Logistic regression for NBA clutch free throws

Chapter 4.3.3: Generalized linear models

The NBA clutch free throws data set has three variables for player $i = 1, \ldots, 10$:

1. Y_i is the number clutch free throws made
2. N_i is the number clutch free throws attempted
3. q_i is the proportion of the non-clutch free throws made

We model these data as

$$Y_i \sim \text{Binomial}(N_i, p_i),$$

where p_i is the true probability of making a clutch shot. The objective is to explore the relationship between clutch and overall percentages, p_i and q_i. We do this using two logistic regression models:

1. $\text{logit}(p_i) = \beta_1 + \beta_2 \text{logit}(q_i)$
2. $\text{logit}(p_i) = \beta_1 + \text{logit}(q_i)$

In both models we select uninformative priors $\beta_j \sim \text{Normal}(0, 10^2)$.

In the first model, $p_i = q_i$ if $\beta_1 = 0$ and $\beta_2 = 1$; in the second model $p_i = q_i$ if $\beta_1 = 0$. Therefore, we compare the posteriors of the β_j to these values to analyze the relationship between p_i and q_i.

Load the data

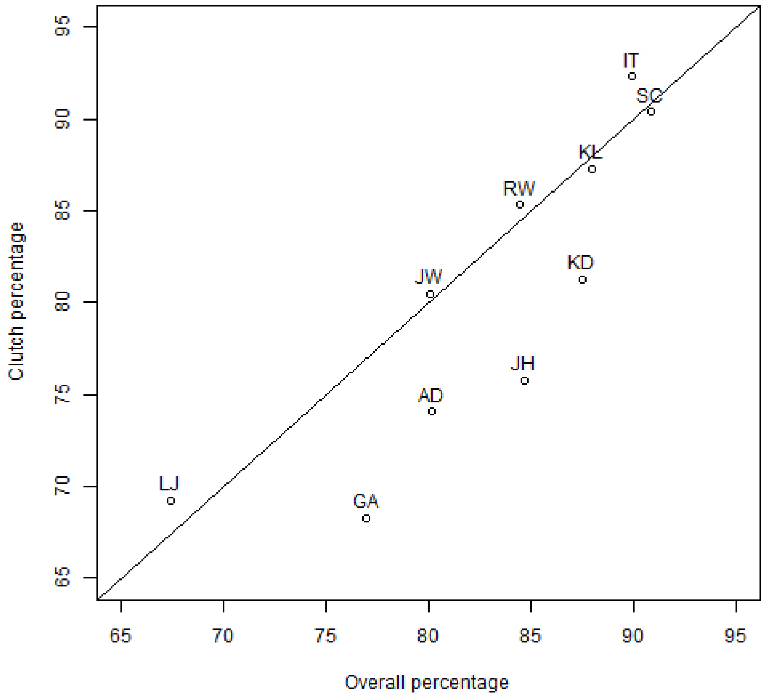
```
set.seed(0820)

Y <- c(64, 72, 55, 27, 75, 24, 28, 66, 40, 13)
N <- c(75, 95, 63, 39, 83, 26, 41, 82, 54, 16)
q <- c(0.845, 0.847, 0.880, 0.674, 0.909, 0.899, 0.770, 0.801, 0.802, 0.875)

X <- log(q)-log(1-q) # X = logit(q)
```

Plot the data

```
inits   <- c("RW","JH","KL","LJ","SC","IT","GA","JW","AD","KD")
plot(100*q,100*Y/N,
     xlim=100*c(0.65,0.95),ylim=100*c(0.65,0.95),
     xlab="Overall percentage",ylab="Clutch percentage")
text(100*q,100*Y/N+1,inits)
abline(0,1)
```



Fit the first model in JAGS

```
library(rjags)
```

```
## Loading required package: coda
```

```
## Linked to JAGS 4.2.0
```

