1.0

$$m_t = \sum_{k=0}^{r} c_k t^k$$

$$Dm_t = m_t - m_{t-1} = \cancel{c_k t^p - c_k (t-1)^p}$$

$$\sum_{k=0}^{p} c_k (t^k - (t-1)^k)$$

$$= \sum_{k=0}^{p} c_k \left[t^k - \sum_{i=0}^{k} C_k^i (t)^i (-1)^{k-i} \right]$$

$$= \sum_{k=0}^{p} c_k \left[- \sum_{i=0}^{k-1} C_k^i (t)^i (-1)^{k-i} \right]$$

$$D^{p+1} m_t = D^p (D m_t) = D^p (f^{p-1}(t))$$

$$= D^{p-1} (f^{p-2}(t))$$

$$= D \cancel{f(t)} c = 0$$

1.11

$$a_j = (2q+1)^{-1}$$

a, $m_t = c_0 + c_1 t$

$$\sum_{j=-q}^{q} a_j m_{t-j} = \sum_{j=-q}^{q} (2q+1)^{-1} (c_0 + c_1 (t-j))$$

$$= \sum_{j=-q}^{q} \frac{c_0 + c_1 t - c_1 j}{(2q+1)}$$

$$= \frac{1}{2q+1}\left(c_0+c_1t\right)+\sum_{j=-q}^{q} \frac{-c_1 j}{2q+1}$$

$$= c_0+c_1t$$

b, $$A_t = \sum_{j=-q}^{q} a_j Z_{t-j}$$

$$EA_t = \sum_{j=-q}^{q} E(a_j Z_{t-j})$$

$$= \frac{1}{2q+1} \sum_{j=-q}^{q} E(Z_{t-j}) = 0$$

$$Var(A_t) = \left(\frac{1}{(2q+1)^2}\right) \sum_{j=-q}^{q} Var(Z_{t-j})$$

$$= \frac{1}{(2q+1)^2} (2q+1)\sigma^2 = \frac{\sigma^2}{2q+1}$$

1.12.

a, $$m_t = \sum_{i=0}^{k} c_i t^i$$

CM: $$m_t = \sum_j a_j m_{t-j}$$

$$\Leftrightarrow \begin{cases} \sum_j a_j = 1 \\ \sum_j j^r a_j = 0 \quad r = 1, \dots k \end{cases}$$

Thứ ngày . .

$\Leftarrow$

$\sum_j a_j m_{t-j}$

$= \sum_j a_j \sum_{i=0}^{k} c_i (t-j)^i$

$= \sum_j a_j \sum_{i=0}^{k} c_i \sum_{l=0}^{i} C_l^i (-1)^l j^l t^{i-l}$

~~Xét bài~~ $\cancel{\Rightarrow i = k,\ l = l'}$

$\cancel{= \sum_j a_j c_k C_{l}^k (-1)^{l} j^{l} t^{k-l}}$

$\cancel{= \sum_j a_j c_k C_0^k}$

~~Xét~~ $\cancel{t^k \Rightarrow i = k}$

$\cancel{= \sum_j a_j c_k (t-j)^k}$

$\cancel{= c_k \sum_j a_j (t-j)^k = c_k \sum_j a_j}$

~~Xét~~ $\cancel{t^n \Rightarrow i = n,\ l = 0},\ 0 \le n \le k$

$\cancel{i = n + a,\ l = c_k},$

Xét ~~hệ~~ hệ số của $t^n \Rightarrow i-l=n \quad 0\le n\le k$

$\Rightarrow \begin{cases} l=i-n & \cancel{0\le n\le k}\ n\le i \\ l=0 & , \ n>i \end{cases}$

$\cancel{= \sum_j a_j \sum_{i=0}^{k} c_i C_i^{i-n} (-1)^{i-n} j^{i-n} \cdots C_{i+l}^{i-n}}$

$\cancel{= \sum_j a_j \sum_{i=0}^{k} c_i (-j)^{i-n} C_i^{i-n}}$

$\cancel{= \sum_j a_j \sum_{i=0}^{n} c_i (-j)^{i-n} C_i^{i-n}}$

$= \sum_j a_j \sum_{i=n}^{k} c_i C_k^{i-n} \cdot (-j)^{i-n}$

$= \sum_{i=n}^{k} \sum_j a_j c_i C_k^{i-n} (-j)^{i-n}$

$= \sum_{i=n}^{k} c_i C_k^{i-n} \sum_j a_j (-j)^{i-n}$

$= c_n$

$\Rightarrow \sum_j a_j m_{t-j} = m_t$

$\boxed{\Rightarrow}$

+ cou

Thứ ngày

$$c_n = \sum_{i=n}^{k} c_i C_k^{i-n} \sum_j a_j (-j)^{i-n}$$

$$\Rightarrow \begin{cases} \sum_j a_j = 1 \\ \sum_{i=n+1}^{k} c_i C_k^{i-n} \sum_j a_j (-j)^{i-n} = 0 \quad \forall n \le k \quad \forall c_i \end{cases}$$

$$\Rightarrow \begin{cases} \sum_j a_j = 1 \\ \sum_j a_j (-j)^{i-n} = 0 \end{cases} \text{ hay } \begin{cases} \sum_j a_j = 1 \\ \sum_j a_j (j)^{r} = 0 \end{cases} \quad r = 1, \dots, k$$

b) $m_t = c_0 + c_1 t + c_2 t^2 + c_3 t^3$

$[a_0, \dots, a_7] = \frac{1}{320} [74, 67, 46, 21, 3, -5, -6, -3]$

$a_j = a_{-j} \quad |j| \le 7$

$a_j = 0, \quad |j| > 7$

$$\sum_j a_j = \sum_{-7}^{7} a_j = \frac{74}{320} + 2 \sum_{j=1}^{7} a_j$$

1.13.

