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A# : A20448916

collaborators : 0

① Are A & B are independent : False

i)

A	B	P(A,B)	P(A) * P(B)
T	T	0.448+0.0252+0.0112+0.0588 = 0.14	0.2*0.46 = 0.092
T	F	0.0144+0.0144+0.0096+0.0216 = 0.06	0.2*0.54 = 0.108
F	T	0.1024+0.0576+0.0256+0.1344 = 0.32	0.8*0.46 = 0.368
F	F	0.0155+0.1152+0.0768+0.1728 = 0.48	0.8*0.54 = 0.432

① T F

A = <0.2, 0.8>

B = <0.46, 0.54>

A and B are not independent since $P(A,B) \neq P(A) * P(B)$

ii) Are A & C independent : True

A	C	P(A,C)	P(A) P(C)
T	T	0.0448+0.0252+0.0144+0.0144 = 0.0988 ≅ 0.098	0.4892*0.2 = 0.09784 ≅ 0.098
T	F	0.0112+0.0588+0.0096+0.0216 = 0.1012 ≅ 0.10	0.2*0.5108 = 0.102 ≅ 0.10
F	T	0.1024+0.0576+0.1152+0.1152 = 0.3904 ≅ 0.39	0.8*0.4892 = 0.3913 ≅ 0.39
F	F	0.0256+0.1344+0.0768+0.1728 = 0.4096 ≅ 0.41	0.8*0.5108 = 0.4086 ≅ 0.41

C: T F

C = <0.4892, 0.5108>

A and C are independent since $P(A,C) = P(A)P(C)$

(c) Are A and C are independent given B? True

A and C are independent iff

$P(A, C|B) = P(A|B)\, P(C|B)$

A	B	C	$\frac{\#(A,B,C)}{\#(B)}$	$P(A\|B)\, P(C\|B)$
T	T	T	$\frac{0.0448+0.0252}{0.46} = 0.15$	$\frac{(0.0448+0.0252+0.0112+0.0588)}{0.46} * \frac{0.0448+0.0}{0.46}$ $\frac{52+0.1024+0.0526}{0.46} = 0.015$
T	T	F	$\frac{0.0112+0.0588}{0.46} = 0.15$	$(0.0448+0.0252+0.0112+0.0588)/0.46 * (0.0112+0.0588+0.0256+0.1344)/0.46) = 0.15$
T	F	T	$\frac{0.0144+0.0144}{0.54} = 0.053$	$(0.0144+0.0144+0.0096+0.0216)/0.54 * (0.0144+0.0144+0.1152+0.1152)/0.54) = 0.053$
T	F	F	$\frac{0.0096+0.0216}{0.54} = 0.057$	$(0.0144+0.0144+0.0096+0.0216)/0.54 * ((0.0096+0.0216+0.0768+0.1728)/0.54) = 0.057$
F	T	T	$\frac{0.1024+0.0576}{0.46} = 0.347$	$(0.1024+0.0576+0.256+0.1344)/0.46 * (0.0448+0.0252+0.1024+0.0576)/0.46 = 0.347$
F	T	F	$\frac{0.0256+0.1344}{0.46} = 0.347$	$(0.1024+0.0576+0.0256+0.1344)/0.46 * (0.0112+0.0588+0.0256+0.1344)/0.46 = 0.347$
F	F	T	$\frac{0.1152+0.1152}{0.54} = 0.426$	$(0.1152+0.1152+0.0768+0.1728)/0.54 * (0.0144+0.0144+0.1152+0.1152)/0.54 = 0.426$
F	F	F	$\frac{0.0768+0.1728}{0.54} = 0.462$	$(0.1152+0.1152+0.0768+0.1728)/0.54 * (0.0096+0.0216+0.0768+0.1728)/0.54 = 0.462$

Hence, A and C are independent given B.

(d) Are A and D are independent ? True

A = <0.2, 0.8>, D = <0.4, 0.6>

A	D	P(A,D) = #(A,D) / 4	P(A) p(D)
T	T	0.0448 + 0.0112 + 0.0144 + 0.0096 = 0.08	0.2 * 0.4 = 0.08
T	f	0.0252 + 0.0588 + 0.0144 + 0.0216 = 0.12	0.2 * 0.6 = 0.12
F	T	0.1024 + 0.0256 + 0.1152 + 0.0768 = 0.32	0.8 * 0.4 = 0.32
f	F	0.0576 + 0.1344 + 0.1152 + 0.1728 = 0.48	0.8 * 0.6 = 0.48

A and D are independent since $p(A, D) = p(A) * p(D)$

(1) (2) Are A and D independent given C? TRUE (4)

A and D are independent iff $p(A, D/c) = p(A/c)\,p(D/c)$

A	C	D	$\frac{P(A \wedge C \wedge D)}{P(C)}$	$P(A/c)\,P(D/c)$
T	T	T	$\frac{0.0448+0.0144}{0.4892} = 0.12$	$(0.0448+0.0252+0.0144+0.0144/0.4892)$ $(0.0448+0.0144+0.1024+0.1152/0.4892) = 0.11$ $= 0.11$
T	T	F	$\frac{0.0252+0.0144}{0.4892} = 0.08$	$(0.0448+0.0252+0.0144+0.0144)/0.4892$ $(0.0252+0.0144+0.0576+0.1152)/0.4892$ $= 0.08$
T	F	T	$\frac{0.0112+0.0096}{0.5108} = 0.04$	$(0.0112+0.0588+0.0096+0.0216)/0.5108 \neq ($ $0.0112+0.0096+0.0256+0.0768/0.5108) = 0.04$
T	F	F	$\frac{0.0588+0.0216}{0.5108} = 0.16$	$(0.0112+0.0588+0.0096+0.0216/0.5108 * (0.0588$ $+0.0216+0.1344+0.1728)/0.5108 = 0.15$
F	T	T	$\frac{0.1024+0.1152}{0.4892} = 0.45$	$(0.1024+0.0576+0.1152+0.1152)/0.4892 * (0.0448+$ $0.0144+0.1024+0.1152)/0.4892$ $= 0.46$
F	T	F	$\frac{0.0576+0.1152}{0.4892} = 0.35$	$(0.1024+0.0576+0.1152+0.1152)/0.4892 *$ $(0.0252+0.0144+0.0576+0.1152)/0.4892$ $= 0.35$
F	F	T	$\frac{0.0256+0.0768}{0.5108} = 0.20$	$(0.0256+0.1344+0.0768+0.1728/0.5108) * (0.0112+0.0096+0.0256+0.0768))/$ $0.5108 = 0.20$
F	F	F	$\frac{0.1344+0.1728}{0.5108} = 0.60$	$(0.0256+0.1344+0.0768+0.1728)/$ $0.5108 * (0.0588+0.0216+0.1344$ $+0.1152)/0.5108$ $= 0.61$

A and D are independent given C.