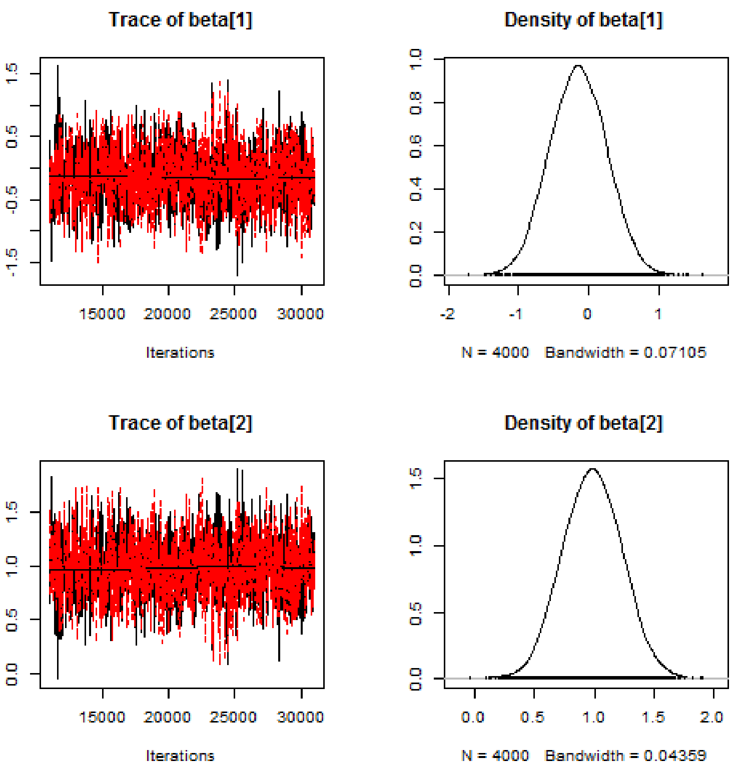
```
## Loaded modules: basemod,bugs
```

```
data   <- list(Y=Y,N=N,X=X)
params <- c("beta")

model_string <- textConnection("model{
  # Likelihood
   for(i in 1:10){
     Y[i]         ~ dbinom(p[i],N[i])
     logit(p[i]) <- beta[1] + beta[2]*X[i]
   }
  # Priors
   beta[1] ~ dnorm(0,0.01)
   beta[2] ~ dnorm(0,0.01)
}")

model <- jags.model(model_string,data = data, n.chains=2,quiet=TRUE)
update(model, 10000, progress.bar="none")
samples1 <- coda.samples(model, variable.names=params, thin=5, n.iter=20000, progress.bar="none")

plot(samples1)
```




```
summary(samples1)
```

```
##
## Iterations = 11005:31000
## Thinning interval = 5
## Number of chains = 2
## Sample size per chain = 4000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##            Mean     SD Naive SE Time-series SE
## beta[1] -0.1486 0.4045 0.004522        0.01294
## beta[2]  0.9826 0.2481 0.002774        0.00787
##
## 2. Quantiles for each variable:
##
##            2.5%     25%     50%    75%  97.5%
## beta[1] -0.9239 -0.4265 -0.1490 0.1275 0.6359
## beta[2]  0.5071  0.8132  0.9827 1.1517 1.4694
```

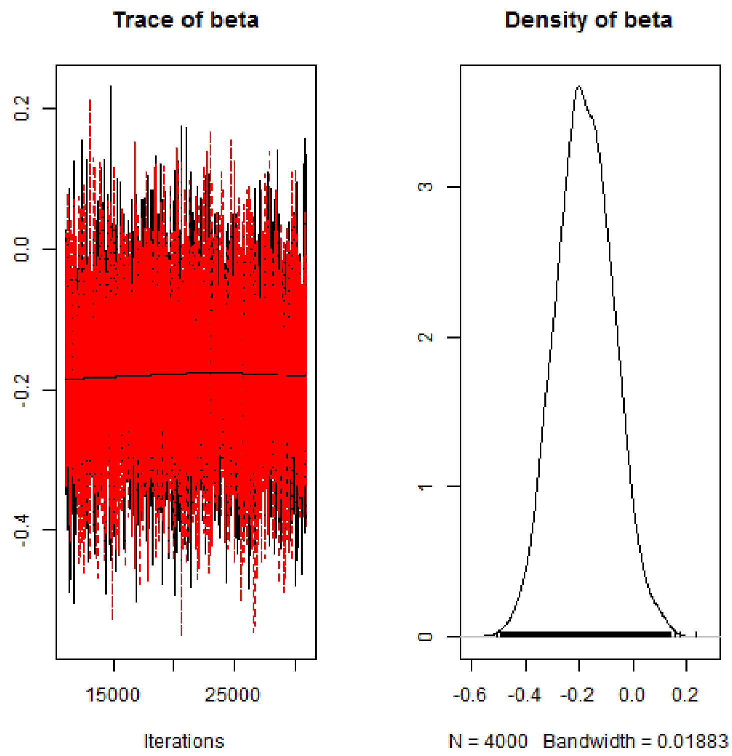
```
b1 <- c(samples1[[1]][,1],samples1[[2]][,1])
b2 <- c(samples1[[1]][,2],samples1[[2]][,2])
```

