Department of Electrical, Computer & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Number	ELE 532
Course Title	Signal and Systems 1
Semester/Year	Winter 2024
Instructor	Dr. Alagan Anpalagan

Lab/Tutorial Report No.	**4**

Report Title	The Fourier Transform: Properties and Applications

Submission Date	2024-04-11
Due Date	2024-04-12

Name	**Student ID**	**Signature***
Magsud Allahyarov	xxxx84937	Magsud

(Note: remove the first 4 digits from your student ID)

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<u>Table of Contents:</u>

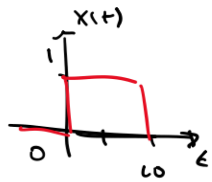
<u>Problem A.1:</u>

For the signal x(t) shown in Figure (1), compute and plot $z(t) = x(t) * x(t)$.

ELE 532 Lab4

Wednesday, November 22, 2023 9:31 PM

Problem A.1 :

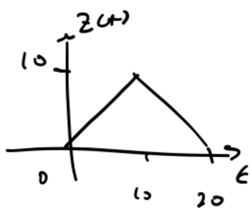


$x(t) = u(t) - u(t-10)$

$z(t) = x(t) * x(t) = (u(t) - u(t-10)) * (u(t) - u(t-10))$

$= u(t) * u(t) - u(t) * u(t-10) - u(t) * u(t-10) + u(t-10) * u(t-10)$

$= \gamma(t) - \gamma(t-10) - \gamma(t-10) + \gamma(t-20)$



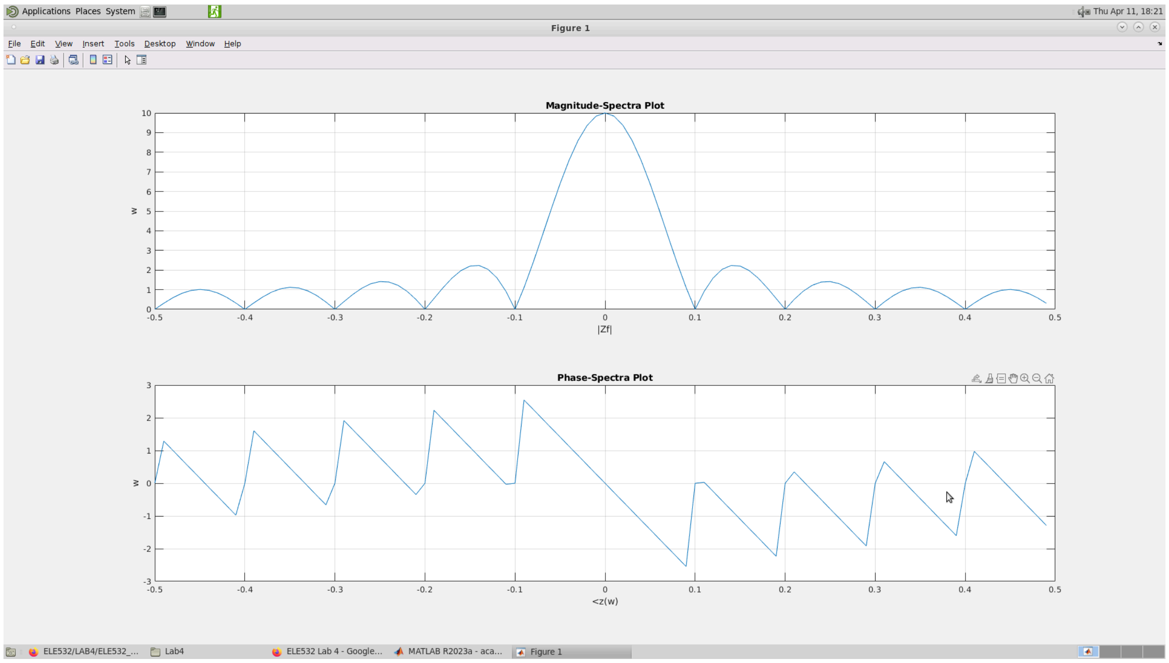
Problem A.2:

Using Matlab, calculate Z(ω) = X(ω)X(ω).

