

## CS 480 – Assignment 5 solutions

1. Convert each of the following FOL sentences into CNF form.

a.  $\forall x P(x) \Rightarrow Q(x)$

$$\neg P(x) \vee Q(x)$$

b.  $\forall x \forall y P(x,y) \Rightarrow Q(x)$

$$\forall x \forall y (\neg P(x,y) \vee Q(x))$$

$$\neg P(x,y) \vee Q(x)$$

OR

$$\forall x \neg (\forall y P(x,y) \vee Q(x))$$

$$\forall x \exists y \neg P(x,y) \vee Q(x)$$

$$\neg P(x, F(x)) \vee Q(x)$$

c.  $\exists x P(x) \wedge Q(x)$

$$P(A) \wedge Q(A)$$

d.  $\exists x \exists y P(x,y) \wedge Q(y,x)$

$$P(A,B) \wedge Q(B,A)$$

e.  $\exists x \forall y P(x,y)$

$$P(A,y)$$

f.  $\forall x \exists y P(x,y)$

$$P(x, F(x))$$

g.  $\forall x \forall y \exists z P(x,y,z)$

$$P(x, y, F(x,y))$$

h.  $\exists x \forall y \forall z P(x, y, z)$

**$P(A, y, z)$**

i.  $\forall x (\exists y P(x, y) \wedge Q(y)) \Rightarrow R(x)$

$\forall x (\neg \exists y (P(x, y) \wedge Q(y)) \vee R(x)$

$\forall x (\forall y \neg (P(x, y) \wedge Q(y)) \vee R(x)$

**$\neg P(x, y) \vee \neg Q(y) \vee R(x)$**

j.  $\forall x (\forall y P(x, y) \Rightarrow Q(y)) \Rightarrow R(x)$

$\forall x (\neg \forall y P(x, y) \Rightarrow Q(y)) \vee R(x)$

$\forall x (\neg \forall y \neg P(x, y) \vee Q(y)) \vee R(x)$

$\forall x (\neg \neg \exists y \neg (\neg P(x, y) \vee Q(y))) \vee R(x)$

$\forall x (\exists y P(x, y) \wedge \neg Q(y)) \vee R(x)$

$(P(x, F(x)) \wedge \neg Q(F(x))) \vee R(x)$

**$(P(x, F(x)) \vee R(x)) \wedge (\neg Q(F(x)) \vee R(x))$**

2. We are given the following pairs of FOL sentences. For each pair of sentences, provide a substitution to unify the sentences. If no such substitution exists, please write so.

a.  $P(x)$

b.  $P(A)$

**Sol:  $\{x/A\}$**

c.  $P(x) \vee Q(x, A)$

d.  $P(B) \vee Q(x, A)$

**Sol:  $\{x/B\}$**

e.  $P(x) \vee Q(A, x)$

f.  $P(x) \vee Q(A, B)$

**Sol:  $\{x/B\}$**

g.  $P(x, A) \vee Q(A, x)$

h.  $P(B, y) \vee Q(y, B)$

**Sol:  $\{x/B, y/A\}$**

i.  $P(x) \vee Q(F(x))$

j.  $P(A) \vee Q(F(A))$

**Sol:  $\{x/A\}$**

k.  $P(x, A) \vee Q(F(x), x)$

l.  $P(B, y) \vee Q(F(B), B)$

**Sol:  $\{x/B, y/A\}$**

m.  $P(x, A) \vee Q(F(x), x)$

n.  $P(B, y) \vee Q(F(A), A)$

**Sol: Fail**

o.  $P(x, y) \vee Q(F(A), B)$

p.  $P(x, y) \vee Q(x, y)$

**Sol:  $\{x/F(A), y/B\}$**

q.  $P(x, y) \vee Q(F(A), A)$

r.  $P(x, y) \vee Q(x, y)$

**Sol:  $\{x/F(A), y/A\}$**

s.  $P(x, y) \vee Q(F(x), y)$

t.  $P(z, y) \vee Q(z, y)$

**Sol: Fail**

3. We are given the following joint distribution for variables A, B, and C. Please compute the requested probabilities. Show each probability distribution as a table/vector. Feel free to use a calculator.

