Gibbs sampling for the two-sample t-test

Chapter 3.2.1: Gibbs sampling

In this exercise we will use Gibbs sampling to test whether two populations have the same mean. Let the data from the first population be

$$Y_1, \ldots, Y_n \sim \text{Normal}(\mu_Y, \sigma^2)$$

and the second population be

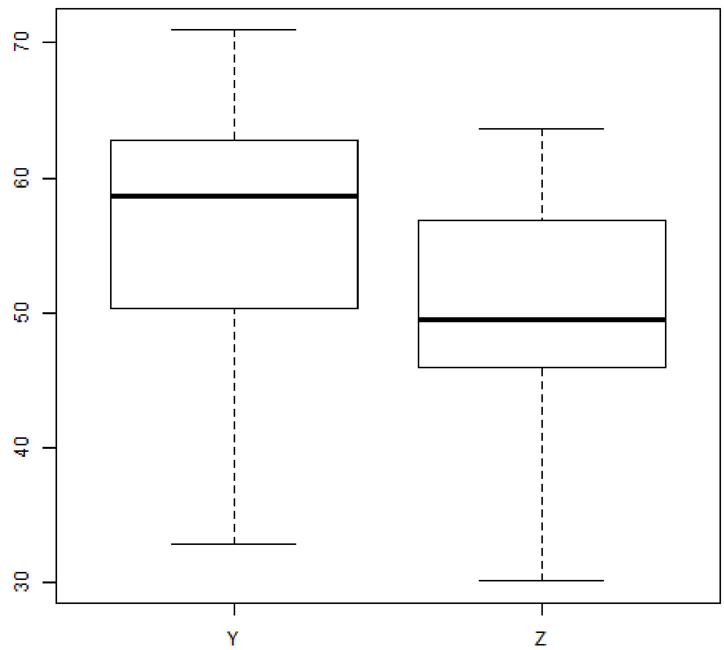
$$Z_1, \ldots, Z_m \sim \text{Normal}(\mu_Z, \sigma^2).$$

The priors are $\mu_Y, \mu_Z \sim \text{Normal}(\mu_0, \sigma_0^2)$ and $\sigma^2 \sim \text{InvGamma}(a, b)$.

Our goal is to compute the posterior of $\Delta = \mu_Y - \mu_Z$ and determine if $\Delta = 0$ (the populations have the same mean) or not. To do these, we compute Δ at each iteration and then compute the posterior 95% credible set. To test out the code we will simulate data so we know the true values.

Simulate data

```
set.seed(1)

n         <- 20
m         <- 20
muY_true  <- 55
muZ_true  <- 50
sig2_true <- 100

Y <- rnorm(n,muY_true,sqrt(sig2_true))
Z <- rnorm(n,muZ_true,sqrt(sig2_true))

boxplot(cbind(Y,Z))
```



Set the priors

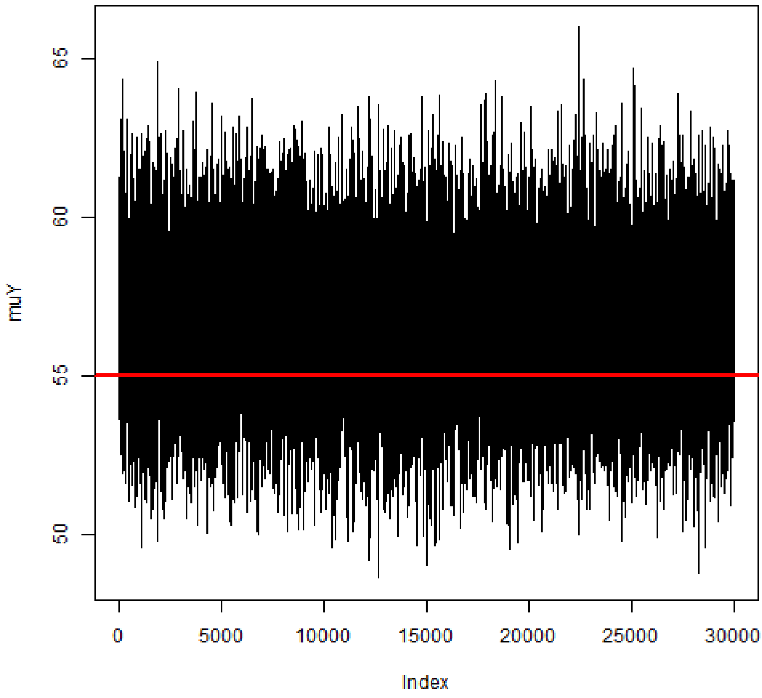
```r
mu0 <- 0
s20 <- 1000
a   <- 0.01
b   <- 0.01
```

