1.1.

a, $E[(Y-c)^2] = E(Y^2) - 2cE(Y) + E(c^2)$

$= c^2 - 2\mu c + a$

$\Rightarrow$ min $\Leftrightarrow$ $c = \mu$

b, $E[(Y-f(X))^2 \mid X]$

$= E(Y^2|X) - 2E(Y f(X) \mid X) + E(f^2(X)|X)$

$= a - 2f(X)E(Y|X) + f^2(X)$

$\Rightarrow$ min $\Leftrightarrow$ $f(X) = E(Y|X)$

c, $E[Y - f(X)]^2$

$= E(Y^2) - 2E(Y f(X)) + E(f^2(X))$

$= E[E[(Y - f(X))^2 | X]]$

min $\Leftrightarrow$ $E[(Y - f(X))^2 | X]$ min

$\Rightarrow$ $f(X) = E(Y|X)$

1.2.

a, tương tự câu b bài 1.1

b, tương tự câu c bài 1.1.

c) $\min E\left[X_{n+1} - f(X_1, X_2, \ldots, X_n) \mid X_1, X_2, \ldots, X_n\right]$

$= \cancel{E\left[E\left(X_{n+1} \mid X_1, \ldots, X_n\right)\right.}$

$= E\left\{\left(X_{n+1} - E(X_{n+1} \mid X_1, \ldots, X_n)\right)^2 \mid X_1, \ldots, X_n\right)$

$= E\left[\left(X_{n+1} - \alpha\right)^2 \mid X_1, \ldots X_n\right]$

$= E\left(X_{n+1}^2 \mid X_1, \ldots X_n\right) - 2\alpha E(X_{n+1} \mid X_1, \ldots X_n) + \alpha^2$

$= E(X_{n+1}^2 \mid X_1, \ldots, X_n) - \alpha E(X_{n+1} \mid X_1, \ldots, X_n)^2$

$= V(X_{n+1}^2 \mid X_1, \ldots, X_n)$

d) $f(X_1, \ldots, X_n) = E(X_{n+1} \mid X_1, \ldots, X_n)$

$\cancel{= \sum_{i=1}^{n} \frac{1}{n} X_i}$

d) Đặt $\overline{X} = \sum_{i=1}^{n} c_i X_i$

$\cancel{V}(\overline{X} - \mu) = E\left(\overline{X} - E(\overline{X} - \mu)\right)^2$

$= E\left(\overline{X} + \mu - \mu \sum c_i\right)^2$

$= E\left(\overline{X} + \mu(1 - \sum c_i)\right)^2$

$V(\bar{X})$ ~~$= V(a$~~

$= E(\bar{X} - E(\bar{X}))^2$

$= E(\bar{X} - \mu)^2$

$= E(\bar{X}^2) - \mu^2$

$= E((\sum c_i X_i)^2) - \mu^2$

$= E\left(\sum_{i=1}^{n} c_i^2 X_i^2 + 2 \sum_{1 \le j < k \le n} c_j X_j c_k X_k\right) - \mu^2$

$= E\left(\sum_{1}^{n} c_i^2 X_i^2\right) + 2E\left(\sum_{1 \le j < k \le n} c_j X_k c_k X_k\right) - \mu^2$

~~$= \sum_{1}^{n} c_i^2 E(X_i^2) +$~~

~~$\frac{\partial V(\bar{X})}{\partial c_j} = 2c_j E(X_j^2) + 2E\left(X_j \sum_{k=1, k \ne j}^{n} c_k X_k\right)$~~

$\frac{\partial V(\bar{X})}{\partial c_j} = 2c_j E(X_j^2) + 2E\left(X_j \sum_{k=1, k \ne j}^{n} c_k X_k\right)$

