ASSIGNMENT - 6
SOLUTIONS

QUESTION 1

a) MLE ESTIMATE : $\langle 10/30, 20/30 \rangle$

$= \langle 1/3, 2/3 \rangle$

b) $P(X_{next}|D) = \left\langle \frac{10+1}{30+2}, \frac{20+1}{30+2} \right\rangle$

$= \left\langle \frac{11}{32}, \frac{21}{32} \right\rangle$

ii) $p(\theta|D) = \text{Beta}(11, 21)$

c) i) $P(X_{next}|D) = \left\langle \frac{10+2}{30+5}, \frac{20+3}{30+5} \right\rangle$

$= \left\langle \frac{12}{35}, \frac{23}{35} \right\rangle$

ii) $p(\theta|D) = \text{Beta}(12, 23)$

— X —

QUESTION 2

a) MLE ESTIMATE $= \left\langle \frac{10}{30}, \frac{20}{30}, \frac{0}{30} \right\rangle$

$= \langle 1/3, 2/3, 0 \rangle$

QUESTION 2

b) i) $P(X_{next}|D) = \left\langle \frac{11}{33}, \frac{21}{33}, \frac{1}{33} \right\rangle$

ii) $p(\theta|D) = \text{dir}(11, 21, 1)$

c) i) $P(X_{next}|D) = \left\langle \frac{12}{39}, \frac{23}{39}, \frac{4}{39} \right\rangle$

ii) $p(\theta|D) = \text{dir}(12, 23, 4)$

— X —

QUESTION 3

a) MLE of $X = \left\langle \frac{60}{150}, \frac{90}{150} \right\rangle$

MLE for $Y|X$

↳ $Y|X=T = \left\langle \frac{30}{60}, \frac{0}{60}, \frac{30}{60} \right\rangle$

$= \langle 1/2, 0, 1/2 \rangle$

↳ $Y|X=F = \left\langle \frac{40}{90}, 0, \frac{50}{90} \right\rangle$

$= \langle 4/9, 0, 5/9 \rangle$

MLE for $Z|Y \Rightarrow$

↳ $Z|Y=R = \left\langle \frac{10}{70}, \frac{60}{70} \right\rangle$

↳ $Z|Y=G$ = undefined.

↳ $Z|Y=B = \left\langle \frac{80}{80}, \frac{0}{80} \right\rangle = \langle 1, 0 \rangle$

3(b) $P(X_{next}) = \left\langle \frac{61}{152}, \frac{91}{152} \right\rangle$

$P(Y_{next} | X=T) = \left\langle \frac{31}{63}, \frac{1}{63}, \frac{31}{63} \right\rangle$

$P(Y_{next} | X=F) = \left\langle \frac{41}{93}, \frac{1}{93}, \frac{51}{93} \right\rangle$

$P(Z_{next} | Y=R) = \left\langle \frac{11}{72}, \frac{61}{72} \right\rangle$

$P(Z_{next} | Y=G) = \left\langle \frac{1}{2}, \frac{1}{2} \right\rangle$

$P(Z_{next} | Y=B) = \left\langle \frac{81}{82}, \frac{1}{82} \right\rangle$

Q3(c) $|D'| = 12$ & P' is uniform

$P(X_{next}) = \left\langle \frac{60+6}{150+12}, \frac{90+6}{150+12} \right\rangle$

$= \left\langle \frac{66}{162}, \frac{96}{162} \right\rangle$

$P(Y_{next} | X=T) = \left\langle \frac{30+2}{60+6}, \frac{0+2}{60+6}, \frac{30+2}{60+6} \right\rangle$

$= \left\langle \frac{32}{66}, \frac{2}{66}, \frac{32}{66} \right\rangle$

$P(Y_{next} | X=F) = \left\langle \frac{40+2}{90+6}, \frac{0+2}{90+6}, \frac{50+2}{90+6} \right\rangle$

$= \left\langle \frac{42}{96}, \frac{2}{96}, \frac{52}{96} \right\rangle$

Q3(c) contd.

