# Signals and System-II

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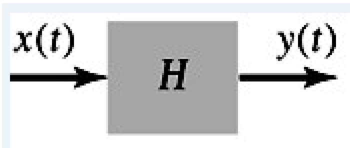
# Properties of System

- Stability
- Memory
- Causality
- Invertibility
- Time invariance
- Linearity

# Properties of Systems

***Stabililty***

A system is said to be bounded input, bounded output(BIBO) stable iff every bounded input results in a bounded output.




Operator H is BIBO stable if the o/p signal satisfies the condition

$$|y(t)| \le M_y \le \infty \quad \text{for all 't'}$$

Whenever input signal x(t) satisfies the condition

$$|x(t)| \le M_x \le \infty \quad \text{for all 't'}$$

$M_y$ and $M_x$ finite positive numbers

# Properties of Systems

Example of unstable system

- Tacoma Narrows suspension bridge
- Collapsed on November 7, 1940.
- Collapsed due to wind induced vibrations

# Properties of Systems

# Example1

Consider the moving average system with input output relation

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

Show that the system is BIBO stable.

**Ans**:

Assume that $|x[n]| < M_x < \infty$ for all 'n'

$$|y[n]| = \frac{1}{3}|x[n] + x[n-1] + x[n-2]|$$

$$= \frac{1}{3}(M_x + M_x + M_x)$$

$$= M_x$$

Bounded input results in bounded o/p

# Example2

Consider a DT system whose i/p-o/p relation is defined by

$$y[n] = r^n x[n]$$

Where $r > 1$. Show that the system is unstable.

**Ans**:

Assume that the input signal x[n] satisfies the condition

$$|x[n]| \leq M_x \leq \infty$$

$$|y[n]| = |r^n x[n]|$$

$$= |r^n||x[n]|$$

$r^n$ diverges as ‘n’ increases.

# Properties of Systems

***Memory***

A system is said to posses memory if its o/p signal depends on past or future values of the input signal.

A system is said to be *memoryless* if its output signal depends only on present value of input signal.

Eg.

Resistor

$$i(t) = \frac{1}{R} v(t) \quad \text{memoryless}$$

Inductor

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v(\tau) d\tau \quad \text{memory}$$

# Properties of Systems

$y[n] = x^2[n]$ memoryless

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

Memory

$$y[0] = \frac{1}{3}(x[0] + x[-1] + x[-2])$$

# Properties of Systems

***Causality***

A system is said to be causal if its present value of the o/p signal depends only on the present or past values of the input signal.

A system is said to be non causal if its output signal depends on one or more future values of the input signal.

Eg. Moving average system

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

causal

Moving average system

$$y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n-2])$$

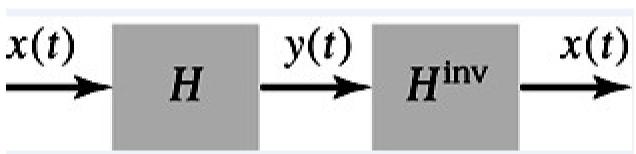
Non causal

$$y[0] = \frac{1}{3}(x[0] + x[1] + x[-2])$$

# Properties of Systems

***Invertibility***

A system is said to be invertible if the input of the system can be recovered  from the o/p.




Note: necessary condition for invertibility distinct i/p must produce distinct o/p

# Properties of Systems

Show that a square law system described by the input-output relation $y(t) = x^2(t)$ is not invertible.

Ans:

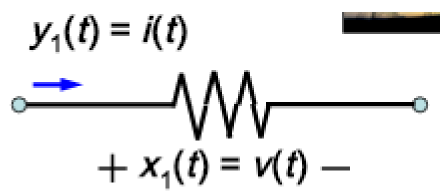
Distinct inputs x(t) and x(-t) produce the same output

***Time invariance***

A system is said to be time invariant if a delay or time advance of the input signal lead to an identical time shift in the output signal.

# Properties of Systems

Eg. Thermistor




Input output relation of thermistor

$$y_1(t) = \frac{x_1(t)}{R(t)}$$

Response $y_2(t)$ of the thermistor to input $x(t - t_0)$

$$y_2(t) = \frac{x_1(t-t_0)}{R(t)}$$

# Properties of Systems

Shifting the output $y_1(t)$

$$y_1(t - t_0) = \frac{x_1(t-t_0)}{R(t-t_0)}$$

For a thermistor $R(t - t_0) \neq R(t)$ for $t \neq 0$

$$y_1(t - t_0) \neq y_2(t) \text{ for } t \neq 0$$

Thermistor is time variant

***Linearity***

A system is linear in terms of the system input x(t) and the system output y(t) if it satisfies two properties.

# Properties of Systems

1. ***Superposition*** :

$x(t) = x_1(t) \Rightarrow y(t) = y_1(t)$

$x(t) = x_2(t) \Rightarrow y(t) = y_2(t)$

$\Rightarrow \quad x(t) = x_1(t) + x_2(t) \Rightarrow y(t) = y_1(t) + y_2(t)$

2. ***Homogeneity*** :

$x(t) \Rightarrow y(t) \quad \Rightarrow \quad ax(t) \Rightarrow ay(t)$

*a* = constant factor

$$a_1x_1(t) + a_2x_2(t) = a_1y_1(t) + a_2y_2(t)$$

# Properties of Systems

Consider a discrete time system described by the input-output relation $y[n] = nx[n]$. Show that the system is linear.

Ans:

Let the input signal x[n] be expressed as the weighted sum

$$x[n] = \sum_{i=1}^{N} a_i x_i[n]$$

Output of the system

$$= n \sum_{i=1}^{N} a_i \, x_i[n]$$

$$= \sum_{i=1}^{N} a_i n \, x_i[n]$$

$$= \sum_{i=1}^{N} a_i \, y_i[n]$$

The system is linear

# Tutorial

$y[n] = 2x[n] + 3$ check for linearity

Ans:

$$y_1[n] = 2x_1[n] + 3$$
$$y_2[n] = 2x_2[n] + 3$$

Let $x_3[n] = x_1[n] + x_2[n]$

$$\begin{aligned} y_3[n] &= 2(x_1[n] + x_2[n]) + 3 \\ &= 2x_1[n] + 2x_2[n] + 3 \\ &\neq ay_1[n] + by_2[n] \end{aligned}$$

# Tutorial

$y(t) = x\left(\frac{t}{2}\right)$

Memory or memoryless system?

**Ans:** $y(1) = x(0.5)$

Output depends on past value of i/p

Memory system

$y(t) = \cos\big(x(t)\big)$ invertible or non-invertible?

**Ans**: Distinct i/p must give distinct o/p.

$x(t) and\ x(t) + 2\pi)$ gives same o/p.