(2) Random variables $X_2, X_3 \cdots X_n$ and $Y_2, Y_3 \cdots Y_m$

a) For all $\forall X_i$ is binary

$P(X_2, X_3 \cdots X_n, Y_2, Y_3 \cdots Y_m)$

$= 2^{n+m-2} - 1$ independent parameters

b) Assuming every variable has 3 values

$P(X_2, X_3 \cdots X_n, Y_2, Y_3 \cdots Y_m)$

$= 3^{n+m-2} - 1$ independent params

c) Assuming X_i has i possible values, Y_i has i possible values

$P(X_2, X_3 \cdots X_n, Y_2, Y_3 \cdots Y_m)$

$2, 3, \cdots n \quad 2 \quad 3 \cdots m$

~~$= 2^{n+m-2} - 1$ independent parameters~~

$= n!m! - 1$

d) Assuming every val is binary

$P(Y_2, Y_3 \cdots Y_m \mid X_2, X_3 \cdots X_n)$

$= (2^{m-1} - 1)$ values $(2^{n-1}$ tables$)$

$= (2^{m-1} - 1)(2^{n-1})$ independent params

(e) Every variables has 3 values

$P(Y_2, Y_3 \cdots Y_m \mid X_2, X_3 \cdots X_n)$

$= (3^{m-1} - 1)(3^{n-1})$ indepedent params

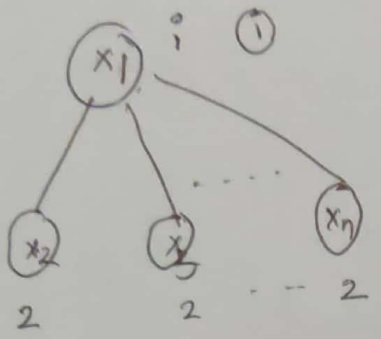
(f) Assuming x_i and y_i has i values

$P(Y_2, Y_3 \cdots Y_m \mid X_2, X_3 \cdots X_n)$

~~$(i^{m-1} - 1)(i^{n-1})$ independent parameters.~~

$= (m! - 1)(n!)$ indepedyt parameters

(3) Naive bayes network is



$= 2(n-1)+1$

$= 2n-2+1$

$= 2n-1$ independent params.

(4)

a) $P(A, B, C, D, E, F, G, H)$

$= P(A)\, P(B)\, P(C|A,B)\, P(D|C)\, P(E)\, P(F|C)\, P(G|D,E,F)$

$P(H|G)$

(b) if each variable takes n possible values,

$P(A, B, C, D, E, F, G, H)$

$= P(A)\, P(B)\, P(C|A,B)\, P(D|C)\, P(E)\, P(F|C)\, P(G|D,E,F) P(H|G)$

$(n-1)\ (n-1)\ [(n-1)(n \cdot n)]\ [(n-1)*n]\ (n-1)\ [(n-1)(n)]\ [(n-1)(n*n*n)]\ [(n-1)*n]$

~~$(n-1)(n-1)$~~

$= (n-1) + (n-1) + [(n-1)n^2] + [(n-1)*n] + (n-1) + [(n-1)(n)]$

$+ [(n-1)(n^3)] + [(n-1)(n)]$

$= (n-1)[1+1+n^2+n+1+n+n^3+n]$

$= (n-1)[n^3+n^2+3n+3]$ independent parameters

$= (n-1)[n^2(n+1)+3(n+1)]$

$= (n-1)(n+1)(n^2+3) = (n^2-1)(n^2+3)$

$= (n^2-1)(n^2+3)$ independent parameters

(c)

i. $A \perp B$ true

ii $A \perp B \mid C$ False

iii) $A \perp B \mid F$ False

iv) $A \perp B \mid G$ False

v) $A \perp B \mid E$ True

vi) $A \perp B \mid H$ False

vii) $A \perp H$ False

viii) $A \perp H \mid F$ False

ix) $A \perp H \mid D, F$ True

x) $D \perp F$ False

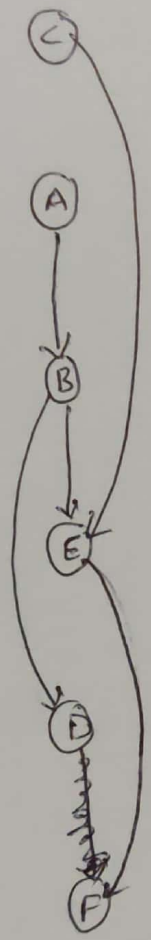
x;) $B \perp E$ True

x11) $B \perp E \mid F$ True

xii) $B \perp E \mid F, H$ False

(5)

a) Given order C, A, B, E, D, F



step 1:

$U \subseteq \{C\}$ such that

$A \perp \{C\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow C \perp A$ ✓

step 2:

$U \subseteq \{C, A\}$ such that

$B \perp \{C, A\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow B \perp C, A$ X

$U = \{C\} \Rightarrow B \perp A \mid C$ X

$U = \{A\} \Rightarrow B \perp C \mid A$ ✓

step 3:

$U \subseteq \{C, A, B\}$ such that

$E \perp \{C, A, B\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow E \perp C, A, B$ ✗

$U = \{C\} \Rightarrow E \perp A, B \mid C$ ✗

$U = \{A\} \Rightarrow E \perp B, C \mid A$ ✗

$U = \{B\} \Rightarrow E \perp A, C \mid B$ ✗

$U = \{B, C\} \Rightarrow E \perp A \mid B, C$ ✓

step 4:

$U \subseteq \{C, A, B, E\}$ such that $D \perp \{C, A, B, E\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow D \perp C, A, B, E$ ✗

$U = \{C\} \Rightarrow D \perp A, B, E \mid C$

$U = \{B\} \Rightarrow D \perp A, E, C \mid B$ ✓

Step 5:

$U \subseteq \{C, A, B, E, D\}$ such that $F \perp \{C, A, B, E, D\} \setminus U \mid U$

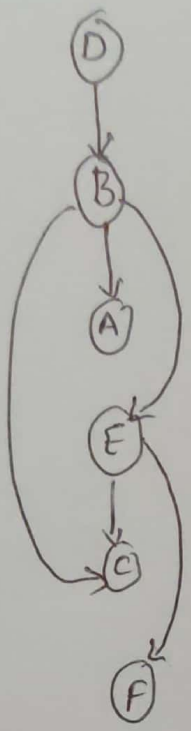
$U = \{\phi\} \Rightarrow F \perp C, A, B, E, D$ ✗

