Gibbs sampling for a one sample t-test

Chapter 3.2.1: Gibbs sampling

Assume $Y_i|\mu,\sigma^2 \sim \text{Normal}(\mu,\sigma^2)$ for $i = 1,\ldots,n$ and let the prior distributions be $\mu \sim \text{Normal}(0,\sigma^2/m)$ and $\sigma^2 \sim \text{InvGamma}(a,b)$. It can be shown (Chapter 2) that the full conditional distributions are

$$\mu\,|\,\sigma^2, Y_1,\ldots,Y_n \sim \text{Normal}\left(\frac{\sum_{i=1}^n Y_i}{n+m}, \frac{\sigma^2}{n+m}\right)$$

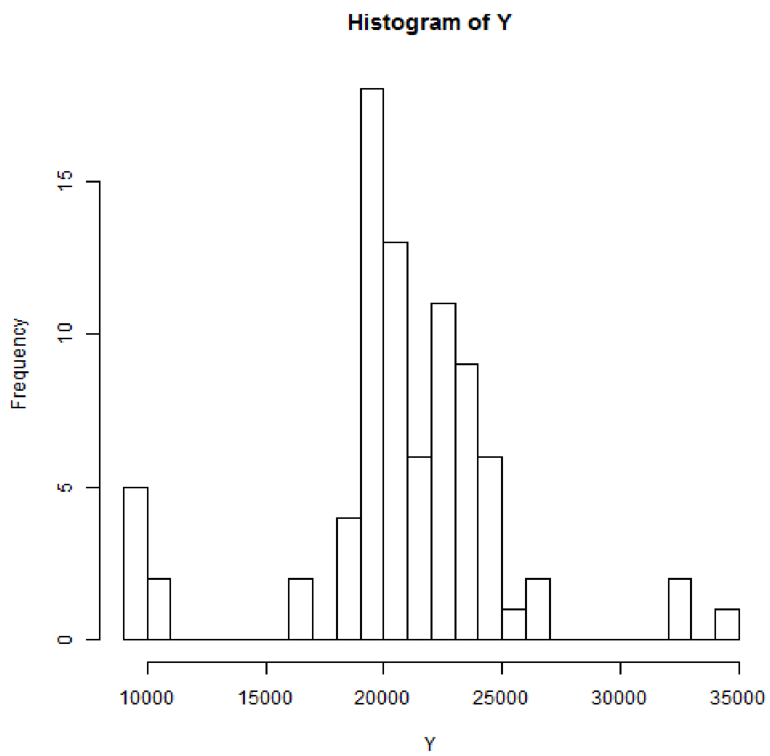
and

$$\sigma^2\,|\,\mu, Y_1,\ldots,Y_n \sim \text{InvGamma}\left(a+n/2, b+\sum_{i=1}^n (Y_i-\mu)^2/2\right).$$

Gibbs sampling iterates between drawing from these two (univariate) full conditional distributions to produce samples from the joint (bivariate) posterior distribution.

Load the galaxy data

```
library(MASS)
Y <- galaxies
n <- length(Y)
hist(Y,breaks=25)
```

