Signals and Systems

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References

1. Haykin S; Signals and Systems, Wiley, 1999
2. Oppenheium, Willisky and Nawab; Signals and Systems (2e), PUI, 1997.

What is a signal?

A signal is formally defined as a *function of one or more variable that conveys information on the nature of a physical phenomenon*.

One dimensional signal: function of a single variable.

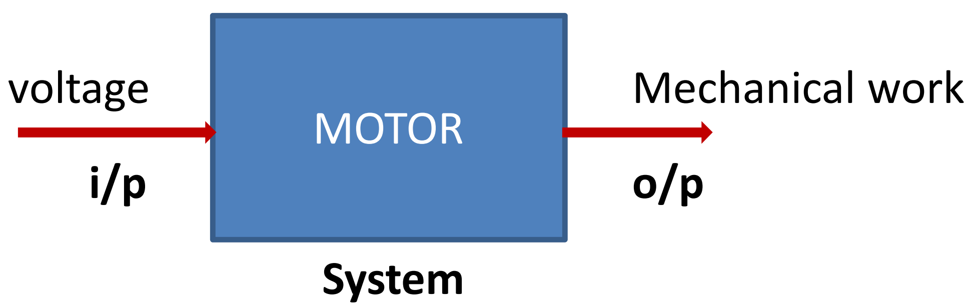
Eg. Speech signal

Two dimensional signal: functions of two variables.

Eg. Image

What is a system?

A system is formally defined as *an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals*.



Application Areas

- Control
- Communications
- Signal Processing

Control Applications

- Industrial control and automation (Control the velocity or position of an object)
- Examples: Controlling the position of a valve or shaft of a motor
- Important Tools:
  - Time-domain solution of differential equations
  - Transfer function (Laplace Transform)
  - Stability

Communication Applications

- Transmission of information (signal) over a channel
- The channel may be free space, coaxial cable, fiber optic cable
- A key component of transmission: Modulation (Analog and Digital Communication)

Signal Processing Applications

- Signal processing: Application of algorithms to modify signals in a way to make them more useful.
- Goals:
  - Efficient and reliable transmission, storage and display of information
  - Information extraction and enhancement
- Examples:
  - Speech and audio processing
  - Multimedia processing (image and video)
  - Underwater acoustic
  - Biological signal analysis

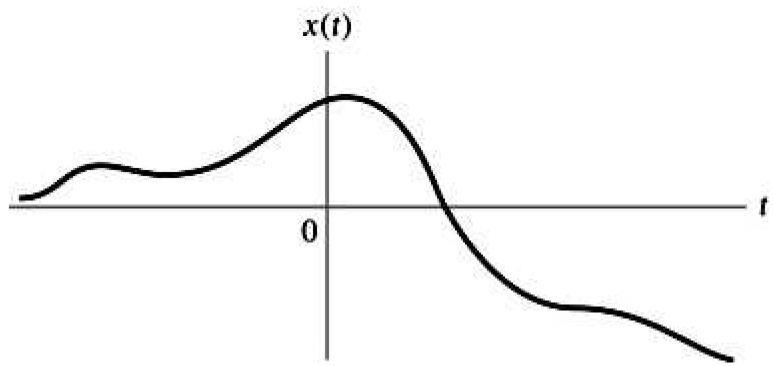
Classification of signals

1. Continuous Time (CT)and Discrete Time (DT)signals
2. Even and odd signals
3. Periodic signals and non periodic signals
4. Deterministic signals and random signals
5. Energy and Power signals

CT and DT signals

CT signal

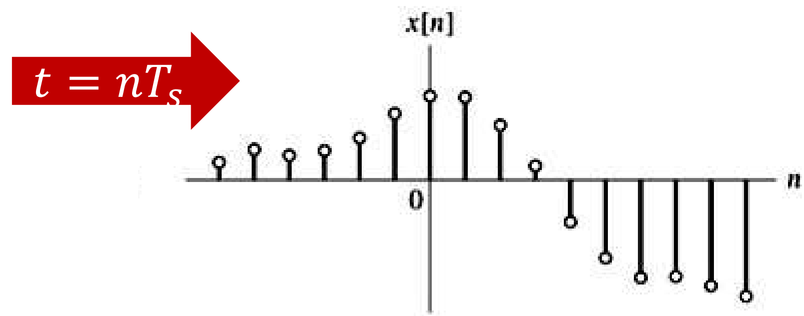
A signal is said to be continuous time signal if it is defined for all time t.



CT and DT signals

DT signals

A discrete time signal is defined only at discrete instants of time.



(b)

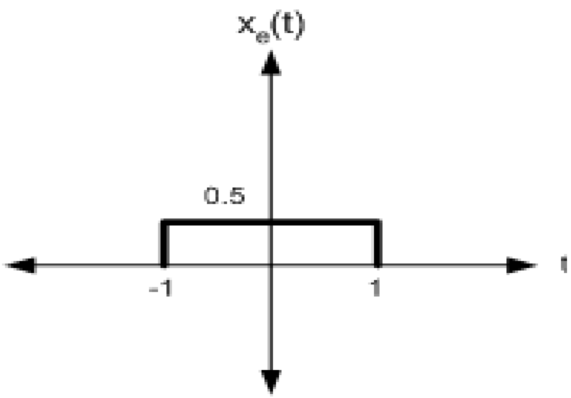
$$x(n) = x(nT_s), \qquad n = 0, \pm 1, \pm 2, \pm 3$$

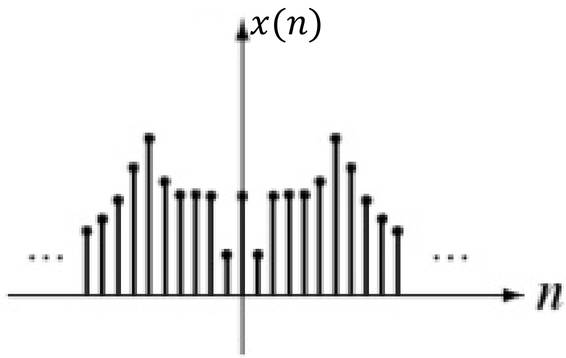
T_s *is the sampling period*

Even and Odd signals

Even signals:

$$x(-t) = x(t) \;\; for\ all\ t$$
$$x(-n) = x(n) \;\; for\ all\ n$$





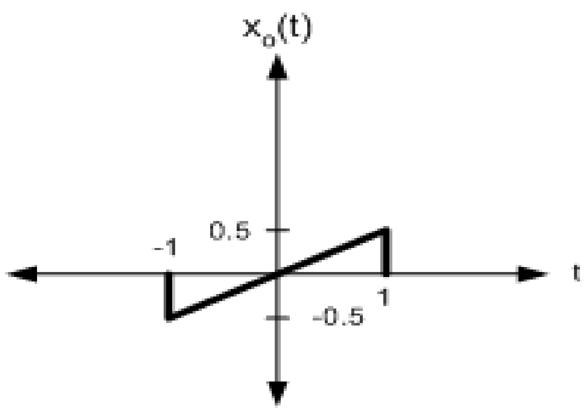
Symmetric about vertical axis

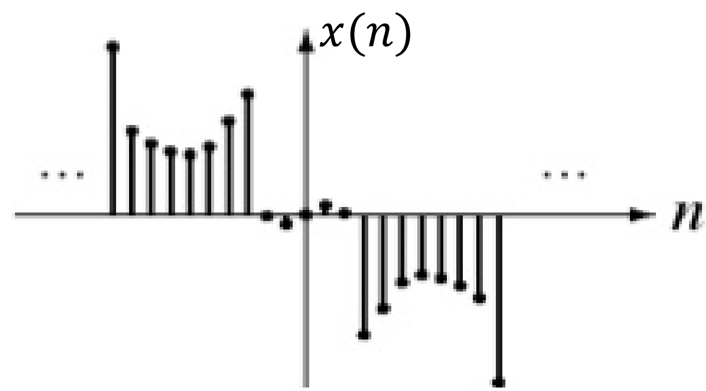
Even and Odd signal

Odd signals:

$$x(-t) = -x(t) \;\; for\ all\ t$$

$$x(-n) = -x(n)\ for\ all\ n$$





Antisymmetric about origin

Example(even and odd signal)

Consider the signal $x(t) = \begin{cases} \sin\frac{\pi t}{T}, -T \leq t \leq T \\ 0, otherwise \end{cases}$.

Is the signal x(t) an even or an odd function of time *t?*

Ans:

Replacing *t* with *–t* yields.

$$x(-t) = \begin{cases} \sin\frac{-\pi t}{T}, -T \leq t \leq T \\ 0, otherwise \end{cases}$$

$$= \begin{cases} -\sin\frac{\pi t}{T}, -T \leq t \leq T \\ 0, otherwise \end{cases}$$

$$= -x(t) \quad for \;\; all \; t$$

x(t) is an odd signal

Even-Odd decomposition of a signal

Consider an arbitrary signal x(t)

Let $x(t) = x_e(t) + x_0(t)$ (1)

$$x_e(-t) = x_e(t)$$

$$x_0(-t) = -x_0(t)$$

Substitute t = -t in (1)

$$x(-t) = x_e(-t) + x_0(-t)$$

$$= x_e(t) - x_0(t) \quad (2)$$

Solving for $x_e(t)$ [(1) + (2)]

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

Solving for $x_0(t)$ [(1) – (2)]

$$x_0(t) = \frac{1}{2}[x(t) - x(-t)]$$

Even-Odd decomposition of a signal

Hyperbolic sine and cosine function

$$sinhx = \frac{e^{x} - e^{-x}}{2}$$

$$coshx = \frac{e^{x}+e^{-x}}{2}$$

Odd and even functions

$$\sinh(-x) = -sinhx$$

$$\cosh(-x) = coshx$$

Even-Odd decomposition of a signal

Find the even and odd components of the signal

$$x(t) = e^{-2t} \cos t$$

Ans:

Replacing t with $-t$ yields

$$x(-t) = e^{2t} \cos(-t)$$
$$= e^{2t} \cos(t)$$

Even component

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$
$$= \frac{1}{2}\left(e^{-2t} \cos t + e^{2t} \cos(t)\right)$$
$$= \cos t \left(\frac{e^{-2t} + e^{2t}}{2}\right)$$
$$= \cosh(2t) \cos t$$

Even-Odd decomposition of a signal

Odd component

$$x_0(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$= \frac{1}{2}(e^{-2t}\cos t - e^{2t}\cos(t))$$

$$= \cos t\left(\frac{e^{-2t} - e^{2t}}{2}\right)$$

$$= -\sinh(2t)\cos t$$

Even-Odd decomposition of a signal

Complex valued signal?

Conjugate Symmetry

A complex valued signal x(t) is said to be conjugate symmetric if

$$x(-t) = x^*(t) \qquad (1)$$

Let

$$x(t) = a(t) + jb(t) \qquad (2)$$

$$x^*(t) = a(t) - jb(t) \qquad (3)$$

Even-Odd decomposition of a signal

Substitute (2) and (3) in (1)

$$a(-t) + jb(-t) = a(t) - jb(t)$$

Equating real part

$$a(-t) = a(t)$$

Equation imaginary part

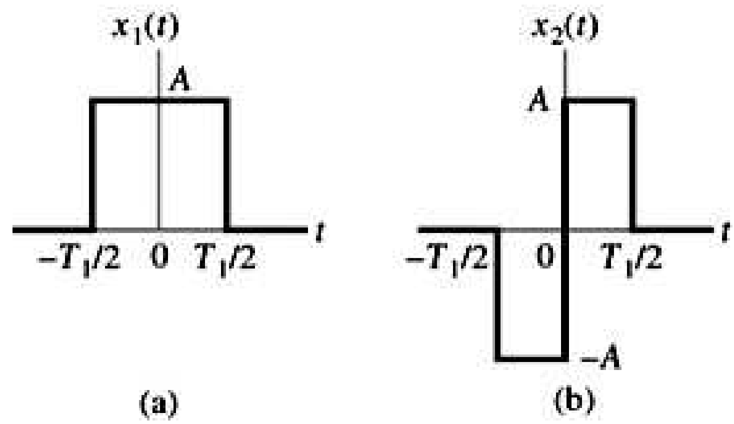
$$b(-t) = -b(t)$$

Inference:

A complex valued signal x(t) is conjugate symmetric if its real part is even and its imaginary part is odd.