$= 2c_j E(X_j^2) + 2\mu^2 \sum_{k=1, k \ne j}^{n} c_k$

$= 2c_j E(X_j^2) + 2\mu^2 (1 - c_j)$

$= 2c_j (E(X^2) - \mu^2) + 2\mu^2 = 0$

$\Rightarrow$ ~~$c_j = \frac{\mu^2}{E(X^2) - \mu^2}$~~ $c_j = \frac{\mu^2}{E(X^2) + \mu^2}$

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$$V(\overline{X}) = E(\overline{X}^2) - E(\overline{X})^2$$

$$= E\left((\overline{X} - E(\overline{X}))^2 \right)$$

$$\cancel{= E(\overline{X} - \frac{1}{n}}$$

$$= E(a_i \Sigma$$

$$E(\overline{X}) = \sum_1^n c_i E(X_i) = c^T X = \mu$$

$$V(\overline{X}) = E\left(\left(\sum_{i=1}^n c_i X_i - E\left(\sum_{i=1}^n c_i X_i\right)\right)^2 \right)$$

$$= E\left[(c^T x - c^T E(x))^2 \right]$$

$$= E\left[c^T [x - E(x)][x - E(x)]^T c \right]$$

$$= E(c^T A c) = c^T E A c$$

Xét $\mathcal{L} = c^T EAc + \lambda (c^T \mathbb{1} - 1)$

$$\frac{\partial \mathcal{L}}{\partial a} = 2EAc + \lambda S$$

$$\Rightarrow c = \frac{-\lambda}{2} EA^{-1}$$

$$\Rightarrow c^T . 1 = \frac{-\lambda}{2} 1^T EA^{-1} c = 1$$

$$\Rightarrow \frac{-\lambda}{2} = \frac{1}{c^T EA^{-1} c}$$

1.3,

Do tính dừng chặt

$\Rightarrow P(X_{t_1} \le x_{t_1}) = P(X_{t_1+h} \le x_{t_1+h})$

$\Rightarrow X_{t_1}, X_{t_2}, \dots, X_{t_n}$ có là iid

$\Rightarrow E(X_t) = \mu_X \quad \forall t$

• X_t và X_{t+h} có cùng pp

$\Rightarrow Cov(X_t, X_{t+h}) = E(X_t X_{t+h}) - \mu^2 = E(X^2) - \mu^2$
$= Var(X)$

$\Rightarrow \cancel{E(X_t X_{t+h}) = Cov(X, X) + \mu^2}$
$\cancel{= Var(X) + \mu^2}$

1.4.

a, $E(X_t) = a + bE(Z_t) + cE(Z_{t-2})$
$= a$

$\cancel{E(X_t X_{t+h}) = Cov(}$

$Cov(X_t, X_{t+h}) = E[(X_t - a)(X_{t+h} - a)]$

$= E(X_t X_{t+h}) - a^2$

$= E[(a + bZ_t + cZ_{t-2})(a + bZ_{t+h} + cZ_{t+h-2})] - a^2$

$= E[a^2 + \dots$

Thứ ngày

$= \cancel{a} + b^2 E(Z_t Z_{t+h}) + bc E(Z_{t} Z_{t+h}) + c^2 E(Z_{t-2} Z_{t+h-2}) + bc E(Z_t Z_{t+h-2})$

$h = 0$:

$Cov(X_t, X_t) = b^2 E(Z_t^2) + bc E(Z_{t-2} Z_t) + bc E(Z_t Z_{t-2})$

$= b^2 (\sigma^2) + c^2 \sigma^2$

$h = \pm 2$:

$Cov(X_t, X_{t \pm 2}) = ab\,\sigma^2$

$h \neq \{\pm 2, 0\}$: $Cov(X_t, X_{t+h}) = 0$

$\Rightarrow \{X_t\}$ là chuỗi dừng

b) $X_t = Z_1 \cos(ct) + Z_2 \sin(ct)$

$E(X_t) = 0$

$Cov(X_t, X_{t+h}) = E[(Z_1 \cos(ct) + Z_2 \sin(ct))(Z_1 \cos(c(t+h)) + Z_2 \sin(c(t+h)))]$