$U = \{C\} \Rightarrow F \perp A, B, E, D \mid C$ ✗

$U = \{B\} \Rightarrow F \perp C, A, E, D \mid B$ ✗

$U = \{E\} \Rightarrow F \perp C, A, B, D \mid E$ ✓

⑤⑥ Given order is D, B, A, E, C, F



step1:

$U \subseteq \{D\}$ such that

$B \perp \{D\} \mid U \mid U$

$U = \{\phi\} \Rightarrow D \perp B$ ✗

step2:

$U \subseteq \{D, B\}$ such that

$A \perp \{D, B\} \mid U \mid U$

$U = \{\phi\} \Rightarrow A \perp D, B$ ✗

$U = \{D\} \Rightarrow A \perp B \mid D$ ✗

$U = \{B\} \Rightarrow A \perp D \mid B$ ✓

step3:

$U \subseteq \{D, B, A\}$ such that

$E \perp \{D, B, A\} \mid U \mid U$

$U = \{\phi\} \Rightarrow E \perp D, B, A$ ✗

$U = \{D\} \Rightarrow E \perp B, A \mid D$ ✗

$U = \{B\} \Rightarrow E \perp D, A \mid B$ ✓

step4:

$U \subseteq \{D, B, A, E\}$ such that

$U = \{\phi\} \Rightarrow C \perp D, B, A, E$ ✗

$U = \{D\} \Rightarrow C \perp B, A, E \mid D$ ✗

$U = \{B\} \Rightarrow C \perp D, A, E \mid B$ ✗

$U = \{A\} \Rightarrow C \perp D, A, B \mid E$ ✗

$U = \{E\} \Rightarrow C \perp B, B, A \mid E$ ✗

~~$U = \{B, E\} \Rightarrow \emptyset \perp \{A, D\}$~~

$U = \{B, E\} \Rightarrow C \perp A, D \mid B, E$ ✓

steps:

$U \subseteq \{D, B, A, E, C\}$ such that $F \perp \{D, B, A, E, C\} \setminus V \mid U$

$U = \{\phi\} \Rightarrow F \perp D, B, A, E, C \mid \phi$ ✗

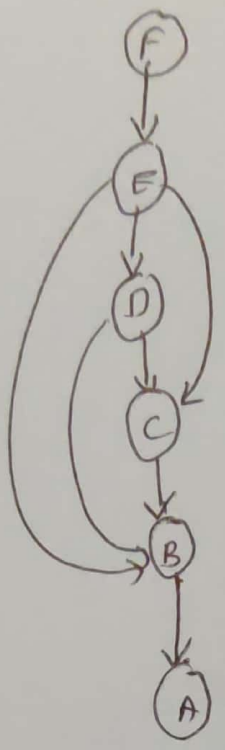
$U = \{D\} \Rightarrow F \perp B, A, E, C \mid D$ ✗

$U = \{B\} \Rightarrow F \perp D, A, E, C \mid B$ ✗

$U = \{A\} \Rightarrow F \perp D, B, E, C \mid A$ ✗

$U = \{E\} \Rightarrow F \perp D, B, A, C \mid E$ ✓

5.



Given order is

F, E, D, C, B, A

step1:

$U \subseteq \{F\}$ such that

$E \perp \{F\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow E \perp F$ X

step2:

$U \subseteq \{F, E\}$ such that

$D \perp \{F, E\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow D \perp F, E$ X

$U = \{F\} \Rightarrow D \perp E \mid F$ X

$U = \{E\} \Rightarrow D \perp F \mid E$ ✓

step3:

$U \subseteq \{F, E, D\}$ such that

$C \perp \{F, E, D\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow C \perp F, E, D$ X

$U = \{F\} \Rightarrow C \perp E, D \mid F$ X

$U = \{E\} \Rightarrow C \perp F, D \mid E$ X

$U = \{D\} \Rightarrow C \perp E, F \mid D$ X

$U = \{D, E\} \Rightarrow C \perp F \mid D, E$

step4:

$U \subseteq \{F, E, D, C\}$ such that

$B \perp \{F, E, D, C\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow B \perp F, E, D, C \mid \phi$ X

$U = \{F\} \Rightarrow B \perp E, D, C \mid F$ X

$U = \{E\} \Rightarrow B \perp F, D, C \mid E$ X

$U = \{D\} \Rightarrow B \perp F, E, C \mid D$ X

$U = \{C\} \Rightarrow B \perp F, E, D \mid C$ X

$U = \{F, E\} \Rightarrow B \perp D, C \mid F, E$ X

$U = \{E, D\} \Rightarrow B \perp F, C \mid E, D$ X

$U = \{C, F\} \Rightarrow B \perp E, D \mid C, F$ X

$U = \{F, D\} \Rightarrow B \perp E, C \mid F, D$ X

$U = \{E, C\} \Rightarrow B \perp F, D \mid E, C$ X

$U = \{D, C\} \Rightarrow B \perp F, E \mid D, C$ X

$U = \{F, E, D\} \Rightarrow B \perp C \mid F, E, D$ ✗

$U = \{E, D, C\} \Rightarrow B \perp F \mid E, D, C$ ✓

Step 5:

$U \subseteq \{F, E, D, C, B\}$ such that $A \perp \{F, E, D, C, B\} \setminus U \mid U$

$U = \{\phi\} \Rightarrow A \perp F, E, D, C, B \mid \phi$ ✗

$U = \{F\} \Rightarrow A \perp E, D, C, B \mid F$ ✗

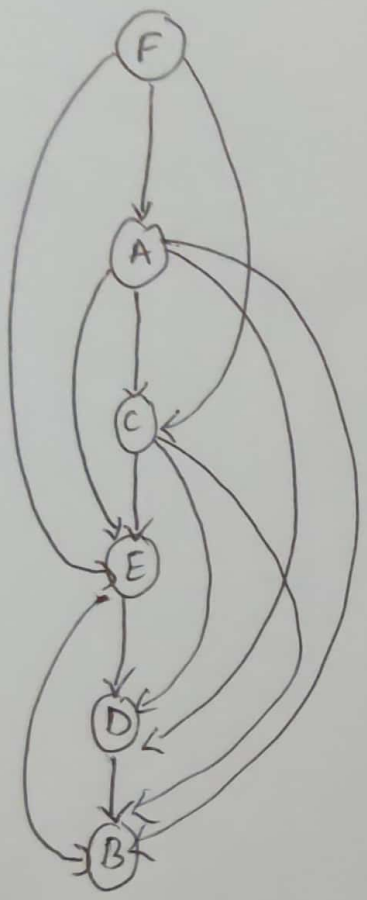
$U = \{D\} \Rightarrow A \perp F, E, C, B \mid D$ ✗

