Metropolis + Gibbs sampling for simulated data

Chapter 3.2.3: Metropolis sampling

Here we use simulate data to verify that the code is working properly. We generate a synthetic dataset and run MCMC to estimate the parameters. Unlike a real data analysis, for a simulation study we know the true values of the parameters and so this provides a check that the code is functioning well.

The data are generated from the logistic regression model

$$Y_i \sim \text{Bernoulli}(p_i) \text{ where } p_i = \frac{1}{1 + \exp(-\sum_{j=1}^{p} X_{ij}\beta_j)}.$$

The covariates are generated as $X_{i1} = 1$ for the intercept and $X_{ij} \sim \text{Normal}(0, 1)$ for $j > 1$. The true values of β_j are also sampled from a normal distribution.

We fit the model with prior $\beta_j \sim \text{Normal}(0, \sigma^2)$ and $\sigma^2 \sim \text{Gamma}(a, b)$. The MCMC code using Metropolis steps to update the β_j and a Gibbs step to update σ^2.

Simulate data

```
n          <- 100
p          <- 20
X          <- cbind(1,matrix(rnorm(n*(p-1)),n,p-1)) # first column for the intercept
true_beta  <- rnorm(p,0,.5)
prob       <- 1/(1+exp(-X%*%true_beta))
Y          <- rbinom(n,1,prob)
```

Initialize

```
# Create matrix to store the samples

 S                <- 25000
 samples          <- matrix(NA,S,p+1)

# Initial values

 beta    <- rep(0,p)
 sigma   <- 1

# priors:

 a      <- 0.1
 b      <- 0.1

# candidate standard deviation:

 can_sd <- 0.1
```

Define the log posterior as a function

```
log_post_beta <- function(Y,X,beta,sigma){
    prob   <- 1/(1+exp(-X%*%beta))
    like   <- sum(dbinom(Y,1,prob,log=TRUE))
    prior <- dnorm(beta[1],0,10,log=TRUE) +        # Intercept
             sum(dnorm(beta[-1],0,sigma,log=TRUE)) # Slopes
return(like+prior)}
```

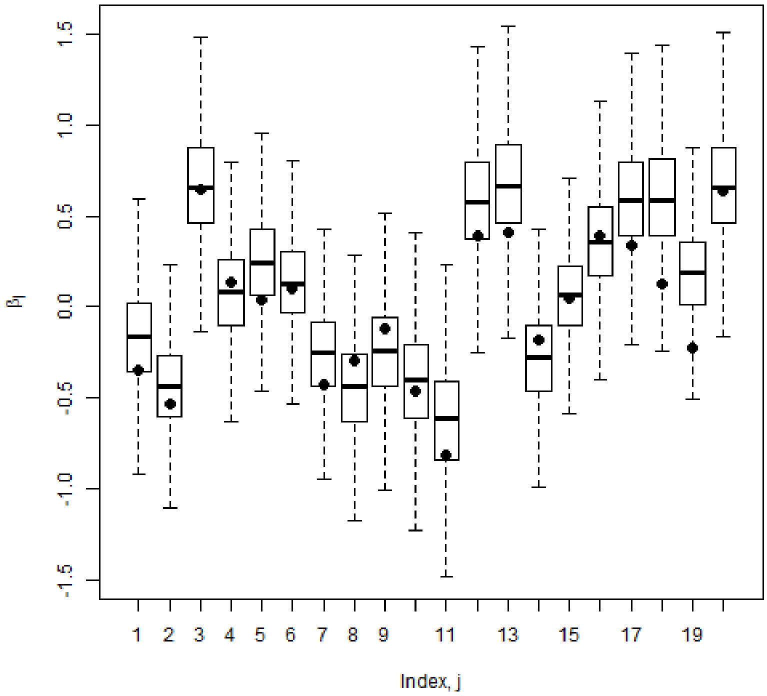
Metropolis sampling

```r
for(s in 1:S){

  # Metropolis for beta
  for(j in 1:p){
    can     <- beta
    can[j] <- rnorm(1,beta[j],can_sd)
    logR   <- log_post_beta(Y,X,can,sigma)-
              log_post_beta(Y,X,beta,sigma)
    if(log(runif(1))<logR){
      beta <- can
    }
  }

  # Gibbs for sigma
  sigma <- 1/sqrt(rgamma(1,(p-1)/2+a,sum(beta[-1]^2)/2+b))

  samples[s,] <- c(beta,sigma)
}
```

Plot the results

The boxplots are the posterior distributions for the β_j, and the points are the true values used to generate the data. The posteriors include the truth for most of the parameters indicating the alorithm is working well.

```r
boxplot(samples[,1:p],outline=FALSE,xlab="Index, j",ylab=expression(beta[j]))
points(true_beta,pch=19,cex=1.25)
```



Loading [MathJax]/jax/output/HTML-CSS/jax.js