi(B) \cdot \phi(A, B) \tau(A, B)$$

A	B	$\Phi(B) \cdot \Phi(A, B) \cdot \tau(A, B)$	$\tau(A)$
T	T	$1 * 5 * 86 = 430$	702
T	F	$4 * 1 * 68 = 272$	
F	T	$1 * 1 * 481 = 481$	1641
F	F	$4 * 5 * 58 = 1160$	

$$P(A) \propto \phi(A) \cdot \tau(A)$$

A	$\tau(A) \cdot \Phi(A)$
T	$702 * 2 = 1404$
F	$1641 * 1 = 1641$
	$\Sigma = 3045$

$$P(A) = \left\langle \frac{1404}{3045}, \frac{1641}{3045} \right\rangle$$

$$P(A) = \langle 0.46, 0.53 \rangle$$

b

$$P(A|B=t) \propto \phi(A) \cdot \phi(B=t) \cdot \phi(A, B=t) \sum_C \phi(C) \cdot \phi(A, C) \cdot \phi(B=t, C)$$

A	C	$\psi(A, B=t, C)$	$\tau(A, B=t)$
T	T	$1 * 1 * 6 = 6$	86
T	F	$8 * 10 * 1 = 80$	
F	T	$1 * 1 * 1 = 1$	481
F	F	$8 * 10 * 6 = 480$	

$$P(A|B=t) \propto \phi(A) \cdot \phi(B=t) \cdot \phi(A, B=t) \cdot \tau(A, B=t)$$

A	$\Phi(A) \cdot \Phi(B=t) \cdot \Phi(A, B=t) \cdot \tau(A, B=t)$
T	$2 * 1 * 5 * 86 = 860$
F	$1 * 1 * 1 * 481 = 481$
	$\Sigma = 1341$

$$\begin{aligned}
 P(A|B = t) &= < \frac{860}{1341}, \frac{481}{1341} > \\
 P(A) &= < 0.64, 0.36 >
 \end{aligned}$$

c

$$P(A|B = t, C = f) \propto \phi(A) \cdot \phi(B) \cdot \phi(C) \cdot \phi(A, B) \cdot \phi(A, C) \cdot \phi(B, C) = \tau(A)$$

A	$\tau(A)$
T	$2 * 1 * 8 * 5 * 10 * 1 = 800$
F	$1 * 1 * 8 * 1 * 10 * 6 = 480$
	$\Sigma = 1280$

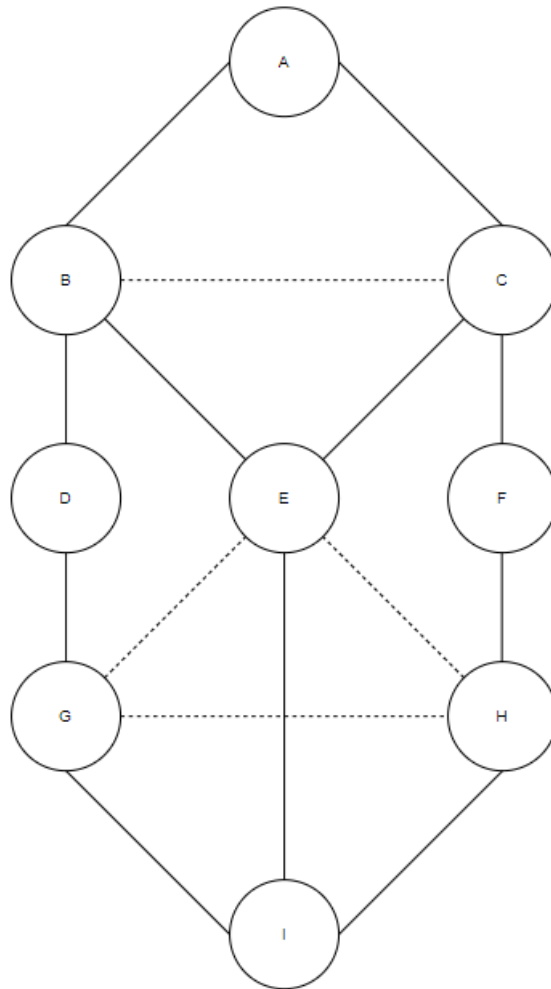
$$\begin{aligned}
 P(A|B = t, C = f) &= < \frac{800}{1280}, \frac{480}{1280} > \\
 P(A) &= < 0.625, 0.375 >
 \end{aligned}$$