Example

The signals $x_1(t)$and $x_2(t)$ shown in Figs constitute the real and imaginary parts, respectively, of a complex valued signal x(t). What form of symmetry does x(t) have?



Ans:

Conjugate symmetry

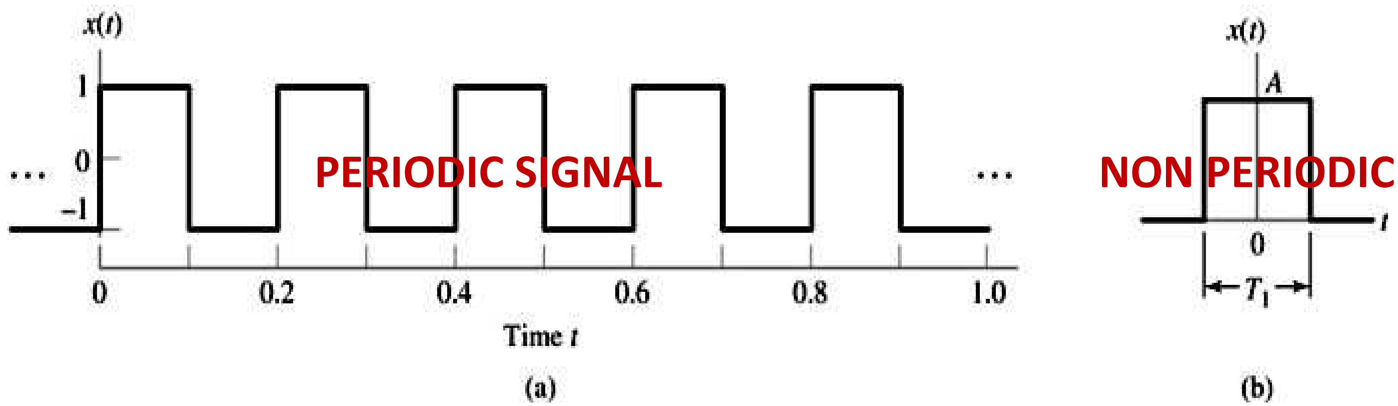
Periodic and non-periodic signals

A CT periodic signal is a function of time that satisfies the condition

$$x(t) = x(t + T) \;\; for\ all\ t$$

Where T is a positive constant

Angular frequency $\omega = 2\pi f$ rad/sec



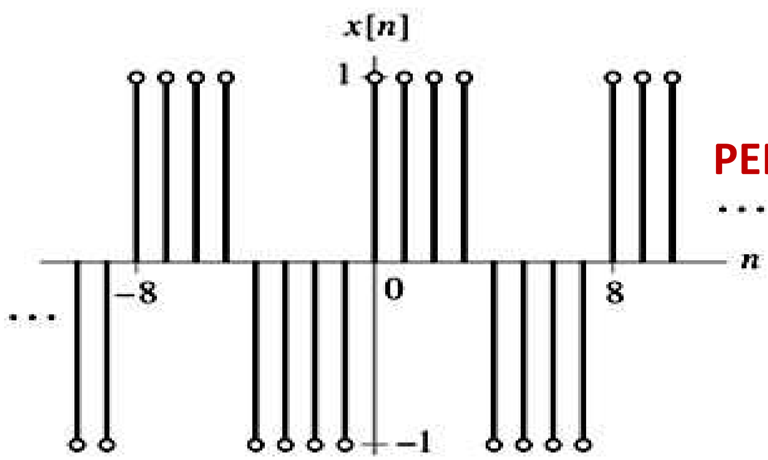
Periodic and non-periodic signals

A DT periodic signal is a function of time that satisfies the condition

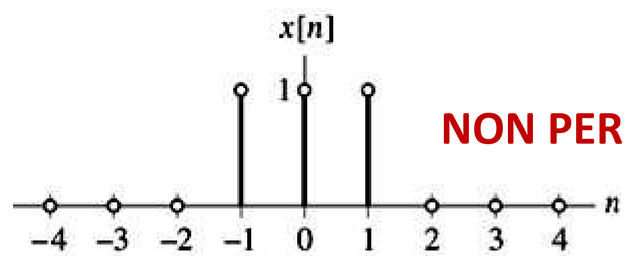
$$x(n) = x(n+N) \;\; for\ integer\ n$$

Where N is a +ve integer

Fundamental angular frequency $\Omega = \frac{2\pi}{N}$ rad



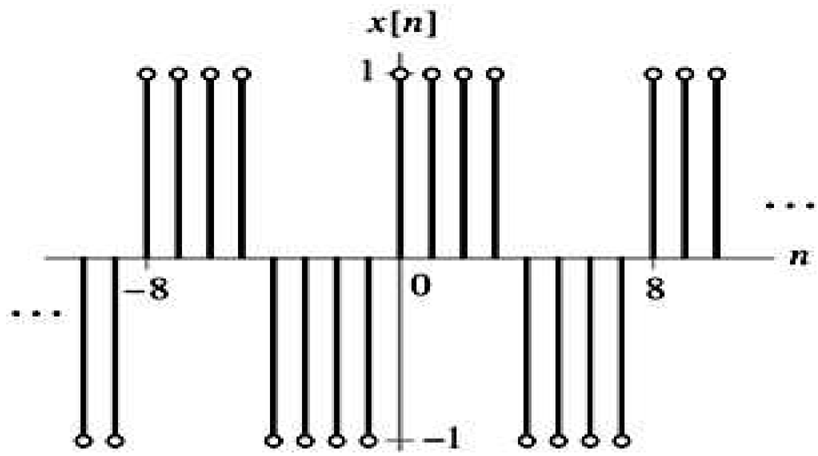
PERIODIC



NON PERIODIC

Example1

Find the fundamental frequency of the DT square wave shown in Fig.



Ans:

$$\Omega = \frac{2\pi}{8}$$
$$= \frac{\pi}{4} rad$$

A DT signal can be periodic iff $\frac{\Omega}{2\pi}$ is a rational number

Example2

Determine if the sinusoidal DT sequence is periodic.

$$x(n) = \cos(0.5n + \theta)$$

Ans:

$$x(n) = \cos(\Omega n + \theta)$$

$$\frac{\Omega}{2\pi} = \frac{0.5}{2\pi}$$

$$= \frac{1}{4\pi}$$

irrational

Aperiodic signal

Linear combination of two CT signals

Let $g(t) = x_1(t) + x_2(t)$

$x_1(t)$ is periodic with fundamental period T_1

$x_2(t)$is periodic with fundamental period T_2

g(t) periodic?

g(t) will be periodic iff $\frac{T_1}{T_2}$ is rational.

The fundamental period of g(t)

$$nT_1 = mT_2$$

Example

Determine if the g(t) is periodic. If yes find the fundamental period.

$$g(t) = 3\sin(4\,\pi t) + 7\cos(3\pi t)$$

Ans:

$$T_1 = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2}sec$$

$$T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}sec$$

$$\frac{T_1}{T_2} = \frac{m}{n} = \frac{3}{4}$$

$\frac{T_1}{T_2}$ is rational –periodic

Fundamental period

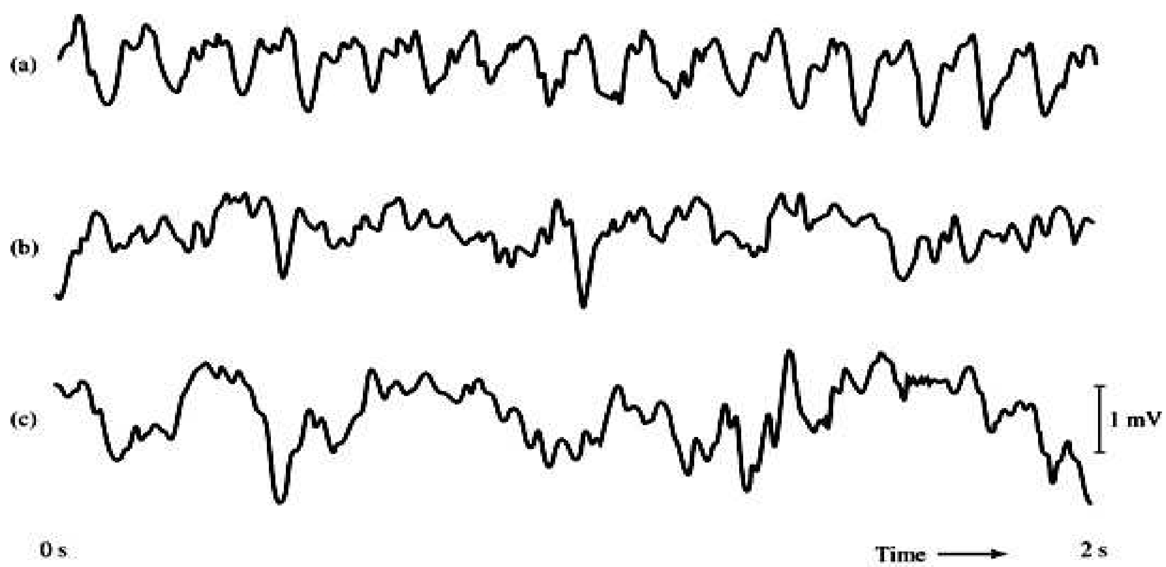
$$nT_1 or\ mT_2$$

2sec

Deterministic and Random signals

A *deterministic signal* is a signal about which there is no uncertainty with respect to its value at any time.

A *random signal* is a signal about which there is uncertainty before it occurs.



EEG signal

Energy signal and power signals

Instantaneous power

$$p(t) = \frac{v^2(t)}{R} \quad or\ p(t) = Ri^2(t)$$

If R=1Ω and x(t) represents current or voltage then instantaneous power is

$$p(t) = x^2(t)$$

Average power?

Average w.r.t to time- time averaged power

Total energy of a CT signal x(t)

$$E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

$$= \int_{-\infty}^{\infty} x^2(t)dt$$

Energy signal and power signals

Time averaged power or average power

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

If the signal is periodic with period T

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

DT signal

Total energy of a DT signal

$$E = \sum_{-\infty}^{\infty} x^2[n]$$

Energy signal and power signals

Average power

$$P = \lim_{N \to \infty} \frac{1}{2N} \sum_{-N}^{N} x^2[n]$$

Average power in a periodic signal x[n] with fundamental period N

$$P = \frac{1}{2N} \sum_{-N}^{N} x^2[n]$$

Energy signal iff $0 < E < \infty$

Power signal iff $0 < P < \infty$

Usually Periodic Signals are viewed as Power Signals and Non Periodic Signals as Energy Signals

Power signal has infinite energy

Energy signal has zero average power

Example1

Find the energy of the signal

$$x(n) = \begin{cases} n, 0 \leq n < 5 \\ 10 - n, 5 \leq n \leq 10 \\ 0, otherwisw \end{cases}$$

Ans:

$$E = \sum_{-\infty}^{\infty} x^2[n]$$
$$= \sum_{n=0}^{4} n^2 + \sum_{n=5}^{10} (10-n)^2$$
$$= [0 + 1 + 4 + 9 + 16] +$$
$$[5^2 + 4^2 + 3^2 + 2^2 + 1^2]$$
$$= 85J$$

Example2

Determine if x[n] is power or energy signal.

$$x(n) = \begin{cases} \sin \pi n, -4 \leq n \leq 4 \\ 0, otherwise \end{cases}$$

Ans:

$$E = \sum_{-\infty}^{\infty} x^2[n]$$

$$= \sum_{-4}^{4} sin^2 \pi n$$

$$= \sum_{-4}^{4} \frac{1 - \cos 2\pi n}{2}$$

$$= 0$$

P=0 aperiodic

Neither energy nor power signal

Example3

$$x(t) = \begin{cases} t, 0 \leq t \leq 1 \\ 2 - t, 1 \leq t \leq 2 \\ 0, otherwise \end{cases}$$

Energy or power signal?

