Analysis of the 2016 US Presidential Election

Chapter 5.5: Model selection criteria

The data for this analysis come from Tony McGovern. The response variable, Y_i, is the percentage change in Republican (GOP) support from 2012 to 2016, i.e.,

$$100\left(\frac{\%\text{ in }2016}{\%\text{ in }2012} - 1\right),$$

in county $i = 1, \ldots, n$.

The $p = 10$ covariates X_{ij} are county-level census variables obtained from Kaggle are:

- Population, percent change - April 1, 2010 to July 1, 2014
- Persons 65 years and over, percent, 2014
- Black or African American alone, percent, 2014
- Hispanic or Latino, percent, 2014
- High school graduate or higher, percent of persons age 25+, 2009-2013
- Bachelor's degree or higher, percent of persons age 25+, 2009-2013
- Homeownership rate, 2009-2013
- Median value of owner-occupied housing units, 2009-2013
- Median household income, 2009-2013
- Persons below poverty level, percent, 2009-2013

For a county in state s, we assume the linear model

$$Y_i = \beta_{0s} + \sum_{j=1}^{p} X_{ij}\beta_{sj} + \varepsilon_i,$$

where β_{js} is the effect of covariate j in state s. We compare three models for the β_{js}

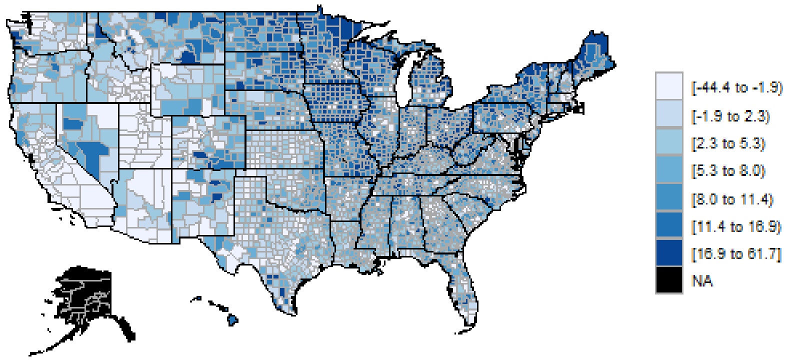
1. Constant slopes: $\beta_{js} \equiv \beta_j$ for all counties
2. Varying slopes with uninformative priors: $\beta_{js} \sim \text{Normal}(0, 100)$
3. Varying slopes with informative priors: $\beta_{js} \sim \text{Normal}(\mu_j, \sigma_j^2)$

In the third model, the means μ_j and variances σ_j^2 are given prior and estimated from the data, therefore information is pooled across states via the prior. The three methods are compared using DIC, and final results are compared across models.

Load the data

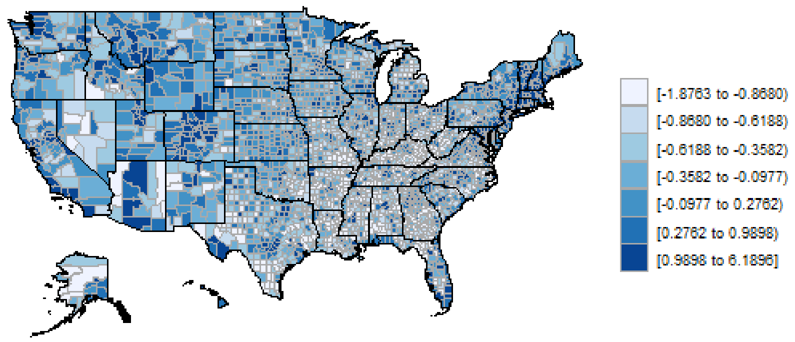
```
load("election_2008_2016.RData")
X     <- scale(X)    # Standardize the covariates
X     <- cbind(1,X)
short <- c("Intercept", "Pop change", "65+","African American",
           "Hispanic","HS grad","Bachelor's",
           "Homeownership rate","Home value",
           "Median income", "Poverty")
names <- c("Intercept", as.character(names[1:11,2]))
colnames(X) <- short

# Define a function to make county maps
county_plot <- function(fips,Y,main="",units=""){
  library(choroplethr)
  temp  <- as.data.frame(list(region=fips,value=Y))
  suppressWarnings(county_choropleth(temp,title=main,legend=units))
}
county_plot(fips,Y,main="Percent increase in GOP support",units="")
```