Code:

```
N = 100;
PWiN = 100;
PWidth = 10;
t = [0:1:(N-1)];
x = [ones(1, PWidth), zeros(1, N-PWidth)];
z = conv(x, x);
f = [-(N/2):1:(N/2)-1]*(1/N);
Xf = fft(x);
zf=Xf.*Xf;
subplot(211); plot(f,fftshift( abs(Xf))); grid on;
title('Magnitude-Spectra Plot');
xlabel('|Zf|'); ylabel('w');
subplot(212); plot(f,fftshift(angle(Xf))); grid on;
title('Phase-Spectra Plot');
```

```
xlabel('<z(w)'); ylabel('w');
```

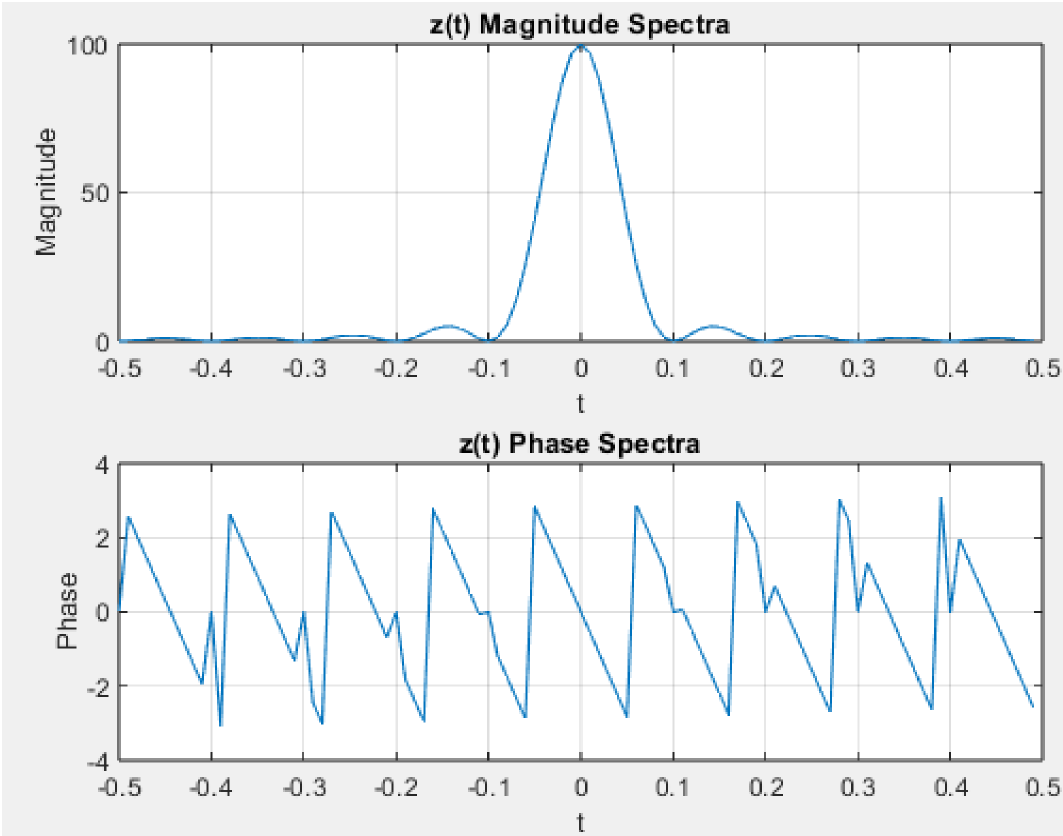


Problem A.3:

Plot the magnitude- and phase-spectra of z(t).

Code:

```
N = 100;
PWidth = 10;
t = [0:1:(N-1)];
x = [ones(1, PWidth), zeros(1, N-PWidth)];
z = conv(x, x);
f = [-(N/2):1:(N/2)-1]*(1/N);
Xf = fft(x);
zf=Xf.*Xf;
subplot(2,1,1);
plot(f,fftshift(abs(zf))); grid on;
title('z(t) Magnitude Spectra');
xlabel('t');ylabel('Magnitude');
subplot(2,1,2); plot(f,fftshift(angle(zf)));
grid on; title('z(t) Phase Spectra');
xlabel('t'); ylabel('Phase');
```



<u>Problem A.4:</u>

Compute z(t) using time-domain and frequency-domain operations implemented in Matlab. Plot both results and compare with the analytic result you determined in Problem A.1. Determine the appropriate time indices for proper labeling of the time-domain plots of z(t). How do the results you generated in Matlab using time- and frequency-domain operations compare with the analytic result you computed in Problem A.1? Explain which property of the Fourier Transform you have demonstrated.

