Normal/normal model using Gibbs sampling

Let Y_i be the change in support for the GOP from 2012 to 2016 in county $i = 1, \ldots, n$. We model

$$Y_i|\mu, \sigma^2 \sim \text{Normal}(\mu, \sigma^2).$$

Our objective is to estimate the joint posterior distribution of (μ, σ^2). We select priors

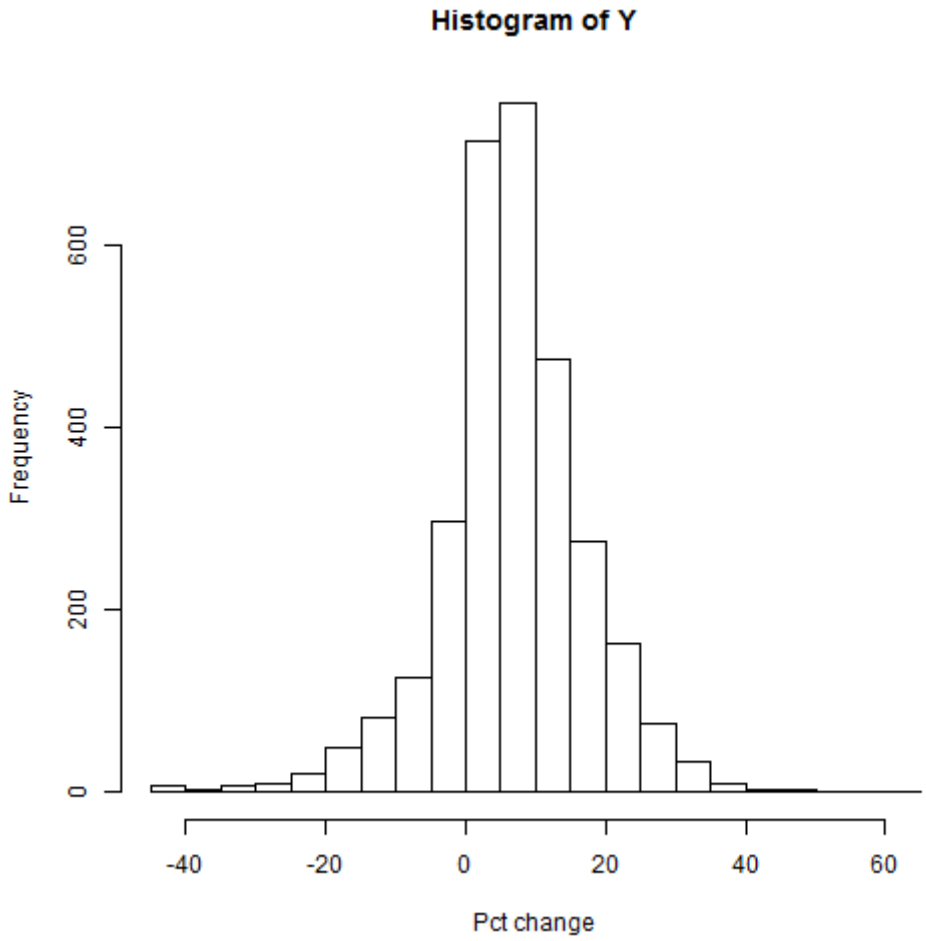
$$\mu \sim \text{Normal}(\mu_0, \sigma_0^2) \quad \sigma^2 \sim \text{InvGamma}(a, b).$$

We'll use at three ways to summarize the posterior

1. Grid approximation
2. Gibbs sampling in R
3. Gibbs sampling in JAGS (later)

2. Gibbs sampling in R

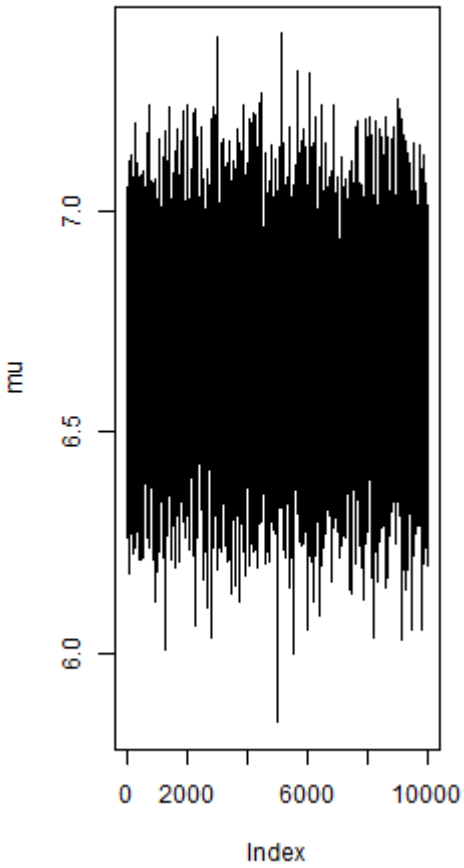
```
load("S:\\Documents\\www\\ABA\\code\\election_2008_2016.RData")

Y <- Y[!is.na(Y)]
n <- length(Y)

hist(Y,breaks=20,xlab="Pct change")
```

