Edge Disjoint Spanning Trees in an Undirected Graph

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Introduction

Given an undirected , unweighted graph G with n vertices and 2n-2 edges, one of which is a double edge. Find if G admits two edge-disjoint spanning trees and if yes output them.

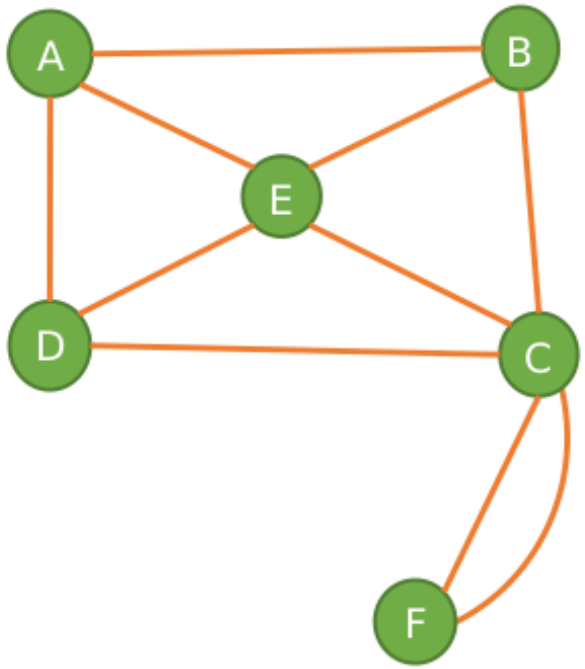

Figure: A Graph with $2n-2$ edges including a double edge

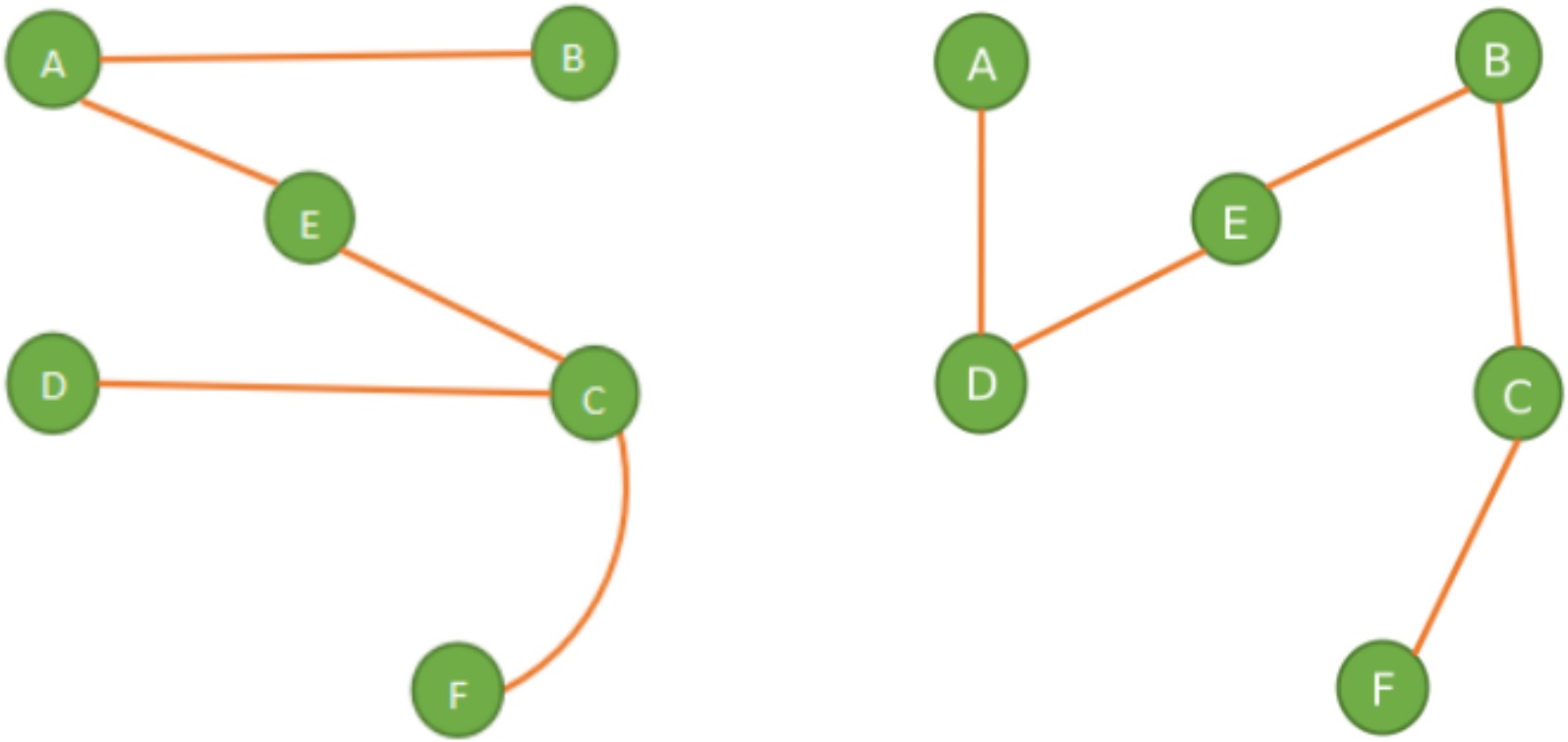

Figure: One of the possible solutions edge-disjoint spanning trees

Earlier Research

Algorithm for finding 2 edge-disjoint spanning trees in a directed graph was first proposed by Prof. R. Tarjan in his paper titled Edge-disjoint spanning trees and depth-first search with running time of O(m+n). The case of the undirected graph was however left open. An algorithm was proposed by Dr. H.N. Gabow for a general undirected graph. Given restrictive settings of $2n-2$ edges, We aim to propose an O(n) algorithm for computing two trees.