Code:

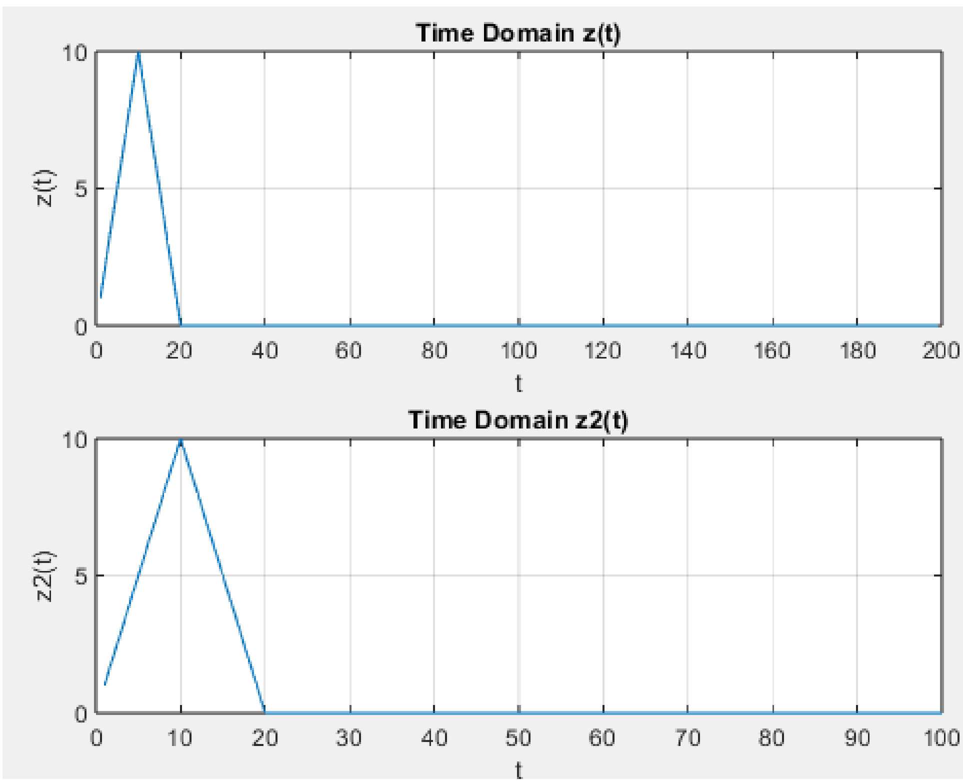
```
N = 100; PWidth = 10;
t = [0:1:(N-1)];
x = [ones(1, PWidth), zeros(1, N-PWidth)];
z = conv(x, x);

Xf = fft(x);
Zf = Xf.*Xf;
z2 = ifft(Zf);
```

```
subplot(2,1,1);
plot(z); grid on;
title('Time Domain z(t)');
xlabel('t'); ylabel('z(t)');

subplot(2,1,2);
plot(z2); grid on;
title('Time Domain z2(t)');
xlabel('t'); ylabel('z2(t)');
```

The results we generated from MATLAB are the exact same as the ones we calculated in Problem A.1. The convolution property was demonstrated in this problem, with the convolution equal of two signals being equal to the product of their Fourier Transforms.