Non-invertible

# Tutorial

$y[n] = x[-n]$ time invariant or time variant?

**Ans**:

Delay in input $x[n-m]$

Corresponding output $y_1[n] = x[-n-m]$

Delay the output

$$y_2[n] = y[n-m] = x[-(n-m)]$$
$$= x[-n+m]$$

$$y_2[n] \neq y_1[n]$$

Time variant

# Tutorial

$y(t) = x\left(\frac{t}{2}\right)$

Linearity, time invariance, memory, causality, stability?

**Ans**:

Memory system

***Linearity***

$$x_1(t) \rightarrow y_1(t) = x_1\left(\frac{t}{2}\right)$$

$$x_2(t) \rightarrow y_2(t) = x_2\left(\frac{t}{2}\right)$$

Let $x_3(t) = ax_1(t) + bx_2(t)$

$$y_3(t) = ax_1\left(\frac{t}{2}\right) + bx_2\left(\frac{t}{2}\right)$$
$$= ay_1(t) + by_2(t)$$

Linear as it satisfies principle superposition and homogeneity

# Tutorial

***Causality***

$$y[-1] = x[-0.5]$$

o/p depends on future value of input

Non-causal

*Stability*

Let $|x(t)| \leq B_x \leq \infty$

Then $|y(t)| = \left|x\left(\frac{t}{2}\right)\right| \leq B_y \leq \infty$

The system is stable.

# Tutorial

$y(t) = tx(t)$ stable or unstable?

Let $|x(t)|$ be bounded.

$y(t)$ will be unbounded , 't' is unbounded

Unstable system

$y(t) = e^{x(t)}$

Check for linearity, Stability, Time Invariance, Memory.

$$x_1(t) \to y_1(t) = e^{x_1(t)}$$

$$x_2(t) \to y_2(t) = e^{x_2(t)}$$

Let $x_3(t) = ax_1(t) + bx_2(t)$

# Tutorial

$$y_3(t) = e^{ax_1(t)+bx_2(t)}$$
$$\neq ay_1(t) + by_2(t)$$

The system is Non-linear

***Time Invariance***

delay i/p $x(t - t_0)$

$$y_1(t) = e^{x(t-t_0)} \qquad (1)$$

Delay o/p

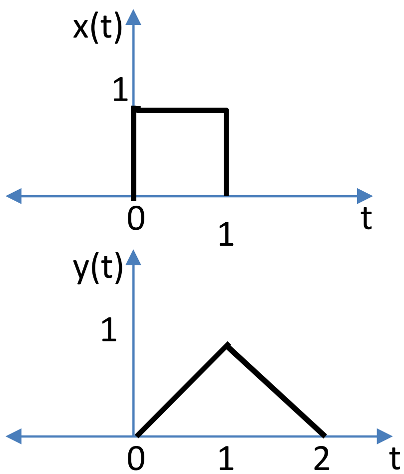
$$y_2(t - t_0) = e^{x(t-t_0)} \qquad (2)$$

(1)=(2) system is time invariant

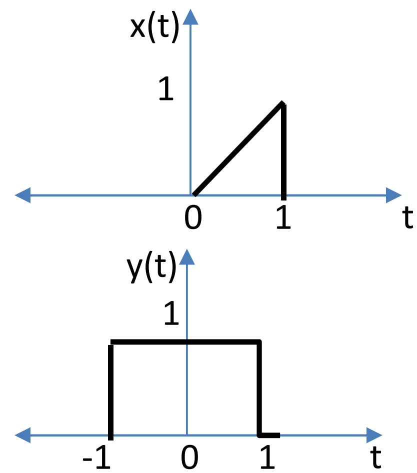
The system is memoryless.

# Tutorial

The i/p and o/p of the system is shown. Determine if the system could be memoryless & causal.




Memory, Causal




Non-Causal

Memory

# Tutorial

$y(t) = [\sin 6t]x(t)$ Check for Memory, Time invariance, Linearity, Stability and Causality.

**Ans**:

Present value of i/p depends on present value of o/p

Memoryless System

Delay in i/p $x(t - t_0)$

$$y_1(t) = [\sin 6t]x(t - t_0)$$

Delay o/p

$$y_2(t - t_0) = [\sin 6(t - t_0]x(t - t_0)$$

$$y_1(t) \neq y_2(t)$$

Time variant system

# Tutorial

$$x_1(t) \rightarrow y_1(t) = [\sin 6t]x_1(t)$$
$$x_2(t) \rightarrow y_2(t) = [\sin 6t]x_2(t)$$

Let $x_3(t) = ax_1(t) + bx_2(t)$

$$y_3(t) = [\sin 6t]x_3(t)$$
$$= a \sin 6t \, x_1(t) + b \sin 6t \, x_2(t)$$
$$= ay_1(t) + by_2(t)$$

Linear System

Causal – o/p doesn't depend on future value of i/p.

# Tutorial

Let $|x(t)| \leq B_x \leq \infty$

$|y(t)| = |\sin 6t||x(t)|$

$|\sin 6t| \leq 1$

$y(t)$ is bounded , system is stable.