Ans:

Aperiodic

$$E = \int_{-\infty}^{\infty} x^2(t)dt$$
$$= \int_0^1 t^2 dt + \int_1^2 (2-t)^2 dt$$
$$= \frac{1}{3} + \int_1^2 (4 - 4t + t^2) \quad dt$$
$$= \frac{2}{3} J$$

Finite energy –Energy signal

Tutorial

Find the even and odd components of each of the following signals.

a) $x(t) = \cos t + \sin t + \sin t \cos t$

b) $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$

c) $x(t) = 1 + t \cos t + t^2 \sin t + t^2 \sin t \cos t$

d) $x(t) = (1 + t^2)cos^3(10t)$

Tutorial

$$a)\quad x(t) = \cos t + \sin t + \sin t \cos t$$
$$= \cos t + \sin t\,(1 + \cos t)$$
$$x(-t) = \cos t - \sin t\,(1 + \cos t)$$
$$x_e(t) = \cos t$$
$$x_0(t) = \sin t\,(1 + \cos t)$$

$$b)\quad x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$$
$$x(-t) = 1 - t + 3t^2 - 5t^3 + 9t^4$$
$$x_e(t) = 1 + 3t^2 + 9t^4$$
$$x_0(t) = t + 5t^3$$

Tutorial

c) $x(t) = 1 + t\cos t + t^2 \sin t + t^3 \sin t \cos t$

$x(-t) = 1 - t\cos t - t^2 \sin t + t^3 \sin t \cos t$

$x_e(t) = 1 + t^3 \sin t \cos t$

$x_0(t) = t\cos t + t^2 \sin t$

d) $x(t) = (1 + t^3)cos^3(10t)$

$= cos^3(10t) + t^3 cos^3(10t)$

$x(-t) = cos^3(10t) - t^3 cos^3(10t)$

$x_e(t) = cos^3(10t)$

$x_0(t) = t^3 cos^3(10t)$

Tutorial

For each of the following signals, determine whether it is periodic, and if it is, find the fundamental period.

a) $x(t) = cos^2(2\pi t)$

b) $x(t) = sin^3(2t)$

c) $x(t) = e^{-2t}\cos(2\pi t)$

a) $x(t) = cos^2(2\pi t)$

$$x(t) = \frac{1+\cos 4\pi t}{2}$$

$$= 0.5 + 0.5\cos 4\pi t \quad \text{periodic}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2}\, s$$

Tutorial

b) $x(t) = sin^3(2t)$

$$sin^3(x) = \frac{3\sin x - \sin 3x}{4}$$

$$x(t) = \frac{3\sin 2t - \sin 6t}{4}$$

$$T_1 = \frac{2\pi}{2} = \pi s$$

$$T_2 = \frac{2\pi}{6} = \frac{\pi}{3} s$$

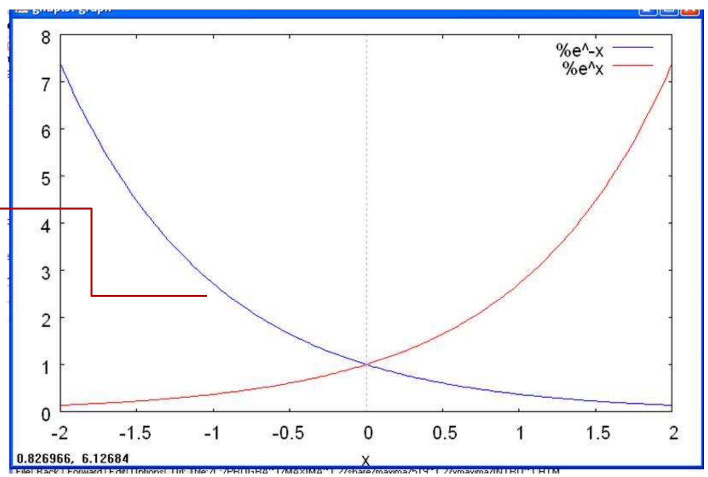
$\frac{T_1}{T_2} = \frac{m}{n} = \frac{3}{1}$ rational- x(t)periodic

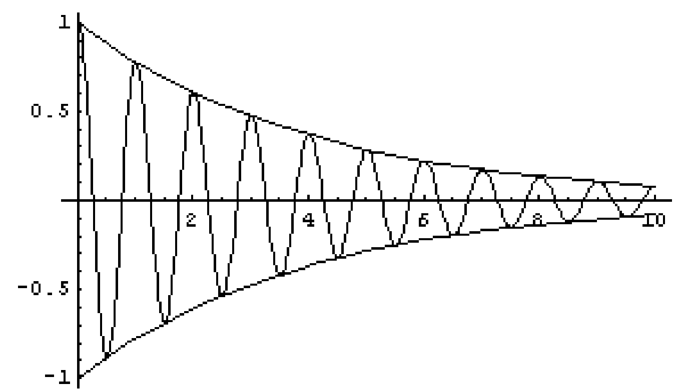
$$T = \pi s$$

Tutorial

c) $x(t) = e^{-2t} \cos 2\pi t$

e^{-2t}





Non-periodic

Tutorial

$$x[n] = (-1)^n$$

n	x[n]
0	1
1	-1
2	1
3	-1

Periodic with N=2 samples

Tutorial

1. $x[n] = (-1)^{n^2}$

n	n^2	x[n]
0	0	1
1	1	-1
2	4	1
3	9	-1
4	16	1

Periodic with N=2 samples

2. $x[n] = \cos 2n$

$\frac{\Omega}{2\pi} = \frac{1}{\pi}$ irrational –non periodic

Tutorial

Categorize each of the following signals as an energy signal or a power signal, and find the energy or time-averaged power.

1.$x(t) = 5\cos\pi t + \sin 5\pi t, -\infty < t < \infty$

Periodic or aperiodic?

$T_1 = 2$ and $T_2 = \frac{2}{5}$

$\frac{T_1}{T_2} = 5$ periodic

Fundamental period T=2

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

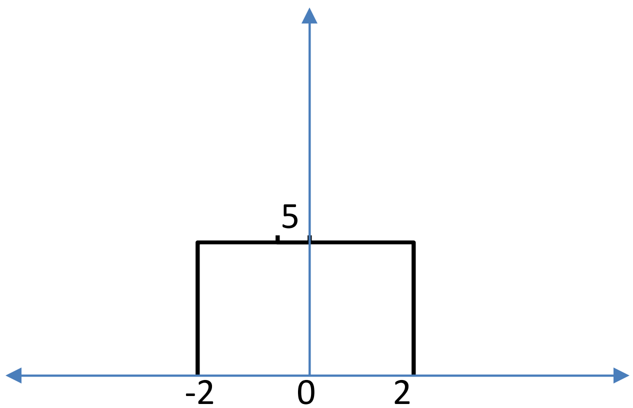
$$= \frac{1}{2}\int_{-1}^{1}(25cos^2\pi t + 10\cos\pi t \sin 5\pi t + sin^2 5\pi t)\, dt$$

Tutorial

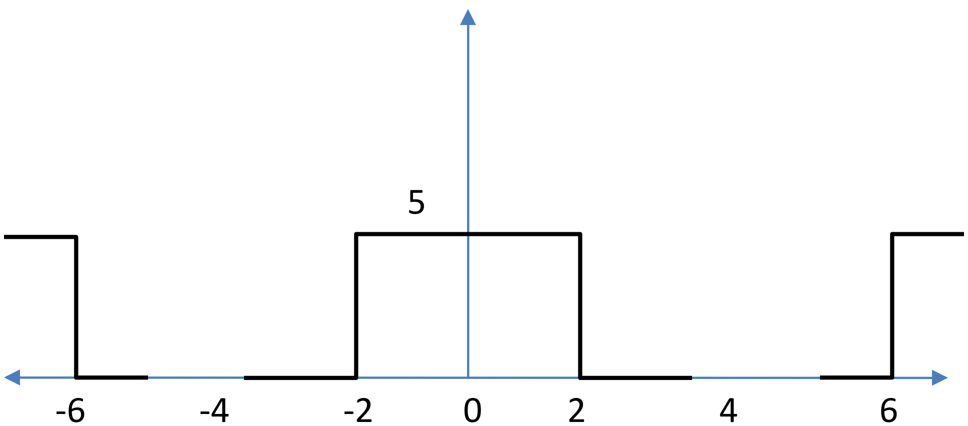
$$= \frac{25}{4}\int_{-1}^{1}(1+\cos 2\pi t)\,dt + 5\int_{-1}^{1}\cos \pi t \sin 5\pi t\,dt$$
$$+ \frac{1}{4}\int_{-1}^{1}(1-\cos 10\pi t)dt$$
$$= \frac{25}{2} + \frac{1}{2} + \frac{5}{2}\int_{-1}^{1}(\sin 4\pi t + \sin 6\pi t)dt$$
$$= \frac{25}{2} + \frac{1}{2}$$
$$= 13W$$

Tutorial

Classify the signals as energy or power signals



(a)



(b)

Tutorial

a)Aperiodic

$$E = \int_{-2}^{2} 5^2 dt$$
$$= 25 * 4 = 100J$$

Finite energy – energy signal

b)Periodic- T=8s

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$
$$= \frac{1}{8} \int_{-2}^{2} 5^2 dt$$
$$= \frac{100}{8} = 12.5W$$

Finite power- power signal

Basic operation on signals

Operations on dependent variables

1. Amplitude scaling
2. Addition
3. Multiplication
4. Differentiation
5. Integration

Operations in independent variables

1. Time scaling
2. Time shifting

Operations on dependent variables

Amplitude scaling

$$y(t) = cx(t)$$
$$y[n] = cx[n]$$

C is the scaling factor.

Eg. Amplifier

Addition

$$y_1(t) = x_1(t) + x_2(t)$$
$$y[n] = x_1[n] + x_2[n]$$

Eg. Audio mixer

Multiplication

$$y_1(t) = x_1(t)\, x_2(t)$$
$$y[n] = x_1[n]x_2[n]$$

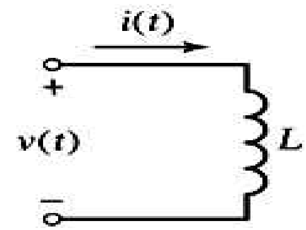
Eg. AM radio signal

Operations on dependent variables

Differentiation

$$y(t) = \frac{d}{dt}x(t)$$

Eg.



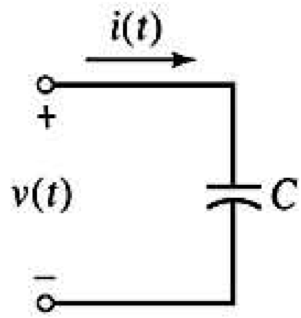
$$v(t) = L\frac{d}{dt}i(t)$$

Integration

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Operations on dependent variables

Eg.