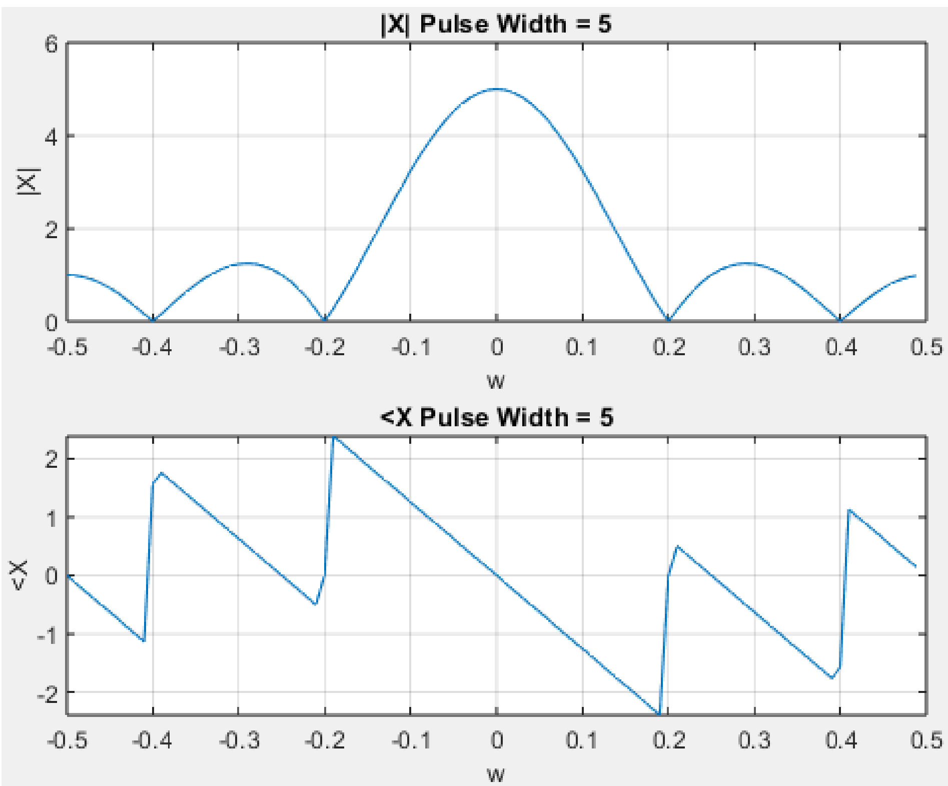


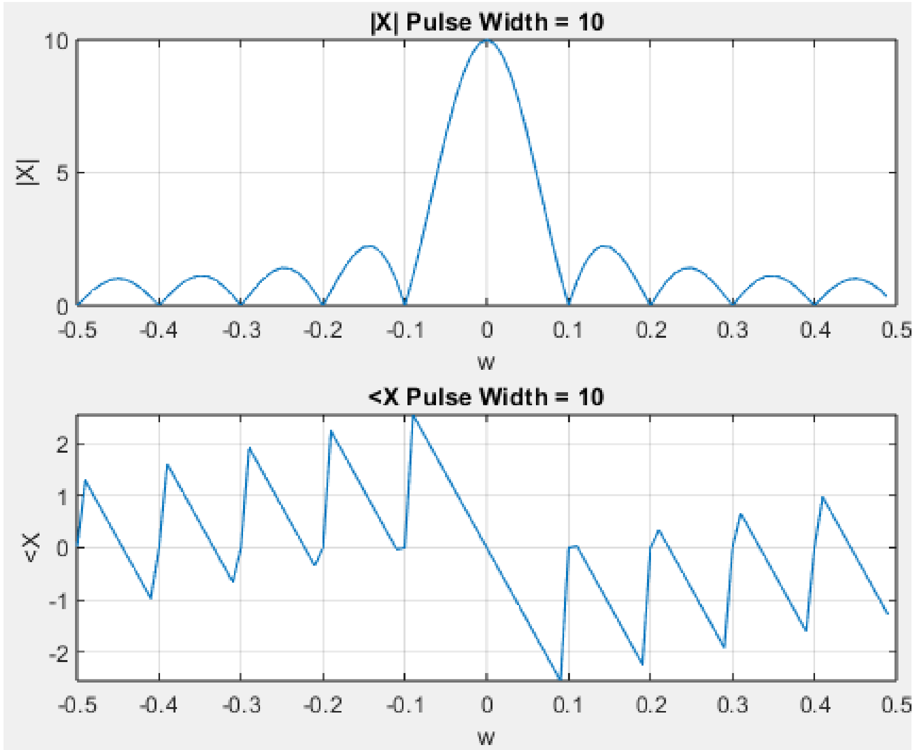
Problem A.5:

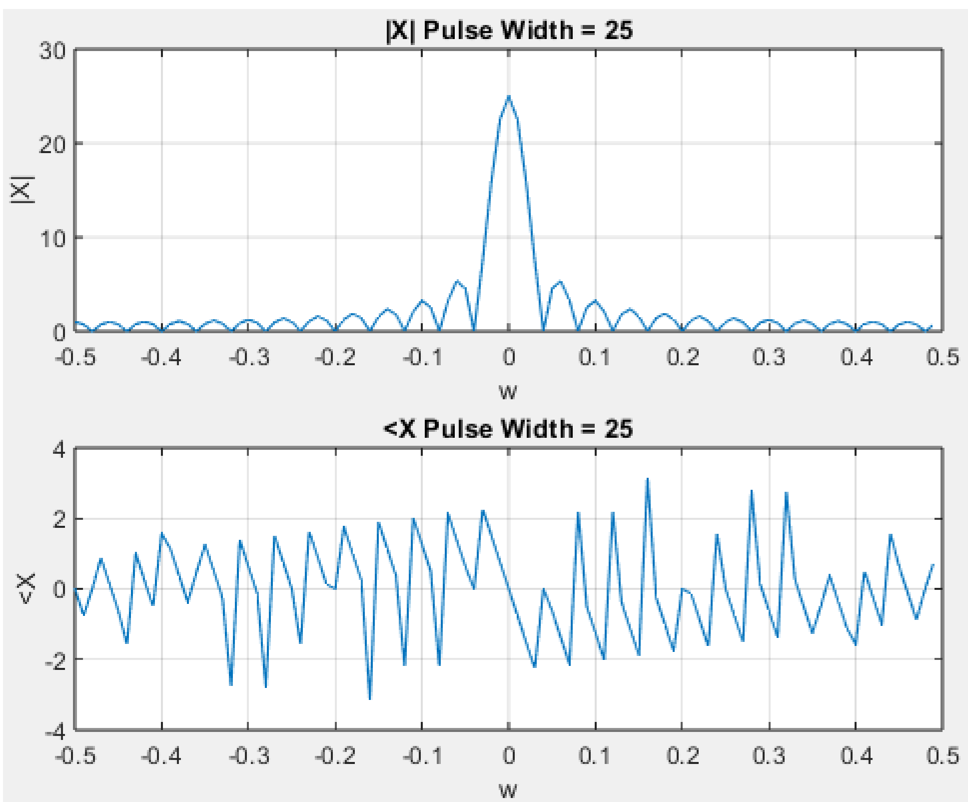
Change the width of the pulse x(t) to 5 while keeping the total length at N = 100. Compute the Fourier Transform of the narrower pulse and plot the corresponding magnitude- and phase-spectra. Repeat for a pulse width of 25. Explain the observed differences from the comparison of the frequency spectra generated by the three pulses with different pulse widths. Explain which property of the Fourier Transform you have demonstrated.

Code:

```
N = 100;
PWidth = 5;
t = [0:1:(N-1)];
x = [ones(1, PWidth), zeros(1, N-PWidth)];
f = [-(N/2):1:(N/2)-1]*(1/N);
Xf = fft(x);
subplot(2,1,1);
plot(f,fftshift(abs(Xf)));
grid on;
title('|X| Pulse Width = 5');
xlabel('w');ylabel('|X|');
subplot(2,1,2); plot(f,fftshift(angle(Xf)));
grid on; title('<X Pulse Width = 5');
xlabel('w'); ylabel('<X');
```