$= \sigma^2 (\cos(ct) \cos(ct + ch) + \sin(ct) \sin(ct + ch))$

$= \sigma^2 \cos(ch)$

$\Rightarrow \{X_t\}$ là chuỗi dừng

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c, $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$

$E(X_t) = 0$

$Cov(X_t, X_{t+h}) = E[(Z_t \cos(ct) + Z_{t-1} \sin(ct))$
$(Z_{t+h} \cos(ct+ch) + Z_{t+h-1} \sin(ct$

Với $h=1 \Rightarrow cov(X_t, X_{t+1}) = \sigma^2 \cos(ct-c) \sin(c$

$\Rightarrow \{X_t\}$ là chuỗi dừng khi c là bội của π

d, $X_t = a + bZ_0$

$E(X_t) = a$

$Cov(X_t, X_{t+h}) = E[(a+bZ_0 - a)(a+bZ_0 - a)]$

$= E(a+bZ_0)^2 - a^2$

$= a^2 + 2abE(Z_0) + b^2E(Z_0^2) - a^2$

$= b^2\sigma^2$

$\Rightarrow \{X_t\}$ là chuỗi dừng

e, $X_t = Z_0 \cos(ct)$

$E(X_t) = 0$

$Cov(X_t, X_{t+h}) = \cancel{Cov}[Z_0^2 \cos(ct) \cos(ct+ch)]$

$= \frac{\sigma^2}{2} E[\cos(2\alpha t + ch) + \cos(ch)]$

$\Rightarrow \{X_t\}$ dừng $\Leftrightarrow c = k\pi$

f) $X_t = Z_t Z_{t-1}$

$E(X_t) = 0$

$Cov(X_t, X_{t+h}) = E(Z_t Z_{t-1} Z_{t+h} Z_{t-1+h})$

$h = 0$: $\cancel{E(Z_t^2 Z_{t-1}^2)} = E(Z_t^2 Z_{t-1}^2)$
$= \sigma^4$

$h = 1$: $E(Z_t \cancel{Z_{t-1}} \cancel{Z_{t+1}} Z_t) = 0$

$h \neq 0$: $Cov(X_t, X_{t+h}) = 0$

$\Rightarrow$ chuỗi dừng.

1.5.

a, $E(X_t) = E(Z_t + \theta Z_{t-2}) = E(Z_t) + \theta E(Z_{t-2}) = 0$

$E(X_t^2) = E(Z_t^2 + \theta^2 Z_{t-2}^2 + 2 Z_t \theta Z_{t-2})$

$= 1 + \theta^2$

(Do $Cov(Z_t, Z_{t+h}) = 0 = E(Z_t Z_{t+h})$ $\forall t \forall h$)

$\Rightarrow E(X_t, X_{t+h}) = E(Z_t Z_{t+h}) + \theta(E(Z_t Z_{t+h-2})$

$+ E(Z_{t-2} Z_{t+h})) + \theta^2 E(Z_{t-2} Z_{t+h-2})$

Thứ ngày . .

$$\Rightarrow \gamma_h = \begin{cases} 1+\theta^2 & \text{nếu } h=0 \\ \theta & \text{nếu } h=\pm 2 \\ 0 & \text{còn lại} \end{cases}$$

$$\rho_h = \begin{cases} 1, & h=0 \\ \dfrac{\theta}{1+\theta^2}, & h=\pm 2 \\ 0, & \text{còn lại} \end{cases}$$

b, $Var(\bar{X}_4) = \frac{1}{16} Var(X_1 + X_2 + X_3 + X_4)$

$= \frac{1}{16} \sum_{i=1}^{4} \sum_{j=1}^{4} cov(X_i, X_j)$

$= \frac{1}{16} (4(1+\theta^2) + 4\theta)$