Percent increase in GOP support




```
county_plot(fips,X[,7],main=names[7],units="")
```

Bachelor's degree or higher, percent of persons age 25+, 2009-2013



```r
# Remove AK, HI and DC due to missing data
state <- as.character(all_dat[,3])
AKHI  <- state=="AK" | state=="HI" | state=="DC"
fips  <- fips[!AKHI]
Y     <- Y[!AKHI]
X     <- X[!AKHI,]
state <- state[!AKHI]

# Assign a numeric id to the counties in each state
st    <- unique(state)
id    <- rep(NA,length(Y))
for(j in 1:48){
  id[state==st[j]]<-j
}

n <- length(Y) # number of counties
N <- 48        # number of states
p <- ncol(X)   # number of covariates

set.seed(0820)
iters <- 50000
burn  <- 10000
```

(1) Constant slopes

```
model1_string <- "model{

  # Likelihood
  for(i in 1:n){
     Y[i]    ~ dnorm(mu[i],taue)
     mu[i] <- inprod(X[i,],beta[])
  }
  # Priors
  for(j in 1:p){beta[j] ~ dnorm(0,0.01)}
  taue ~ dgamma(0.1,0.1)
  sig <- 1/sqrt(taue)

  # WAIC calculations
  for(i in 1:n){
    like[i]    <- dnorm(Y[i],mu[i],taue)
  }
}"

library(rjags)

# Load the model
dat    <- list(Y=Y,n=n,X=X,p=p)
init   <- list(beta=rep(0,p))
model1 <- jags.model(textConnection(model1_string),n.chains=2,
                     inits=init,data = dat,quiet=TRUE)
# Generate samples
update(model1, burn, progress.bar="none")
samp1    <- coda.samples(model1,
           variable.names=c("beta"),
           n.iter=iters, progress.bar="none")

# Compile results
ESS1     <- effectiveSize(samp1)
out1     <- summary(samp1)$quantiles
rownames(out1)<-short

# Compute DIC
dic1     <- dic.samples(model1,n.iter=iters,progress.bar="none")

# Compute WAIC
waic1    <- coda.samples(model1,
           variable.names=c("like"),
           n.iter=iters, progress.bar="none")
like1    <- waic1[[1]]
fbar1    <- colMeans(like1)
P1       <- sum(apply(log(like1),2,var))
WAIC1    <- -2*sum(log(fbar1))+2*P1
```

(2) Slopes as fixed effects

```
model2_string <- "model{

  # Likelihood
  for(i in 1:n){
     Y[i]    ~ dnorm(mnY[i],taue)
     mnY[i] <- inprod(X[i,],beta[id[i],])
  }

  # Slopes
  for(j in 1:p){for(i in 1:N){
      beta[i,j] ~ dnorm(0,0.01)
  }}

  # Priors
  taue ~ dgamma(0.1,0.1)


  # WAIC calculations
  for(i in 1:n){
    like[i]    <- dnorm(Y[i],mnY[i],taue)
  }
 }"


 library(rjags)

 # Load the model
 dat    <- list(Y=Y,n=n,N=N,X=X,p=p,id=id)
 init   <- list(beta=matrix(0,N,p))
 model2 <- jags.model(textConnection(model2_string),n.chains=2,
                      inits=init,data = dat,quiet=TRUE)

 # Generate samples
 update(model2, burn, progress.bar="none")
 samp2  <- coda.samples(model2,
           variable.names=c("beta"),
           n.iter=iters, progress.bar="none")

 # Compile results
 ESS2     <- effectiveSize(samp2)
 sum      <- summary(samp2)$stat
 post_mn2 <- matrix(sum[,1],N,p)
 post_sd2 <- matrix(sum[,2],N,p)

 # Compute DIC
 dic2    <- dic.samples(model2,n.iter=iters,progress.bar="none")

 # Compute WAIC
 waic2    <- coda.samples(model2,
            variable.names=c("like"),
            n.iter=iters, progress.bar="none")
 like2    <- waic2[[1]]
 fbar2    <- colMeans(like2)
 P2       <- sum(apply(log(like2),2,var))
 WAIC2    <- -2*sum(log(fbar2))+2*P2
```