|X| Pulse Width = 10
|X|
w
<X Pulse Width = 10
<X
w

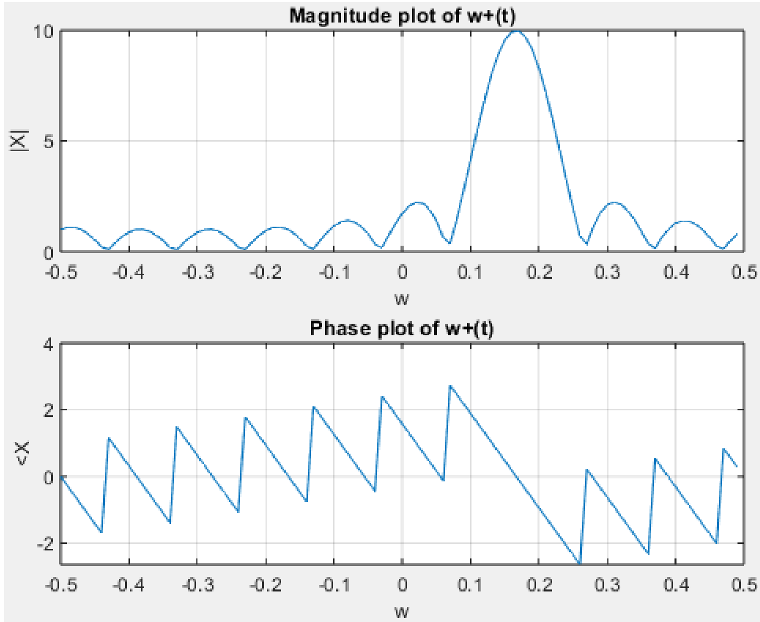

|X| Pulse Width = 25
|X|
w
<X Pulse Width = 25
<X
w

Problem A.6:

Let w+(t) = x(t)e j(π/3)t where x(t) is the original pulse of pulsewidth 10 shown in Figure (1). Using Matlab, compute and plot the magnitude and phase spectra of w+(t). Compare the frequency spectra result with those you generated in Problem A.3 and comment on the observed differences. Repeat for w−(t) = x(t)e −j(π/3)t and wc(t) = x(t) cos(π/3)t. Explain which property of the Fourier Transform you have demonstrated.

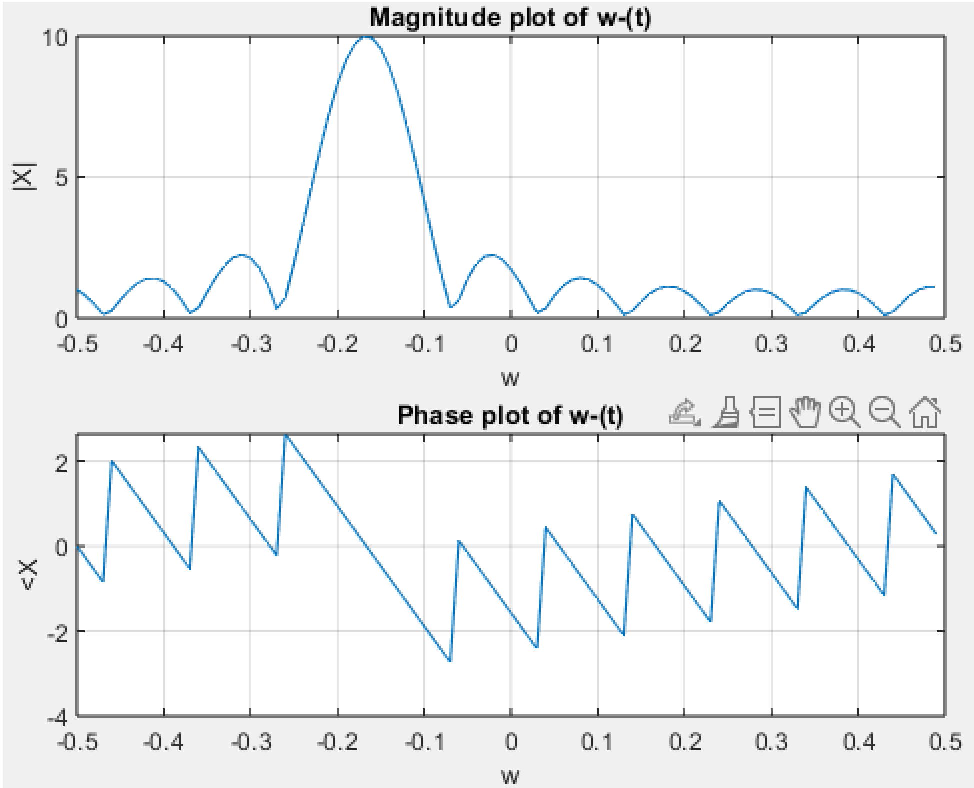
Code:

```
N = 100;
PWidth = 10;
t = [0:1:(N-1)];
x = [ones(1, PWidth), zeros(1, N-PWidth)];
f = [-(N/2):1:(N/2)-1]*(1/N);
wt = x.*exp(j*t*pi/3);
Xw = fft(wt);
subplot(2,1,1);
plot(f,fftshift(abs(Xw)));
grid on;
title('Magnitude plot of w+(t)');
xlabel('w');ylabel('|X|');
subplot(2,1,2); plot(f,fftshift(angle(Xw)));
grid on; title('Phase plot of w+(t)');
xlabel('w'); ylabel('<X');
```