A	B	C	P(A,B,C)
T	T	T	0.014
T	T	F	0.126
T	F	T	0.012
T	F	F	0.048
F	T	T	0.392
F	T	F	0.168
F	F	T	0.144
F	F	F	0.096

a.  $P(A, C)$

A	C	P(A,C)
T	T	$0.014 + 0.012 = \mathbf{0.026}$
T	F	$0.126 + 0.048 = \mathbf{0.174}$
F	T	$0.392 + 0.144 = \mathbf{0.536}$
F	F	$0.168 + 0.096 = \mathbf{0.264}$

b.  $P(C)$

C	P(C)
T	$0.014 + 0.012 + 0.392 + 0.144 = \mathbf{0.562}$
F	$0.126 + 0.048 + 0.168 + 0.096 = \mathbf{0.438}$

c.  $P(A|C)$

A	$P(A   C = T)$
T	$0.026 / 0.562 = \mathbf{0.046}$
F	$0.536 / 0.562 = \mathbf{0.954}$

A	$P(A   C = F)$
T	$0.174 / 0.438 = \mathbf{0.397}$
F	$0.264 / 0.438 = \mathbf{0.603}$

d.  $P(A, B \mid C)$

A	B	$P(A, B \mid C = T)$
T	T	$0.014 / 0.562 = \mathbf{0.025}$
T	F	$0.012 / 0.562 = \mathbf{0.021}$
F	T	$0.392 / 0.562 = \mathbf{0.698}$
F	F	$0.144 / 0.562 = \mathbf{0.256}$

A	B	$P(A, B \mid C = F)$
T	T	$0.126 / 0.438 = \mathbf{0.288}$
T	F	$0.048 / 0.438 = \mathbf{0.110}$
F	T	$0.168 / 0.438 = \mathbf{0.384}$
F	F	$0.096 / 0.438 = \mathbf{0.219}$

e.  $P(B \mid A, C)$

B	$P(B \mid A = T, C = T)$
T	$0.014 / 0.026 = \mathbf{0.538}$
F	$0.012 / 0.026 = \mathbf{0.462}$

B	$P(B \mid A = T, C = F)$
T	$0.126 / 0.174 = \mathbf{0.724}$
F	$0.048 / 0.174 = \mathbf{0.276}$

B	$P(B \mid A = F, C = T)$
T	$0.392 / 0.536 = \mathbf{0.731}$
F	$0.144 / 0.536 = \mathbf{0.269}$

B	$P(B \mid A = F, C = F)$
T	$0.168 / 0.264 = \mathbf{0.636}$
F	$0.096 / 0.264 = \mathbf{0.364}$

4. We are given random variables  $X_2, X_3, \dots, X_n$ , where  $n > 2$ . (There is no  $X_1$ ). Please answer the following questions.

a) Assuming all variables are binary, how many independent parameters are needed to represent

i.  $P(X_2)$ ?

**Sol: 1**

ii.  $P(X_n)$ ?

**Sol: 1**

iii.  $P(X_2, X_3, \dots, X_n)?$

**Sol:  $2^{n-1} - 1$**

iv.  $P(X_2 \mid X_3, \dots, X_n)?$

**Sol:  $2^{n-2}$**

v.  $P(X_2, X_3, \dots, X_{n-1} \mid X_n)?$

**Sol:  $2 * (2^{n-2} - 1)$**

b) Assuming the size of the domain of  $X_i$  is  $i$  for all  $i \in \{2, 3, \dots, n\}$ , how many independent parameters are needed to represent

i.  $P(X_2)?$

**Sol: 1**

ii.  $P(X_n)?$

**Sol:  $n - 1$**

iii.  $P(X_2, X_3, \dots, X_n)?$

**Sol:  $n! - 1$**

iv.  $P(X_2 \mid X_3, \dots, X_n)?$

**Sol:  $n! / 2$**

v.  $P(X_2, X_3, \dots, X_{n-1} \mid X_n)?$

**Sol:  $n * ((n-1)! - 1)$**