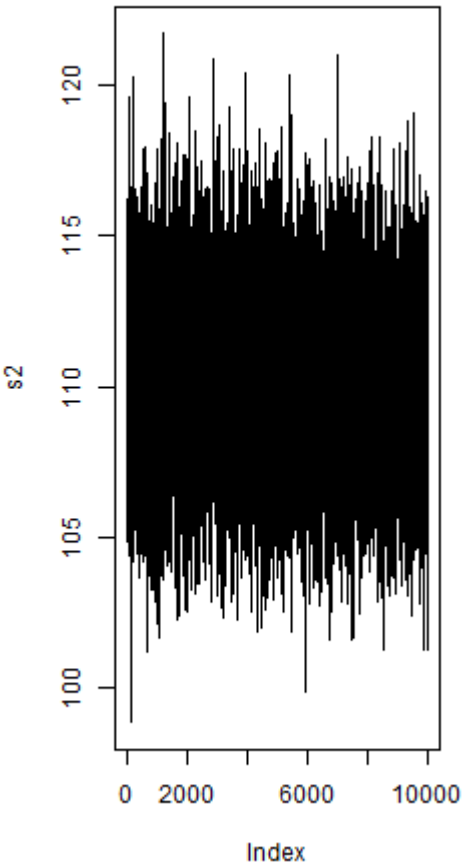


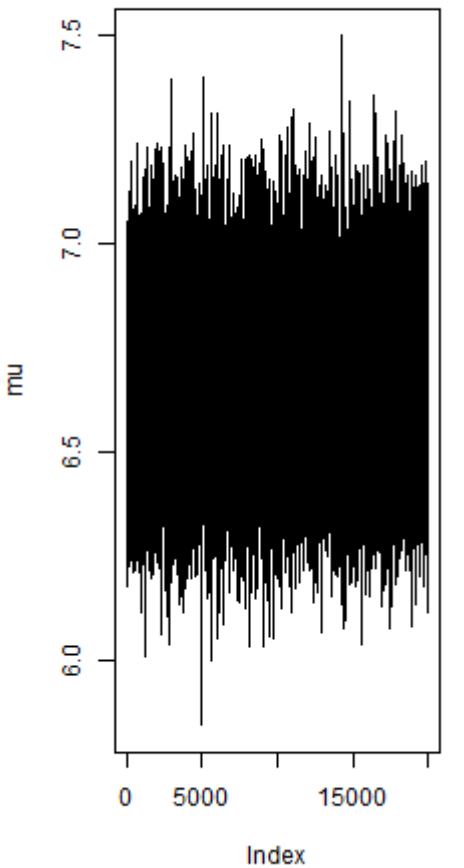

```
### Priors

  mu0 <- 0
  s20 <- 1000
  a   <- 0.01
  b   <- 0.01
```

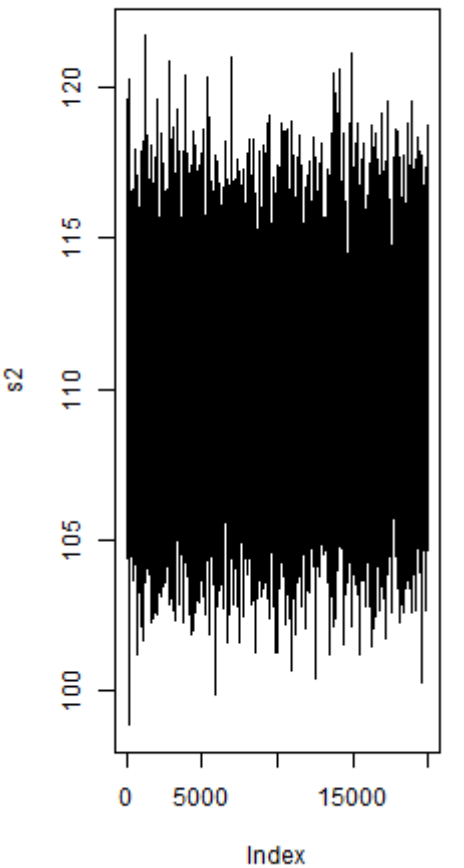
MCMC!