$U = \{E\} \Rightarrow A \perp F, D, C, B \mid E$ ✗

$U = \{C\} \Rightarrow A \perp F, E, D, C \mid B$ ✗

$U = \{B\} \Rightarrow A \perp F, E, D, C \mid B$ ✓

5) (d)



Given order is F, A, C, E, D, B

step1:

$U \subseteq \{F\}$ such that $A \perp \{F\} \setminus V | U$

$U = \{\phi\} \Rightarrow A \perp F$ X

step2:

$U \subseteq \{F, A\}$ such that $C \perp \{F, A\} \setminus V | U$

$U = \{\phi\} \Rightarrow C \perp F, A$ X

$U = \{F\} \Rightarrow C \perp A | F$ X

$U = \{A\} \Rightarrow C \perp F | A$ X

step3:

$U \subseteq \{F, A, C\}$ such that

$E \perp \{F, A, C\} \setminus V | U$

$U = \{\phi\} \Rightarrow E \perp F, A, C$ X

$U = \{F\} \Rightarrow E \perp A, C | F$ X

$U = \{A\} \Rightarrow E \perp C, F | A$ X

$U = \{C\} \Rightarrow E \perp F, A | C$ X

$U = \{F, A\} \Rightarrow E \perp C | F, A$ X

$U = \{A, C\} \Rightarrow E \perp F | C, A$ X

$U = \{F, C\} \Rightarrow E \perp A | F, C$ X

step4: $U \subseteq \{F, A, C, E\}$ such that

$D \perp \{F, A, C, E\} \setminus V | U$

$U = \{\phi\} \Rightarrow D \perp F, A, C, E$ X

$U = \{F\} \Rightarrow D \perp A, C, E$ X

$U = \{A\} \Rightarrow D \perp F, C, E | A$ X

$U = \{C\} \Rightarrow D \perp F, A, E | C$ X

$U = \{E\} \Rightarrow D \perp F, A, C | E$ X

$U = \{F, A\} \Rightarrow D \perp C, E | F, A$ X

$U = \{F, C\} \Rightarrow D \perp A, E | F, C$ X

$U = \{F, E\} \Rightarrow D \perp A, C | F, E$ X

$U = \{A, C\} \Rightarrow D \perp F, E \mid A, C$ ✗

$U = \{A, E\} \Rightarrow D \perp F, C \mid A, E$ ✗

$U = \{C, E\} \Rightarrow D \perp F, A \mid C, E$ ✗

$U = \{F, A, C\} \Rightarrow D \perp E \mid F, A, C$ ✗

$U = \{A, C, E\} \Rightarrow D \perp F \mid A, C, E$ ✓

Step 5: $U \subseteq \{F, A, C, E, D\}$ such that $B \perp \{F, A, C, E, D\} \setminus U$

$U = \{\phi\} \Rightarrow B \perp F, A, C, E, D$ ✗

$U = \{F\} \Rightarrow B \perp A, C, E, D \mid F$ ✗

$U = \{A\} \Rightarrow B \perp F, C, E, D \mid A$ ✗

$U = \{C\} \Rightarrow B \perp F, A, E, D \mid C$ ✗

$U = \{E\} \Rightarrow B \perp F, A, C, D \mid E$ ✗

$U = \{D\} \Rightarrow B \perp F, A, C, E \mid D$ ✗

$U = \{F, A\} \Rightarrow B \perp C, E, D \mid F, A$ ✗

$U = \{F, C\} \Rightarrow B \perp A, E, D \mid F, C$ ✗

$U = \{F, E\} \Rightarrow B \perp A, C, D \mid F, E$ ✗

$U = \{F, D\} \Rightarrow B \perp A, C, E \mid F, D$ ✗

$U = \{A, C\} \Rightarrow B \perp F, E, D \mid A, C$ ✗

$U = \{A, E\} \Rightarrow B \perp F, C, D \mid A, E$ ✗

$U = \{A, D\} \Rightarrow B \perp F, C, E \mid A, D$ ✗

$U = \{C, E\} \Rightarrow B \perp F, A, D \mid C, E$ ✗

$U = \{C, D\} \Rightarrow B \perp F, A, E \mid C, D$ ✗

$U = \{E, D\} \Rightarrow B \perp F, A, C \mid E, D$ ✗

$U = \{F, A, C\} \Rightarrow B \perp E, D \mid F, A, C$ ✗

$U = \{A, C, F\} \Rightarrow B \perp F, D \mid A, C, E$ ✗

$U = \{C, E, D\} \Rightarrow B \perp F, A \mid C, E, D$ ✗

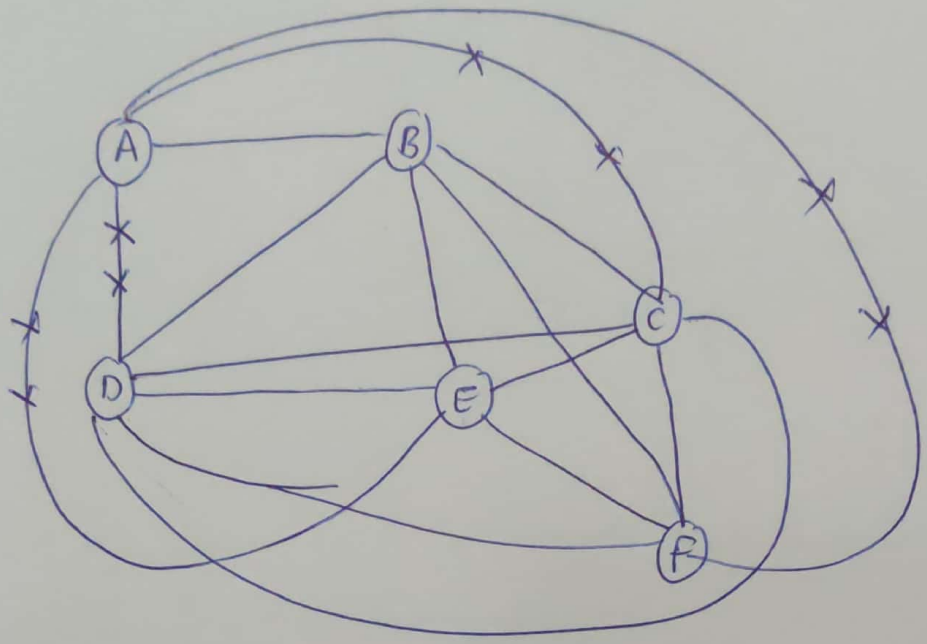
$U = \{E, D, F\} \Rightarrow B \perp A, C \mid E, D, F$ ✗