Check for linearity, causality, time-invariance, stability

1.$y[n] = n^2 x[n]$

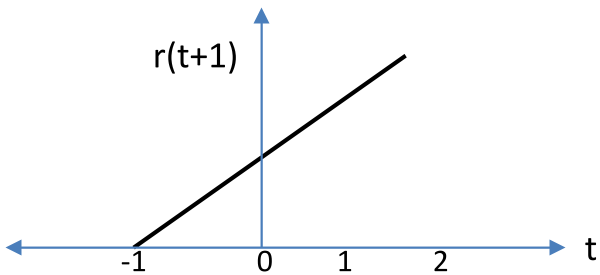
Linear , timevariant, causal, unstable

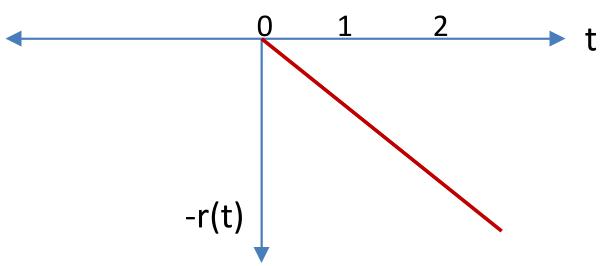
2. $y[n] = x[2n]$

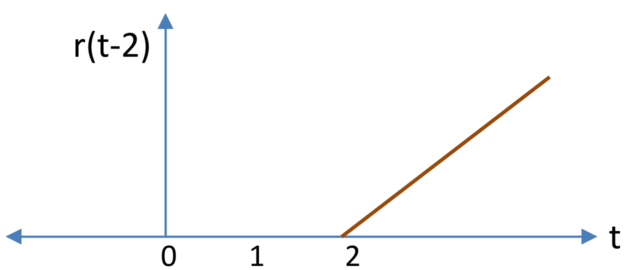
Linear, time-variant, non-causal

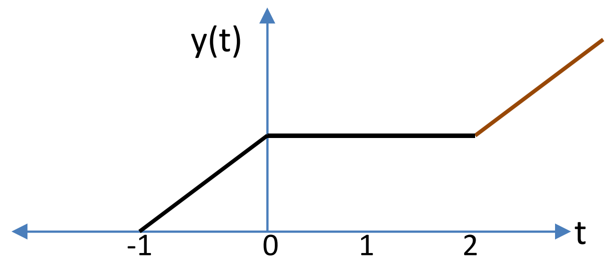
# Sketch the waveform

$$y(t) = r(t+1) - r(t) + r(t+2)$$



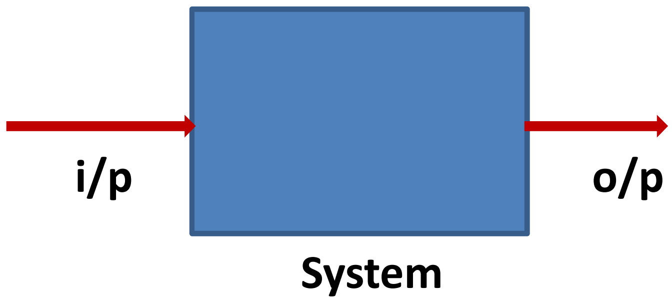



# LTI Systems

- Linear Time Invariant Systems
- Tools for analyzing and predicting the behavior of LTI systems.




- Characterizing an LTI system
  - Impulse Response
  - Linear constant-coefficient differential equation or difference equation
- Gives insight into system's behavior.
- Useful in analyzing and implementing the system

# Impulse Response

Impulse response is defined *as the o/p of an LTI system due to a unit impulse signal input applied at time t=0 or n=0.*

*Symbol* $h(t), h[n]$

Impulse response completely characterizes the behavior of any LTI system.

Given the impulse response, output can be determined for any arbitrary input signal.

*Convolution Sum* – DT LTI systems

*Convolution Integral* – CT LTI systems

# Convolution Sum

Consider an LTI system

$$\delta[n] \rightarrow h[n]$$

Apply time invariance property

$$\delta[n-k] \rightarrow h[n-k]$$

Apply principle of Homogeneity

$$x[k]\delta[n-k] \rightarrow x[k]h[n-k]$$

Apply principle of superposition

$$\sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \quad \rightarrow \quad \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Input is weighted superposition of time shifted impulse

Output is weighted super position of time shifted impulse responses.

# Convolution Sum

$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$ - Convolution Sum

$y[n] = x[n] * h[n]$

# Example

$x[n] = \{-1, 0, 1, 2, 3\}$

$h[n] = \{1, -0.5\}$

Change of variable $n \rightarrow k$

Starting index of o/p (-1+0=-1)

$$y[-1] = \sum_{k=-\infty}^{\infty} x[k]h[-1-k]$$
$$= -1$$

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k]$$
$$= 0.5$$

$y[1] = 1, y[2] = 1.5, y[3] = 2,$
$y[4] = -1.5$
$y[n] = \{-1, 0.5, 1, 1.5, 2, -1.5\}$

| -3 | -2 | -1 | 0 | 1 | 2 | 3 | **k** |
|---|---|---|---|---|---|---|---|
| 0 | 0 | -1 | 0 | 1 | 2 | 3 | x[k] |
| 0 | 0 | 0 | 1 | -0.5 | 0 | 0 | h[k] |
| 0 | 0 | -0.5 | 1 | 0 | 0 | 0 | h[-k] |
| 0 | -0.5 | 1 | 0 | 0 | 0 | 0 | h[-1-k] |
| 0 | 0 | 0 | -0.5 | 1 | 0 | 0 | h[1-k] |
| 0 | 0 | 0 | 0 | -0.5 | 1 | 0 | h[2-k] |
| 0 | 0 | 0 | 0 | 0 | -0.5 | 1 | h[3-k] |
| 0 | 0 | 0 | 0 | 0 | 0 | -0.5 | h[4-k] |

# Example

Assuming that the impulse response of an LTI system is given by $h[n] = 0.5^n u[n]$. Determine the o/p response y[n] to the i/p sequence.$x[n] = 0.8^n u[n]$.

Ans:

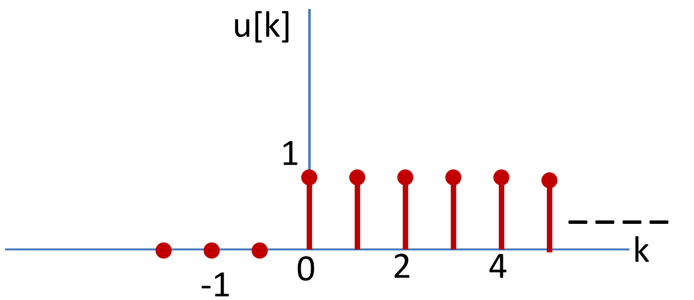
Change of variable x[k] and h[k]

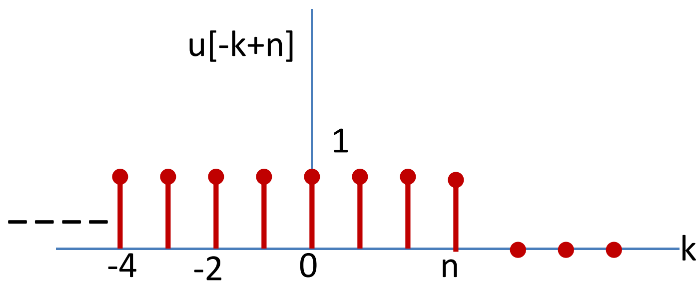
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} 0.8^k u[k] 0.5^{n-k} u[n-k]$$

$$= 0.5^n \sum_{k=-\infty}^{\infty} (\frac{0.8}{0.5})^k u[k]u[n-k]$$

# Example




For $n < 0$

$$u[k]u[n-k] = 0$$

$$y[n] = 0$$

For $n \geq 0$

$$y[n] = 0.5^n \sum_{k=0}^{n} \left(\frac{0.8}{0.5}\right)^k$$

GP series

$$\sum_{n=0}^{N} r^n = \frac{1-r^{N+1}}{1-r}$$

$$y[n] = 0.5^n \left[\frac{1-\left(\frac{0.8}{0.5}\right)^{n+1}}{1-\frac{0.8}{0.5}}\right]$$