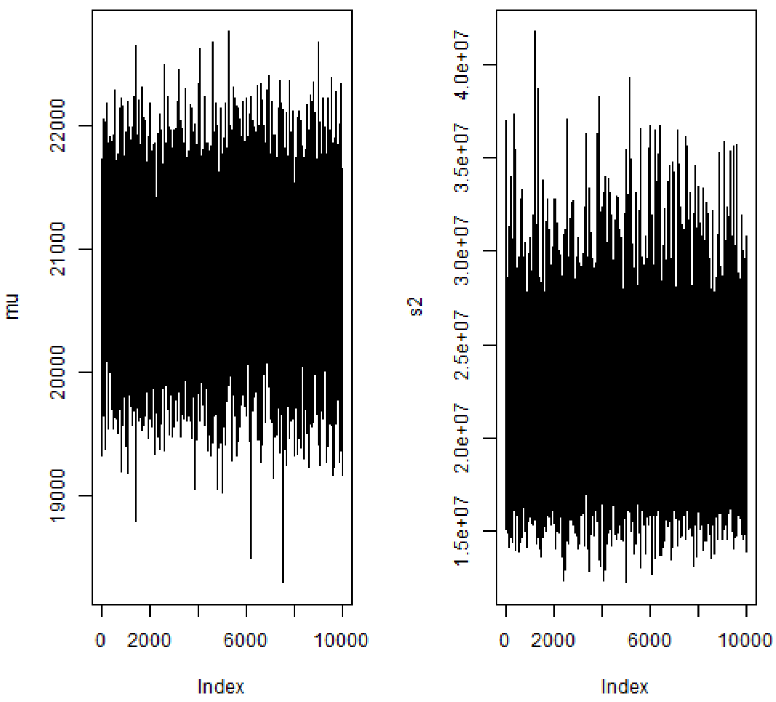


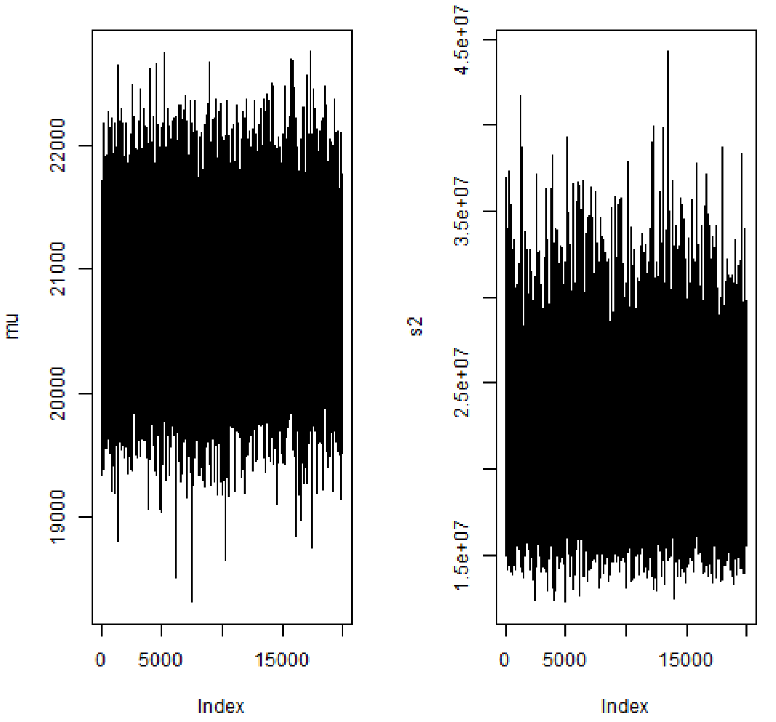
Fix the priors

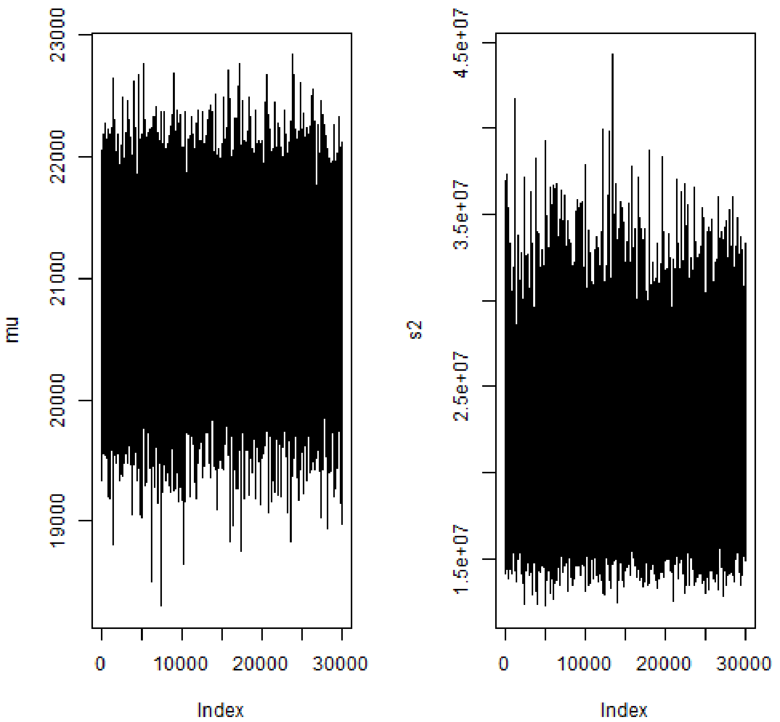
```
m <- 0.01
a <- 0.01
b <- 0.01
```

Gibbs sampling

```
n.iters <- 30000
keep.mu <- rep(0,n.iters)
keep.s2 <- rep(0,n.iters)

# Initial values
mu          <- mean(Y)
s2          <- var(Y)
keep.mu[1] <- mu
keep.s2[1] <- s2

for(iter in 2:n.iters){

  # sample mu|s2,Y

   MN  <- sum(Y)/(n+m)
   VR  <- s2/(n+m)
   mu <- rnorm(1,MN,sqrt(VR))

  # sample s2|mu,Y

   A  <- a + n/2
   B  <- b + sum((Y-mu)^2)/2
   s2 <- 1/rgamma(1,A,B)

  # keep track of the results
   keep.mu[iter] <- mu
   keep.s2[iter] <- s2

  # Plot the samples every 10000 iterations
   if(iter%%10000==0){
     par(mfrow=c(1,2))
     plot(keep.mu[1:iter],type="l",ylab="mu")
     plot(keep.s2[1:iter],type="l",ylab="s2")
   }
}
```