$U = \{F, A, C, E\} \Rightarrow B \perp D \mid F, A, C, E$ ✗

$U = \{A, C, E, D\} \Rightarrow B \perp F \mid A, C, E, D$ ✓

(6) (16)

The skeleton of the fully connected graph of the given structure is



Lets take the edge A – B

A ⊥ B X

A ⊥ B | D, E, F, C X

Lets consider neighbours of A

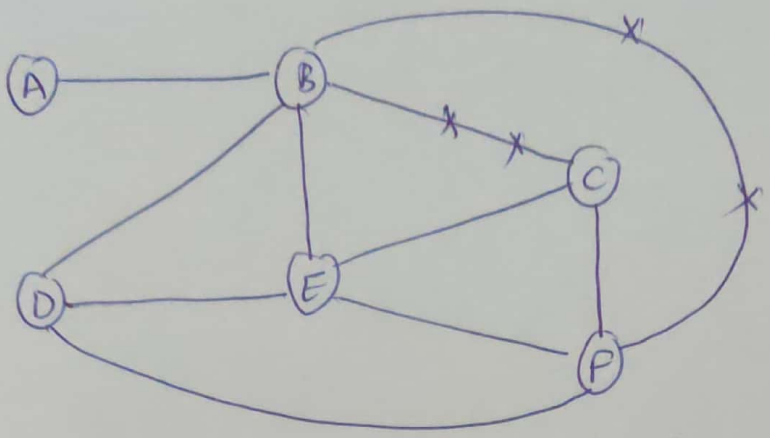
A ⊥ C | ϕ ✓

A ⊥ D | B ✓

A ⊥ E | B ✓

A ⊥ F | B ✓

Note: X represents the deletion of the edge



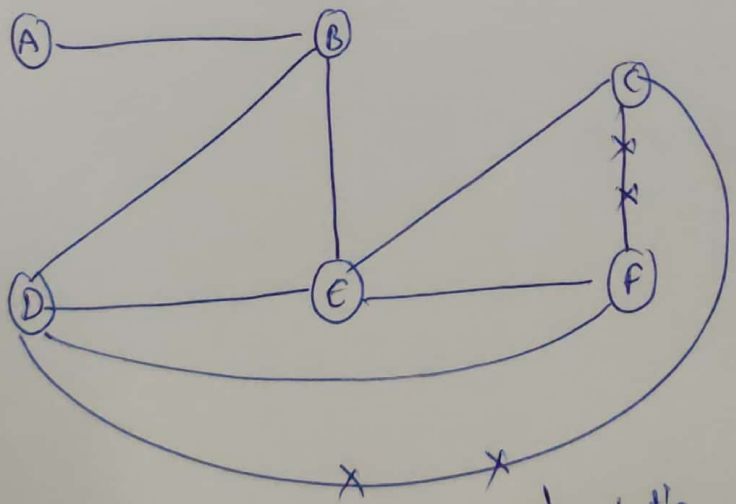
Let's consider the neighbours of B.

$B \perp C \mid \phi$ ✓

$B \perp D \mid A, E, C, f$ ✗

$B \perp E$ ✗

$B \perp F \mid E$ ✓



Let's take C-D edge

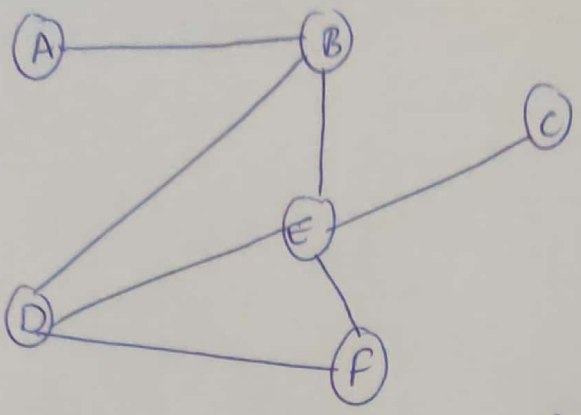
$C \perp D$ ✓

Let's take neighbours of C:

$C \perp E \mid \phi$ ✗

$C \perp E \mid A, B, D, E, f$ ✗

$C \perp F \mid E$ ✓

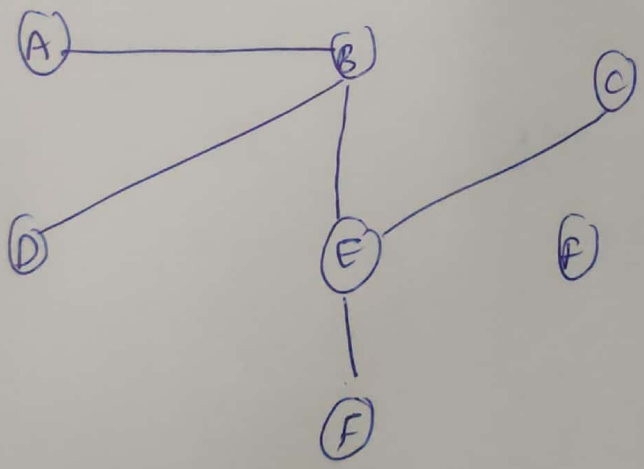


Lets take neighbours of D:

