Department of Electrical, Computer & Biomedical Engineering
Faculty of Engineering & Architectural Science

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Instructor	Dr. Alagan Anpalagan

Lab/Tutorial Report No.	**2**

Report Title	System Properties and Convolution

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Impulse Response

Problem A.1: Complete Lathi, Section 2.7-1 Script Files, page 213. Use Matlab command poly to generate the characteristic polynomial from the characteristic values specified by lambda.

Code:

```
R = [1e4, 1e4, 1e4];
C = [1e-9, 1e-6];
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
lambda = roots(A);
poly(lambda);
```

Result:

```
lambda =

   1.0e+03 *

  -0.1500 + 3.1587i
  -0.1500 - 3.1587i
```

```
>> poly(A)

ans =

   1.0e+09 *

    0.0000   -0.0100    3.0100   -3.0000
```

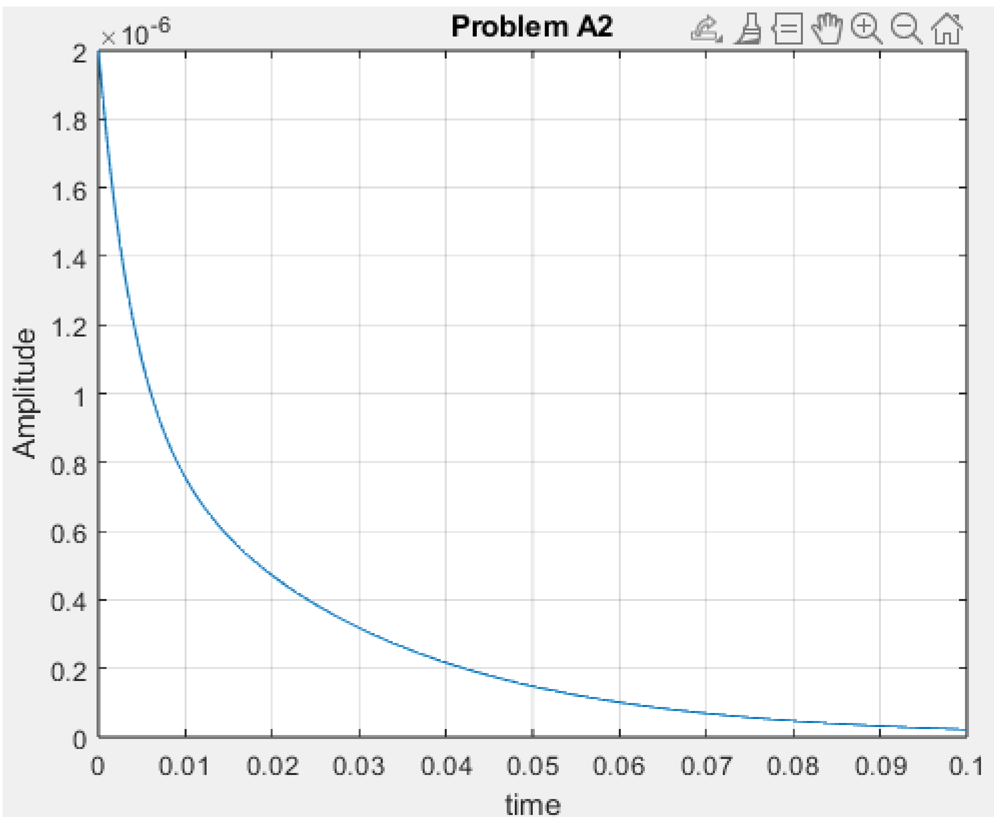
Problem A.2: Plot the impulse response of the system in Problem A.1 for t = [0:0.0005:0.1].

Code:

```
t = (0:0.0005:0.1);
R = [1e4, 1e4, 1e4];
C = [1e-6, 1e-6];
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
```

```
lambda = roots(A);
poly(lambda);
u = @(t) 1 * (t>=0);
h = @(t) (C(1).* exp(lambda(1).*t) + C(2).* exp(lambda(2).*t)).*(u(t));
figure;
plot (t, h(t));
xlabel('time'); ylabel('Amplitude'); title('Problem A2'); grid;
hold off;
```

Result:



<u>Problem A.3:</u> Complete Lathi, Section 2.7-2 Function M-Files, page 214

Code:

```
function [lambda] = CH2MP2(R,C)
% CH2MP2.m : Chapter 2, MATLAB Program 2
% Function M-file finds characteristic roots of op-amp circuit.
% INPUTS: R = length-3 vector of resistances
```

```
% C = length-2 vector of capacitances
% OUTPUTS: lambda = characteristic roots
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);
```