Question 4

- a. A, B, C, D, F, G, H, I, J, K, L
- b. A, B, C, D, F, G, H, L
- c. A, B, D, F, G, H
- d. F, H, I, J, K, L
- e. F, H, I, K, L

Question 5

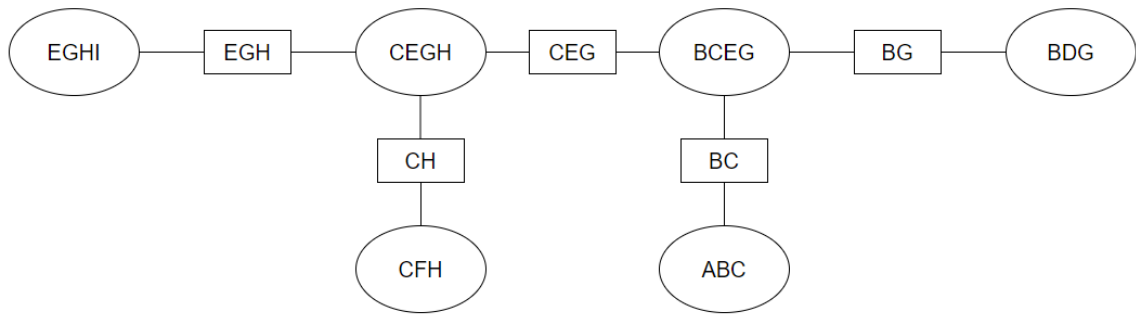
The moralized graph is :



Variable elimination order(as per min fill heuristic):

Variable	Maximal Clique
A	ABC
I	EGHI
D	BDG
F	CFH
B	BCEG
C	CEGH

Join Tree



Question 6 a

i

$$\delta_{AB \rightarrow BC} = \phi(B) \sum_A \phi(A) \phi(A, B)$$

B	A	$\psi(A, B)$	$\tau(B)$
T	T	$2 * 4 = 8$	9
T	F	$1 * 1 = 1$	
F	T	$2 * 1 = 2$	6
F	F	$1 * 4 = 4$	

$$\delta_{AB \rightarrow BC} = \phi(B) \tau(B)$$

Multiply & Normalize $\delta_{AB \rightarrow BC} = \langle 9, 6 \rangle$

iv

$$\delta_{CD \rightarrow BC} = \sum_D \phi(D) \phi(C, D)$$

C	D	$\psi(C, D)$	$\tau(C)$
T	T	$1 * 4 = 4$	6
T	F	$2 * 1 = 2$	
F	T	$1 * 1 = 1$	9
F	F	$2 * 4 = 8$	

$$\delta_{CD \rightarrow BC} = \langle 6, 9 \rangle$$

v

$$\delta_{CE \rightarrow BC} = \sum_E \phi(E) \phi(C, E)$$

C	E	$\psi(C, E)$	$\tau(C)$
T	T	$1 * 4 = 4$	6
T	F	$2 * 1 = 2$	
F	T	$1 * 1 = 1$	9
F	F	$2 * 4 = 8$	

$$\delta_{CE \rightarrow BC} = \langle 6, 9 \rangle$$

ii

$$\delta_{BC \rightarrow CD} = \sum_B \phi(C) \phi(B, C) \delta_{AB \rightarrow BC} \delta_{CE \rightarrow BC}$$