$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

Operations on independent variables

Time scaling

$$y(t) = x(at)$$

If a>1 – signal compression

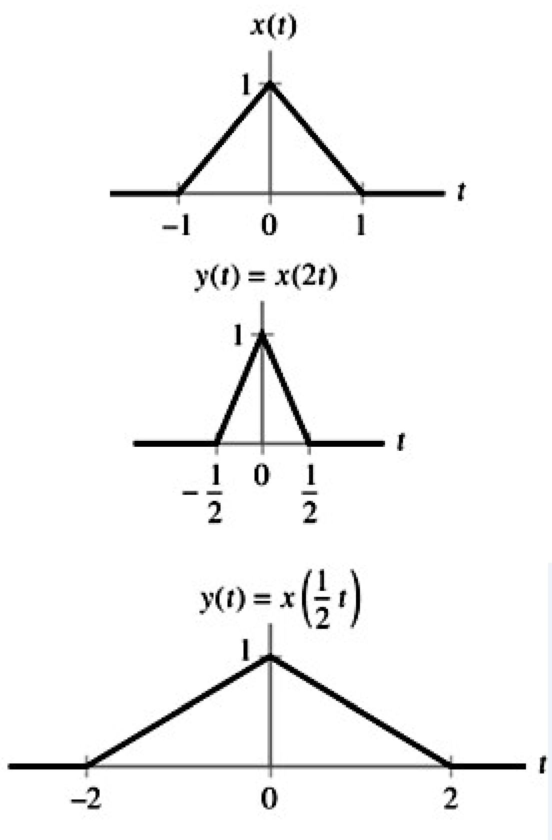
If 0<a<1- signal expansion

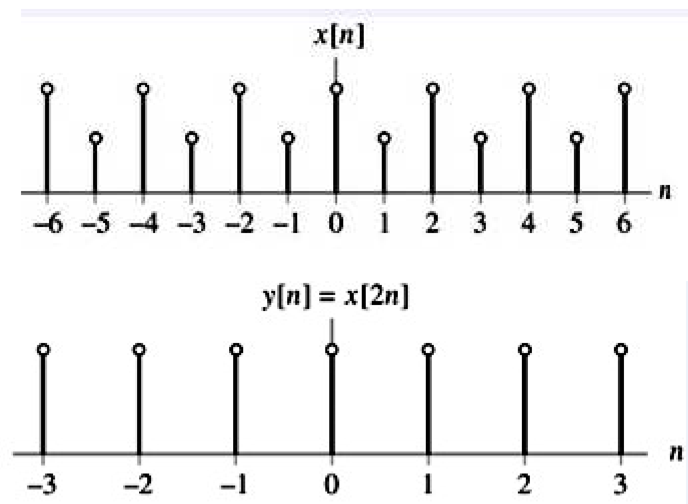
$$y[n] = x[kn], k > 0$$

k can have only integer values

k>1 –signal compression

Operations on independent variables





Compression of DT signal results in some loss of information

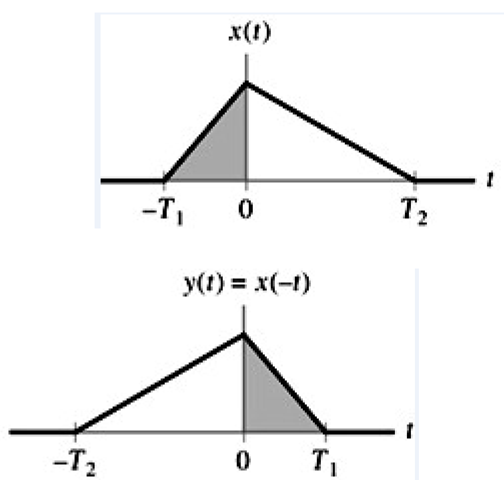
Operations on independent variables

Reflection

$$y(t) = x(-t)$$

$y(t)$ represents a reflected version of $x(t)$about t=0

$$y[n] = x[-n]$$



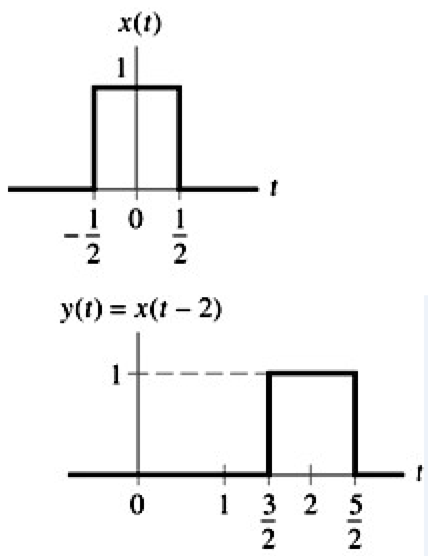
Operations on independent variables

Time shifting

$$y(t) = x(t - t_0)$$

$t_0 > 0$ right shift and $t_0 < 0$ left shift

Figure shows a rectangular pulse x(t) of unit amplitude and unit duration. Find $y(t) = x(t - 2)$.



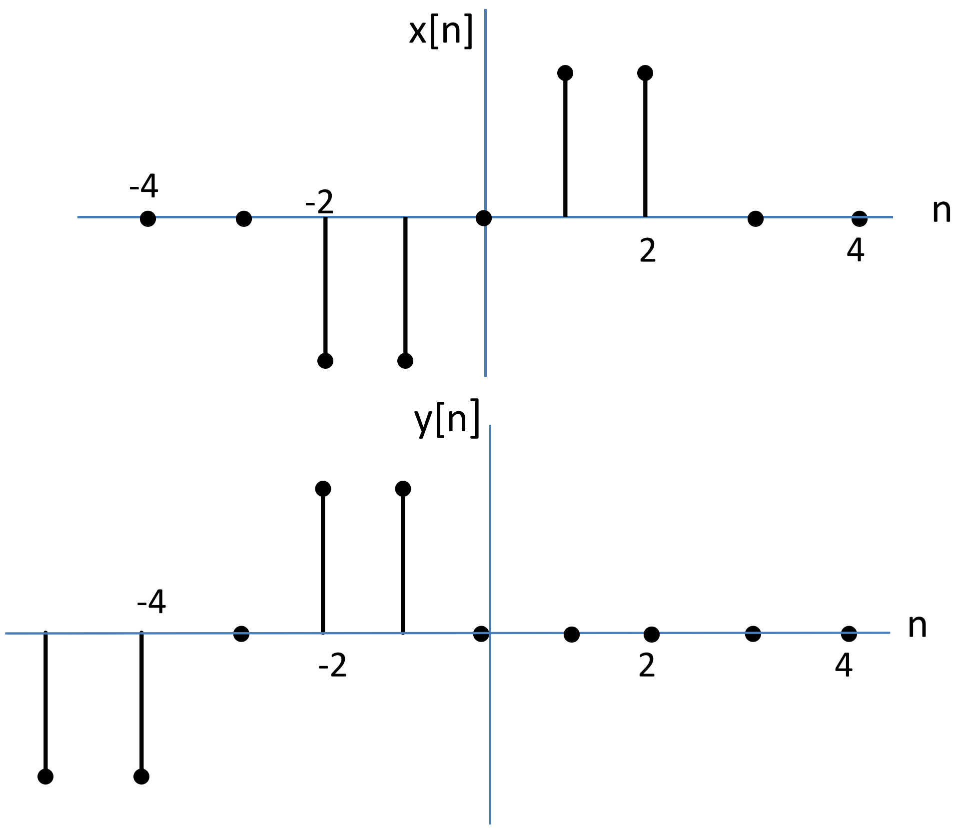
Operations on independent variables

In the case of DT signal

$$y[n] = x[n-m]$$, m is an integer

The DT signal $x[n] = \begin{cases} 1, n = 1,2 \\ -1, n = -1,-2 \\ 0, n = 0 and |n| > 2 \end{cases}$ .Find the time-shifted signal $y[n] = x[n+3]$.

Operations on independent variables



n	x[n]	y[n]=x[n+3]
-5	0	y[-5]=x[-2]
-4	0	y[-4]=x[-1]
-3	0	y[-3]=x[0]
-2	-1	y[-2]=x[1]
-1	-1	y[-1]=x[2]
0	0	y[0]=x[3]
1	1	y[1]=x[4]
2	1	y[2]=x[5]

Operations on independent variables

Precedence Rule For Time Shifting And Time Scaling

Let $y(t) = x(at - b)$

To obtain y(t) from x(t) time-shifting and time scaling must be performed in the correct order.

1.*Perform time-shifting first*

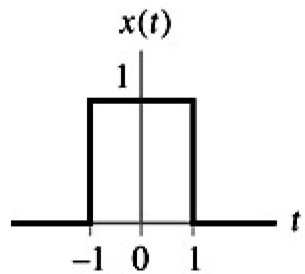
$v(t) = x(t - b)$ – intermediate signal

2.*Perform Scaling*

$$y(t) = v(at)$$
$$= x(at - b)$$

Example1

Consider the rectangular pulse x(t) of unit amplitude and a duration of 2 time units, depicted in Fig.Find $y(t) = x(2t+3)$.



Ans: b=-3, a=2

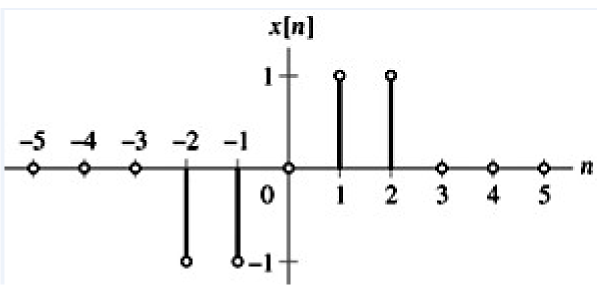
Example2

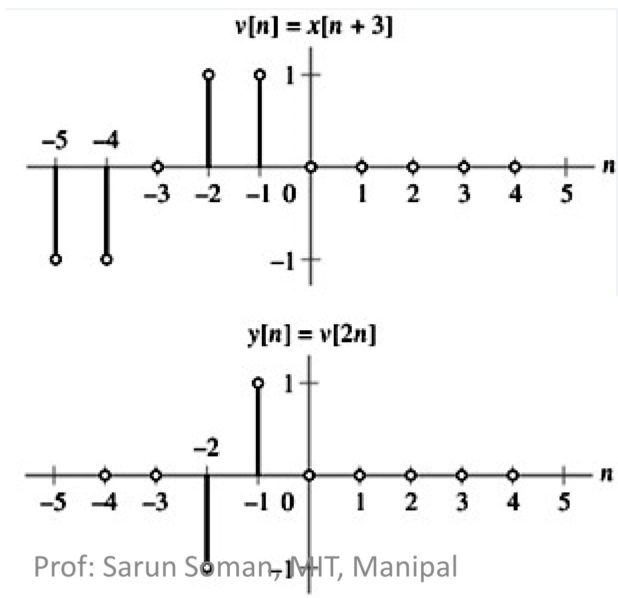
A DT signal is defined by $x(n) = \begin{cases} 1, n = 1,2 \\ -1, n = -1,-2 \\ 0, n = 0 and |n| > 2 \end{cases}$.

Find $y[n] = x[2n + 3]$.

Ans: $y[n] = x[kn - m]$

k=2,m=-3





Elementary Signals

Exponential Signals

1. Exponential Signals
2. Sinusoidal Signals
3. Step function
4. Ramp function

Serve as building blocks for the construction of more complex signals.

Can be used to model many physical signals that occur in nature.