$P(Z_{next} | Y=R) = \left\langle \frac{10+2}{70+4}, \frac{60+2}{70+4} \right\rangle$

$= \left\langle \frac{12}{74}, \frac{62}{74} \right\rangle$

$P(Z_{next} | Y=G) = \left\langle \frac{0+2}{0+4}, \frac{0+2}{0+4} \right\rangle$

$= \left\langle \frac{1}{2}, 1/2 \right\rangle$

$P(Z_{next} | Y=B) = \left\langle \frac{80+2}{80+4}, \frac{0+2}{80+4} \right\rangle$

$= \left\langle \frac{82}{84}, \frac{2}{84} \right\rangle$

—— X ——

Question 4

a) counts (X)

$(X=0) \Rightarrow 2 + P(x=0|Y=1, Z=0) + P(x=0|Z=0) + P(x=0|Y=1)$

$(X=1) \Rightarrow 0 + P(x=1|Y=1, Z=0) + P(x=1|Z=0) + P(x=1|Y=1)$

Now,

$P(X|Y=1, Z=0) \propto P(X) \cdot P(Y=1|X) \cdot P(Z=0|Y=1)$

$\Rightarrow P(X|Y=1, Z=0) = \langle 0.143, 0.857 \rangle$

$P(X|Z=0) \propto P(X) \sum_Y P(Y|X) \cdot P(Z=0|Y)$

X	Y	$P(Y\|X) \cdot P(Z=0\|Y) = f(x)$	
0	0	0.8×0.7	$\searrow$ 0.62
0	1	0.2×0.3	$\nearrow$
1	0	0.2×0.7	$\searrow$ 0.38
1	1	0.8×0.3	$\nearrow$

$P(X|Z=0) \propto P(X) \cdot f(x)$

$= P(X|Z=0)$

X	$P(X\|Z=0)$
0	$0.62 \times 0.4 = 0.248$
1	$0.38 \times 0.6 = 0.228$

$P(X|Z=0) = \langle 0.52, 0.48 \rangle$

$P(X|Y=1) \propto P(X) \cdot P(Y=1|X) \cdot \sum_Z P(Z|Y=1)$

$P(X|Y=1) = \langle 0.143, 0.857 \rangle$

counts(X=0) $= 2 + 0.143 + 0.52 + 0.143$

$= 2.806$

counts(X=1) $= 0.857 + 0.48 + 0.857$

$= 2.194$

counts(Y, X)

↳ for $X=0, Y=0 \Rightarrow$

$1 + P(Y=0, X=0|X=0, Z=1) + P(Y=0, X=0|Z=0)$

$= 2.101$

↳ for $X=0, Y=1 \Rightarrow$

$P(Y=1, X=0|X=0, Z=1) + P(X=0, Y=1|Y=1, Z=0) + P(X=0, Y=1|Z=0) + P(X=0, Y=1|Y=1)$

$= 0.704$

↳ for $X=1, Y=0 \Rightarrow$

$P(X=1, Y=0|Z=0) = 0.176$

↳ for $X=1, Y=1$

$P(X=1, Y=1|Y=1, Z=0) + P(X=1, Y=1|Z=0) + P(X=1, Y=1|Y=1)$

$= 2.017$

counts(Z, Y)

↳ For $Y=0, Z=0 \Rightarrow$

$P(Z=0, Y=0 \mid X=0, Y=0) + P(Z=0, Y=0 \mid Z=0)$

$= 1.347$

↳ For $Y=0, Z=1 \Rightarrow$

$P(Y=0, Z=1 \mid X=0, Y=0) + P(Z=1, Y=0 \mid X=0, Z=1)$

$= 0.932$

↳ For $Y=1, Z=0 \Rightarrow$

$1 + P(Y=1, Z=0 \mid Z=0) + P(Y=1, Z=0 \mid Y=1)$

$= 1.653$

↳ For $Y=1, Z=1 \Rightarrow$

$P(Z=1, Y=1 \mid X=0, Z=1) + P(Z=1, Y=1 \mid Y=1)$

$= 1.068$

Q4(b) (Normalize the counts)

$P(X) = \langle 0.5614, 0.4386 \rangle$

$P(Y \mid X=0) = \langle 0.749, 0.251 \rangle$

$P(Y \mid X=1) = \langle 0.080, 0.920 \rangle$

$P(Z \mid Y=0) = \langle 0.59, 0.41 \rangle$

$P(Z \mid Y=1) = \langle 0.61, 0.39 \rangle$