- $m_t = c_0 + c_1 t$

without distortion $\Leftrightarrow m_t = (1 + \alpha B + \beta B^2 + \gamma B^3) m_t$

$\Leftrightarrow \alpha m_{t-1} + \beta m_{t-2} + \gamma m_{t-3} = 0$

$\Leftrightarrow \alpha (c_0 + c_1(t-1)) + \beta (c_0 + c_1(t-2)) + \gamma (c_0 + c_1(t-3)) = 0$

$\Leftrightarrow (c_0 + c_1 t)(\alpha + \beta + \gamma) - c_1(\alpha + 2\beta + 3\gamma) = 0 \quad \forall c_0, c_1, t$

$$\Leftrightarrow \begin{cases} \alpha + \beta + \gamma = 0 \\ \alpha + 2\beta + 3\gamma = 0 \end{cases} \quad (1)$$

Eliminating arbitary seasonal components of $d = 2$

$$\Leftrightarrow \begin{cases} (1 + \alpha B + \beta B^2 + \gamma B^3) s_t = 0, \forall t \\ s_{t+2} = s_t, \; s_1 + s_2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} s_t + \alpha s_{t-1} + \beta s_{t-2} + \gamma s_{t-3} = 0 \\ s_{t+2} = s_t \; ; \; s_1 + s_2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 1 + \beta = \alpha + \gamma \quad (2) \\ s_{t+2} = s_t, \; s_1 + s_2 = 0 \end{cases}$$

$$(1), (2) \rightarrow \begin{cases} \alpha + \beta + \gamma = 0 \\ \alpha + 2\beta + 3\gamma = 0 \\ \alpha - \beta + \gamma = 1 \end{cases} \Leftrightarrow (\alpha, \beta, \gamma) = \left(\frac{1}{4}, \frac{-1}{2}, \frac{1}{4}\right)$$

$[a_{-2}, a_{-1}, a_0, a_1, a_2] = \frac{1}{9}[-1, 4, 3, 4, -1]$

$* \sum_{j=-2}^{2} a_j = 1$

$* \sum_{j=-2}^{2} j^r a_j = 0, \ \forall r \in \{1,2,3\}$

$\Rightarrow$ Passes 3^{rd} degree polynomial trends without distortion

Seasonal component with $d=3$: $\begin{cases} s_{t+3} = s_t, \ \forall t \\ s_1 + s_2 + s_3 = 0 \end{cases}$

$$\sum_{j=-2}^{2} a_j s_{t-j} = \frac{1}{9}\left(-s_{t-2} + 4s_{t-1} + 3s_t + 4s_{t+1} - s_{t+2}\right)$$

$$= \frac{1}{9}\left(-s_{t+1} + 4s_{t-1} + 3s_t + 4s_{t+1} - s_{t+2}\right)$$

$$= \frac{1}{3}\left(s_t + s_{t+1} + s_{t+2}\right) = 0$$

$\Rightarrow$ Eliminate seasonal componants with period 3

1.15.

a) ~~$\nabla\nabla_{12} X_t$~~ $X_t = a + bt + s_t + Y_t$

$\nabla\nabla_{12} X_t$ ~~$= (1-B)(1-B^{12}) X_t$~~

$= \nabla\nabla_{12}(a+bt) + \nabla\nabla_{12} s_t + \nabla\nabla_{12} Y_t$

$= \nabla(1-B)(1-B^{12})(bt) + \nabla(1-B)(1-B^{12}) s_t$
$+ \nabla(1-B)(1-B^{12}) Y_t$

$$\Rightarrow \nabla\nabla_{12} X_t = \nabla\nabla_{12} Y_t$$

$$\gamma_{\nabla\nabla_{12}X}(h) = Cov(\nabla\nabla_{12} X_{t+h}, \nabla\nabla_{12} X_t)$$

$$= Cov(\nabla\nabla_{12} Y_{t+h}, \nabla\nabla_{12} Y_t)$$

$$= Cov(Y_{t+h} - Y_{t+h-1} - Y_{t+h-12} + Y_{t+h-13},$$

$$Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13})$$

$$= Cov(Y_{t+h}, Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13})$$

$$- Cov(Y_{t+h-1}, Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13})$$

$$- Cov(Y_{t+h-12}, Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13})$$

$$+ Cov(Y_{t+h-13}, Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13})$$

$$= \sigma^2 (\chi_{\{0\}} - \chi_{\{-1\}} - \chi_{\{-12\}} + \chi_{\{-13\}})(h)$$

$$- \sigma^2 (\chi_{\{1\}} - \chi_{\{0\}} - \chi_{\{-11\}} + \chi_{\{-12\}})(h)$$

$$- \sigma^2 (\chi_{\{12\}} - \chi_{\{11\}} - \chi_{\{0\}} + \chi_{\{-1\}})(h)$$

$$+ \sigma^2 (\chi_{\{13\}} - \chi_{\{12\}} - \chi_{\{1\}} + \chi_{\{0\}})(h)$$

$$= \sigma^2 (\chi_{\{-13\}} - 2\chi_{\{-12\}} + \chi_{\{-11\}} - 2\chi_{\{-1\}} + 4\chi_{\{0\}}$$

$$- 2\chi_{\{1\}} + \chi_{\{11\}} - 2\chi_{\{12\}} + \chi_{\{13\}})(h)$$

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$$= \begin{cases} \sigma^2 & h = \pm 13;\ \pm 11 \\ -2\sigma^2 & h = \pm 12;\ \pm 1 \\ 4\sigma^2 & h = 0 \\ 0 & \text{, trái lại.} \end{cases}$$