Result:

```
>> lambda = CH2MP2([1e4, 1e4, 1e4], [1e-9, 1e-6])

lambda =

   1.0e+03 *

  -0.1500 + 3.1587i
  -0.1500 - 3.1587i
```

Convolution:

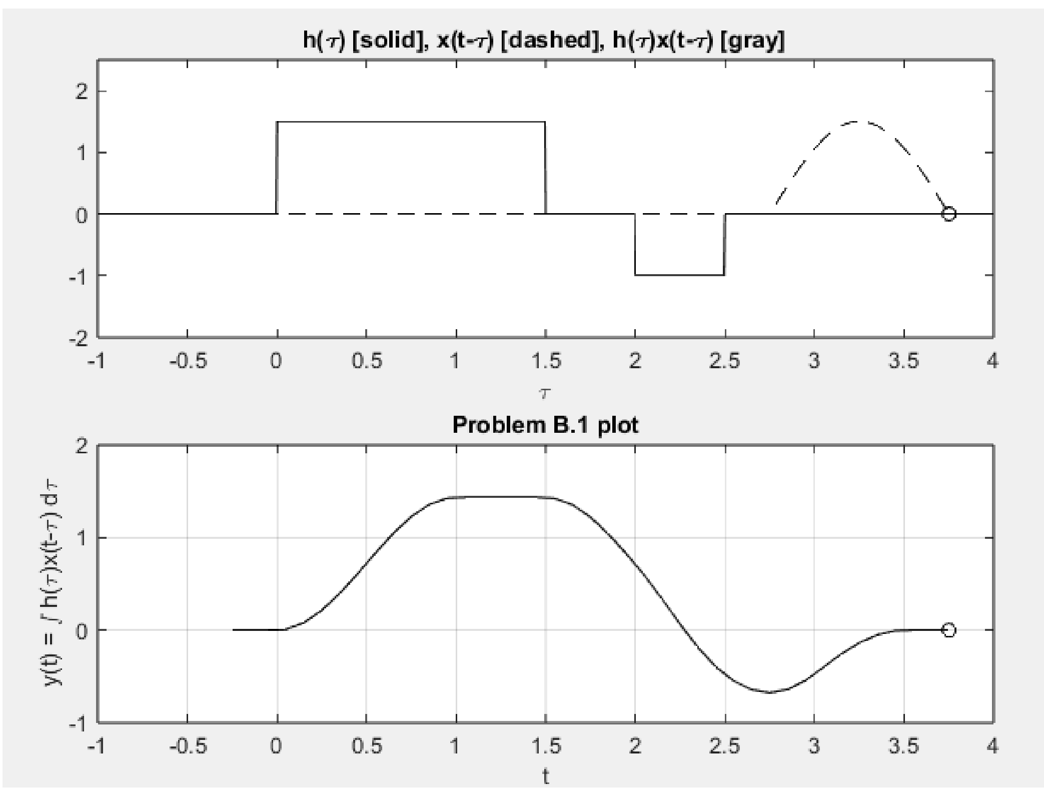
Problem B.1: Lathi, Section 2.7-4 Graphical Understanding of Convolution, page 217. Plot y(t) at step t = 2.25 as shown in Figure 2.28 on page 219. Use the Matlab command pause instead of drawnow to observe the steps of the convolution operation slowly.

Code:

```
figure(1); % Create figure window
u = @(t) 1.0*(t>=0);
x = @(t) 1.5*sin(pi*t).*(u(t)-u(t-1));
h = @(t) 1.5*(u(t)-u(t-1.5))-u(t-2)+u(t-2.5);
dtau = 0.005; tau = -1:dtau:4;
ti = 0; tvec = -.25:.1:3.75;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
   ti = ti+1; % Time index
   xh = x(t-tau).*h(tau); lxh = length(xh);
   y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
   subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
   axis([tau(1) tau(end) -2.0 2.5]);
```

```
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed],
h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); title ('Problem B.1 plot'); ylabel('y(t) = \int
h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    pause;
end
```

Result:

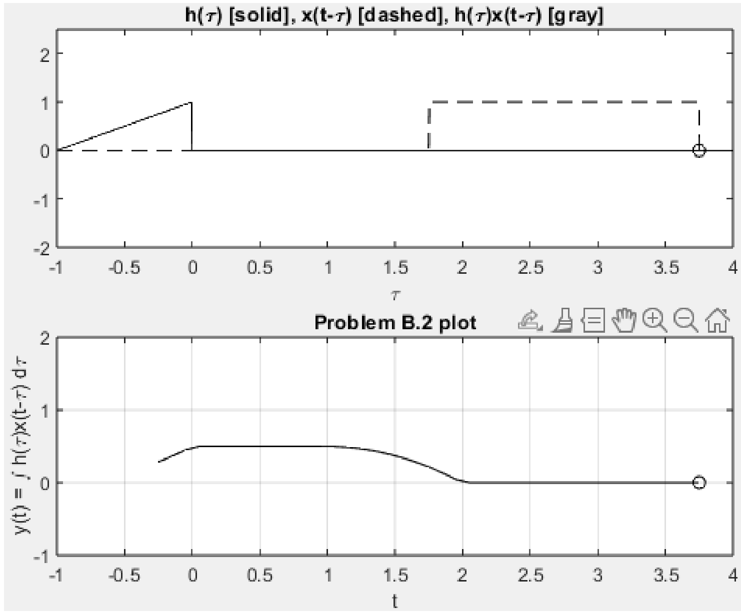


<u>Problem B.2:</u> Perform the convolution of the signal x(t) in Figure P2.4-28 (a) (page 229) with h(t) in Figure P2.4-30 (page 230). Plot all signals and results.

Code:

```
figure(2);
u = @(t) 1.0*(t>=0);
x = @(t) u(t)-u(t-2);
h = @(t) (t+1).*(u(t+1)-u(t));
dtau = 0.005; tau = -1:dtau:4;
ti = 0; tvec = -.25:.1:3.75;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
   ti = ti+1; % Time index
   xh = x(t-tau).*h(tau); lxh = length(xh);
   y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
   subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
   axis([tau(1) tau(end) -2.0 2.5]);
   patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
       [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
       [.8 .8 .8],'edgecolor','none');
   xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed],
h(\tau)x(t-\tau) [gray]');
   c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
   subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
   xlabel('t'); title ('Problem B.2 plot'); ylabel('y(t) = \int
h(\tau)x(t-\tau) d\tau');
   axis([tau(1) tau(end) -1.0 2.0]); grid;
   pause;
end
```

Result:



Problem B.3: Perform the convolution of the signal x1(t) and x2(t) in Figure P2.4-27(a), (b) and (h). Plot all signals and results.

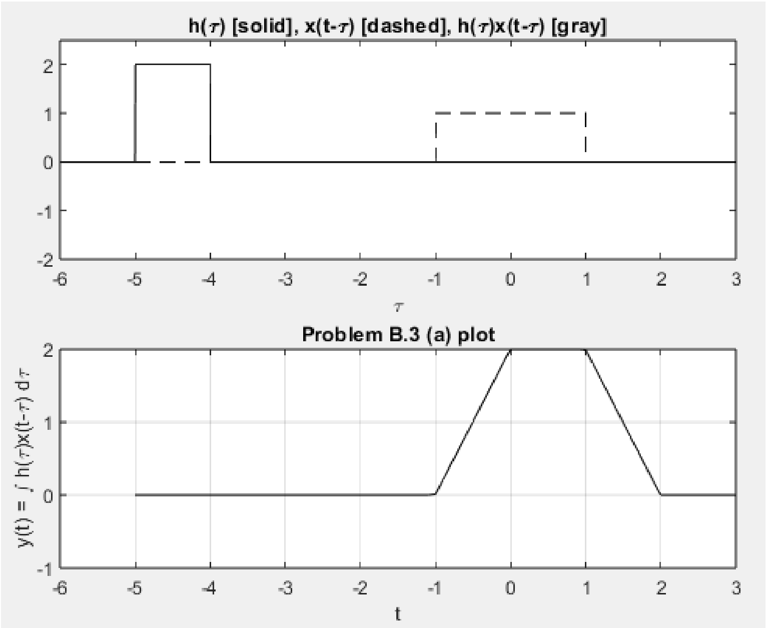
Part (A):