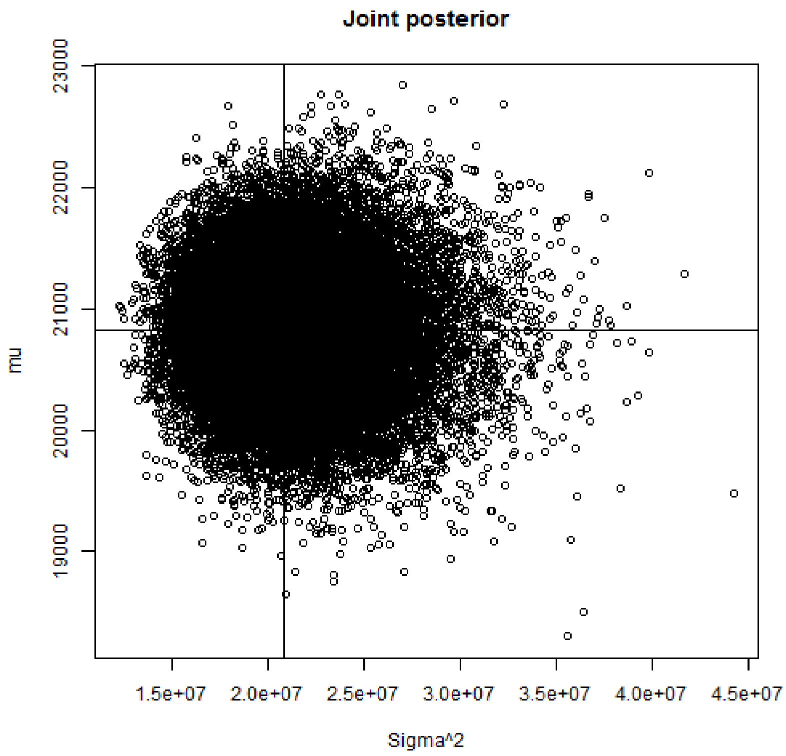






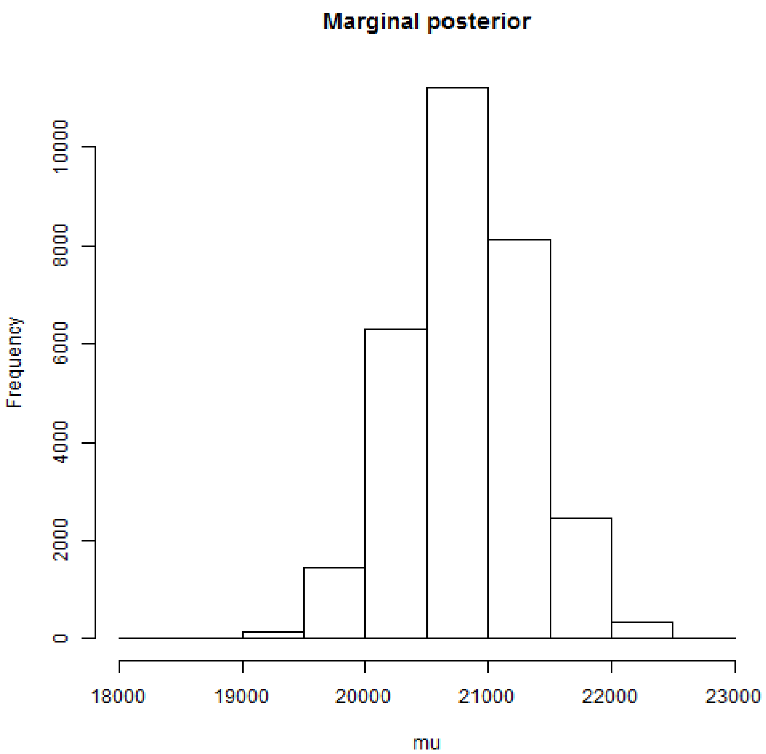
Plot the samples from the joint posterior of μ and σ^2

```
plot(keep.s2,keep.mu,xlab="Sigma^2",ylab="mu",main="Joint posterior")
abline(mean(Y),0)
abline(v=var(Y))
```