# Example

$$= 0.5^n \frac{0.5}{0.5^{n+1}} \frac{1}{0.3} [0.8^{n+1} - 0.5^{n+1}]$$

$$= \frac{10}{3} [0.8^{n+1} - 0.5^{n+1}]$$

$$y[n] = \begin{cases} 0, & n < 0 \\ \frac{10}{3} [0.8^{n+1} - 0.5^{n+1}], & \mathrm{n} \geq 0 \end{cases}$$

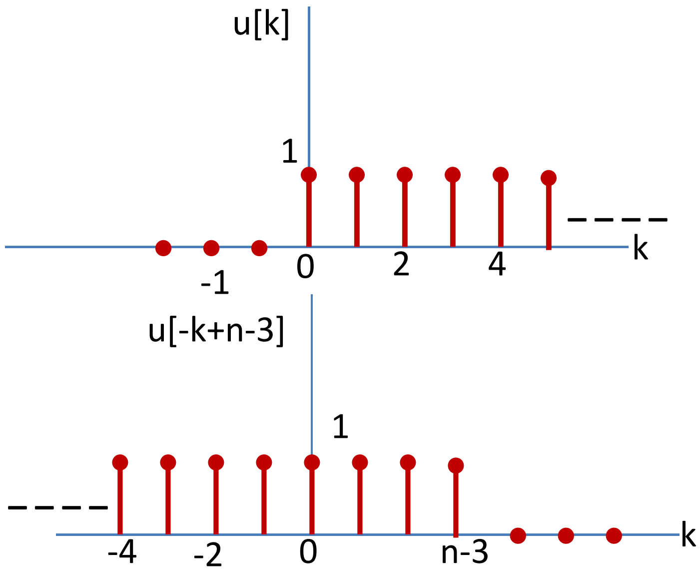
# Example

$$y[n] = u[n] * u[n-3]$$

Ans:

Change of variable

$$x[k] = u[k], h[k] = u[k-3]$$




$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} u[k]u[n-k-3]$$

For $n-3<0$

$$n<3$$

$$y[n]=0$$

# Example

For $n - 3 \geq 0$

$$n \geq 3$$

$$y[n] = \sum_{k=0}^{n-3} 1$$

$$y[n] = n - 3 + 1$$
$$= n - 2$$

$$y[n] = \begin{cases} 0, n < 3 \\ n - 2, n \geq 3 \end{cases}$$

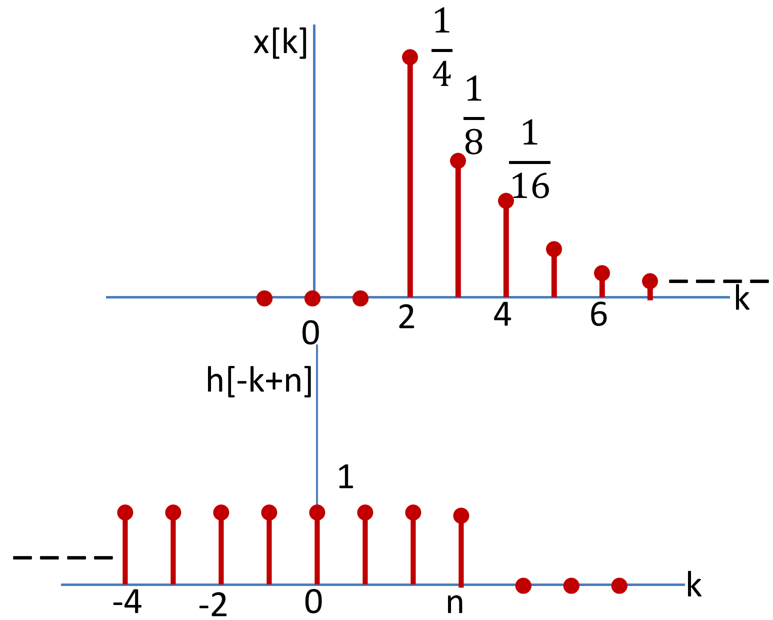
# Example

$$y[n] = (\frac{1}{2})^n u[n-2] * u[n]$$

**Ans:**

Change of variable

$$x[k] = (\frac{1}{2})^k u[k-2]$$
$$h[k] = u[k]$$
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$
$$y[n] = \sum_{k=-\infty}^{\infty} (\frac{1}{2})^k u[k-2]u[n-k]$$




For $n < 2$

$$y[n] = 0$$

# Example

For $n \geq 2$

$$y[n] = \sum_{k=2}^{n} (\frac{1}{2})^k$$

GP series

$$\sum_{n=0}^{N} r^n = \frac{1-r^{N+1}}{1-r}$$

Let k-2=m

$$y[n] = \sum_{m=0}^{n-2} (\frac{1}{2})^{m+2}$$

$$y[n] = (\frac{1}{2})^2 \sum_{m=0}^{n-2} (\frac{1}{2})^m$$

$$y[n] = (\frac{1}{2})^2 \left[\frac{1-(\frac{1}{2})^{n-1}}{1-\frac{1}{2}}\right]$$

$$y[n] = \frac{1}{2} - (\frac{1}{2})^n$$

$$y[n] = \begin{cases} 0, & n < 2 \\ \frac{1}{2} - (\frac{1}{2})^n, & n \geq 2 \end{cases}$$

# Example

$x[n] = 2^n u[-n], h[n] = u[n].$

Find the convolution sum.

Ans:

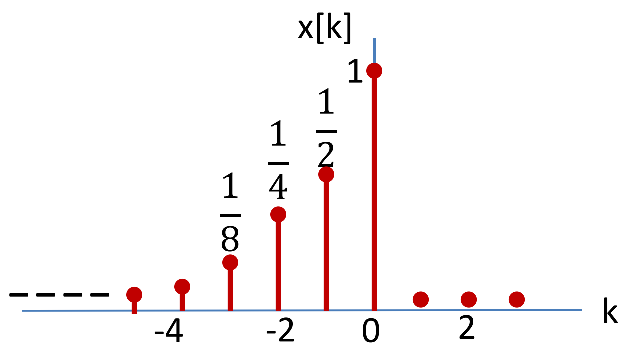
Change of variable

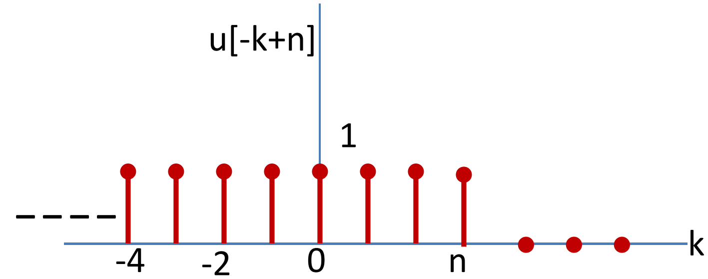
$x[k] = 2^k u[-k], h[k] = u[k]$

$$y[n] = \sum_{k=-\infty}^{\infty} 2^k u[-k]\, u[n-k]$$

For $n < 0$

$$y[n] = \sum_{k=-\infty}^{n} 2^k$$




GP series

$\sum_{n=0}^{N} r^n = \frac{1-r^{N+1}}{1-r}$

# Example

Let $l = -k$

$$y[n] = \sum_{l=\infty}^{-n} 2^{-l}$$

Let $l + n = m$

$$y[n] = \sum_{m=\infty}^{0} 2^{-(m-n)}$$

$$y[n] = \sum_{m=0}^{\infty} 2^{n-m}$$

$$y[n] = 2^n \sum_{m=0}^{\infty} (\frac{1}{2})^m$$

Infinite sum

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}, |a| < 1$$

$$y[n] = 2^n \frac{1}{1 - \frac{1}{2}}$$

$$= 2^{n+1}$$

For $n \geq 0$

$$y[n] = \sum_{k=-\infty}^{0} 2^k$$

# Example

Let $l = -k$

$$y[n] = \sum_{l=0}^{\infty} 2^{-l}$$

$$y[n] = \sum_{l=0}^{\infty} (\frac{1}{2})^{l}$$

$$y[n] = \frac{1}{1 - \frac{1}{2}}$$

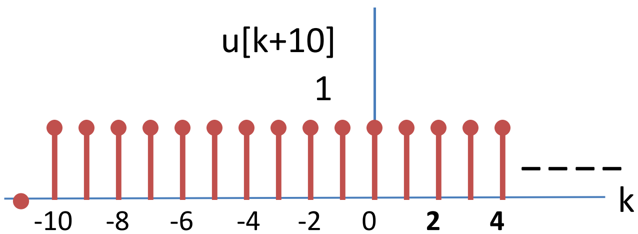
$$= 2$$

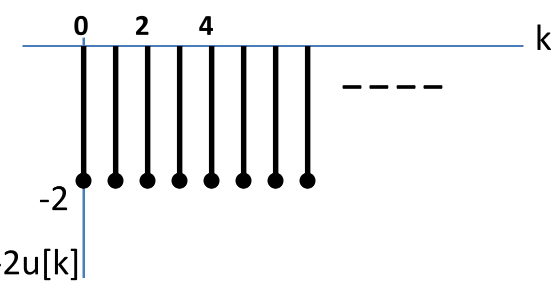
$$y[n] = \begin{cases} 2^{n+1}, n < 0 \\ 2, n \geq 0 \end{cases}$$

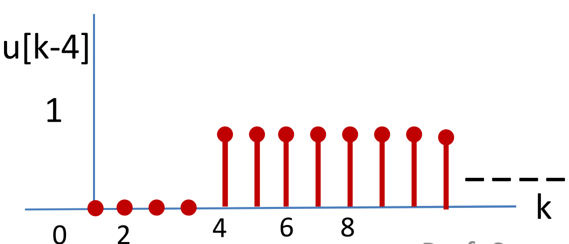
# Example

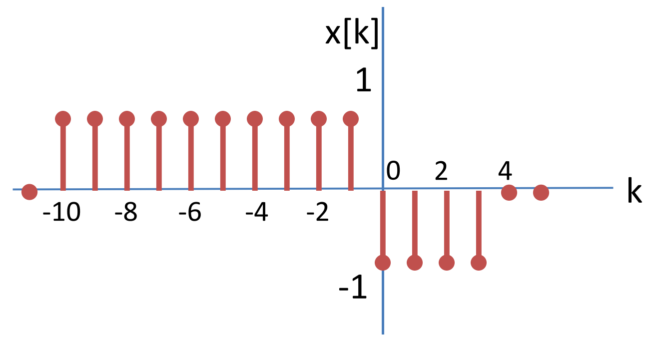
$$y[n] = (u[n+10] - 2u[n] + u[n-4]) * u[n-2]$$

Ans:

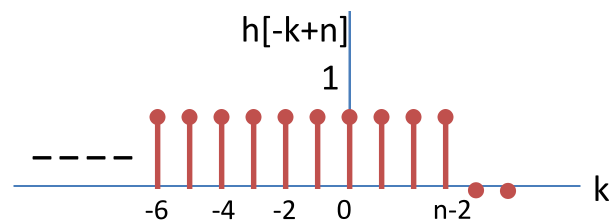


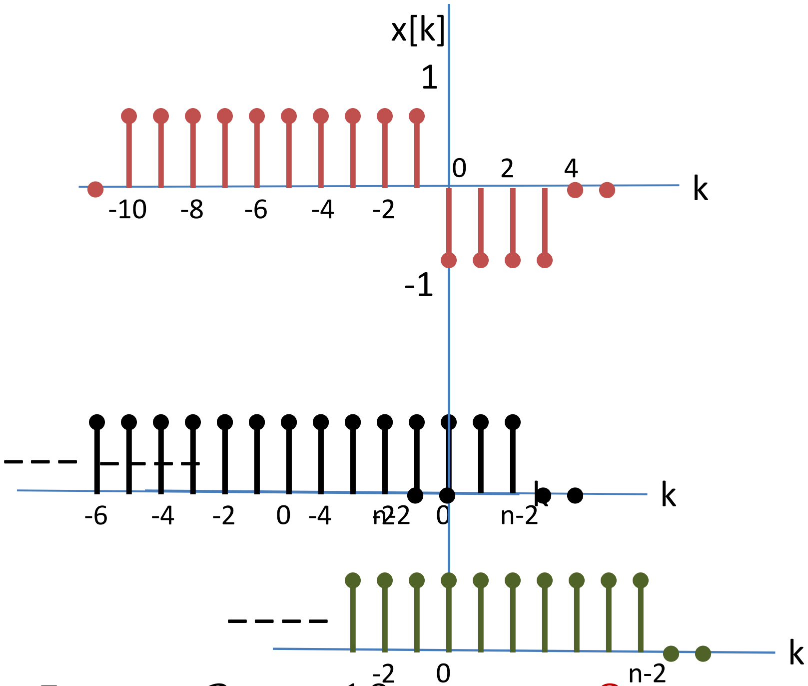




$$h[k] = u[k-2]$$
$$h[-k] = u[-k-2]$$
$$h[n-k] = u[-k-2+n]$$
$$= u[-k+n-2]$$



# Example




For $n-2<-10$ , $\quad n<-8$

$$y[n]=0$$

For $-10\le n-2<0, \quad -8\le n<2$

$$y[n]=\sum_{k=-10}^{n-2}1$$

Let k+10=m

$$y[n]=\sum_{m=0}^{n+8}1=n+9$$

For $0\le n-2<4, \qquad 2\le n<6$

$$y[n]=\sum_{k=-10}^{-1}1+\sum_{k=0}^{n-2}-1$$

$$=\sum_{m=0}^{9}1+\sum_{k=0}^{n-2}-1$$

$$=10-(n-1)=11-n$$

For $n-2\ge 4, \qquad n\ge 6$

$$y[n]=\sum_{k=-10}^{-1}1+\sum_{k=0}^{3}-1$$

$$=10+(-4)=6$$

# Example

Find the convolution sum of two sequences.