(3) Slopes as random effects

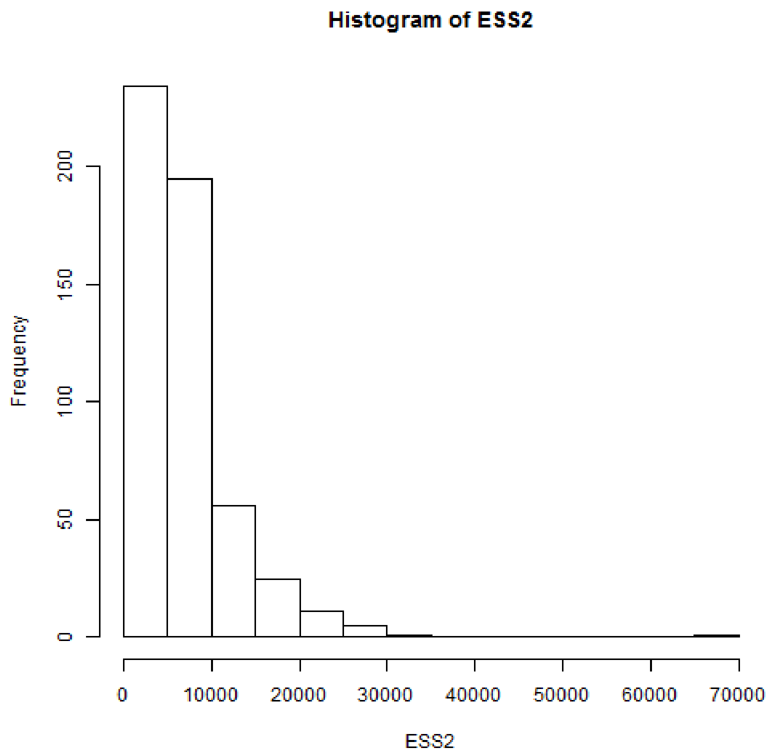
```
model3_string <- "model{

  # Likelihood
  for(i in 1:n){
     Y[i]    ~ dnorm(mnY[i],taue)
     mnY[i] <- inprod(X[i,],beta[id[i],])
  }

  # Random slopes
  for(j in 1:p){
    for(i in 1:N){
      beta[i,j] ~ dnorm(mu[j],taub[j])
    }
    mu[j]    ~ dnorm(0,0.01)
    taub[j]  ~ dgamma(0.1,0.1)
  }

  # Priors
  taue ~ dgamma(0.1,0.1)


  # WAIC calculations
  for(i in 1:n){
    like[i]    <- dnorm(Y[i],mnY[i],taue)
  }
 }"


 library(rjags)

 # Load the model
 dat    <- list(Y=Y,n=n,N=N,X=X,p=p,id=id)
 init   <- list(beta=matrix(0,N,p))
 model3 <- jags.model(textConnection(model3_string),n.chains=2,
                      inits=init,data = dat,quiet=TRUE)

 # Generate samples
 update(model3, burn, progress.bar="none")
 samp3  <- coda.samples(model3,
          variable.names=c("beta"),
          n.iter=iters, progress.bar="none")

 # Compile results
 ESS3     <- effectiveSize(samp3)
 sum      <- summary(samp3)$stat
 post_mn3 <- matrix(sum[,1],N,p)
 post_sd3 <- matrix(sum[,2],N,p)

 # Compute DIC
 dic3    <- dic.samples(model3,n.iter=iters,progress.bar="none")

 # Compute WAIC
 waic3   <- coda.samples(model3,
           variable.names=c("like"),
           n.iter=iters, progress.bar="none")
 like3   <- waic3[[1]]
 fbar3   <- colMeans(like3)
 P3      <- sum(apply(log(like3),2,var))
 WAIC3   <- -2*sum(log(fbar3))+2*P3
```