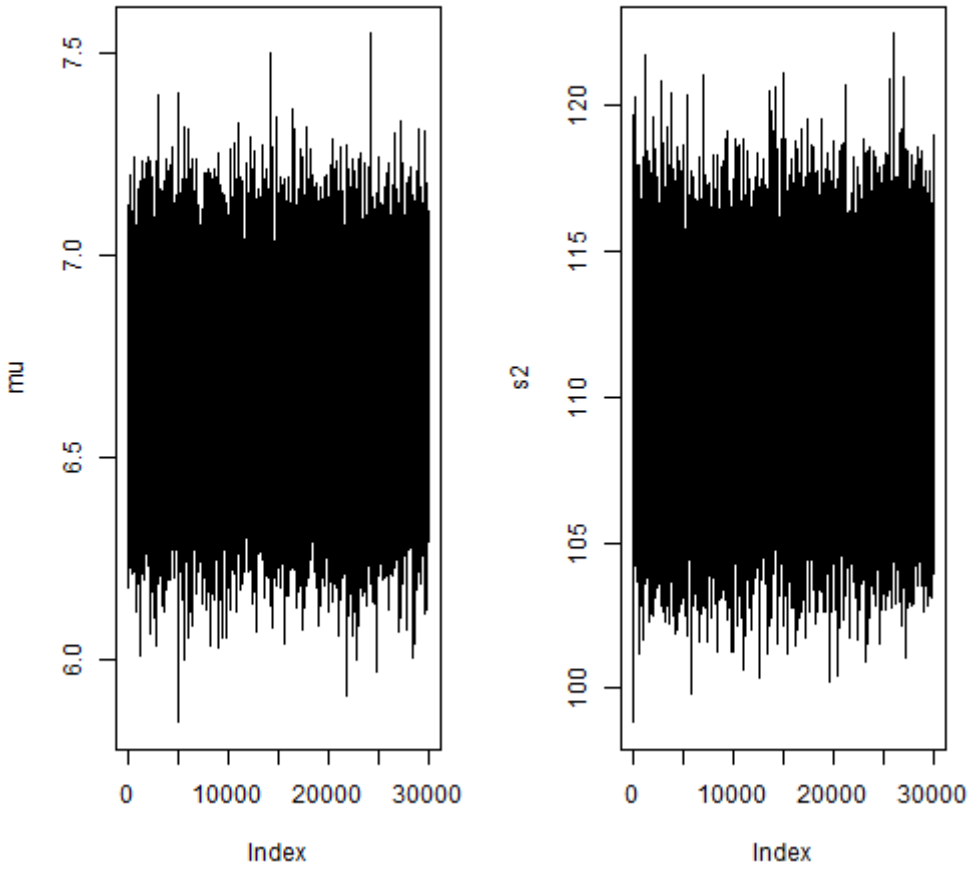
```
n.iters <- 30000
keep.mu <- rep(0,n.iters)
keep.s2 <- rep(0,n.iters)

# Initial values
mu         <- mean(Y)
s2         <- var(Y)
keep.mu[1] <- mu
keep.s2[1] <- s2

for(iter in 2:n.iters){

  # sample mu|s2,Y

   A  <- sum(Y)/s2+mu0/s20
   B  <- n/s2+1/s20
   mu <- rnorm(1,A/B,1/sqrt(B))

  # sample s2|mu,Y

   A  <- n/2+a
   B  <- sum((Y-mu)^2)/2+b
   s2 <- 1/rgamma(1,A,B)

  # keep track of the results
   keep.mu[iter] <- mu
   keep.s2[iter] <- s2

  # Plot the samples every 10000 iterations
   if(iter%%10000==0){
     par(mfrow=c(1,2))
     plot(keep.mu[1:iter],type="l",ylab="mu")
     plot(keep.s2[1:iter],type="l",ylab="s2")
   }
}
```