$x[n] = \{\underset{\uparrow}{1}, 2, 3\}$ $\quad h[n] = \{\underset{\uparrow}{2}, 1, 4\}$

$y[n] = \{\underset{\uparrow}{2}, 5, 12, 11, 12\}$

| x[n] / h[n] | 1 | 2 | 3 |
|---|---|---|---|
| 2 | | | |
| 1 | | | |
| 4 | | | |

# Additional example

For the two signals shown in Fig.Q5. obtain the expression for (a)y(t) in terms of x(t)

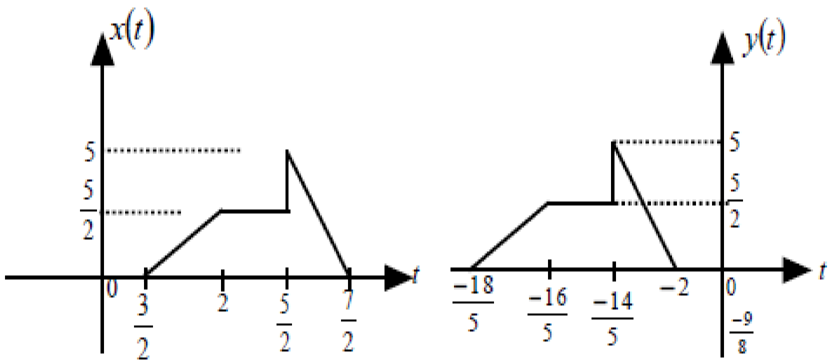


Fig.Q5

(a) $y(t) = x(at - b)$ (1)

a and b are the two unknowns

From Fig.

$$y\left(\frac{3}{2}\right) = x\left(-\frac{18}{5}\right)$$

Using eq.(1)

$$\frac{3}{2} = -a\frac{18}{5} - b \qquad (2)$$

Similarly $y(2) = x(-\frac{16}{5})$

$$2 = -a\frac{16}{5} - b \qquad (3)$$

Solve (2)& (3)

$$a = \frac{5}{4}, b = -6$$

$$y(t) = x\left(\frac{5}{4}t + 6\right)$$

# Additional Example

The o/p y[n] of an LTI system is given as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n+2k]$$

Where x[n] is the i/p and $g[n] = u[n] - u[n-3]$. Determine y[n] when $x[n] = \delta[n-2]$.

Ans:

Change of variable $x[k] = \delta[k-2]$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-2]\ g[n+2k]$$

$\delta[k-2]$ exist only at k=2

$$y[n] = g[n+4]$$

$$g[n+4] = u[n+4] - u[n+1]$$

# Convolution Integral

Consider an LTI system with impulse response $h(t)$

$$\delta(t) \rightarrow h(t)$$

$$\delta(t-\tau) \rightarrow h(t-\tau)$$ time invariance

$$x(\tau)\delta(t-\tau) \rightarrow x(\tau)h(t-\tau)$$ homogeneity

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\,\tau \rightarrow \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$ super position

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$ Convolution integral

# Example

$$y(t) = u(t+1) * u(t-2)$$

Ans:

Change of variable

$$x(\tau) = u(\tau+1), h(\tau) = u(\tau-2)$$

$$h(-\tau) = u(-\tau-2)$$

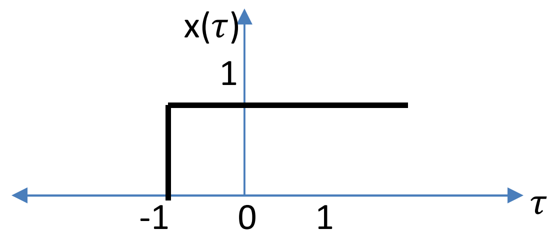
$$h(t-\tau) = u(-\tau+t-2)$$

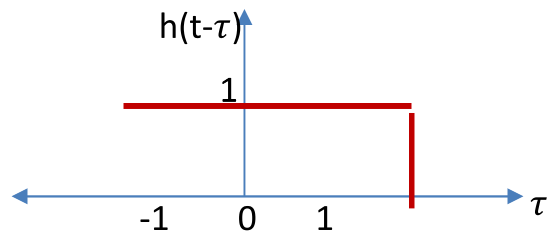
For $t-2 < -1,$ $t < 1$

$$y(t) = 0$$

For $t-2 \geq -1,$ $t \geq 1$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau$$




$$y(t) = \int_{-1}^{t-2} 1\, d\tau$$

$$= t-2+1$$

$$= t-1$$

$$y(t) = \begin{cases} 0, t < 1 \\ t-1, t \geq 1 \end{cases}$$

# Example

$$y(t) = [u(t+2) - u(t-1)] * u(-t+2)$$

Ans:

Change of variable

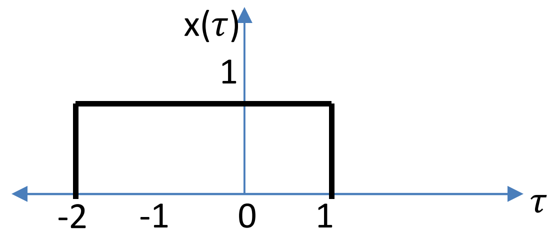
$$x(\tau) = u(\tau+2) - u(\tau-1)$$
$$h(\tau) = u(-\tau+2)$$
$$h(-\tau) = u(\tau+2)$$
$$h(t-\tau) = u(\tau+t+2)$$