Elementary Signals

Exponential Signal

$$x(t) = Be^{-at}$$

a<0 decaying exponential

a>0 growing exponential

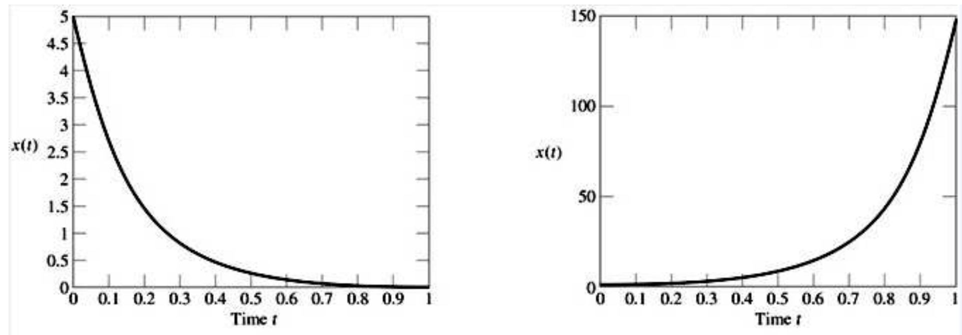
$$x[n] = Br^{n}$$

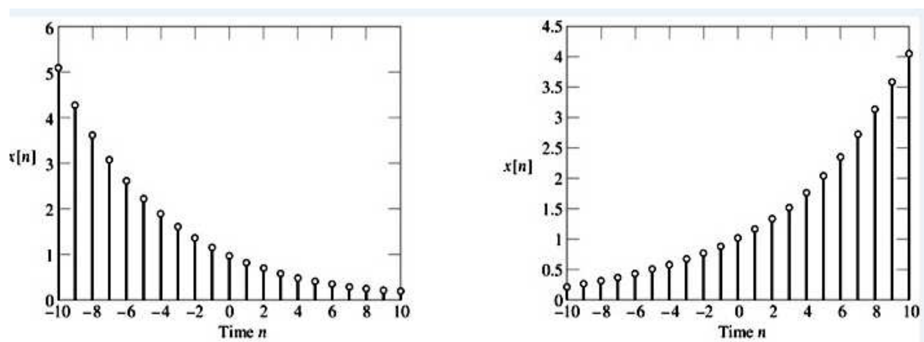
r>1 growing exponential

r<1 decaying exponential

Complex exponential signals

$e^{j\omega t}$ and $e^{j\Omega n}$





(a)   (b)

Elementary Signals

Sinusoidal Signals

$$x(t) = A\cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega}$$

Proof:

$$\begin{aligned} x(t+T) &= A\cos(\omega(t+T) + \phi) \\ &= A\cos(\omega t + \omega T + \phi) \\ &= A\cos(\omega t + 2\pi + \phi) \\ &= A\cos(\omega t + \phi) \\ &= x(t) \end{aligned}$$

Elementary Signals

DT sinusoid

$$x[n] = A\cos(\Omega n + \emptyset)$$

Proof:

$$x[n+N] = A\cos(\Omega(n+N) + \emptyset)$$
$$= A\cos(\Omega n + \Omega N + \emptyset)$$

$x[n] = x[n+N]$ *iff* ΩN *is an integer multiple of* 2π

$$\Omega N = 2\pi m$$
$$\frac{\Omega}{2\pi} = \frac{m}{N}$$

Elementary Signals

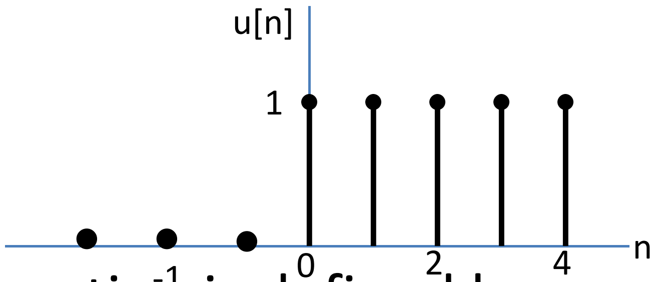
Exponentially Damped Sinusoidal Signals

$$x(t) = Ae^{-\alpha t}\sin(\omega t + \emptyset), \alpha > 0$$

Step function

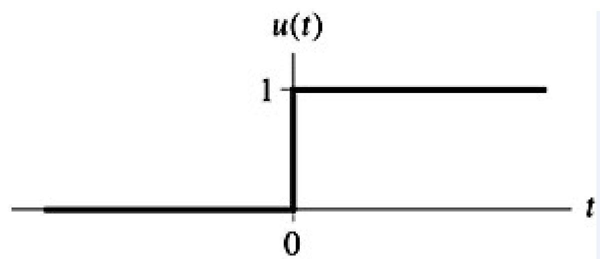
The DT version of the unit step function is defined by

$$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$$



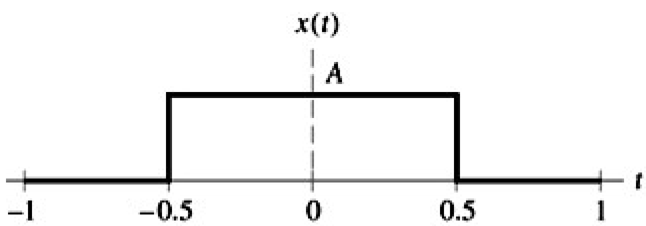
The CT time version of the unit step function is defined by

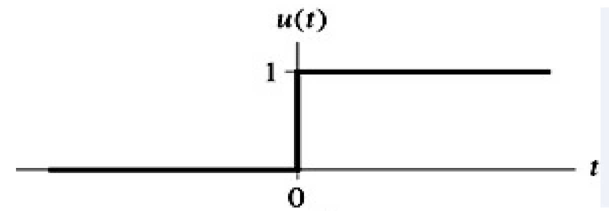
$$u(t) = \begin{cases} 1, t > 0 \\ 0, t < 0 \end{cases}$$



Example 1

Consider the rectangular pulse x(t) shown in Fig. The pulse has an amplitude A and duration of 1s. Express x(t) as a weighted sum of two step function.

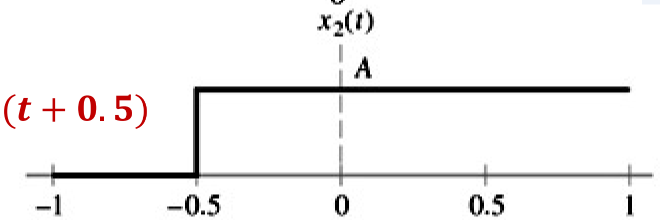




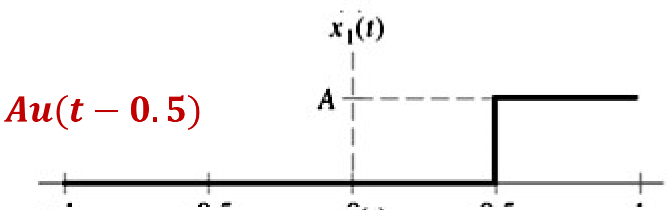
Ans:

$$x(t) = Au(t + 0.5) - Au(t - 0.5)$$

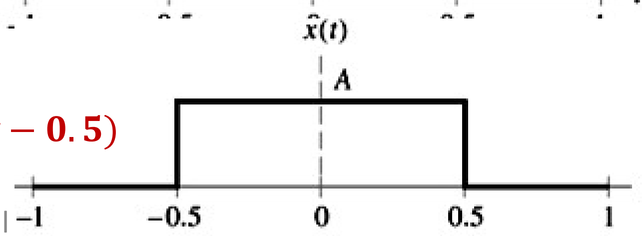
$\boldsymbol{Au(t+0.5)}$



$\boldsymbol{Au(t-0.5)}$



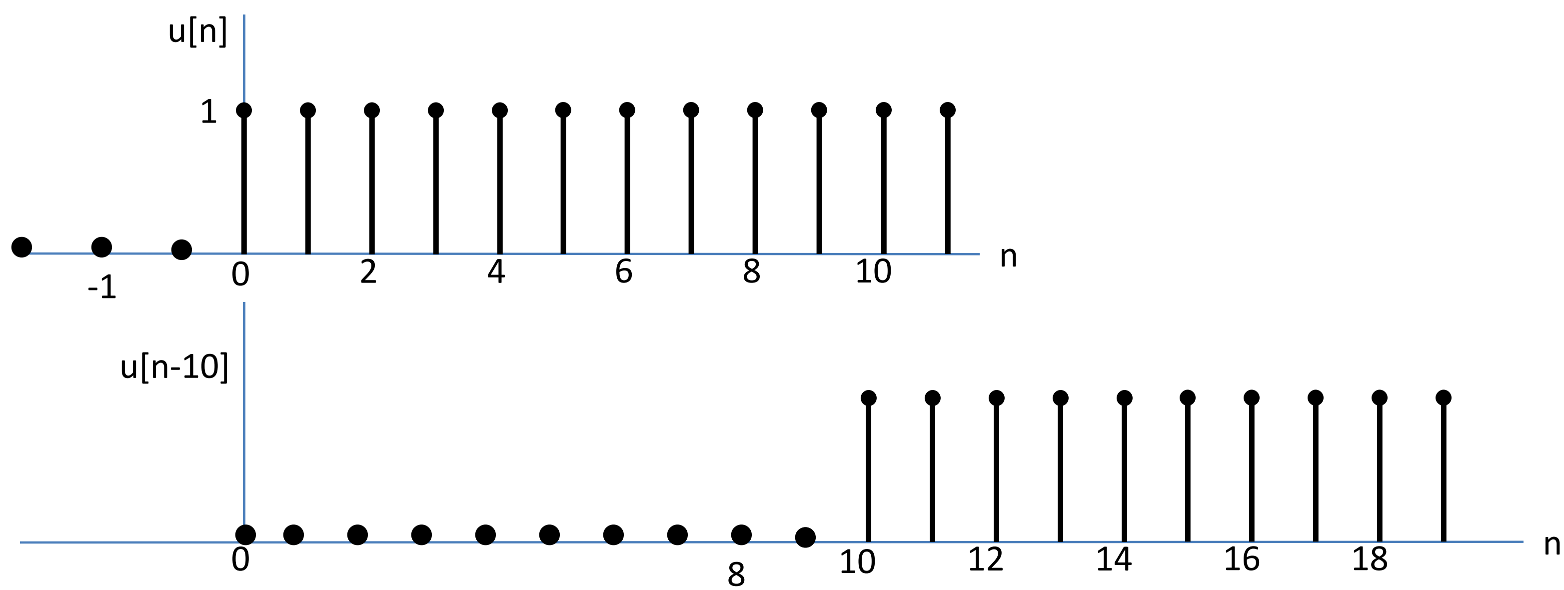
$\boldsymbol{Au(t+0.5)-Au(t-0.5)}$



Example 2

A DT signal $x[n] = \begin{cases} 1, 0 \leq n \leq 9 \\ 0, otherwise \end{cases}$.Using u[n], describe x[n] as superposition of two step functions.

Ans:



$$x[n] = u[n] - u[n-10]$$

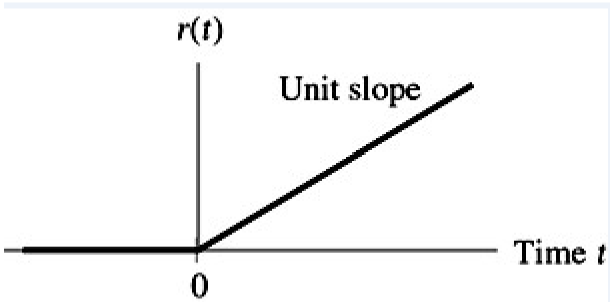
Elementary Signals

Ramp function

$$r(t) = \begin{cases} t, t \geq 0 \\ 0, t < 0 \end{cases}$$

or

$$r[t] = tu[t]$$

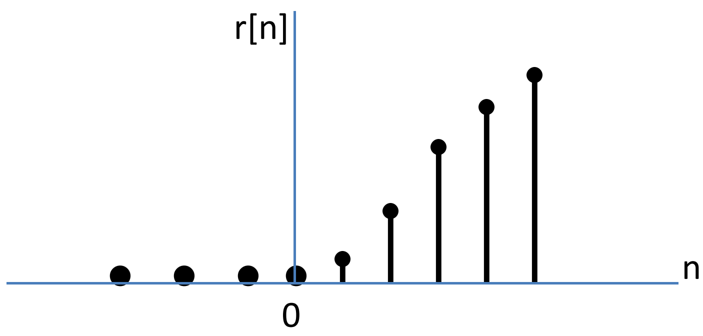


Note: *Integral of unit step function u(t) is a ramp function of unit slope.*

$$r[n] = \begin{cases} n, n \geq 0 \\ 0, n < 0 \end{cases}$$

or

$$r[n] = nu[n]$$

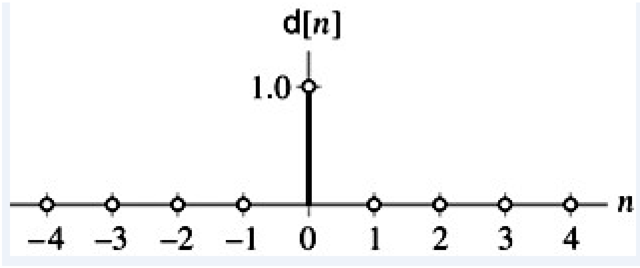


Elementary Signals

Impulse function

DT version of the unit impulse if defined by

$$x[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$$

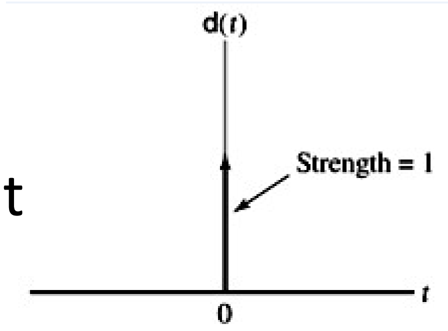


CT version of unit impulse

$$\delta(t) = 0 \; for \; t \neq 0 \qquad (1)$$

and

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \qquad (2)$$



(1) says that impulse is zero every where except at origin.