mu
7.0
6.5
6.0
0
2000
6000
10000
Index


s2
120
115
110
105
100
0
2000
6000
10000
Index

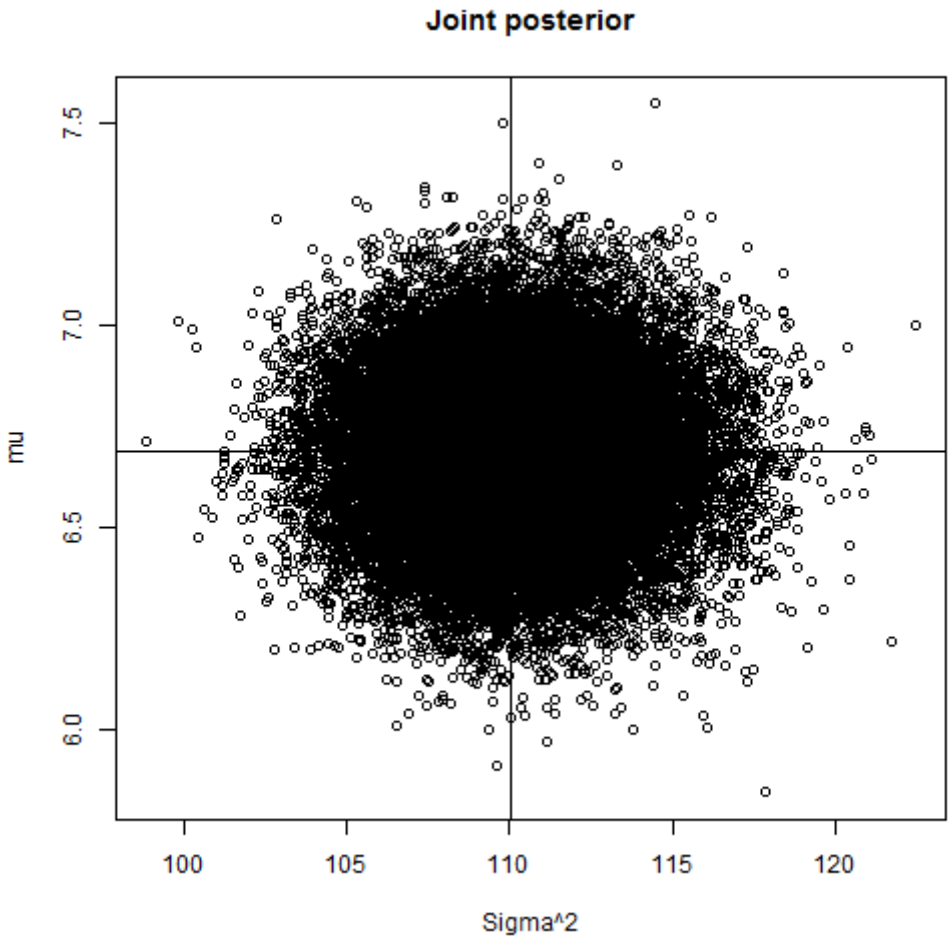

mu
7.5
7.0
6.5
6.0
0
5000
15000
Index


s2
120
115
110
105
100
0
5000
15000
Index



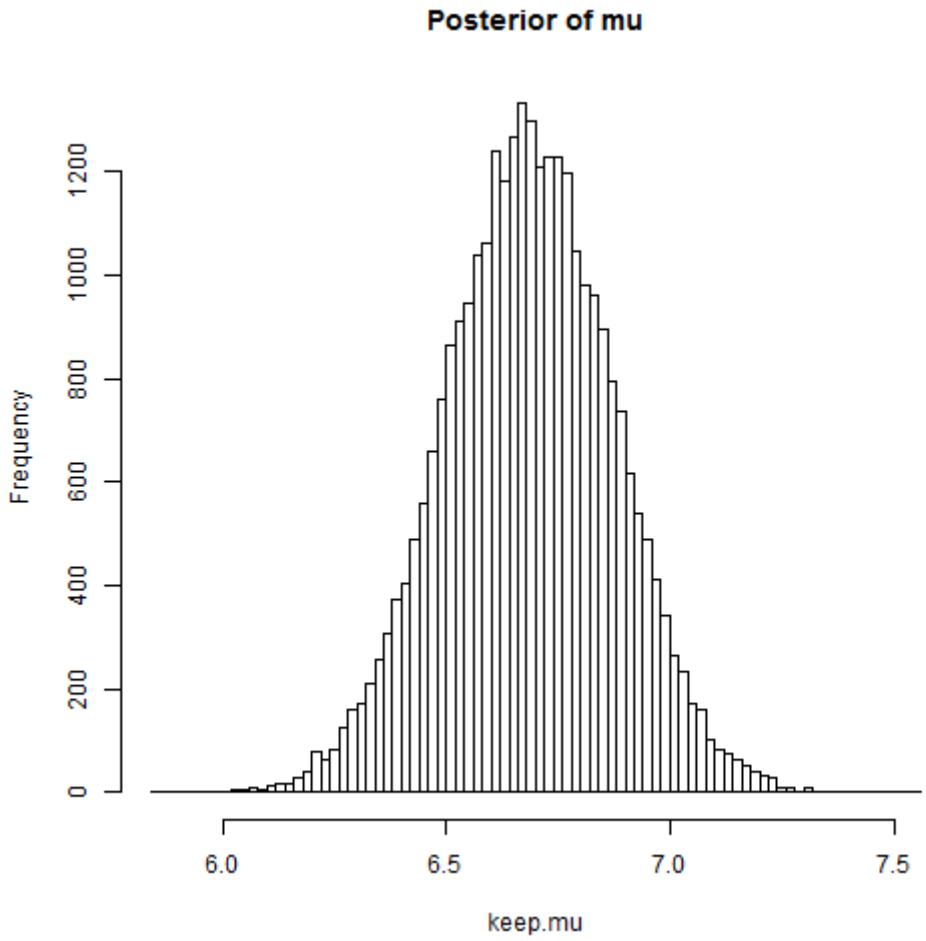
Plots of the joint posterior distribution of (μ, σ^2).