Gibbs sampling

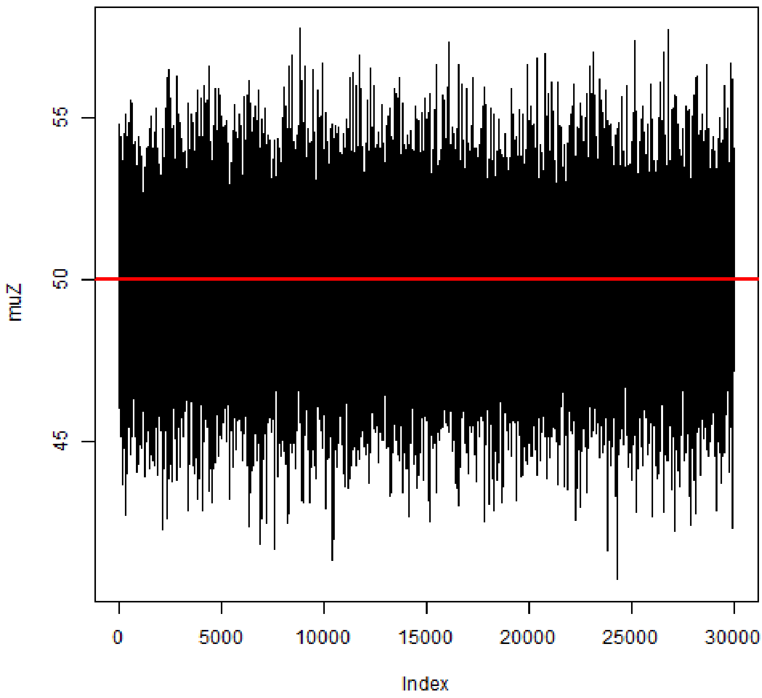
```r
n.iters <- 30000
keepers <- matrix(0,n.iters,4)
colnames(keepers)<-c("muY","muZ","sigma2","Delta")

# Initial values
muY          <- mean(Y)
muZ          <- mean(Z)
s2           <- (var(Y)+var(Z))/2
keepers[1,] <- c(muY,muZ,s2,muY-muZ)

for(iter in 2:n.iters){

  # sample muY|muZ,s2,Y,Z

   A    <- sum(Y)/s2+mu0/s20
   B    <- n/s2+1/s20
   muY  <- rnorm(1,A/B,1/sqrt(B))

  # sample muZ|muY,s2,Y,Z

   A    <- sum(Z)/s2+mu0/s20
   B    <- m/s2+1/s20
   muZ  <- rnorm(1,A/B,1/sqrt(B))

  # sample s2|muY,muZ,Y,Z

   A  <- n/2+m/2+a
   B  <- sum((Y-muY)^2)/2 + sum((Z-muZ)^2)/2+b
   s2 <- 1/rgamma(1,A,B)

  # keep track of the results
   keepers[iter,] <- c(muY,muZ,s2,muY-muZ)

}
```