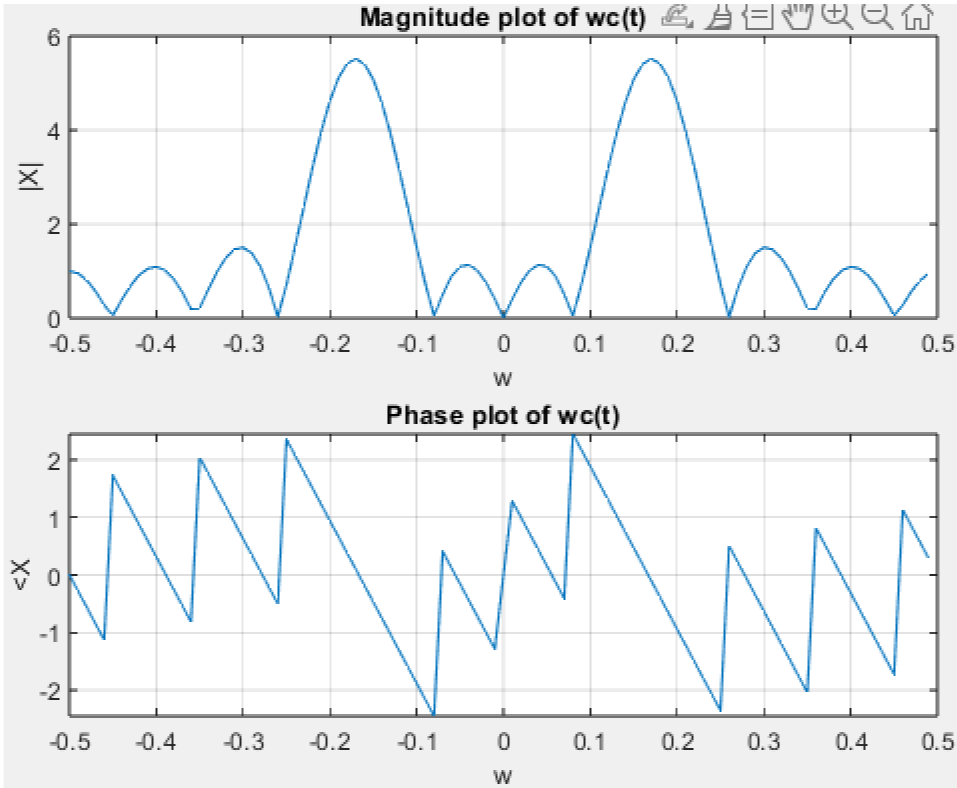
Code:

```
N = 100;
PWidth = 10;
t = [0:1:(N-1)];
x = [ones(1, PWidth), zeros(1, N-PWidth)];
f = [-(N/2):1:(N/2)-1]*(1/N);
wt = x.*exp(-j*t*pi/3);
Xw = fft(wt);
subplot(2,1,1);
plot(f,fftshift(abs(Xw)));
grid on;
title('Magnitude plot of w-(t)');
xlabel('w');ylabel('|X|');
subplot(2,1,2); plot(f,fftshift(angle(Xw)));
grid on; title('Phase plot of w-(t)');
xlabel('w'); ylabel('<X');
```



Code:

```
N = 100;
PWidth = 10;
t = [0:1:(N-1)];
x = [ones(1, PWidth), zeros(1, N-PWidth)];
f = [-(N/2):1:(N/2)-1]*(1/N);
wt = x.*cos(t*pi/3);
Xw = fft(wt);
subplot(2,1,1);
plot(f,fftshift(abs(Xw)));
grid on;
title('Magnitude plot of wc(t)');
xlabel('w');ylabel('|X|');
subplot(2,1,2); plot(f,fftshift(angle(Xw)));
grid on; title('Phase plot of wc(t)');
xlabel('w'); ylabel('<X');
```



The property demonstrated is Frequency Shift, which shows that a shift in the frequency domain can be caused by multiplying the time-domain function by an exponential function, since cosines can be represented in Euler form the above statement holds true.