```
plot(keep.s2,keep.mu,xlab="Sigma^2",ylab="mu",main="Joint posterior")
abline(mean(Y),0)
abline(v=var(Y))
```

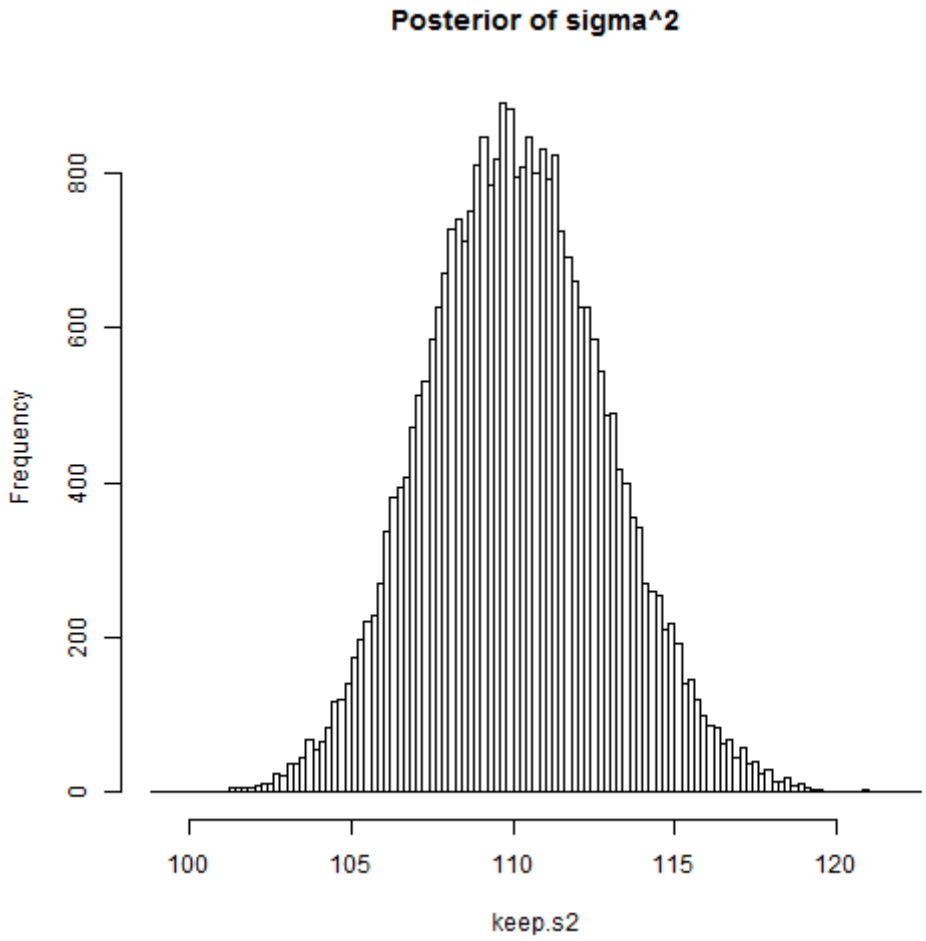


Plots of the marginal distributions of μ and σ^2.

```
hist(keep.mu,main="Posterior of mu",breaks=100)
```