Plot convergence

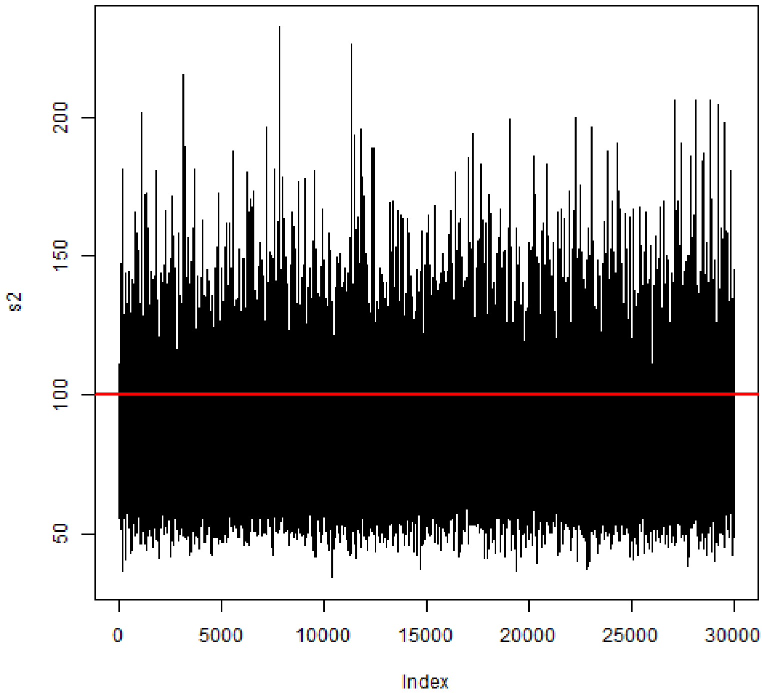
```r
plot(keepers[,1],type="l",ylab="muY")
abline(muY_true,0,col=2,lwd=2)
```



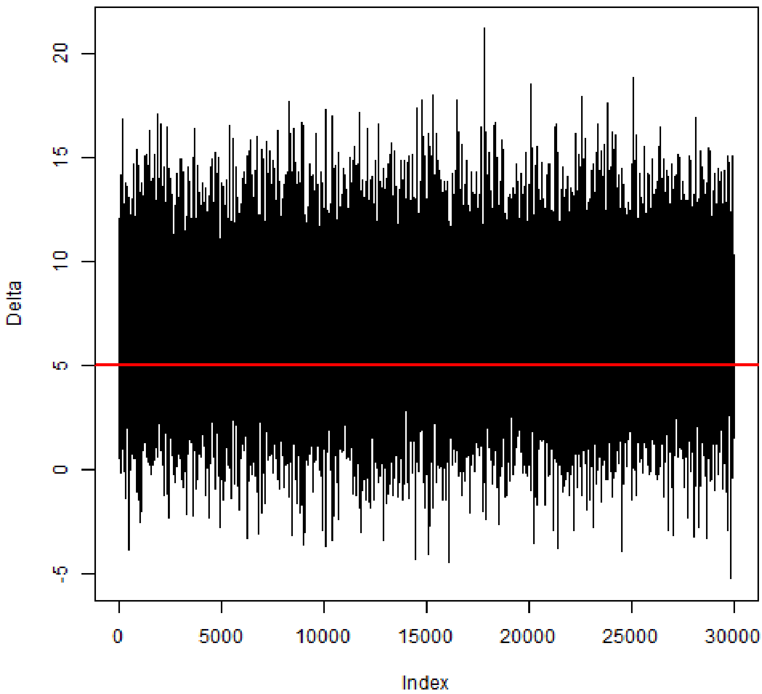

```
plot(keepers[,2],type="l",ylab="muZ")
abline(muZ_true,0,col=2,lwd=2)
```




```
plot(keepers[,3],type="l",ylab="s2")
abline(sig2_true,0,col=2,lwd=2)
```