(2) Says total area under the unit impulse is unity.

Elementary Signals

Evolution of a rectangular pulse of unit area into an impulse of unit strength.

As duration decreases , the rectangular pulse approximates the impulse more closely.

Mathematical relation between impulse and rectangular pulse function.

$$\delta(t) = \lim_{\Delta \to 0} x(t)$$

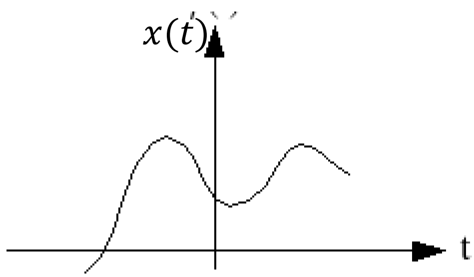
Elementary Signals

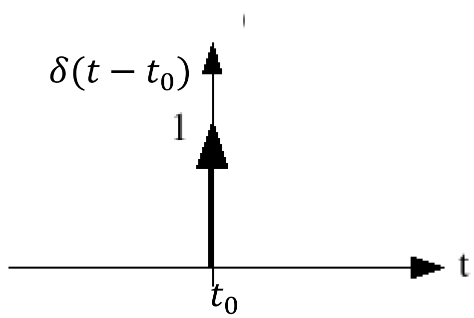
$\delta(t)$ is the derivative of $u(t)$

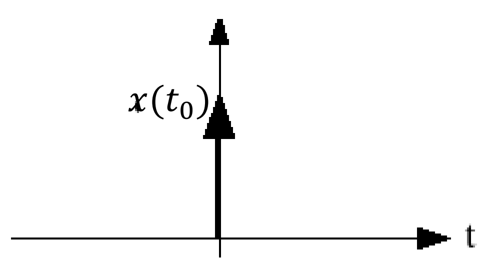
Sifting property

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)\, dt = x(t_0)$$

Sifts out the value of $x(t)$ at time $t = t_0$







Elementary Signals

Time scaling property

$$\delta(at) = \frac{1}{a}\delta(t), a > 0$$

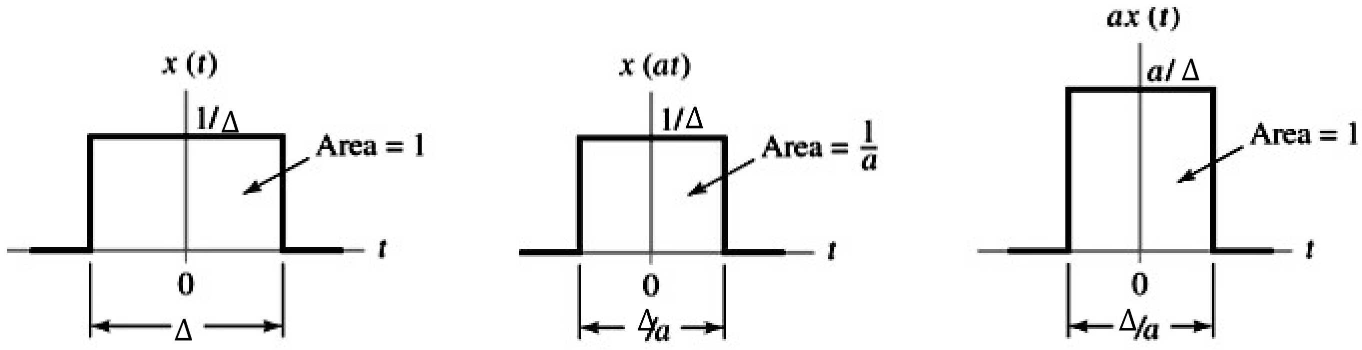
Proof:

$$\delta(t) = \lim_{\Delta \to 0} x(t)$$

Replacing 't' with 'at'

$$\delta(at) = \lim_{\Delta \to 0} x(at) \quad (1)$$

Elementary Signals



(1) becomes

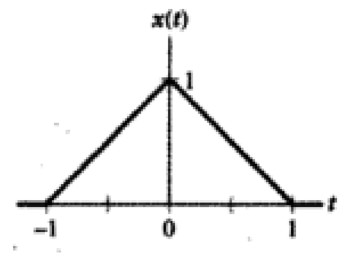
$$\lim_{\Delta \to 0} ax(at) = \delta(t)$$

$$\lim_{\Delta \to 0} x(at) = \frac{1}{a}\delta(t)$$

$$\delta(at) = \frac{1}{a}\delta(t)$$

Tutorials

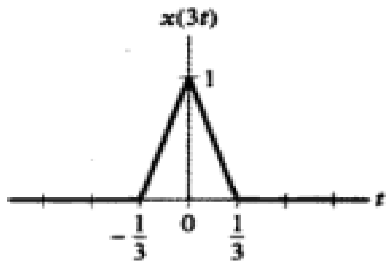
A triangular pulse signal x(t) is depicted in Fig. Sketch each of the following signals derived from x(t).



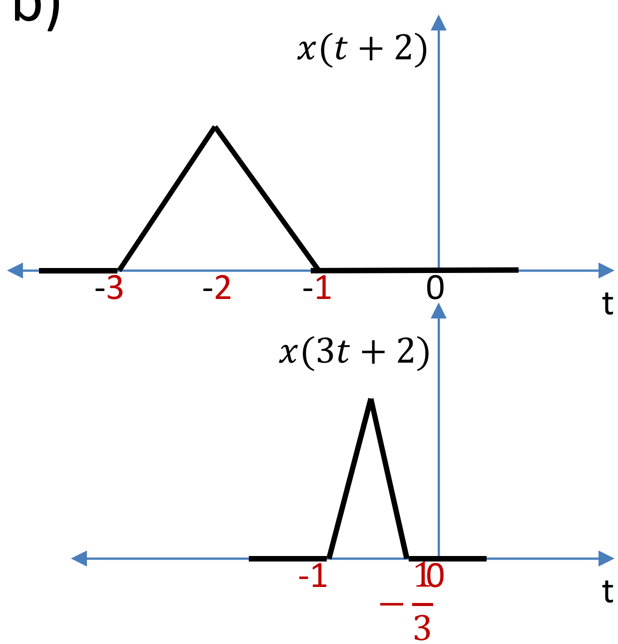
a) x(3t)
b) x(3t+2)
c) x(-2t-1)
d) x(2(t+2))
e) x(2(t-2))
f) x(3t)+x(3t+2)

Tutorials

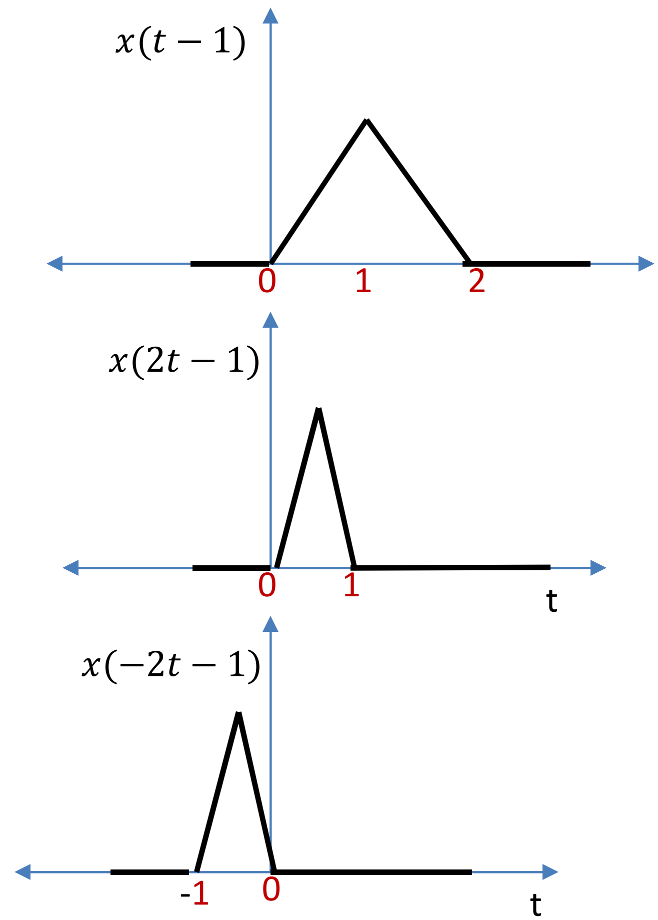
a)x(3t)



b)

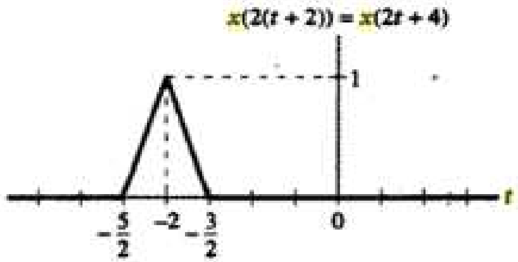


c) x(-2t-1)

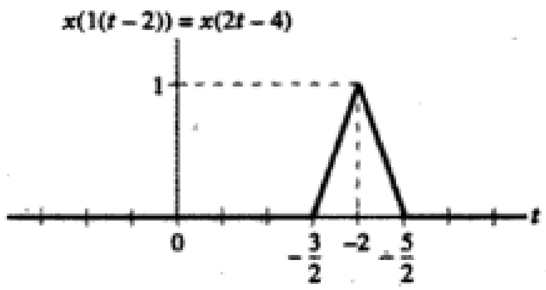


Tutorial

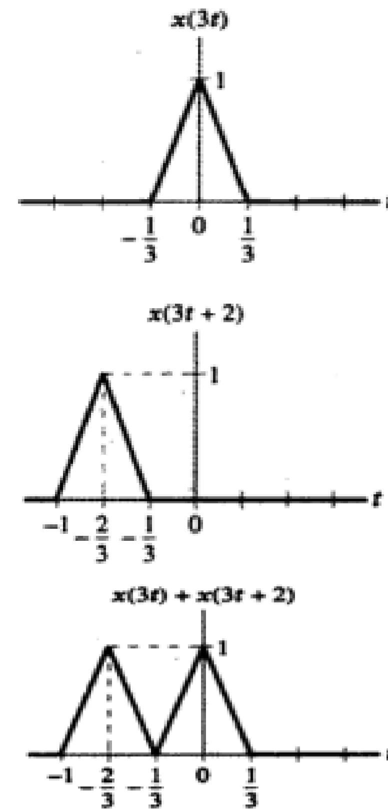
e)



d)