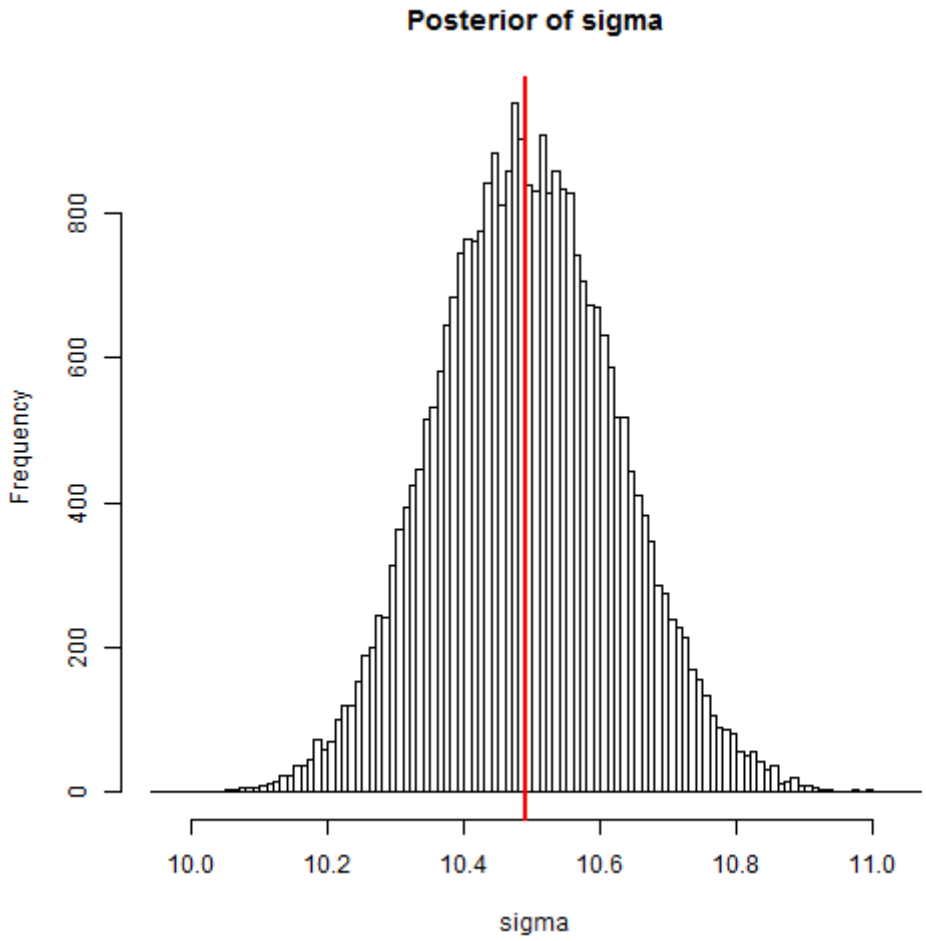



```
hist(keep.s2,main="Posterior of sigma^2",breaks=100)
```



Plots of the marginal distribution of σ.

```
sigma<- sqrt(keep.s2)
hist(sigma,main="Posterior of sigma",breaks=100)
abline(v=sd(Y),col=2,lwd=2)
```



Posterior summaries

```
output <- matrix(0,2,4)
colnames(output)<-c("Mean","SD","Q025","Q975")
rownames(output)<-c("mu","sigma^2")

output[1,1]<-mean(keep.mu)
output[1,2]<-sd(keep.mu)
output[1,3]<-quantile(keep.mu,0.025)
output[1,4]<-quantile(keep.mu,0.975)

output[2,1]<-mean(keep.s2)
output[2,2]<-sd(keep.s2)
output[2,3]<-quantile(keep.s2,0.025)
output[2,4]<-quantile(keep.s2,0.975)

kable(output,digits=2)
```

	Mean	SD	Q025	Q975
mu	6.68	0.19	6.31	7.05
$sigma^2$	110.11	2.79	104.83	115.74