Investigation so far

We tried implementing Tarjan's Algorithm for the directed trees and tied modifying it for the undirected graphs but we haven't achieved any kind of breakthrough related to this. Parallely, We have tried to build the two trees constructively and have successfully achieved the edge allocation for vertices of degree 2, 3 and 4. Based on these we have proposed an algorithm for constructing the trees, through which we have been able to reduce the number of edges to less than **n**

Lemmas Proved

Lemma 1

Given the graph G, for each of it's two degree vertex, we can always randomly put one of it's edge in T1 and the other in T2.Thereby, Obtaining a new smaller instance of the initial problem.

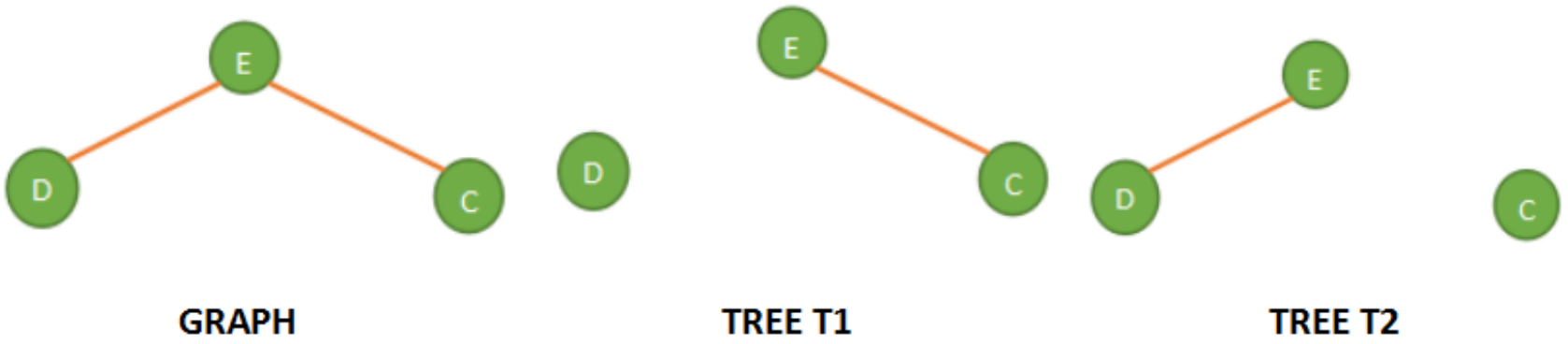

Figure: A vertex with degree 2

Lemmas Proved

Lemma 2

We can randomly exchange two edges of a 3 degree vertex in the given graph in one of the tree and the other edge in other tree.

