Gibbs sampling for the concussions data

Chapter 3.2.1: Gibbs sampling

Let Y_i be the number of concussions (aggregated over all teams and games) in season i (1=2012,…,4=2015). We model these counts as

$$Y_i \sim \text{Poisson}(N\lambda_i) \text{ where } \lambda_i \mid \gamma \sim \text{Gamma}(1, \gamma),$$

λ_i is the concussion rate in year i and N is the number of games in each year. The prior for γ is $\gamma \sim \text{Gamma}(a, b)$. The objective is to deterimine if the concussion rate has changed over time by comparing the posteriors of the λ_i.

Gibbs sampling cycles through the parameters and updates each using a draw from its full conditional distributions. The full conditional distributions are

$$\lambda_i \mid \text{rest} \sim \text{Gamma}(Y_i + 1, N + \gamma)$$

and

$$\gamma \mid \text{rest} \sim \text{Gamma}(a + 4, b + \sum_{i=1}^{4} \lambda_i).$$

This produces draws from the joint posterior of $(\lambda_1, \ldots, \lambda_4, \gamma)$.

To evaluate whether the rate has changed between years i and j, we approximate the posterior probabilities that $\lambda_i > \lambda_j$ using the proportion of the MCMC samples for which this is the case.

Load the data

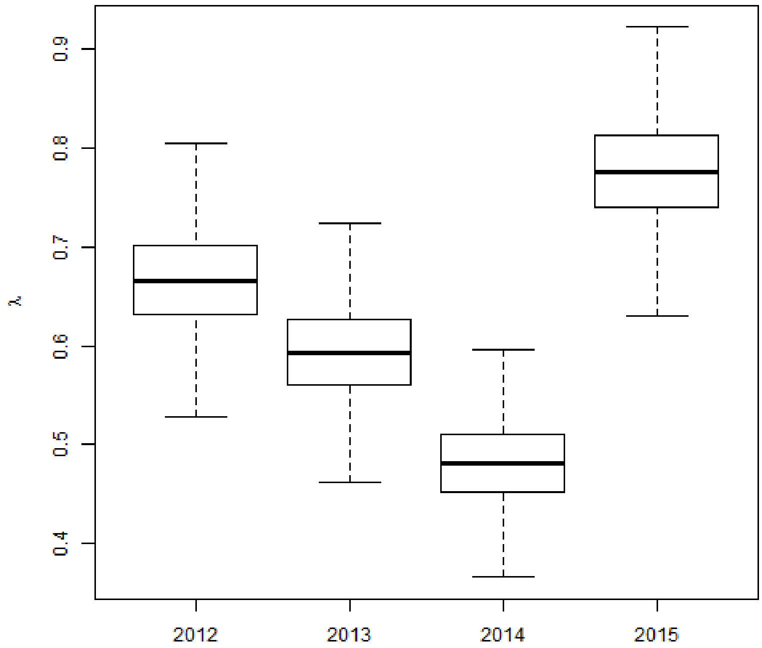
```
Y <- c(171, 152, 123, 199)
n <- 4
N <- 256
```

Gibbs sampling

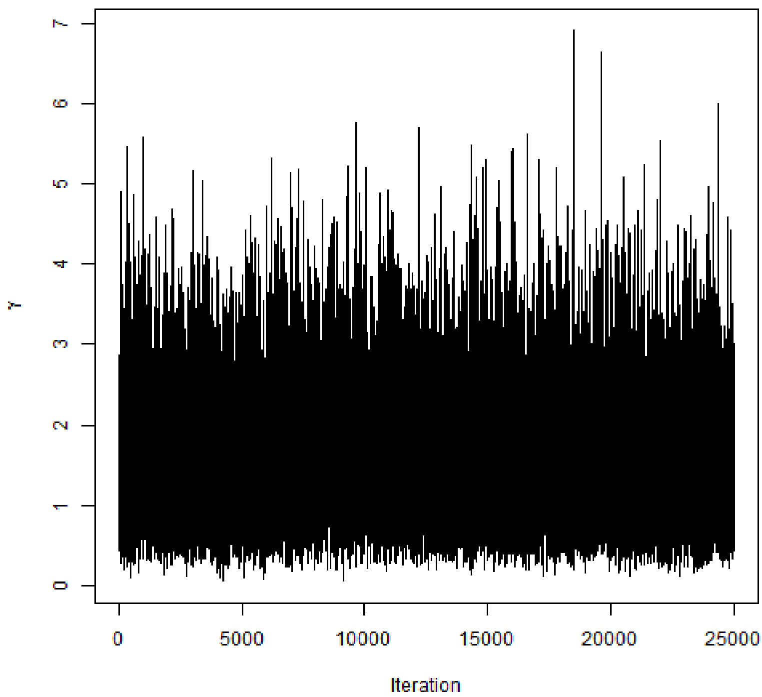
```
# Create an empty matrix for the S MCMC samples

 S                <- 25000
 samples          <- matrix(NA,S,5)
 colnames(samples) <- c("lam1","lam2","lam3","lam4","gamma")

# Initial values

 lambda <- log(Y/N)
 gamma  <- 1/mean(lambda)

# priors: lambda|gamma ~ Gamma(1,gamma), gamma ~ InvG(a,b)

 a      <- 0.1
 b      <- 0.1

# Gibbs sampling

for(s in 1:S){
  for(i in 1:n){
    lambda[i] <- rgamma(1,Y[i]+1,N+gamma)
  }
  gamma       <- rgamma(1,a+4,b+sum(lambda))
  samples[s,] <- c(lambda,gamma)
}
```

Summarize the posterior

```
boxplot(samples[,1:4],outline=FALSE,ylab=expression(lambda),names=2012:2015)
```




```r
plot(samples[,5],type="l",xlab="Iteration",ylab=expression(gamma))
```




```r
# Posterior mean, median and 95% credible intervals
round(apply(samples,2,mean),2)
```

```
##  lam1  lam2  lam3  lam4 gamma
##  0.67  0.59  0.48  0.78  1.56
```

```r
round(apply(samples,2,quantile,c(0.500,0.025,0.975)),2)
```

```
##       lam1 lam2 lam3 lam4 gamma
## 50%   0.67 0.59 0.48 0.78  1.44
## 2.5%  0.57 0.50 0.40 0.67  0.43
## 97.5% 0.77 0.69 0.57 0.89  3.40
```

Approximate $\mathrm{Prob}(\lambda_i > \lambda_j | Y)$ for all pairs of i and j

```
# Is the rate higher in 2015 than 2012?
mean(samples[,4]>samples[,1])
```

```
## [1] 0.92924
```

```
# Is the rate higher in 2014 than 2012?
mean(samples[,3]>samples[,1])
```

```
## [1] 0.00272
```

```
# Is the rate higher in 2013 than 2012?
mean(samples[,4]>samples[,1])
```

```
## [1] 0.92924
```

```
# Is the rate higher in 2015 than 2013?
mean(samples[,4]>samples[,2])
```

```
## [1] 0.99476
```

```
# Is the rate higher in 2014 than 2013?
mean(samples[,3]>samples[,2])
```

```
## [1] 0.04056
```

```
# Is the rate higher in 2015 than 2014?
mean(samples[,4]>samples[,3])
```

```
## [1] 1
```

There appear to be some differences between years, but no clear increasing or decreasing trend.