$D \perp B \mid A, E, F, C$ ✗

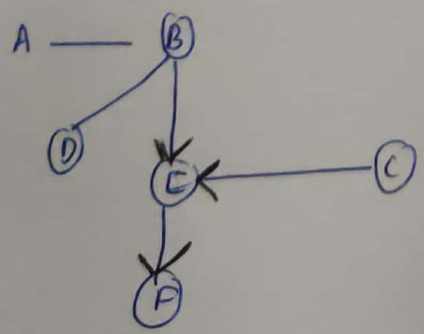
$D \perp E \mid B$ ✓

$D \perp F \mid B$ ✓



$E \perp f$ ✗

So, final skeleton is



A-B-D ✗

D-B-E ✗

B-E-C ✓

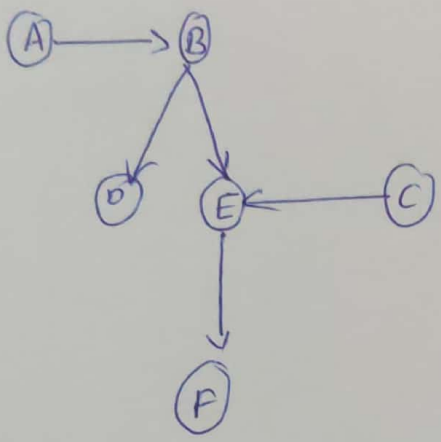
F-E-C ✗

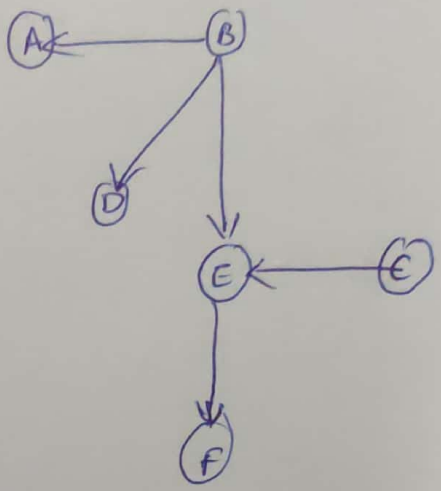
B-E-F ✗

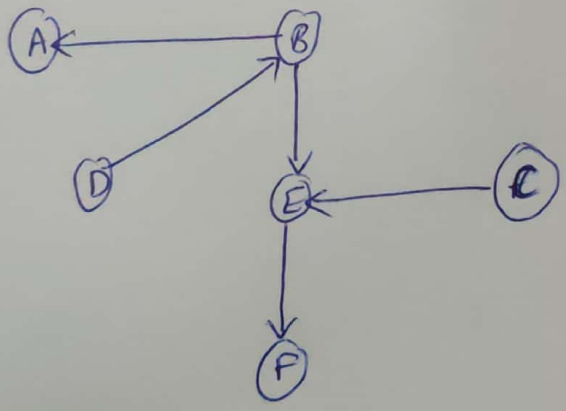
A-B-E ✗

Keeping the immoralities like the given structure.

The possible 1-equivalent structures are three as shown below (2)







7

(a) $P(SAT = low) = 0.725$

(b) $P(SAT = low \mid Difficulty = difficult)$

$= 0.725$

(c) $P(SAT = low \mid Grade = a)$

$= 0.4901$

d) $P(SAT = low \mid Grade = a, Difficulty = difficult)$

$= 0.3419$

e) $P(SAT = low \mid Intelligence = low)$

$= 0.95$

f) $P(SAT = low \mid Grade = a, Difficulty = difficult, Intelligence = low)$

$= 0.95$

8)
DATASET: HEPATITIS

The link for the dataset that I have chosen is**:** **https://www.openml.org/d/55**

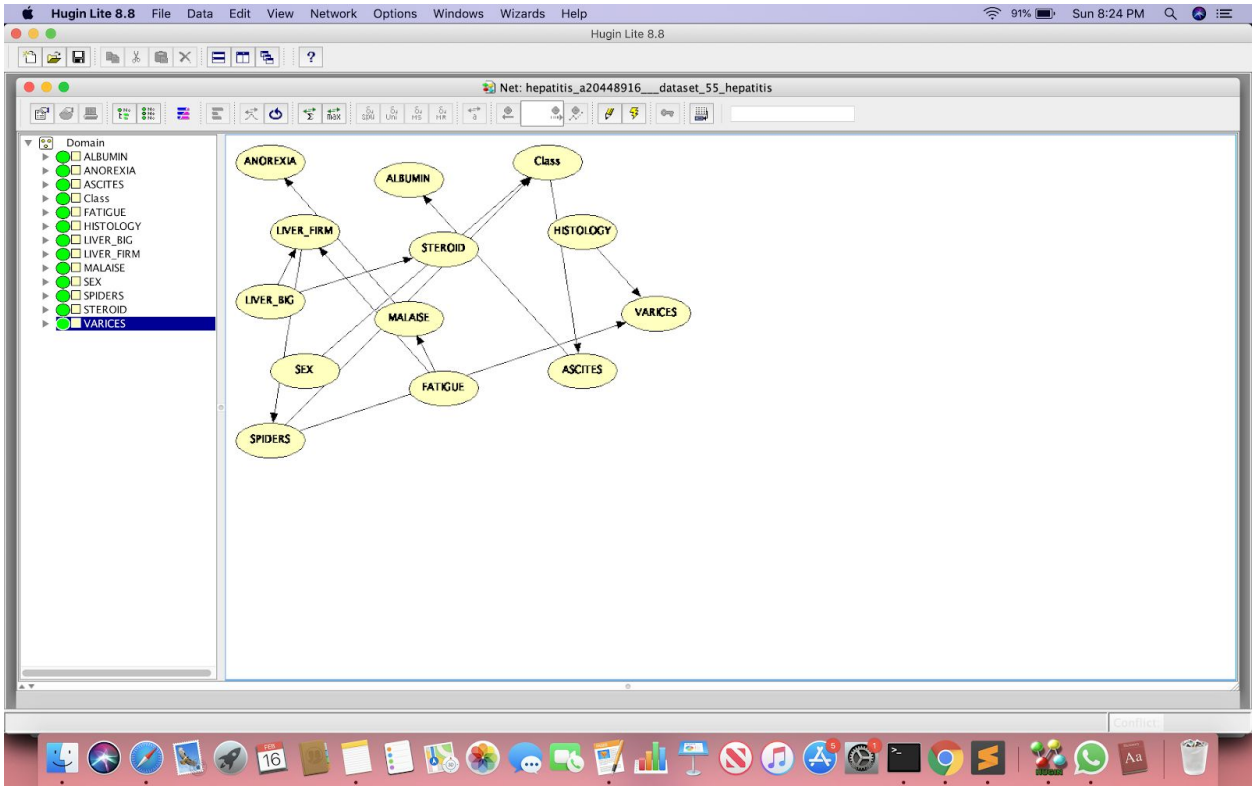
Below columns are deleted from the original set as these attributes have a very wide range of numeric values.

1) Alk_phosphate
2) bilirubin
3) Protime

Few attribute values have been modified as it has a wide range of values.