Fit the second model in JAGS

```
model_string <- textConnection("model{
  # Likelihood
   for(i in 1:10){
     Y[i]        ~ dbinom(p[i],N[i])
     logit(p[i]) <- beta + X[i]
   }
  # Priors
   beta ~ dnorm(0,0.01)
}")

model <- jags.model(model_string,data = data, n.chains=2,quiet=TRUE)
update(model, 10000, progress.bar="none")
samples2 <- coda.samples(model, variable.names=params, thin=5, n.iter=20000, progress.bar="none")
b3 <- c(samples2[[1]],samples2[[2]])

plot(samples2)
```




```
summary(samples2)
```

```
##
## Iterations = 11005:31000
## Thinning interval = 5
## Number of chains = 2
## Sample size per chain = 4000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##           Mean             SD       Naive SE Time-series SE
##      -0.179428       0.107197       0.001199       0.001198
##
## 2. Quantiles for each variable:
##
##     2.5%      25%      50%      75%    97.5%
## -0.38598 -0.25251 -0.18186 -0.10721  0.03334
```

```
mean(b3<0) # Prob(beta_3<0|Y)
```

```
## [1] 0.95275
```

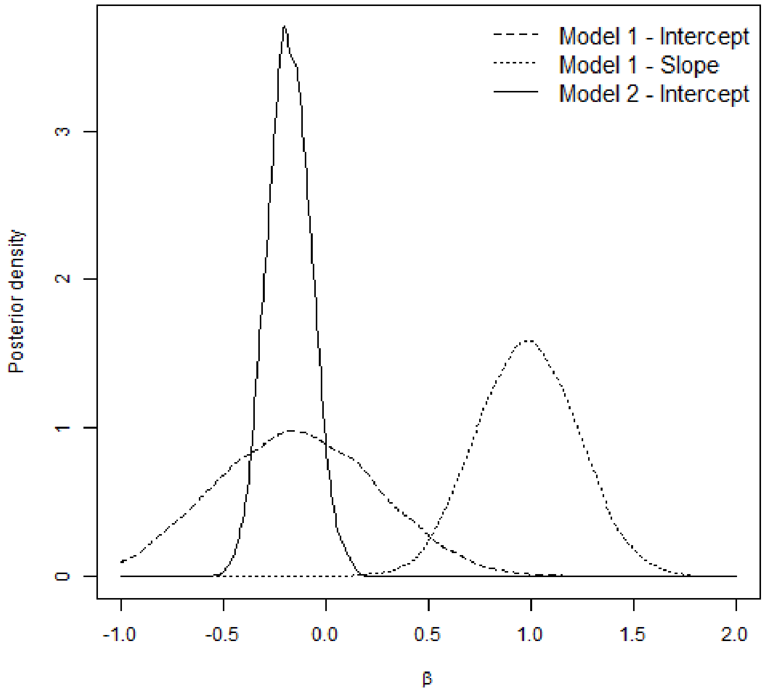
Plot the posterior densities from both models

```
d1 <- density(b1,from=-1,to=2)
d2 <- density(b2,from=-1,to=2)
d3 <- density(b3,from=-1,to=2)

mx <- max(c(d1$y,d2$y,d3$y))

plot(d3$x,d3$y,type="l",xlim=c(-1,2),ylim=c(0,mx),xlab=expression(beta),ylab="Posterior density")
lines(d1$x,d1$y,lty=2)
lines(d2$x,d2$y,lty=3)

legend("topright",c("Model 1 - Intercept","Model 1 - Slope","Model 2 - Intercept"),
       bty="n",lty=c(2,3,1),cex=1.25)
```



Summary: In the second model, we find that β_1 is negative with posterior probability around 0.95. If β_1 is negative this implies that the clutch probability is less than the overall probability. Therefore, there is some evidence that free throw percentage decreases in clutch situations.

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