# Metropolis sampling for the concussions data

### Chapter 3.2.3: Metropolis sampling

Let $Y_i$ be the number of concussions (aggregated over all teams and games) in season $i$ (1=2012,…,4=2015). We model these counts as

$$Y_i \sim \text{Poisson}(N\lambda_i) \text{ where } \lambda_i = \exp(\beta_1 + i\beta_2),$$

$N$ is the number of games played per year and $\lambda_i$ is the rate in year $i$. To complete the Bayesian model, we specify uninformative priors $\beta_1, \beta_2 \sim \text{Normal}(0, \tau^2)$.

The log of the mean concussion rate is linear in time with $\beta_2$ determining the slope. The objective is to determine if the concussion rate is increasing, i.e., $\beta_2 > 0$.

## Load the data

```
Y <- c(171, 152, 123, 199)
t <- 1:4
n <- 4
N <- 256
```

## Initialize

```
# Create an empty matrix for the MCMC samples

 S                  <- 25000
 samples            <- matrix(NA,S,2)
 colnames(samples) <- c("beta1","beta2")
 fitted             <- matrix(0,S,4)

# Initial values

 beta    <- c(log(mean(Y/N)),0)

# priors: beta[j] ~ N(0,tau^2)

 tau     <- 10

# candidate standard deviations

 can_sd <- rep(0.1,2)
```

## Define the log posterior as a function

```
log_post <- function(Y,N,t,beta,tau){
   mn     <- N*exp(beta[1]+beta[2]*t)
   like   <- sum(dpois(Y,mn,log=TRUE))
   prior  <- sum(dnorm(beta,0,tau,log=TRUE))
   post   <- like + prior
return(post)}
```

## Metropolis sampling

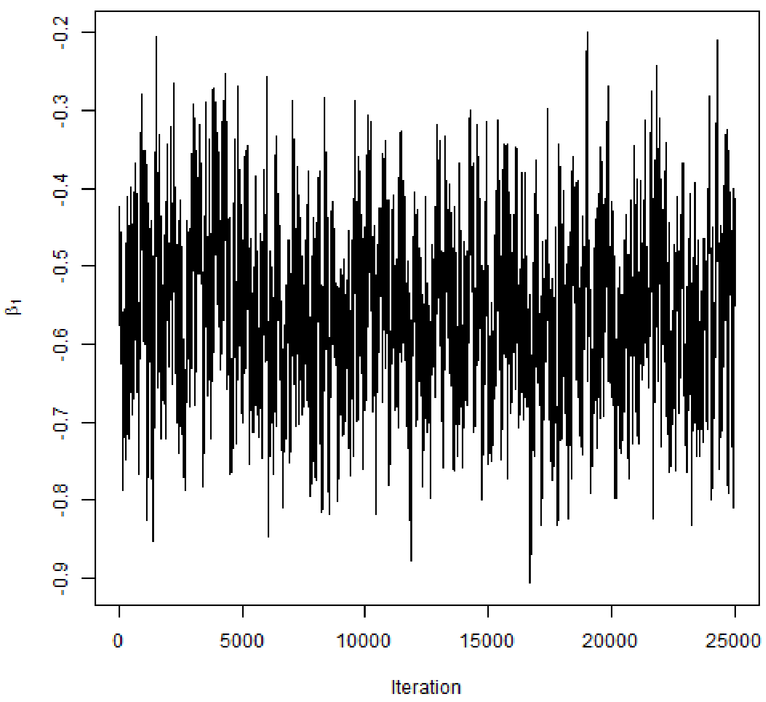
```r
for(s in 1:S){
  for(j in 1:2){
    can    <- beta
    can[j] <- rnorm(1,beta[j],can_sd[j])
    logR   <- log_post(Y,N,t,can,tau)-log_post(Y,N,t,beta,tau)
    if(log(runif(1))<logR){
      beta <- can
    }
  }
  samples[s,] <- beta
  fitted[s,]  <- N*exp(beta[1]+beta[2]*t)
}
```

## Compute the acceptance rates and plot the samples

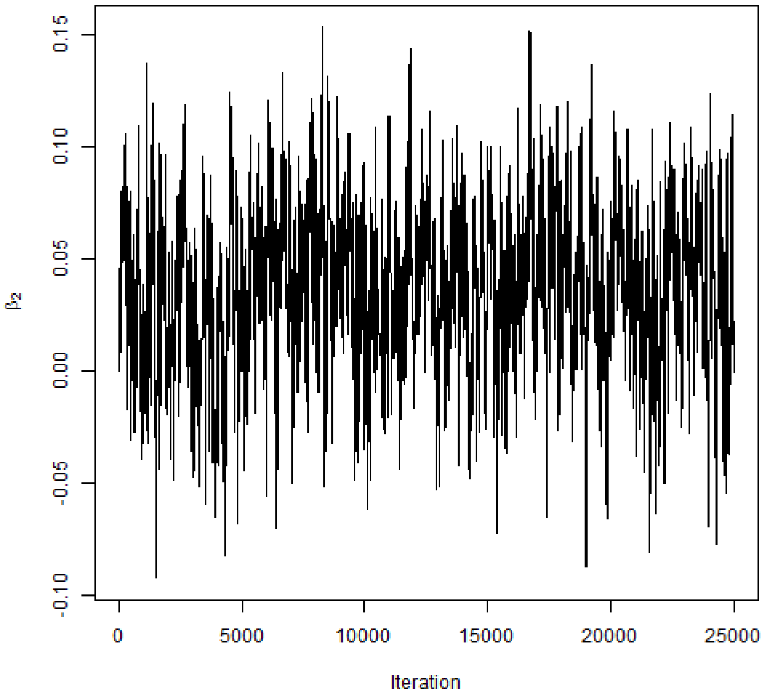
```r
# Acceptance rates
colMeans(samples[1:24999,]!=samples[2:25000,])
```

```
##     beta1     beta2
## 0.4311772 0.1752070
```

