Maximum Power Transfer Theorem

The amount of power received by a load is an important parameter in electrical and electronic applications. In DC circuits, we can represent the load with a resistor having resistance of R_L ohms. Similarly, in AC circuits, we can represent it with a complex load having an impedance of Z_L ohms.

Maximum power transfer theorem states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.

Similarly, **Maximum power transfer theorem** states that the AC voltage source will deliver maximum power to the variable complex load only when the load impedance is equal to the complex conjugate of source impedance.

In this chapter, let us discuss about the maximum power transfer theorem for DC circuits.

Proof of Maximum Power Transfer Theorem

Replace any two terminal linear network or circuit to the left side of variable load resistor having resistance of R_L ohms with a Thevenin's equivalent circuit. We know that Thevenin's equivalent circuit resembles a practical voltage source.

This concept is illustrated in following figures.
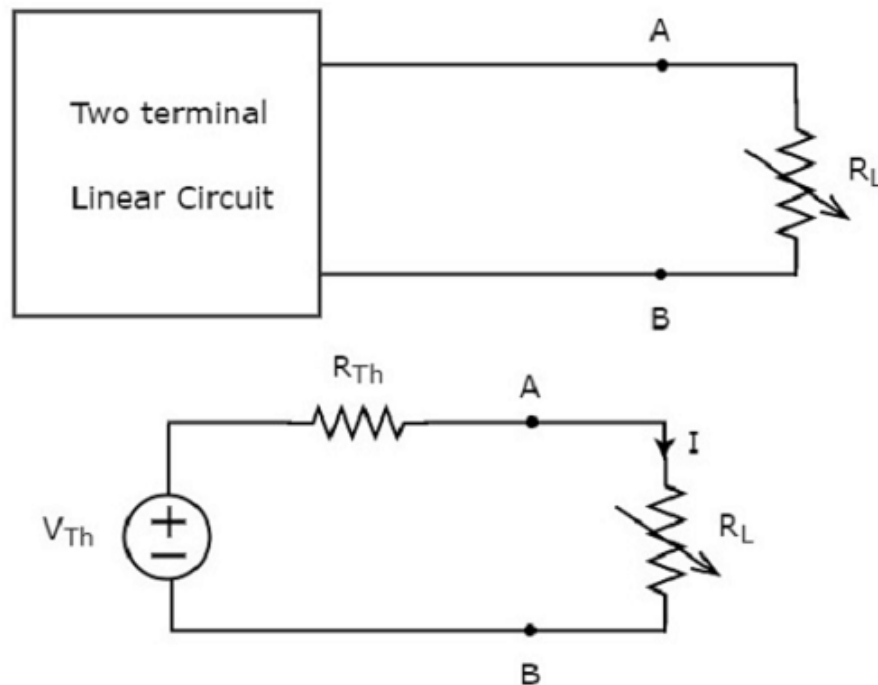
The amount of power dissipated across the load resistor is

$$P_L = I^2 R_L$$

Substitute $I = \dfrac{V_{Th}}{R_{Th} + R_L}$ in the above equation.

$$P_L = \left(\frac{V_{Th}}{(R_{Th} + R_L)} \right)^2 R_L$$

$$\Rightarrow P_L = V_{Th}{}^2 \{ \frac{R_L}{(R_{Th}+R_L)^2} \} \qquad\qquad \textbf{Equation 1}$$

Condition for Maximum Power Transfer

For maximum or minimum, first derivative will be zero. So, differentiate Equation 1 with respect to R_L and make it equal to zero.

$$\frac{dP_L}{dR_L} = V_{Th}{}^2 \{ \frac{(R_{Th}+R_L)^2 \times 1 - R_L \times 2(R_{Th}+R_L)}{(R_{Th}+R_L)^4} \} = 0$$

$$\Rightarrow (R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L) = 0$$

$$\Rightarrow (R_{Th}+R_L)(R_{Th}+R_L-2R_L) = 0$$

$$\Rightarrow (R_{Th}-R_L) = 0$$

$$\Rightarrow R_{Th} = R_L \ or \ R_L = R_{Th}$$

Therefore, the **condition for maximum power** dissipation across the load is $R_L = R_{Th}$. That means, if the value of load resistance is equal to the value of source resistance i.e., Thevenin's resistance, then the power dissipated across the load will be of maximum value.

The value of Maximum Power Transfer

Substitute $R_L = R_{Th} \ \& \ P_L = P_{L,Max}$ in Equation 1.

$$P_{L,Max} = V_{Th}{}^2 \{ \frac{R_{Th}}{(R_{Th}+R_{Th})^2} \}$$

$$P_{L,Max} = V_{Th}{}^2 \{ \frac{R_{Th}}{4R_{Th}{}^2} \}$$

$$\Rightarrow P_{L,Max} = \frac{V_{Th}{}^2}{4R_{Th}}$$

$$\Rightarrow P_{L,Max} = \frac{V_{Th}{}^2}{4R_L}, \ since \ R_L = R_{Th}$$

Therefore, the **maximum amount of power** transferred to the load is

$$P_{L,Max} = \frac{V_{Th}^2}{4R_L} = \frac{V_{Th}^2}{4R_{Th}}$$

Efficiency of Maximum Power Transfer

We can calculate the efficiency of maximum power transfer, η_{Max} using following formula.

$$\eta_{Max} = \frac{P_{L,Max}}{P_S}$$ **Equation 2**

Where,

- $P_{L,Max}$ is the maximum amount of power transferred to the load.

- P_S is the amount of power generated by the source.

The **amount of power generated** by the source is

$$P_S = 2I^2 R_{Th} + I^2 R_L$$

$$\Rightarrow P_S = 2I^2 R_{Th}, \ since \ R_L = R_{Th}$$

- Substitute $I = \frac{V_{Th}}{2R_{Th}}$ in the above equation.

$$P_S = 2\left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th}$$

$$\Rightarrow P_S = 2\left(\frac{V_{Th}^2}{4R_{Th}^2}\right) R_{Th}$$

$$\Rightarrow P_S = \frac{V_{Th}^2}{2R_{Th}}$$

- Substitute the values of $P_{L,Max}$ and P_S in Equation 2.

$$\eta_{Max} = \frac{\left(\frac{V_{Th}^2}{4R_{Th}}\right)}{\left(\frac{V_{Th}^2}{2R_{Th}}\right)}$$

$$\Rightarrow \eta_M \qquad \frac{1}{2}$$

$$\%\eta_M \qquad \eta_M$$

$$\Rightarrow \quad \eta_M \qquad \frac{1}{2}$$

$$\Rightarrow \quad \eta_M$$

$$\Rightarrow \eta_M \qquad \frac{1}{2}$$

$$\%\eta_M \qquad \eta_M$$