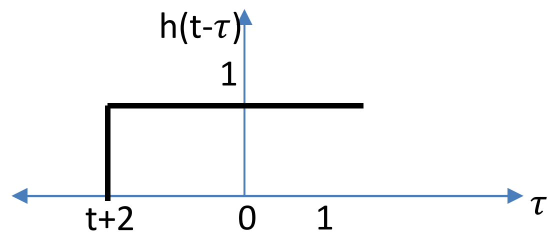
For $t+2 < -2$ , $t < -4$

$$y(t) = \int_{-2}^{1} 1 d\,\tau$$

$$y(t) = 3$$

For $-2 \leq t+2 < 1$, $-4 \leq t < -1$




$$y(t) = \int_{t+2}^{1} 1 d\tau$$
$$= 1 - (t+2)$$
$$= -t - 1$$

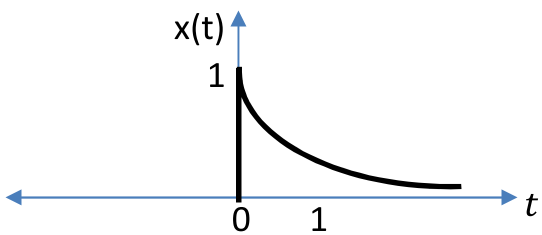
For $t+2 \geq 1$, $t \geq -1$

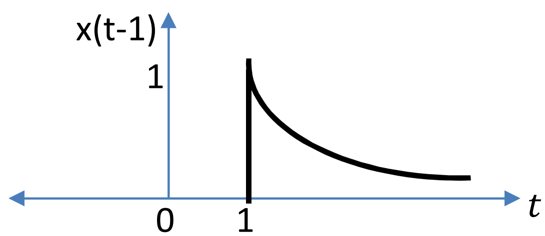
$$y(t) = 0$$

# Example

Exponential Function

$$x(t) = e^{-t}u(t)$$



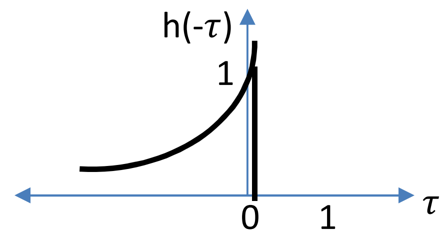



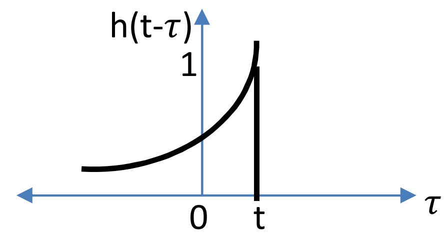

$$x(t-1) = e^{-(t-1)}u(t-1)$$
$$= e^{-t+1}u(t-1)$$

$$h(\tau) = e^{-\tau}u(\tau)$$

Plot $h(t-\tau)$

Ans:



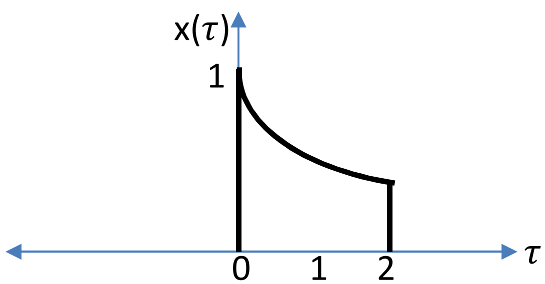

$$h(t-\tau) = e^{-(t-\tau)}u(t-\tau)$$
$$= e^{\tau-t}u(-\tau+t)$$

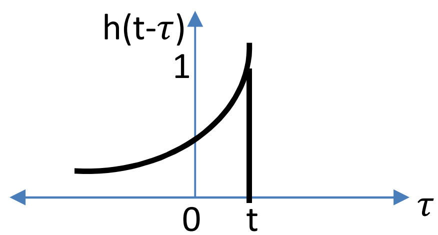
# Example

$$x(t) = e^{-3t}[u(t) - u(t-2)]$$
$$h(t) = e^{-t}u(t)$$

**Ans**:

Change of variable




For $t < 0, y(t) = 0$

For 0$\leq t < 2$

$$y(t) = \int_0^t e^{-3\tau}e^{-(t-\tau)}d\tau$$

$$= e^{-t}\int_0^t e^{-2\tau}d\tau$$

$$= -e^{-t}\left[\frac{e^{-2t}-1}{-2}\right]$$

$$= \left[\frac{e^{-t}-e^{-3t}}{2}\right]$$

For $t > 2$

$$y(t) = e^{-t}\int_0^2 e^{-2\tau}d\tau$$

$$= \frac{e^{-t}}{2}[1-e^{-4}]$$

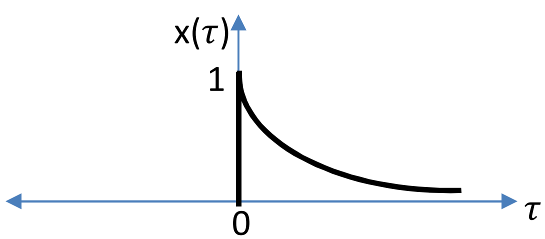
# Example

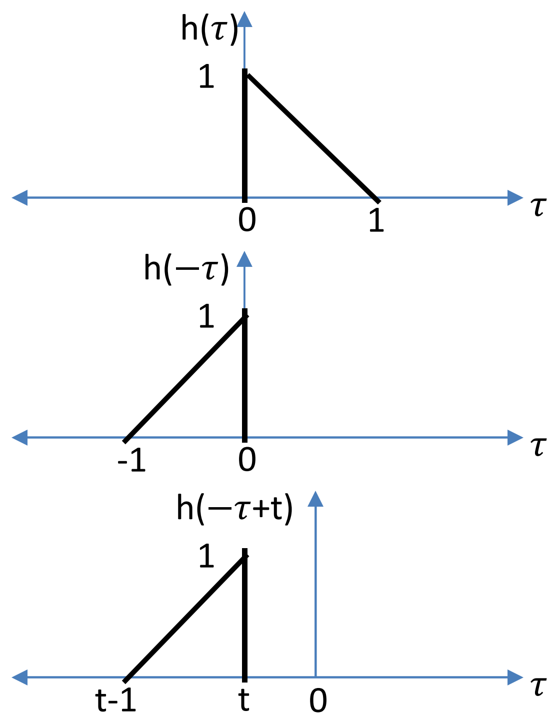
The input signal $x(t) = e^{-t}u(t)$ to an LTI system whose impulse response is given by

$$h(t) = \begin{cases} 1 - t, 0 \le t \le 1 \\ 0, otherwise \end{cases}$$

Calculate the o/p.