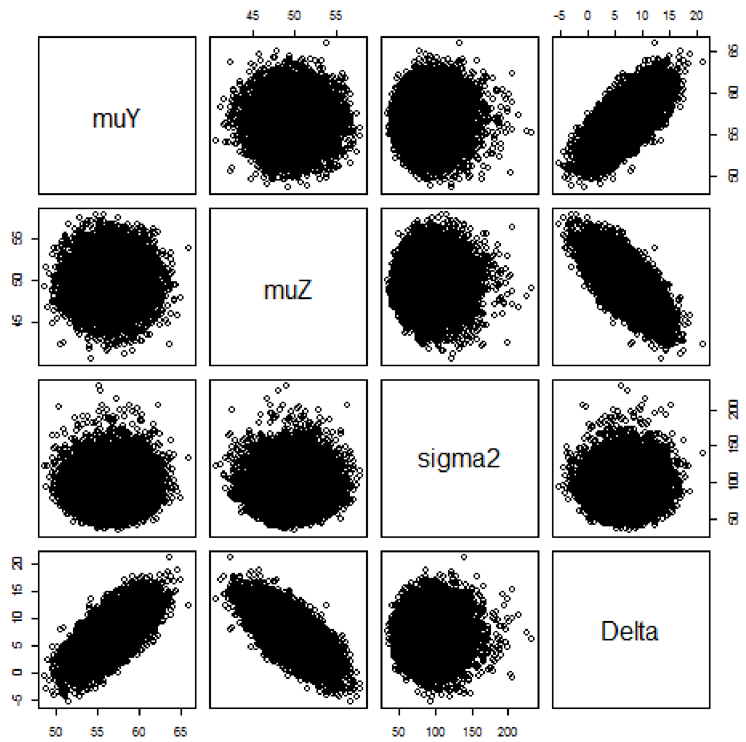



```r
plot(keepers[,4],type="l",ylab="Delta")
abline(muY_true-muZ_true,0,col=2,lwd=2)
```



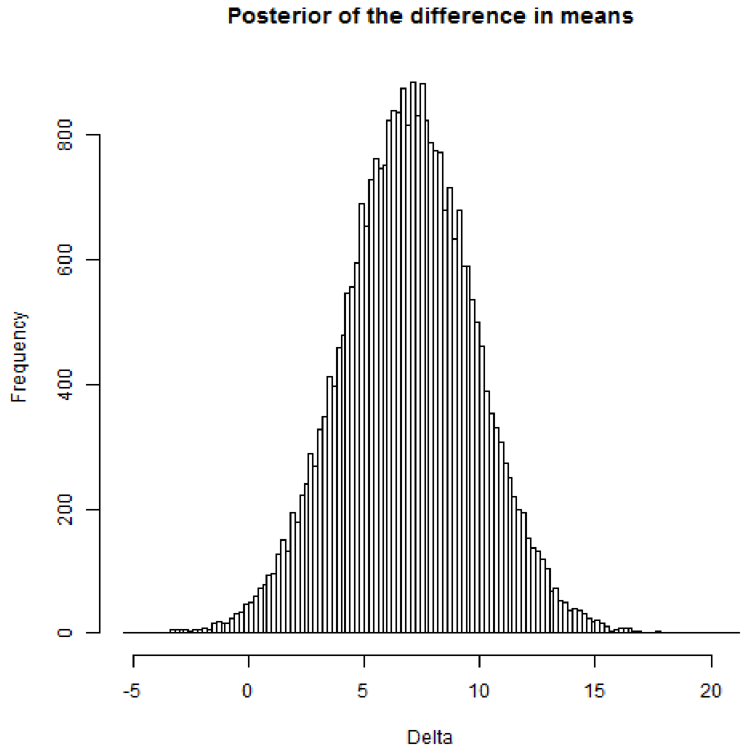
Plot the joint posterior

```r
pairs(keepers)
```



Summarize the marginal posterior of $\Delta = \mu_Y - \mu_Z$, $p(\Delta | Y, Z)$

```r
Delta <- keepers[,4]
hist(Delta,main="Posterior of the difference in means",breaks=100)
```




```r
muY_true;muZ_true;muY_true-muZ_true # True values
```

```
## [1] 55
```

```
## [1] 50
```

```
## [1] 5
```

```r
quantile(Delta,c(0.025,0.975)) # Posterior 95% credible set
```

```
##      2.5%     97.5%
##  1.263061 12.612540
```

```r
mean(Delta>0) # Posterior probability that muY>muZ
```

```
## [1] 0.9911667
```

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