CS480 – Assignment 5
Assigned: Friday, 10/18/2019
Due: 9:59pm (Chicago) on Sunday, 10/27/2019
Please submit your solutions through blackboard assignment page.

1. Convert each of the following FOL sentences into CNF form.

- **a.** $\forall x\ P(x) \Rightarrow Q(x)$
- **b.** $\forall x \forall y\ P(x,y) \Rightarrow Q(x)$
- **c.** $\exists x\ P(x) \wedge Q(x)$
- **d.** $\exists x \exists y\ P(x,y) \wedge Q(y,x)$
- **e.** $\exists x \forall y\ P(x,y)$
- **f.** $\forall x \exists y\ P(x,y)$
- **g.** $\forall x \forall y \exists z\ P(x,y,z)$
- **h.** $\exists x \forall y \forall z\ P(x,y,z)$
- **i.** $\forall x(\exists y\ P(x,y) \wedge Q(y)) \Rightarrow R(x)$
- **j.** $\forall x(\forall y\ P(x,y) \Rightarrow Q(y)) \Rightarrow R(x)$

2. We are given the following pairs of FOL sentences. For each pair of sentences, provide a substitution to unify the sentences. If no such substitution exists, please write so.

- **a.** $P(x)$
- **b.** $P(A)$

- **c.** $P(x) \vee Q(x, A)$
- **d.** $P(B) \vee Q(x, A)$

- **e.** $P(x) \vee Q(A, x)$
- **f.** $P(x) \vee Q(A, B)$

- **g.** $P(x, A) \vee Q(A, x)$
- **h.** $P(B, y) \vee Q(y, B)$

- **i.** $P(x) \vee Q(F(x))$
- **j.** $P(A) \vee Q(F(A))$

- **k.** $P(x, A) \vee Q(F(x), x)$
- **l.** $P(B, y) \vee Q(F(B), B)$

- **m.** $P(x, A) \vee Q(F(x), x)$
- **n.** $P(B, y) \vee Q(F(A), A)$

o. $P(x, y) \vee Q(F(A), B)$

p. $P(x, y) \vee Q(x, y)$

q. $P(x, y) \vee Q(F(A), A)$

r. $P(x, y) \vee Q(x, y)$

s. $P(x, y) \vee Q(F(x), y)$

t. $P(z, y) \vee Q(z, y)$

3. We are given the following joint distribution for variables A, B, and C. Please compute the requested probabilities. Show each probability distribution as a table/vector. Feel free to use a calculator.

A	B	C	P(A, B, C)
T	T	T	0.014
T	T	F	0.126
T	F	T	0.012
T	F	F	0.048
F	T	T	0.392
F	T	F	0.168
F	F	T	0.144
F	F	F	0.096

a. P(A, C)

b. P(C)

c. P(A|C)

d. P(A, B | C)

e. P(B | A, C)

4. We are given random variables X_2, X_3, …, X_n, where n>2. (There is no X_1). Please answer the following questions.

a. Assuming all variables are binary, how many *independent* parameters are needed to represent

i. $P(X_2)$?

ii. $P(X_n)$?

iii. $P(X_2, X_3, \ldots, X_n)$?

iv. $P(X_2 \mid X_3, \ldots, X_n)$?

v. $P(X_2, X_3, \ldots, X_{n-1} \mid X_n)$?

b. Assuming the size of the domain of X_i is i for all $i \in \{2, 3, \ldots, n\}$, how many *independent* parameters are needed to represent

i. $P(X_2)$?
ii. $P(X_n)$?
iii. $P(X_2, X_3, \ldots, X_n)$?
iv. $P(X_2 \mid X_3, \ldots, X_n)$?
v. $P(X_2, X_3, \ldots, X_{n-1} \mid X_n)$?