1) albumin

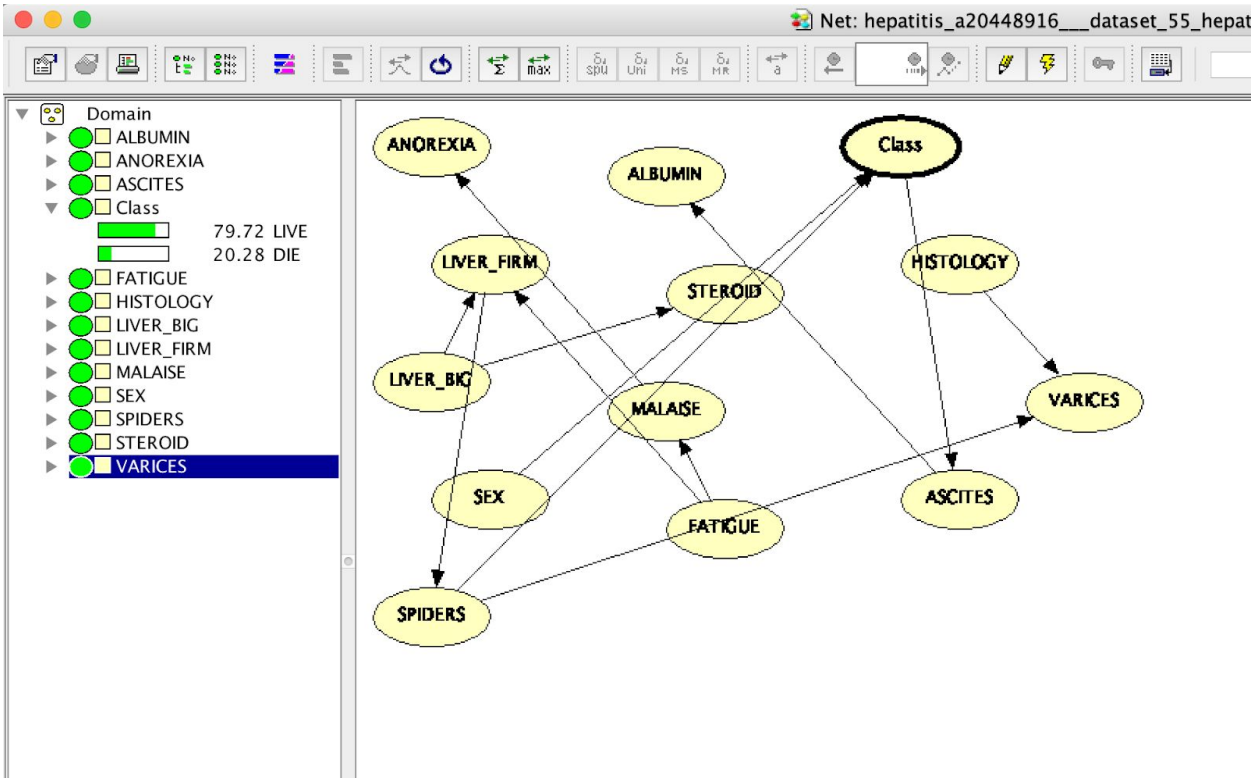
Below is the screenshot of the structure obtained after learning through Hugin.



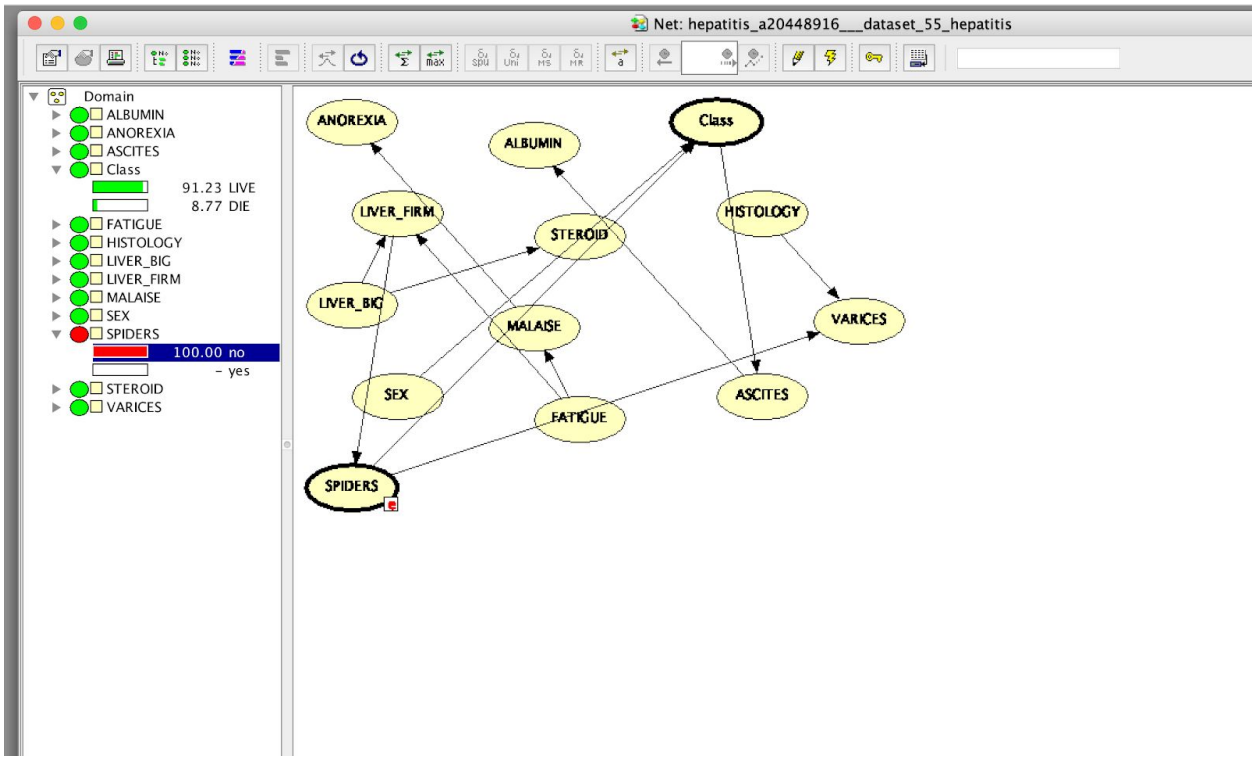
Probability Queries:

1) P(Class=Live) = 0.7972

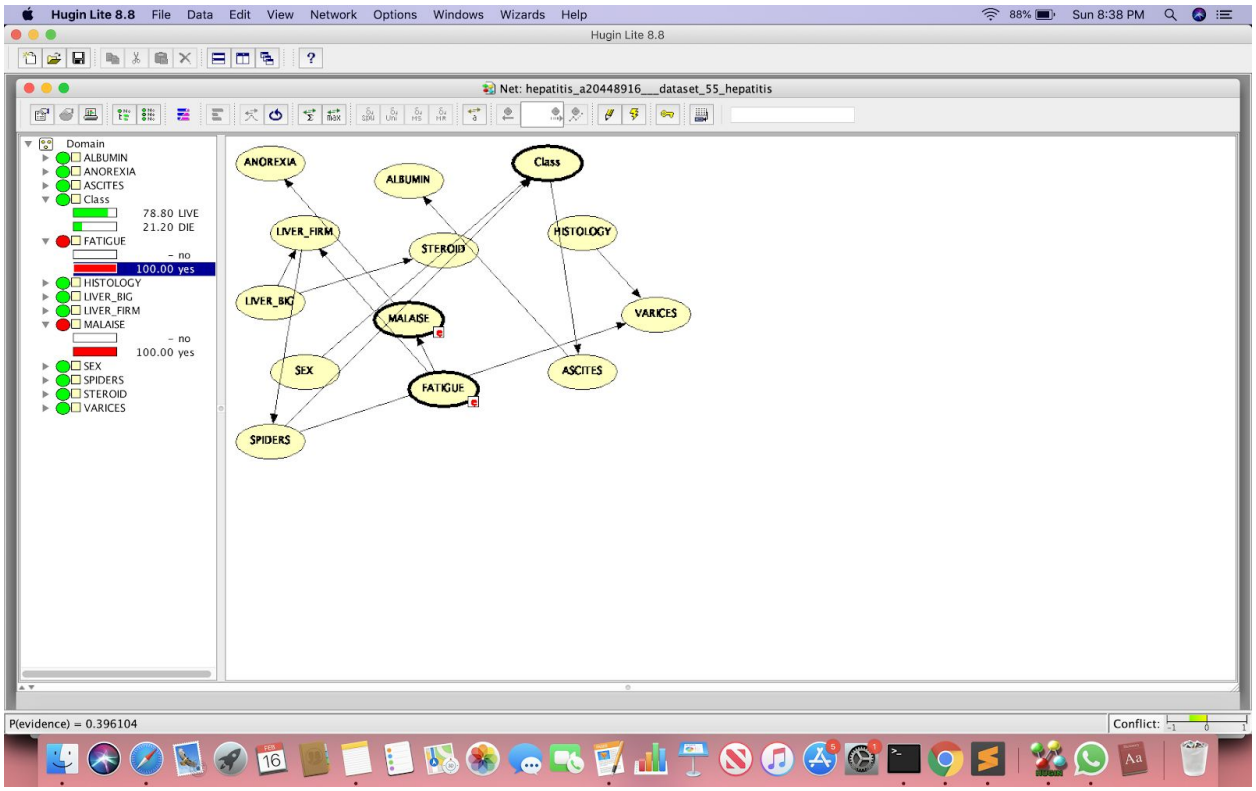
P(Class=Die) = 0.2028



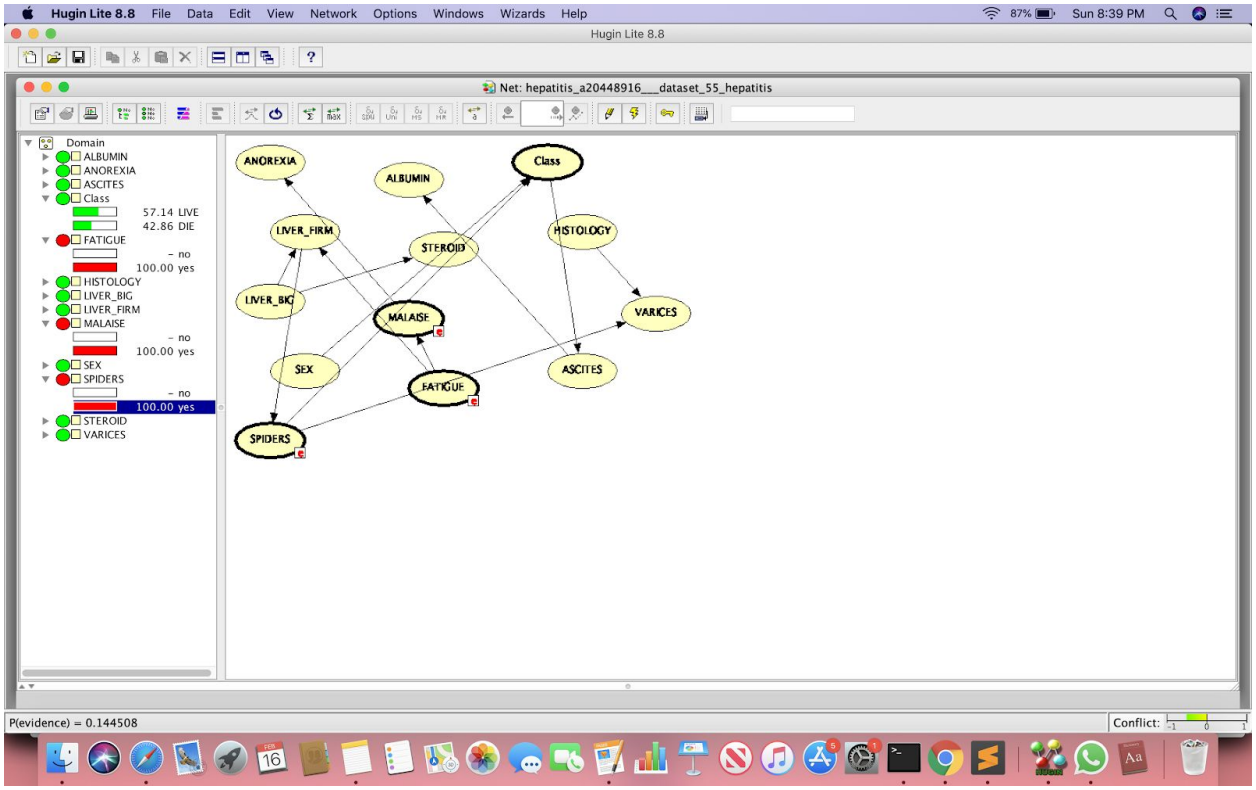
2) P(Class=Live | Spiders=no) = 0.9123



3) P(Class=Live | fatigue=yes, malaise=yes) = 0.788



4) P(Class= Live| fatigue=yes, malaise=yes, spiders= yes) = 0.5714



5) P(Class= Die|anorexia=no, varices=no) = 0.1865