Ans:



# Example

For $t < 0$

$$y(t) = 0$$

For $t \geq 0$ and $t - 1 \leq 0$

$0 \leq t \leq 1$

$$y(t) = \int_0^t x(\tau)h(t-\tau)d\tau$$

$$= \int_0^t e^{-\tau}u(\tau)(\tau + 1 - t)d\tau$$

$$= \int_0^t \tau e^{-\tau}d\tau + (1-t)\int_0^t e^{-\tau}d\tau$$

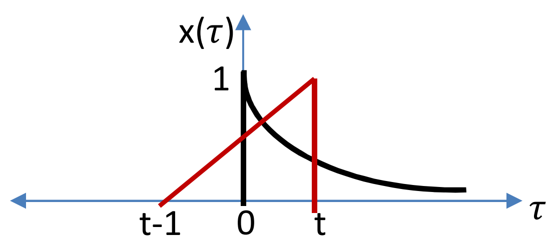
*Indefinite integral*

$$\int te^{at}dt = \frac{e^{at}}{a^2}(at-1)$$

$$= [-\tau e^{-\tau} - e^{-\tau}]| + (1-t)[-e^{-\tau}]|$$

$$= [1 - te^{-t} - e^{-t}] + (1-t)[-e^{-t}]$$

$$y(t) = 2 - t - 2e^{-t}$$



# Example

For $t - 1 > 0$

$$t > 1$$

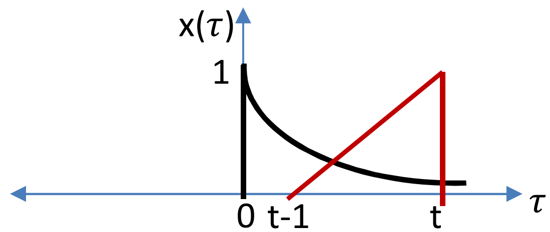
$$y(t) = \int_{t-1}^{t} x(\tau)h(t-\tau)d\tau$$

$$= \int_{t-1}^{t} e^{-\tau}(\tau + 1 - t)d\tau$$

$$= \int_{t-1}^{t} \tau e^{-\tau}d\tau + (1-t)\int_{t-1}^{t} e^{-\tau}d\tau$$

$$= [-\tau e^{-\tau} - e^{-\tau}]| + (1-t)[-e^{-\tau}]|$$

$$y(t) = e^{-(t-1)} - 2e^{-t}$$



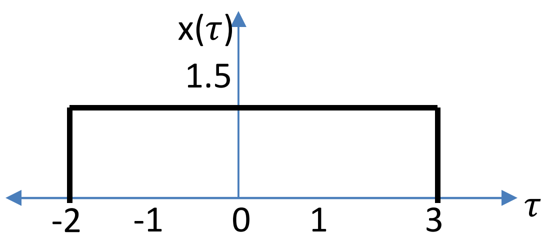
# Example

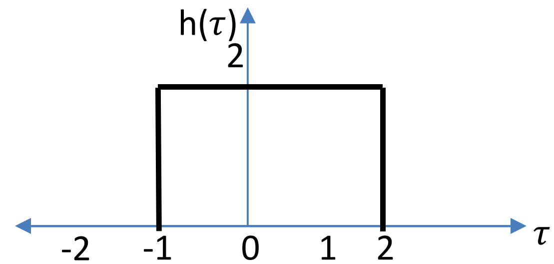
Calculate the o/p for the following i/p signal & impulse

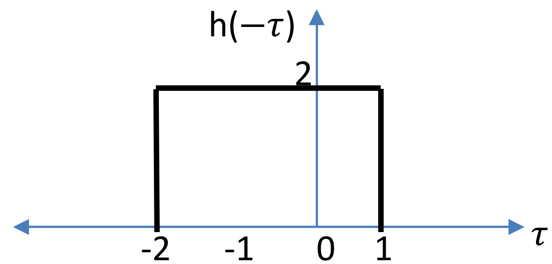
$$x(t) = \begin{cases} 1.5, -2 \le t \le 3 \\ 0, otherwise \end{cases}$$

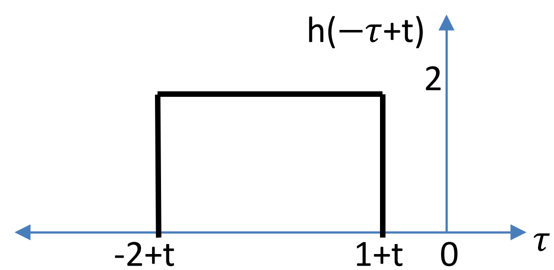
$$h(t) = \begin{cases} 2, -1 \le t \le 2 \\ 0, otherwise \end{cases}$$

Ans:



# Example

For $1 + t > -2$
$t < -3$
$y(t) = 0$

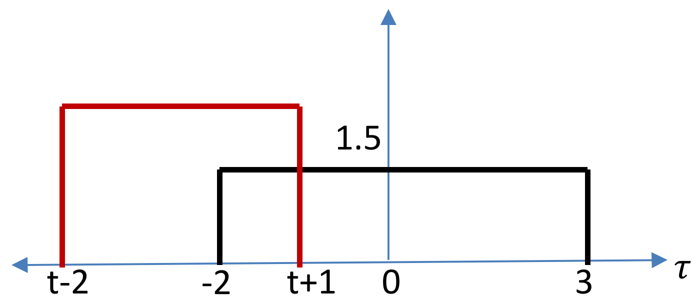
For $t + 1 \geq -2 \;\&\; t + 1 < 0$
$-3 \leq t < -1$

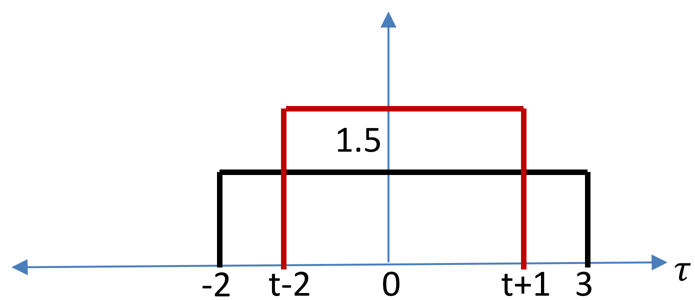
$$y(t) = \int_{-2}^{t+1} x(\tau)h(t-\tau)d\tau$$

$$= \int_{-2}^{t+1} 3d\tau$$

$$= 3(t+3)$$

For $t + 1 \geq 0 \;\& -2 + t < 0$
$-1 \leq t < 2$




$$y(t) = \int_{t-2}^{t+1} 3d\tau$$

$$= 9$$

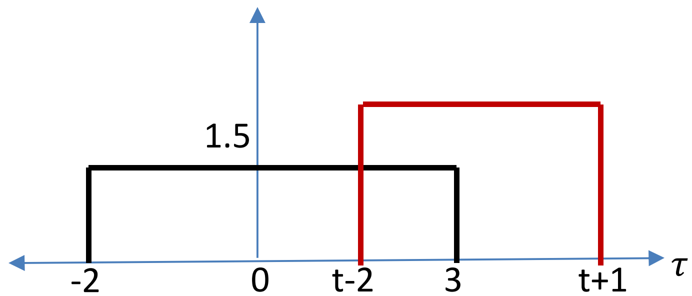
# Example

For $t + 1 \geq 3$ and $t - 2 \leq 3$

$$2 \leq t \leq 5$$

$$y(t) = \int_{t-2}^{3} 3d\tau$$

$$= 3(5 - t)$$




for $t \geq 5$

$$y(t) = 0$$