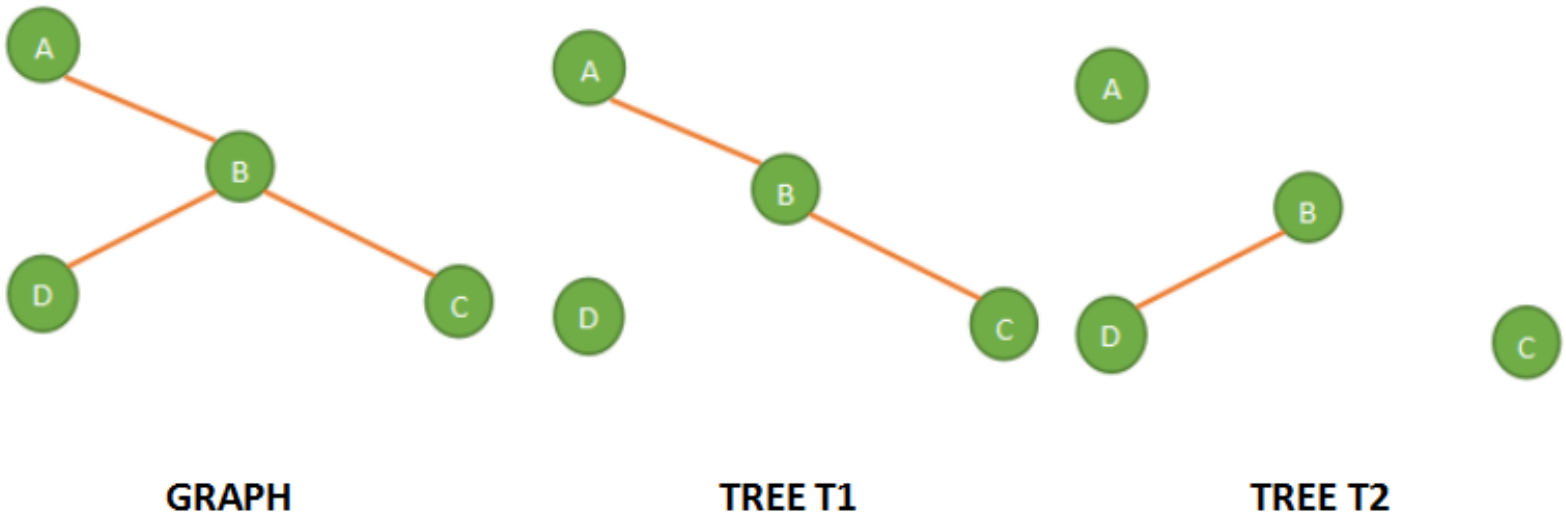

Figure: A vertex with degree 3

Lemmas Proved

Lemma 3

For the graph $G^{'}$ that is formed after removing 2 and 3 degree vertices from the original graph, we can randomly put one of the remaining edge of 4 degree vertices that has been reduced to 3 degree in $G^{'}$, in the tree where it is disconnected.

Algorithm Proposed

The algorithm proposed removes atleast n edges from the experimented original graph and put them in the trees T1 and T2. Following are the list of steps of the algorithm:-

STEP 1:- Sort the vertices in ascending order according to their degree.

STEP 2:- Start removing two edges corresponding to every vertex starting from the first one in the sorted list and put one of them in tree T1 and other in tree T2. If you encounter a vertex that doesnt have 2 edges left, ignore it and move ahead to the next vertex.

Algorithm Validation

To create the input set to check the algorithm, we have used Laman graph that has 2n-3 edges with n vertices. Laman graph has the property that if a DFS tree is constructed and then one of the remaining edge is doubled and the same edge is doubled in the parent graph so that it has 2n-2 edges now, then that graph admits two edge disjoint trees.

Future Work

We will be working with Professor Shashank Mehta on this project during the semester. We will explore ways in which we can reduce three degree vertex to get a smaller instance of the graph. We plan to exploit 2 cut property of the graph to re

Thank You!