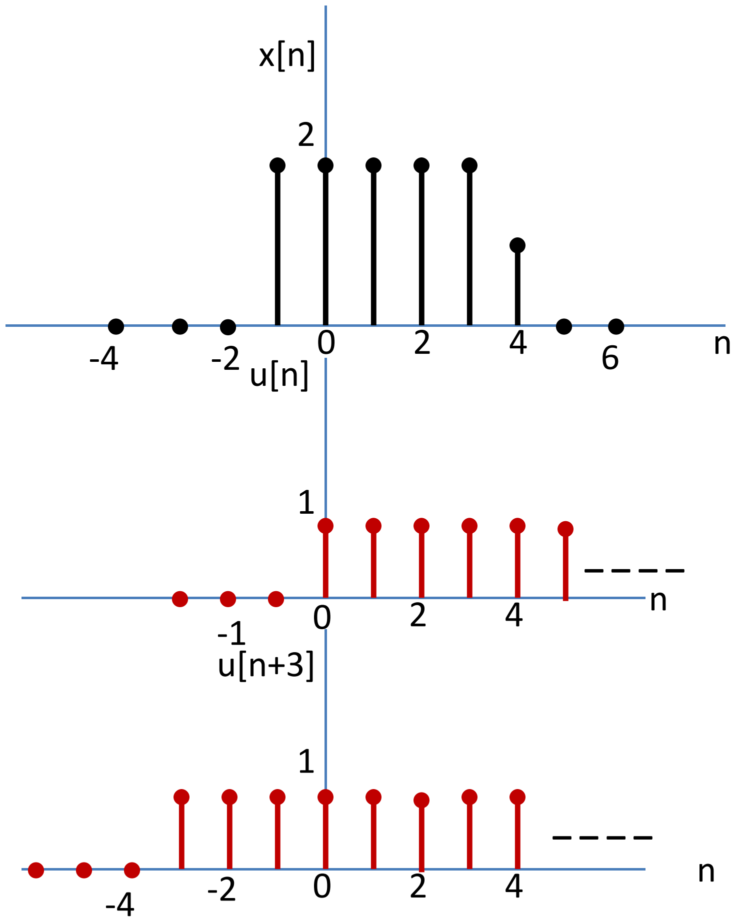


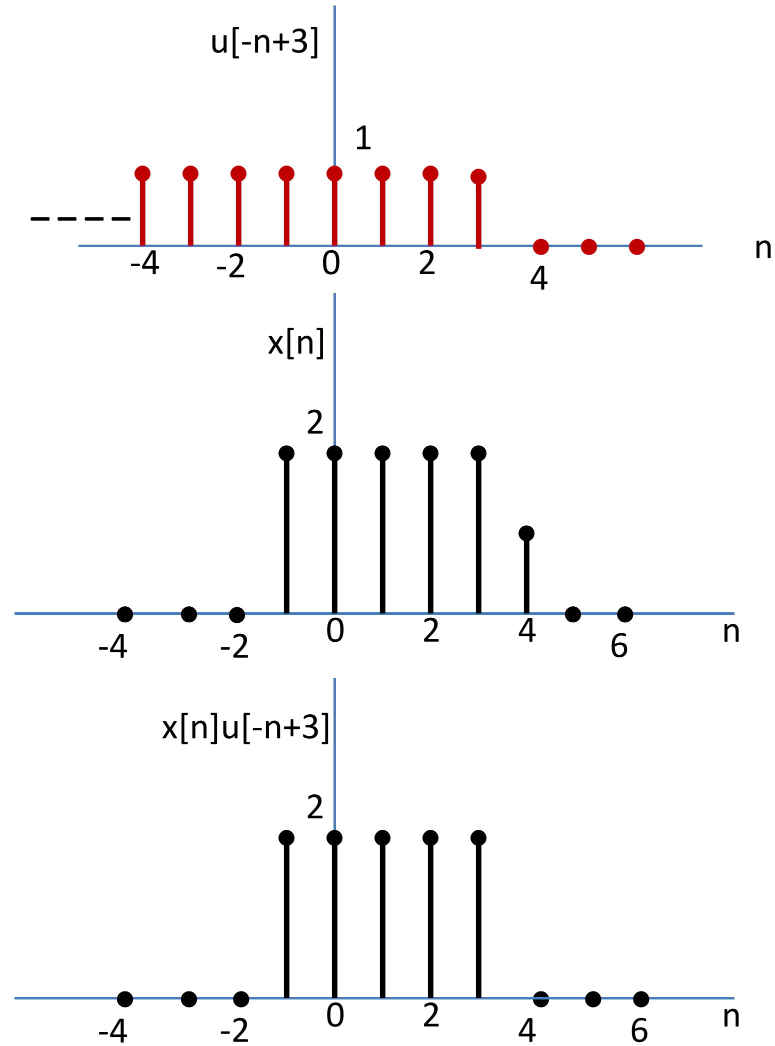
f)



Tutorial

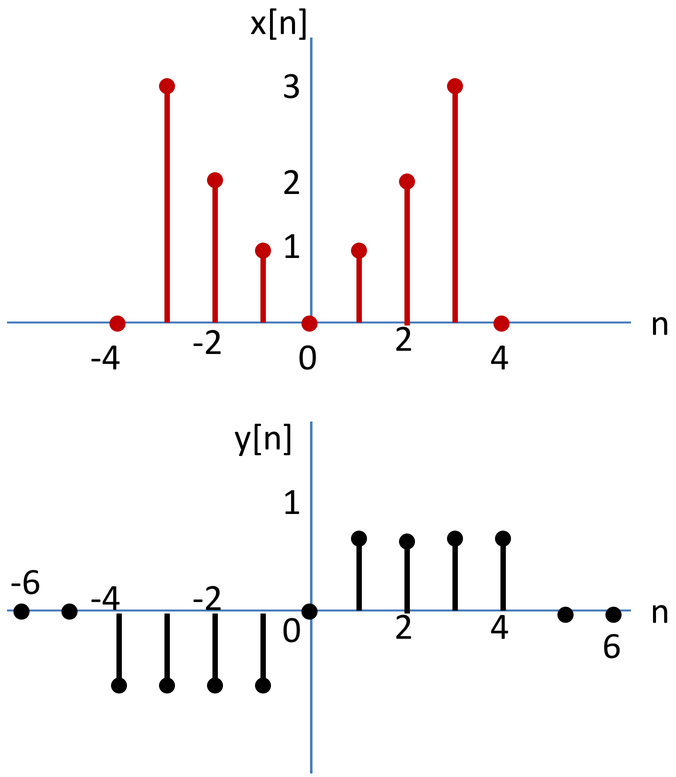
Sketch $x[n]u[3-n]$





Tutorials

Given x[n] & y[n]