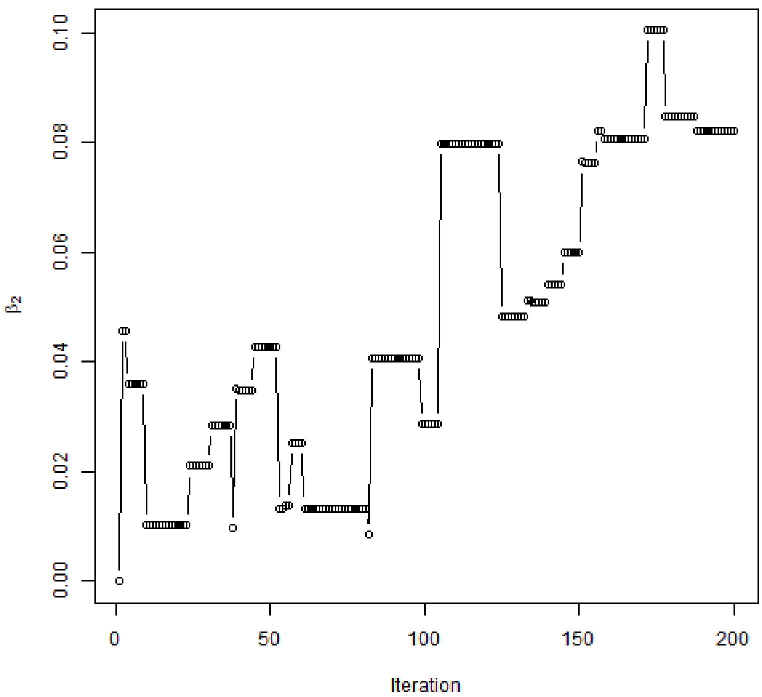
```r
plot(samples[,1],type="l",xlab="Iteration",ylab=expression(beta[1]))
```




```r
plot(samples[,2],type="l",xlab="Iteration",ylab=expression(beta[2]))
```



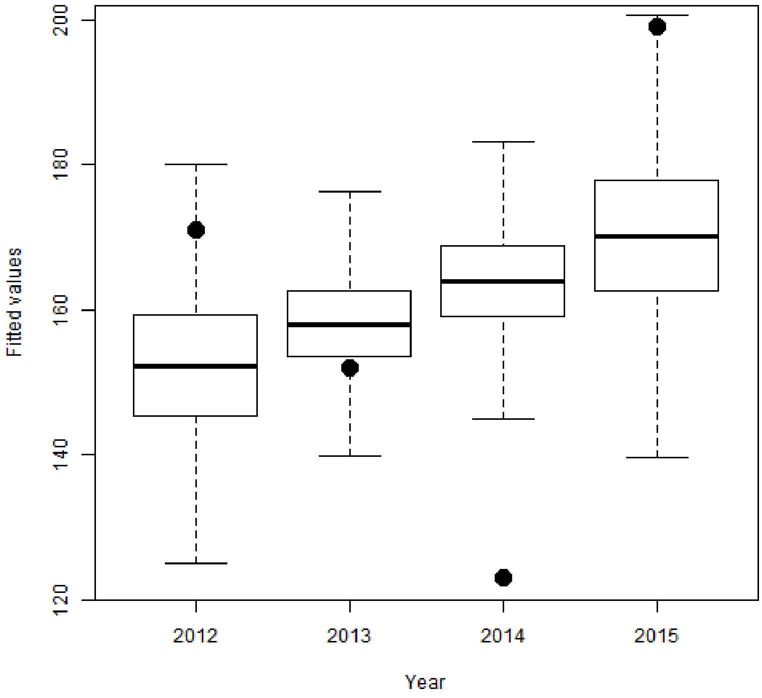

```r
plot(samples[1:200,2],type="b",xlab="Iteration",ylab=expression(beta[2]))
```




## Summarize the fitted values for each year

The boxplots are the posterior distribution of the $N\lambda_i = N\exp(\beta_1 + i\beta_2)$, and the points are the observed counts. The linear trend doesn't fit particularly well.

```r
boxplot(fitted,outline=FALSE,ylim=range(Y),
        xlab="Year",ylab="Fitted values",names=2012:2015)
points(Y,pch=19,cex=2)
```




```r
# Posterior probability that the slope is positive
mean(samples[,2]>0)
```

```
## [1] 0.84052
```

There is some evidence that the rate is increasing, but it seems to be driven only by the last year.