Transformada de Fourier

Wednesday, June 1, 2022 9:24 PM

Números Complejos:

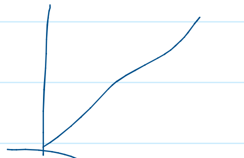
$C = R + jI$ $\quad R$ y $I \in \mathbb{R}$

$j = \sqrt{-1}$

$C^* = R - jI$ (conjugado)

* Pueden ser vistos geométricamente como puntos en un plano.
* Puede ser escrito con coordenadas polares.

$$C = |C|(\cos\theta + j\,\text{sen}\,\theta) \quad \ldots \text{ I}$$



$|C| = \sqrt{R^2+I^2}$ ... tamaño del vector $(0,0) \to (R,I)$

θ, angulo del vector y el eje real, $\theta = \text{arctg}(I/R)$

Euler:

$$e^{j\theta} = \cos\theta + j\,\text{sen}\,\theta$$

Reemplazando en (I)

$$C = |C|\, e^{j\theta}$$

Ejemplo: $1+2j \cong \sqrt{5}\, e^{j\theta}$ $\quad \theta = 64.4°$ aprx.

Funciones:

$$F(u) = R(u) + jI(u)$$

$$|F(u)| = \sqrt{R(u)^2 + I(u)^2}$$

$$\theta(u) = \text{arctg}\,[I(u)/R(u)]$$

Serie de Fourier:

$$f(t) = \sum_{-\infty}^{\infty} C_n e^{j\frac{2\pi n}{T}t}$$

t variable periodica

T periodo

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{j\frac{2\pi n}{T}}\, dt.$$

Transformada Fourier.

$$F(u) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ut}\, dt.$$

$f(t)$: función continua

t: variable continua.

$$f(t) = \int_{-\infty}^{\infty} F(u)\, e^{j2\pi ut}\, du$$

... inversa

Usando Euler

$$F(u) = \int_{-\infty}^{\infty} f(t)\cos(2\pi ut) - j\,\text{sen}(2\pi ut)\, dt.$$

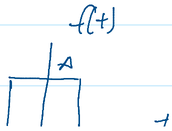
$f(t)$ → parte real.

t = tiempo (seg)

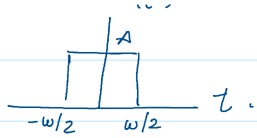
u ciclos. (ciclo/seg)

Ejemplo:

$$F(u) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ut}\, dt$$



Ejemplo:



$$F(u) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi u t} dt$$

$$= \int_{-w/2}^{w/2} A e^{-j2\pi u t} dt$$

$$= \frac{-A}{j2\pi u} \left[e^{-j2\pi u t} \right]_{-w/2}^{w/2}$$

$$= \frac{-A}{j2\pi u} \left[e^{-j\pi u w} - e^{j\pi u w} \right]$$

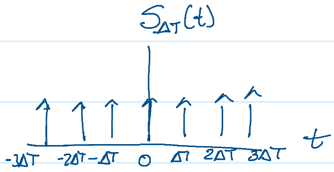
$$= \frac{A}{j2\pi u} \left[e^{j\pi u w} - e^{-j\pi u w} \right]$$

identidad trigonometrica

$$\operatorname{sen}\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$F(u) = AW \frac{\operatorname{sen}(\pi u w)}{\pi u w}$$

Tren de impulsos:



$$F(u) = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi u t} dt.$$

$\delta(t)$: función impulso / delta.

$$= \int_{-\infty}^{\infty} e^{-j2\pi u t} \delta(t) dt. \qquad \text{Si} \quad t=0$$

$$= e^{0}$$

$$= 1$$

Transformada discreta de Fourier DFT.

$$\tilde{F}(u) = \int_{-\infty}^{\infty} \tilde{f}(t) e^{-j2\pi u t} dt.$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t) \delta(t - n\Delta T) e^{-j2\pi u t} dt$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) \delta(t - n\Delta T) e^{-j2\pi u t} dt.$$

$$= \sum_{n=\infty}^{\infty} f_n e^{-j2\pi u n \Delta T}.$$

$$u = \frac{m}{M\Delta T}$$

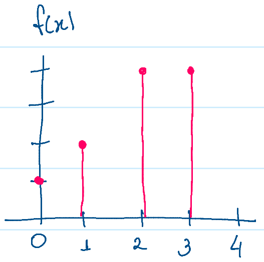
M muestras

$u=0, \ u = 1/\Delta T$

$$F_m = \sum_{n=0}^{M-1} f_n e^{-j2\pi mn/M} \qquad \cong \quad F(u) = \sum_{n=0}^{M-1} f(x) e^{-j2\pi u x/M}$$

$$f_n = \frac{1}{M} \sum_{n=0}^{M-1} F_m e^{j2\pi mn/M} \qquad \cong \quad f(x) = \frac{1}{M} \sum_{n=0}^{M-1} F(u) e^{j2\pi u x/M}$$

Ejemplo :



$$F(0) = \sum_{n=0}^{3} f(x)\, e^{-j2\pi \cancel{u}^{0} x/M} = f(0)+f(1)+f(2)+f(3) = 11$$

$$F(1) = \sum_{n=0}^{3} f(x)\, e^{-j2\pi \cancel{u}^{1} x/M} =$$

$$= 1e^{0} + 2e^{-j\pi/2} + 4e^{-j\pi} + 4e^{-j3\pi/2} = -3+2j$$

$$F(2) = \sum_{n=0}^{3} f(x)\, e^{-j2\pi \cancel{u}^{2} x/M}$$

$$= -1-j$$

$$F(3) = \sum_{n=0}^{3} f(x)\, e^{-j2\pi \cancel{u}^{3} x/M}$$

$$= -3-2j$$

Si tuvieramos $F(u)$ y no $f(x)$ entonces

$$f(0) = \frac{1}{4} \sum_{n=0}^{3} F(u)\, e^{j2\pi u \cancel{x}^{0}/M}$$

$$= \frac{1}{4} \sum_{n=0}^{3} F(u) = \frac{1}{4}\left[11-3+2j-1-3-2j\right]$$

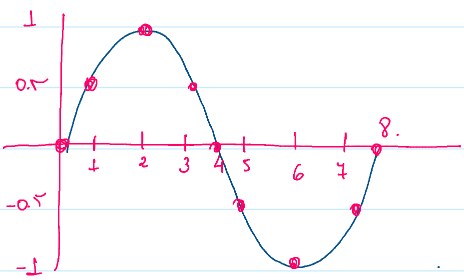
$$= \frac{1}{4}[4] = 1$$

Para 2 variables.

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi (ux/M + vy/N)}$$

$$f(x,y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u,v)\, e^{j2\pi (ux/M + vy/N)}$$

Ejercicio :



$f(0) = 0$	$f(4) = 0$
$f(1) = 0.707$	$f(5) = -0.707$
$f(2) = 1$	$f(6) = -1$
$f(3) = 0.707$	$f(7) = -0.707.$

$F(0) = ?$ .....

?