Find

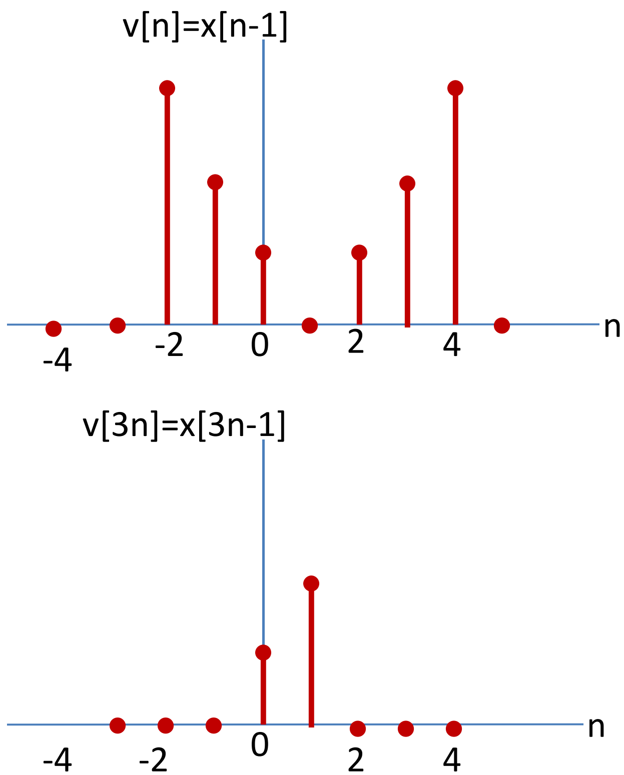
1) $x[3n-1]$

2) $y[2-2n]$

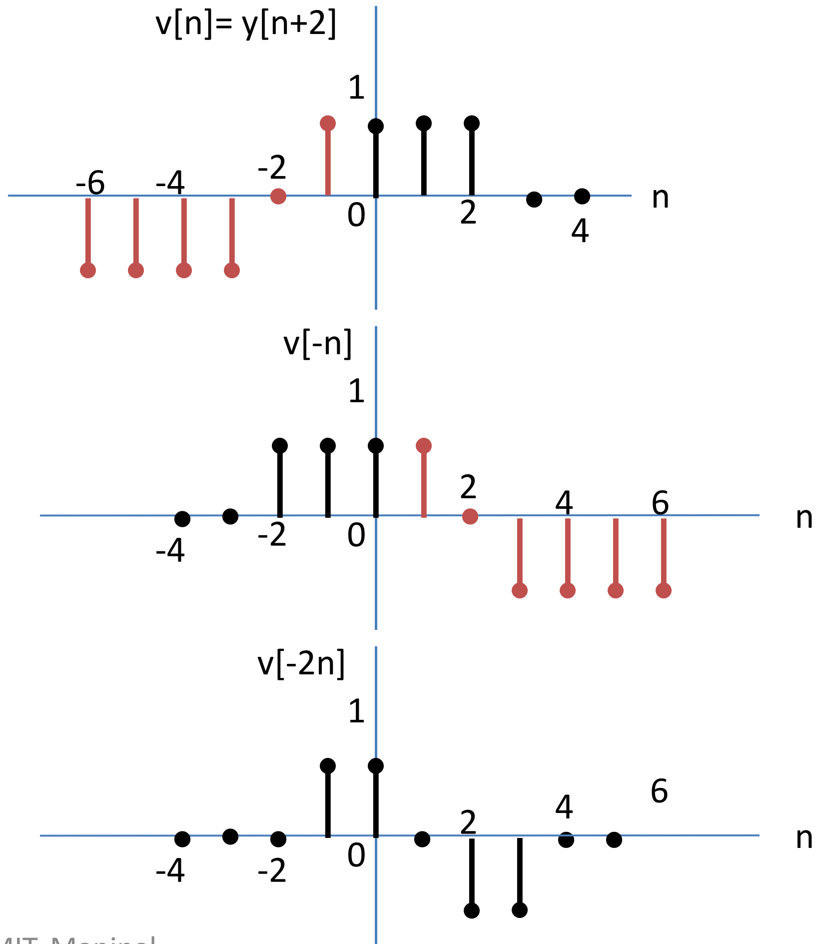
3) $x[2n]+y[n-4]$

Tutorial

1. $x[3n-1]$

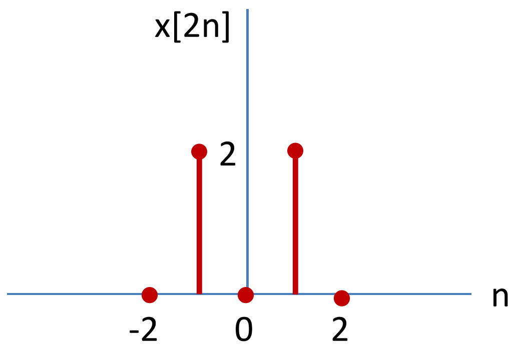


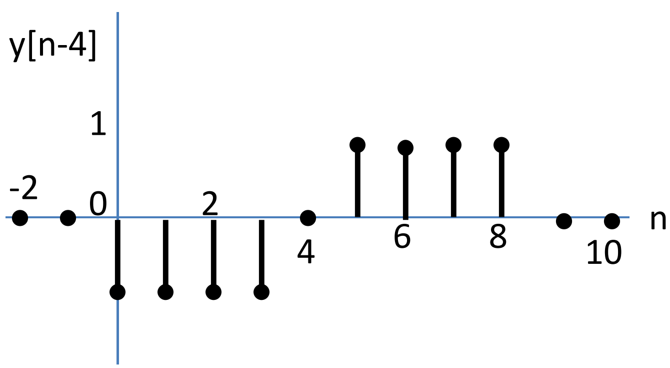
2. $y[-2n+2]$

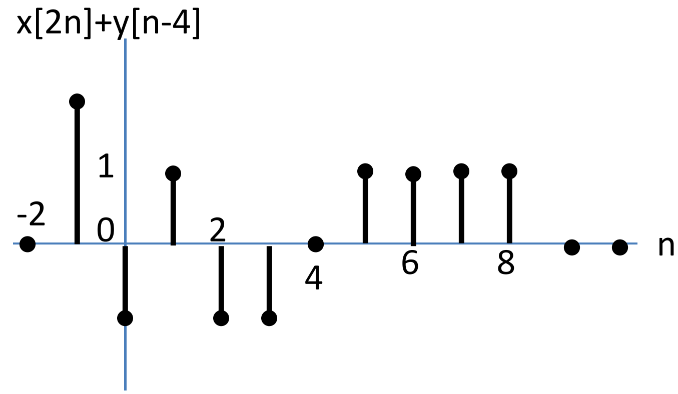


Tutorial

3. $x[2n] + y[n-4]$

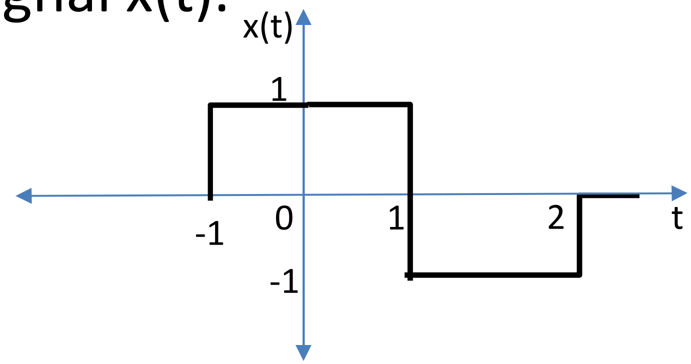


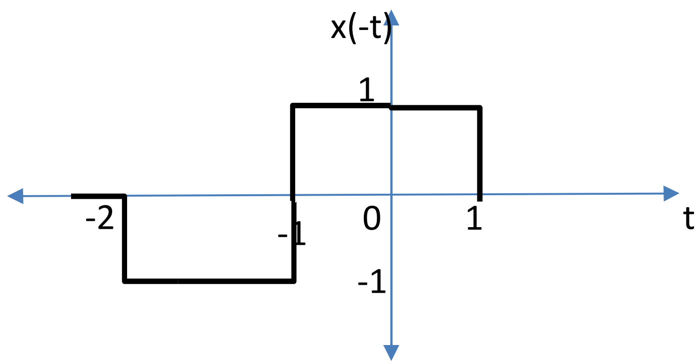




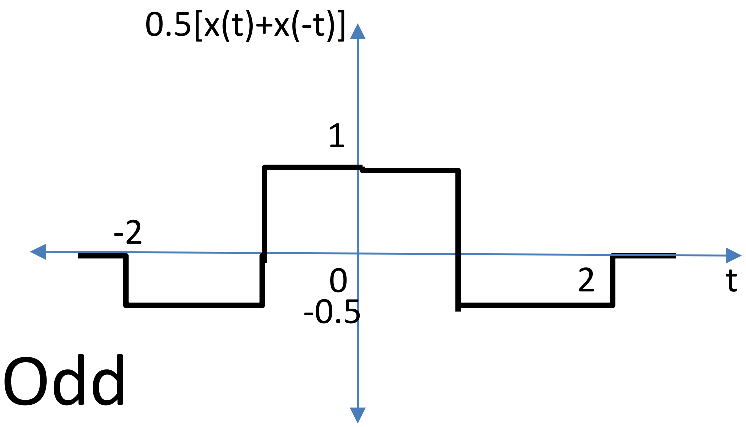
Tutorial

Find the odd and even part of signal x(t).

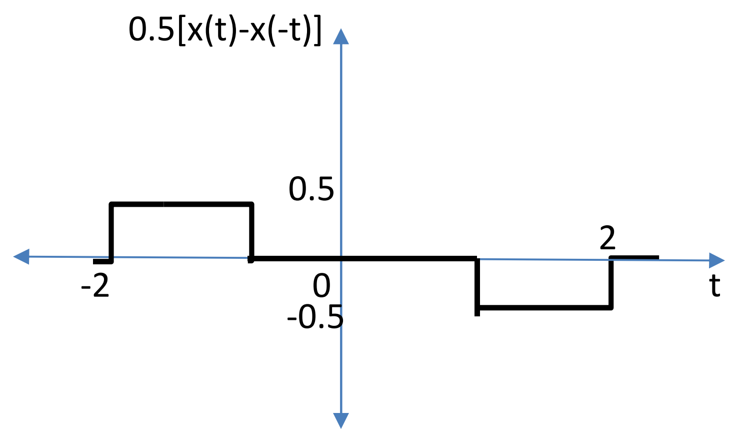




Even



Odd



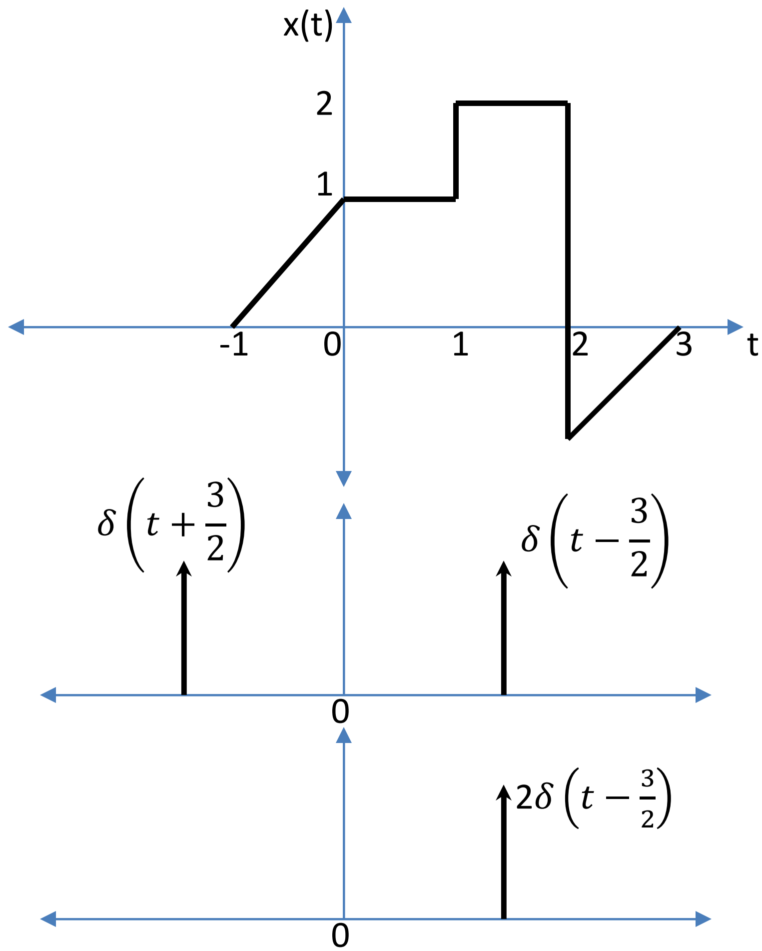
Tutorial

A CT signal x(t) is shown. Find

$$y(t) = x(t)\left[\delta\left(t+\frac{3}{2}\right) - \delta\left(t-\frac{3}{2}\right)\right]$$

Sifting property

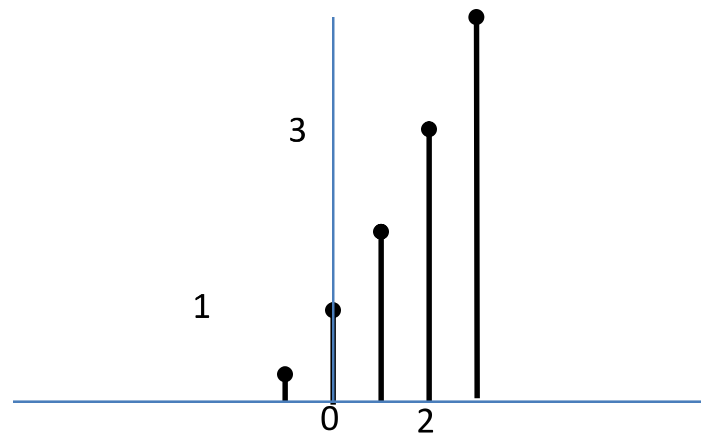
$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)\,dt = x(t_0)$$



Tutorial

Sketch the following signals.

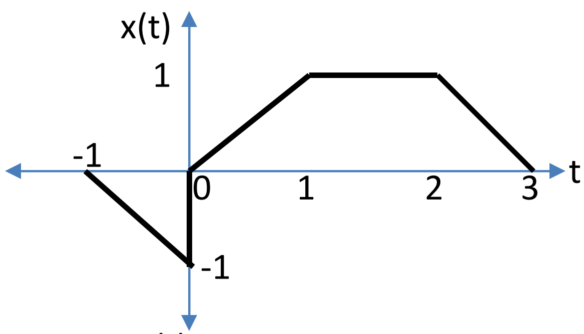
1. $x[n] = \left(\frac{1}{2}, \underset{\uparrow}{1}, \; 2, \; 3, \; 4, \; 8\right)$

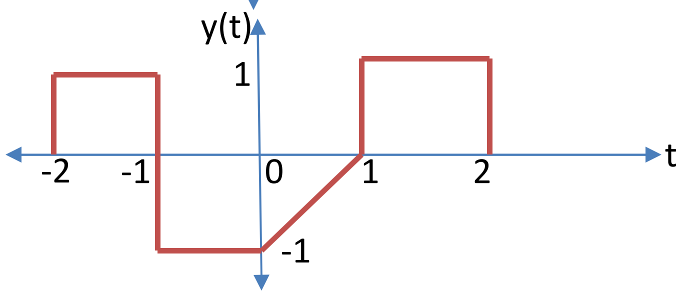


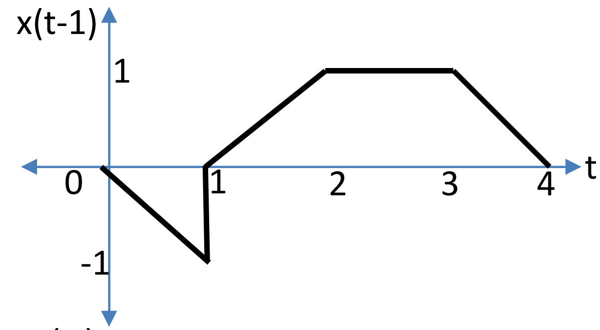
Tutorial

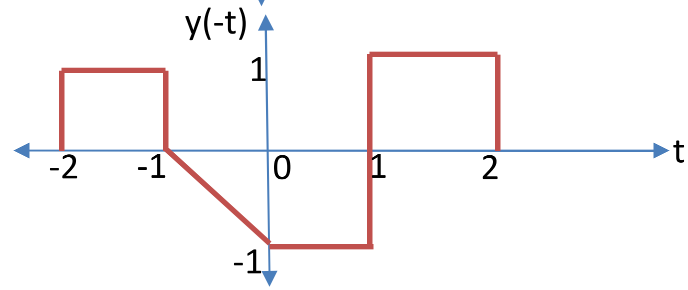
x(t) and y(t) are shown in Fig.
Sketch

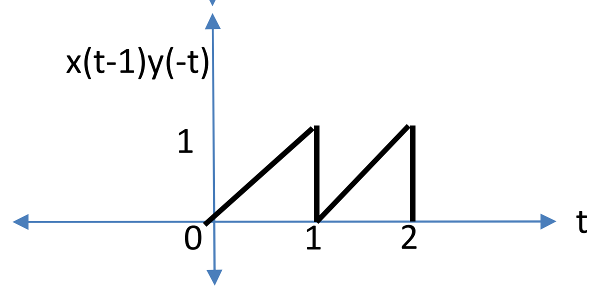
1. $x(t-1)y(-t)$
2. $x(t)y(-t-1)$











Tutorial

$x(t)y(-t-1)$