C	B	$\psi(C,B)$	$\tau(C)$
T	T	$1 * 4 * 9 * 6 = 216$	252
T	F	$1 * 1 * 6 * 6 = 36$	
F	T	$1 * 1 * 9 * 9 = 81$	297
F	F	$1 * 4 * 6 * 9 = 216$	

$$\delta_{BC \rightarrow CD} = \langle 252, 297 \rangle$$

iii

$$\delta_{BC \rightarrow CE} = \sum_B \phi(C) \phi(B, C) \delta_{AB \rightarrow BC} \delta_{CD \rightarrow BC}$$

C	B	$\psi(C,B)$	$\tau(C)$
T	T	$1 * 4 * 9 * 6 = 216$	252
T	F	$1 * 1 * 6 * 6 = 36$	
F	T	$1 * 1 * 9 * 9 = 81$	297
F	F	$1 * 4 * 6 * 9 = 216$	

$$\delta_{BC \rightarrow CE} = \langle 252, 297 \rangle$$

vi

$$\delta_{BC \rightarrow AB} = \sum_C \phi(C) \phi(B, C) \delta_{CD \rightarrow BC} \delta_{CE \rightarrow BC}$$

B	C	$\psi(B,C)$	$\tau(B)$
T	T	$1 * 4 * 6 * 6 = 144$	225
T	F	$1 * 1 * 9 * 9 = 81$	
F	T	$1 * 1 * 6 * 6 = 36$	360
F	F	$1 * 4 * 9 * 9 = 324$	

$$\delta_{BC \rightarrow AB} = \langle 225, 360 \rangle$$

Question 6 b

i

$$P(A) = \sum_B \beta_{AB}$$

A	B	β_{AB}
T	T	$2 * 4 * 1 * 225 = 1800$
T	F	$2 * 1 * 1 * 360 = 720$
F	T	$1 * 1 * 1 * 225 = 225$
F	F	$1 * 4 * 1 * 360 = 1440$

$$\tau(A) = < 2520, 1665 >$$

$$\text{Normalize}$$

$$P(A) = < 0.602, 0.398 >$$

ii

$$P(B) = \sum_A \beta_{AB}$$

From above table directly,

$$\tau(B) = < 2025, 2160 >$$

$$\text{Normalize}$$

$$P(B) = < 0.484, 0.516 >$$

iii

$$P(C) = \delta_{CD \rightarrow BC} \cdot \delta_{BC \rightarrow CD}$$

$$\tau(C) = < 6 * 252, 9 * 297 >$$

$$\tau(C) = < 1512, 2673 >$$

$$P(C) = < \frac{1512}{4185}, \frac{2673}{4185} >$$

$$P(C) = < 0.361, 0.639 >$$

iv

$$P(D) = \sum_C \beta_{CD} = \sum_c \phi(D)\phi(C,D)\delta_{BC \rightarrow CD}$$

D	C	β_{CD}	$\tau(D)$
T	T	$1 * 4 * 252 = 1008$	1305
T	F	$1 * 1 * 297 = 297$	
F	T	$2 * 1 * 252 = 504$	2880
F	F	$2 * 4 * 297 = 2376$	

$$P(D) = < \frac{1305}{4185}, \frac{2880}{4185} >$$

$$P(D) = < 0.312, 0.688 >$$

v

$$P(E) = \sum_C \beta_{CE} = \sum_c \phi(E)\phi(C,E)\delta_{BC \rightarrow CE}$$

E	C	β_{CE}	$\tau(E)$
T	T	$1 * 4 * 252 = 1008$	1305
T	F	$1 * 1 * 297 = 297$	
F	T	$2 * 1 * 252 = 504$	2880
F	F	$2 * 4 * 297 = 2376$	

$$P(E) = < \frac{1305}{4185}, \frac{2880}{4185} >$$

$$P(E) = < 0.312, 0.688 >$$