Check convergence

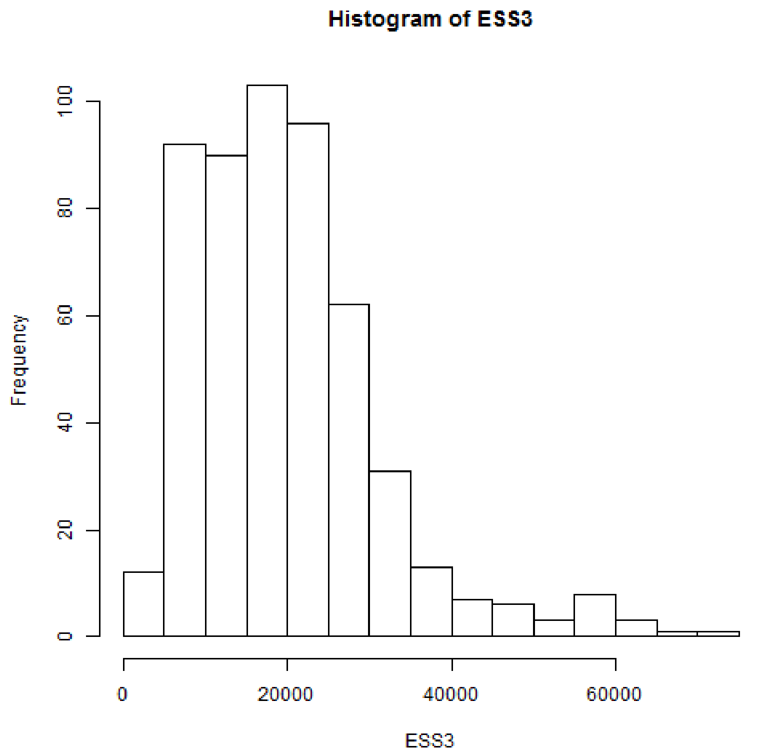
```
ESS1
```

```
##    beta[1]    beta[2]    beta[3]    beta[4]    beta[5]    beta[6]    beta[7]
## 99549.110 55385.021 18406.529 40822.338 30379.744 17363.815 16930.493
##    beta[8]    beta[9]   beta[10]   beta[11]
## 24129.501 17280.572  8596.049 11948.855
```

```
hist(ESS2)
```




```
hist(ESS3)
```



Summary: The effective sample size is large for all parameters and all models, therefore it seems the MCMC algorithm has converged.

Summarize the non-spatial model

```
kable(round(out1,2))
```

	2.5%	25%	50%	75%	97.5%
Intercept	6.41	6.58	6.67	6.76	6.93
Pop change	-1.46	-1.25	-1.13	-1.02	-0.82
65+	0.54	0.79	0.93	1.06	1.32
African American	-1.88	-1.67	-1.56	-1.45	-1.23
Hispanic	-2.40	-2.18	-2.06	-1.95	-1.73
HS grad	1.26	1.58	1.75	1.92	2.25
Bachelor's	-6.71	-6.37	-6.20	-6.02	-5.68
Homeownership rate	-0.38	-0.12	0.01	0.15	0.41
Home value	-1.98	-1.67	-1.51	-1.36	-1.05
Median income	1.14	1.62	1.87	2.13	2.61
Poverty	0.91	1.28	1.48	1.67	2.04

Summary: All but one (home ownership rate) of the covariates have 95% interval that excludes zero. GOP support tended to increase in counties with

- decreasing population
- high proportion of seniors and high school graduates
- low proportions of African Americans and Hispanics
- High income but low home value
- High poverty rate

Compare models with DIC

```
dic1
```

```
## Mean deviance:  21300
## penalty 12.04
## Penalized deviance: 21312
```

```
dic2
```

```
## Mean deviance:  18483
## penalty 454.8
## Penalized deviance: 18938
```

```
dic3
```

```
## Mean deviance:  18604
## penalty 237.7
## Penalized deviance: 18842
```

Summary: The first model with constant slopes is the simplest but fits the data poorly and thus has highest DIC. The second model with different slopes in each state has the best fit (smallest mean deviance), but is too complicated and has large p_D. The final model has fairly small mean deviance and p_D, and thus balances fit and complexity to give the smallest DIC.