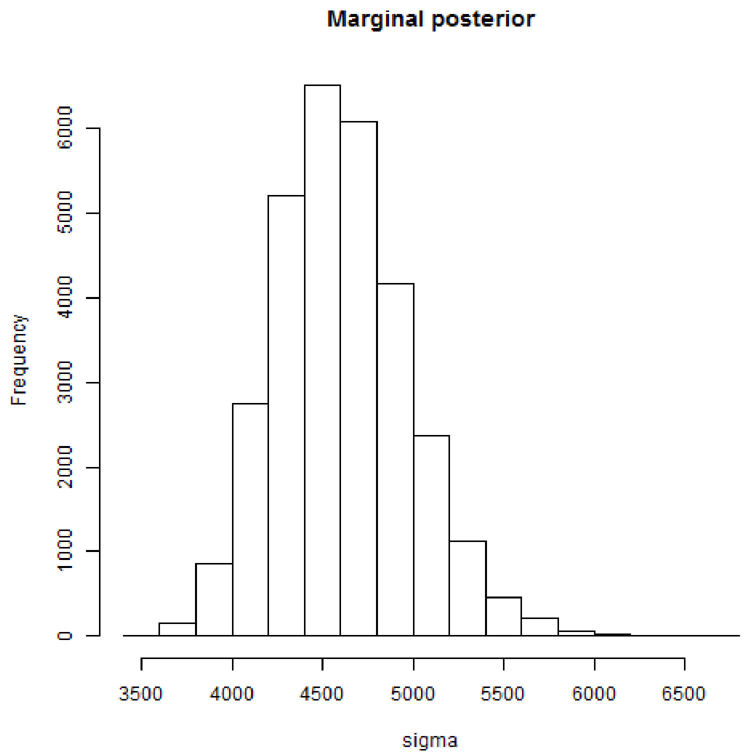
Plot the samples from the marginal (over σ^2) posterior of μ, $p(\mu \mid Y_1, \ldots, Y_n)$

```
hist(keep.mu,xlab="mu",main="Marginal posterior")
```



Plot the samples from the marginal (over μ) posterior of σ, $p(\sigma \mid Y_1, \ldots, Y_n)$

```
keep.s <- sqrt(keep.s2)
hist(keep.s,xlab="sigma",main="Marginal posterior")
```



Compute the approximate marginal means and 95\% credible sets

```
# mu
mean(keep.mu)
```

```
## [1] 20823.19
```

```
quantile(keep.mu,c(0.025,0.975))
```

```
##    2.5%   97.5%
## 19827.0 21827.6
```

```
# sigma^2
mean(keep.s2)
```

```
## [1] 21380683
```

```
quantile(keep.s2,c(0.025,0.975))
```

```
##     2.5%    97.5%
## 15724355 29214747
```

```
# sigma
mean(keep.s)
```

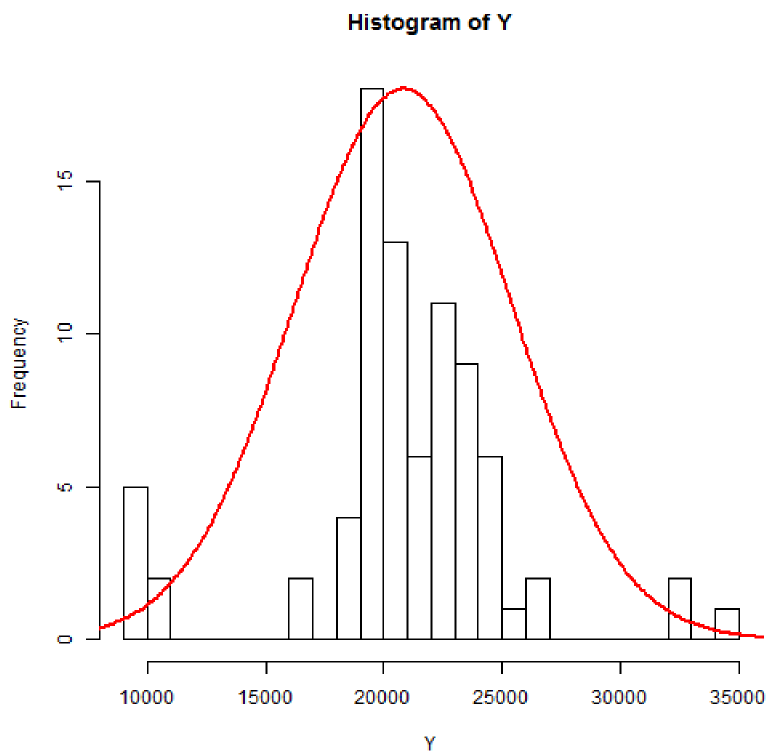
```
## [1] 4609.232
```

```
quantile(keep.s,c(0.025,0.975))
```

```
##     2.5%    97.5%
## 3965.395 5405.067
```

Plot the data versus the fitted model

```
mu_hat  <- mean(keep.mu)
sig_hat <- mean(keep.s)
h       <- hist(Y,breaks=25)

y       <- seq(4000,40000,100)
d       <- dnorm(y,mu_hat,sig_hat)
d       <- max(h$count)*d/max(d)
lines(y,d,lwd=2,col=2)
```



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