Compare models with WAIC

```
WAIC1; P1
```

```
## [1] 21334.98
```

```
## [1] 20.09423
```

```
WAIC2; P2
```

```
## [1] 18970.31
```

```
## [1] 405.2344
```

```
WAIC3; P3
```

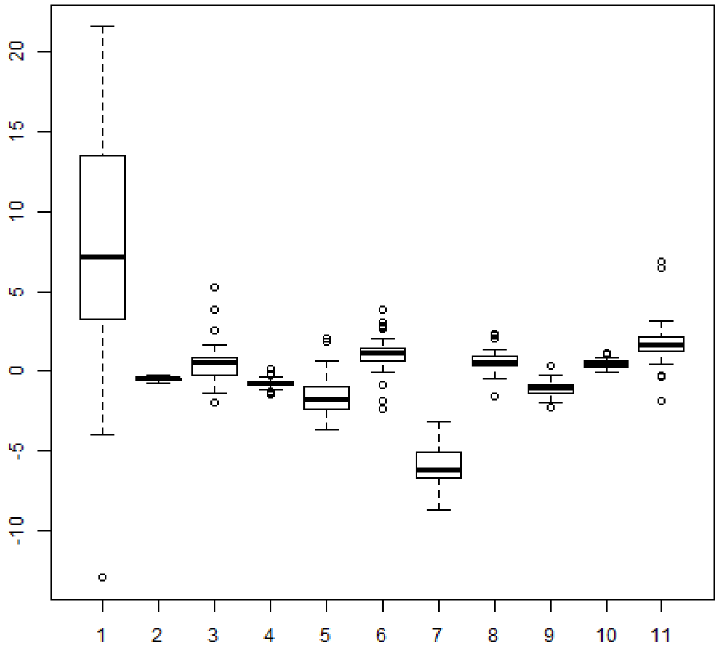
```
## [1] 18910.27
```

```
## [1] 259.3111
```

Summary: WAIC disagrees with DIC. WAIC prefers Model 2 with uninformative priors.

Explore the results of the final model

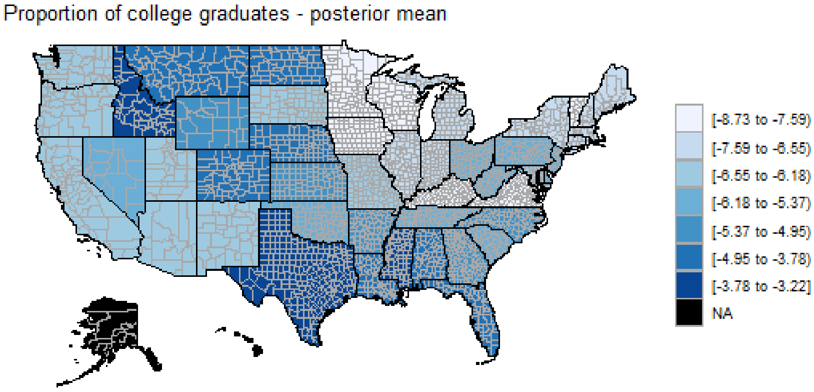
```
boxplot(post_mn3)
```



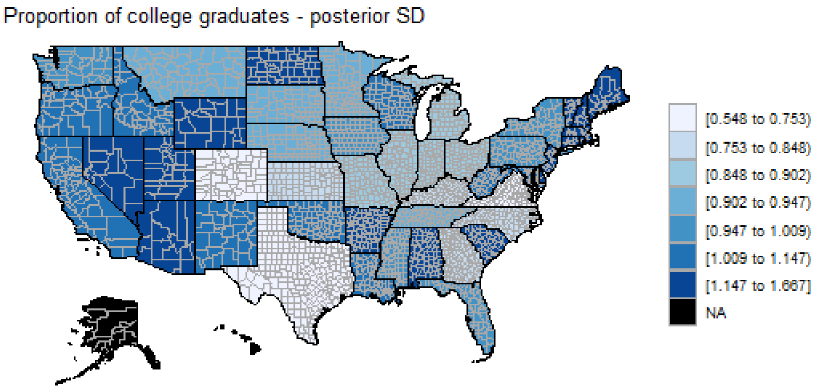
Summary: The effect of the proportion of college graduates varies the most across states

Explore the three estimate of the effects of college graduates

```
# Posterior mean
county_plot(fips,post_mn3[id,7],
            main="Proportion of college graduates - posterior mean")
```




```r
# Posterior sd
county_plot(fips,post_sd3[id,7],
main="Proportion of college graduates - posterior SD")
```



Summary: The effect of the proportion of college graduates is the strongest (most negative) in the midwest.

Processing math: 100%