Code:

```
figure(3);
u = @(t) 1.0*(t>=0);
A = 1; B = 2; %Assumption
x = @(t) A*(u(t-4)-u(t-6));
h = @(t) B*(u(t+5)-u(t+4));
dtau = 0.005; tau = -6:dtau:3;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
```

```
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed],
h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); title('Problem B.3 (a) plot'); ylabel('y(t) = \int
h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Result:

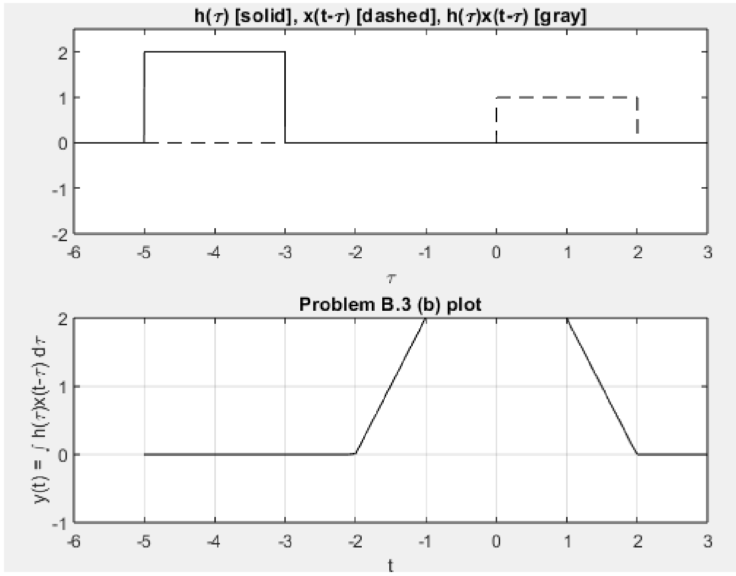


Part (B):

Code:

```
figure(4);
u = @(t) 1.0*(t>=0);
A = 1; B = 2; %Assumption
x = @(t) A*(u(t-3)-u(t-5));
h = @(t) B*(u(t+5)-u(t+3));
```

Result:

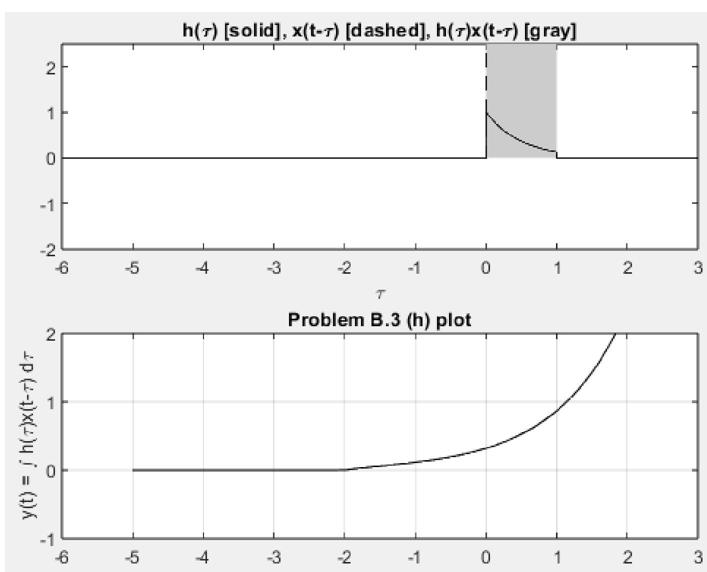


Part (H):

Code:

```
figure(5);
u = @(t) 1.0*(t>=0);
x = @(t) exp(t).*(u(t+2)-u(t-5));
h = @(t) exp(-2*t).*(u(t)-u(t-1));
dtau = 0.005; tau = -6:dtau:3;
```

Result:



System Behavior and Stability:

Problem C.1: Consider the LTI systems S1, S2, S3 and S4 represented by their respective unit impulse response functions given as follows:

$$h_1(t) = e^{t/5}u(t); \ (3)$$

$$h_2(t) = 4e^{-t/5}u(t); \ (4)$$

$$h_3(t) = 4e^{-t}u(t); \ (5)$$

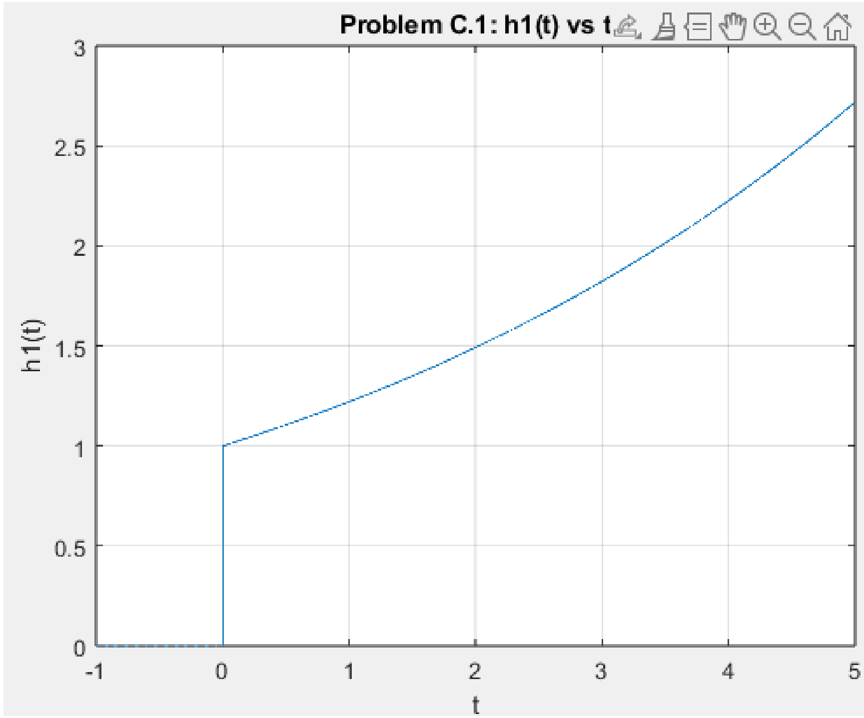
$$h_4(t) = 4(e^{-t/5} - e^{-t})u(t) \ ; \ (6)$$

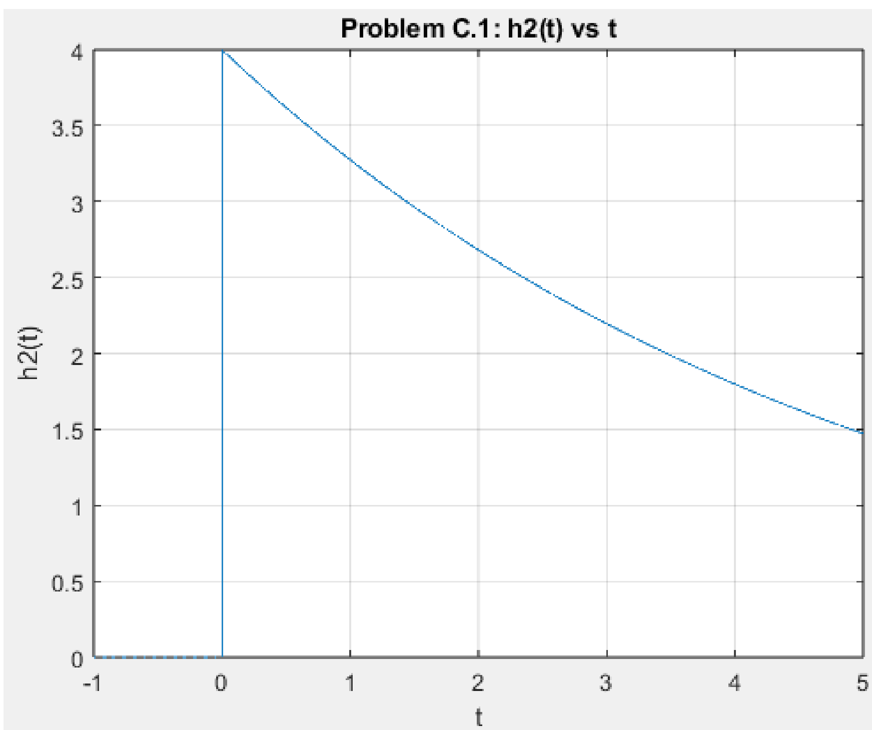
Plot each unit impulse response function for t = [-1:0.001:5].

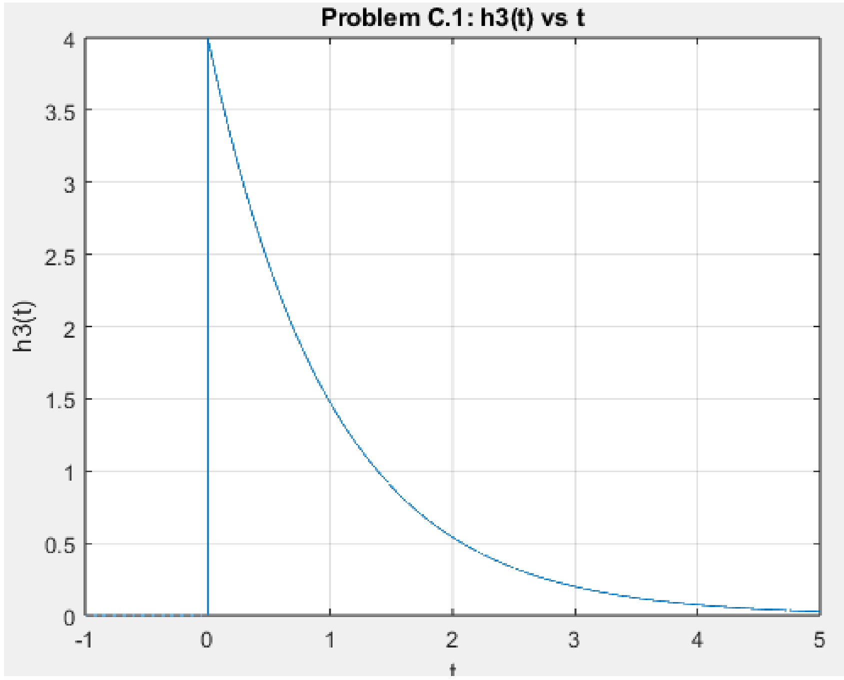
Code:

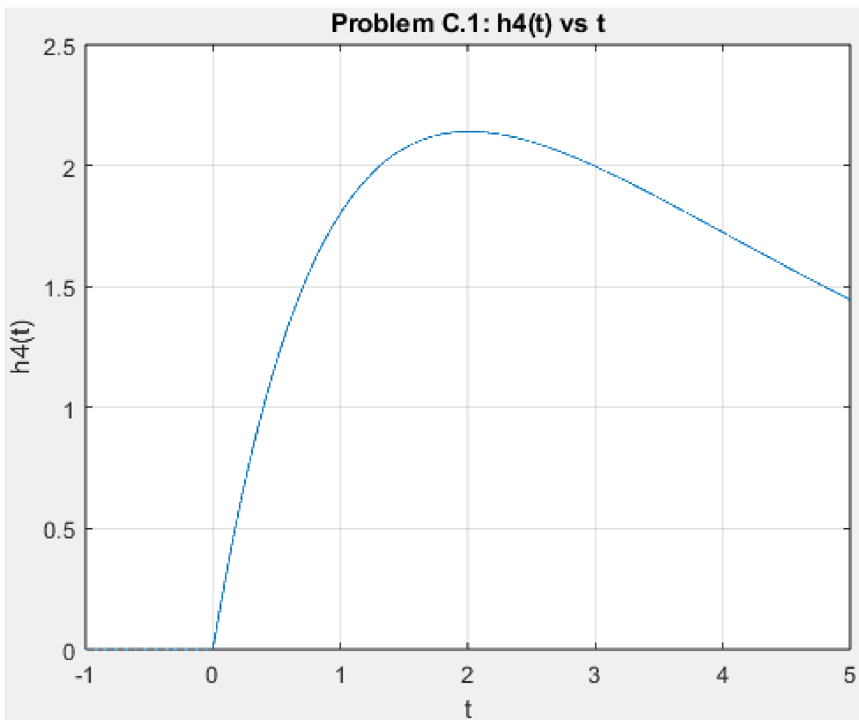
```
u = @(t) 1.0.* (t >= 0);
h1 = @(t) exp(t/5).*u(t);
h2 = @(t) 4*exp(-t/5).*u(t);
h3 = @(t) 4*exp(-t).*u(t);
h4 = @(t) 4*(exp(-t/5) - exp(-t)).*u(t);
t = -1:0.001:5;
plot(t, h1(t));grid;
xlabel('t'); ylabel('h1(t)');title('Problem C.1: h1(t) vs t');
plot(t, h2(t));grid;
xlabel('t'); ylabel('h2(t)');title('Problem C.1: h2(t) vs t');
plot(t, h3(t));grid;
xlabel('t'); ylabel('h3(t)');title('Problem C.1: h3(t) vs t');
plot(t, h4(t));grid;
xlabel('t'); ylabel('h4(t)');title('Problem C.1: h4(t) vs t');
```

Result:


Problem C.1: h1(t) vs t
h1(t)
t
3
2.5
2
1.5
1
0.5
0
-1
0
1
2
3
4
5


Problem C.1: h2(t) vs t
h2(t)
t
4
3.5
3
2.5
2
1.5
1
0.5
0
-1
0
1
2
3
4
5





Problem C.2: Determine the characteristic values (eigenvalues) of systems S1–S4.

$$S1: \lambda = 1/5$$

$$S2: \lambda = -1/5$$

$$S3: \lambda = -1$$

$$S4: \lambda 1 = -1/5, \lambda 2 = -1$$

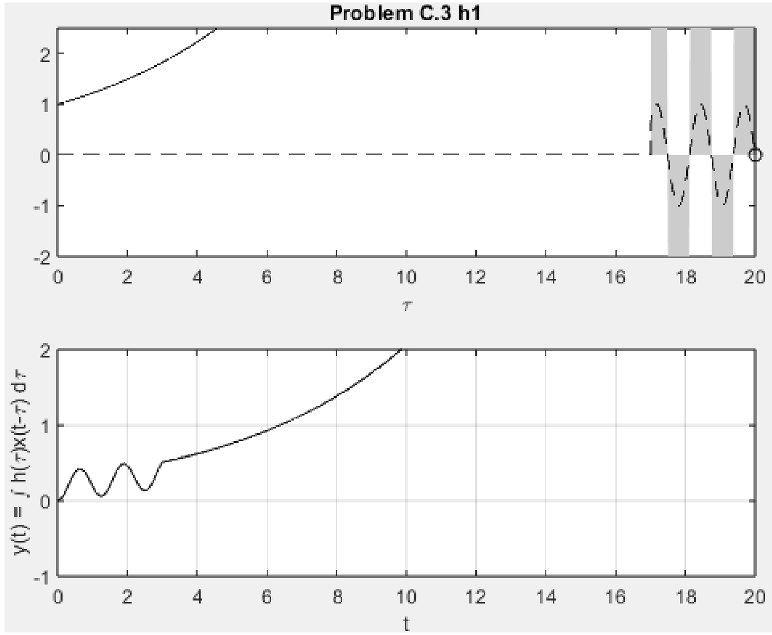
Problem C.3: Truncate the impulse response functions h1(t), . . . , h4(t) such that they are nonzero only for $0 \le t \le 20$. Determine the convolution of the truncated impulse response functions with the input signal $x(t) = [u(t) - u(t - 3)] \sin 5t$ using the M-file in Problem B.1 with the following changes tau = [0:dtau:20] and tvec = [0:0.1:20]. Plot the output of each system. State and explain your observations. Is there any relationship between the outputs of systems S2, S3, and S4? Explain.

Part h1:

Code:

```
figure(6);
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) exp(t/5).*(u(t)-u(t-20));
dtau = 0.005; tau = 0:dtau:20;
ti = 0; tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
   ti = ti+1; % Time index
   xh = x(t-tau).*h(tau); lxh = length(xh);
   y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
   subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
   axis([tau(1) tau(end) -2.0 2.5]);
   patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
       [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
       [.8 .8 .8],'edgecolor','none');
   xlabel('\tau'); title('Problem C.3 h1');
   c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
   subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
   xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
   axis([tau(1) tau(end) -1.0 2.0]); grid;
   drawnow;
end
```

Result:

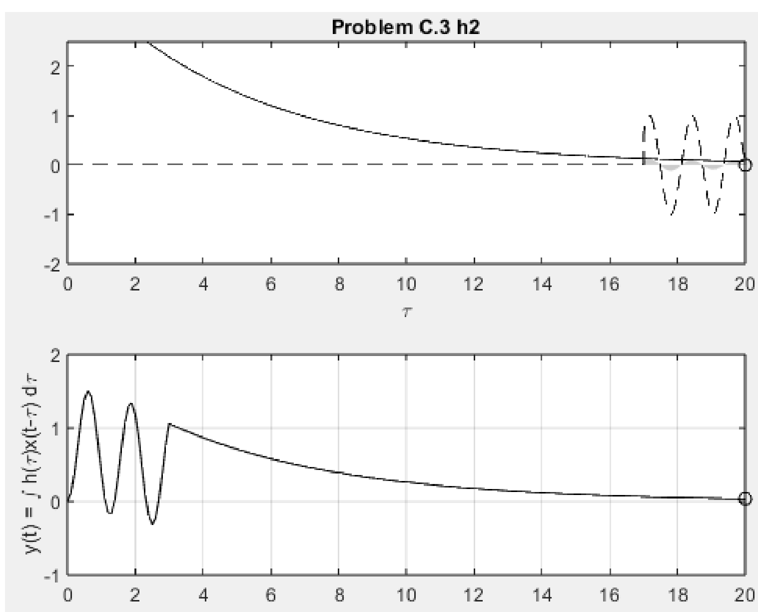


Part h2:

Code:

```
figure(7);
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4*exp(-t/5).*(u(t)-u(t-20));
```

Result:

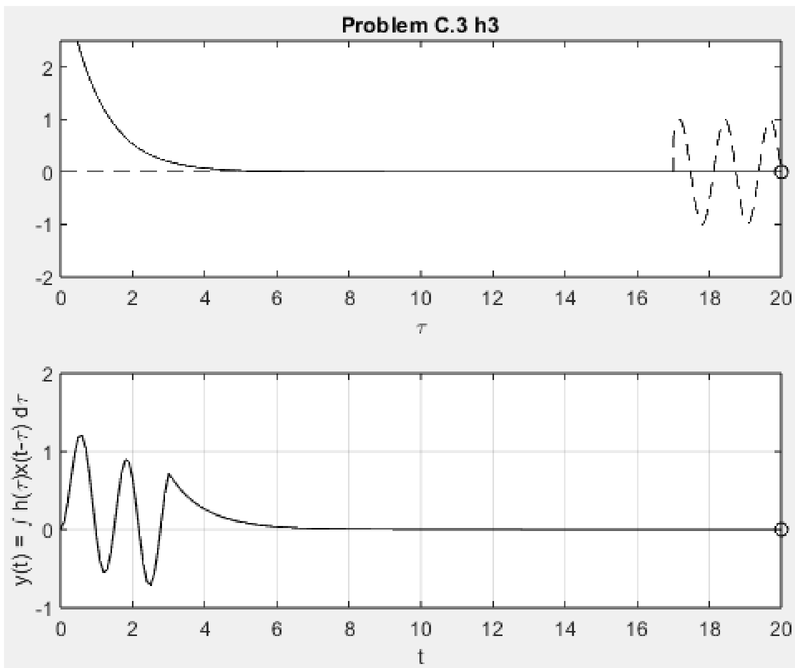


Part h3:

Code:

```
figure(8);
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4*exp(-t).*(u(t)-u(t-20));
```

Result:

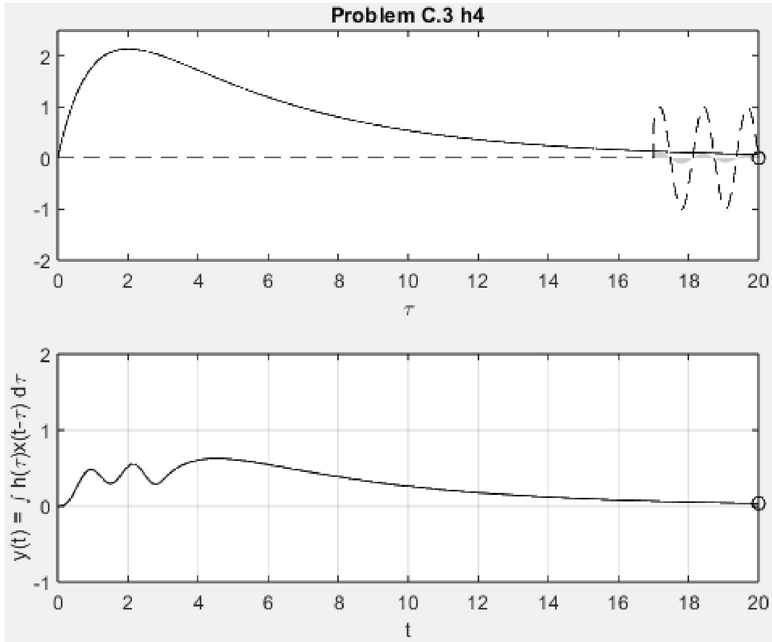


Part h4:

Code:

```
figure(9);
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4*(exp(-t/5)-exp(-t)).*(u(t)-u(t-20));
```

Result:



The plots for the S2, S3, and S4, all produced similar waveforms in the convolution with h(t). The waveforms produced all had a similar structure, initially starting off in some sort of a sine wave, and later on straightening out and infinitely approaching zero.

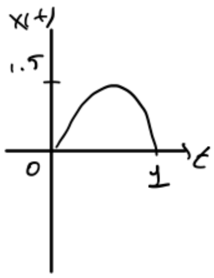
Discussion:

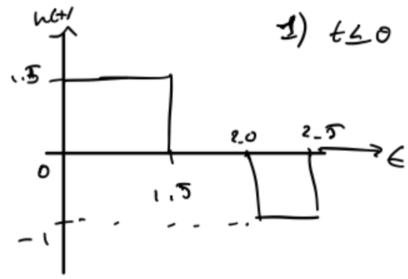
Problem D.1: Calculate the results of Problems B.1, B.2 and B.3 above by hand and compare to those obtained with your Matlab code.

Problem B.1

$x(t) = 1.5 \sin(\pi t)\{u(t) - u(t-1)\}$

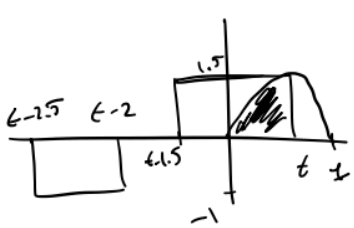
$h(t) = 1.5\,(u(t) - u(t-1.5)) - u(t-2) + u(t-2.5)$





1) $t \leq 0 \Rightarrow y(t) = 0$

2) $0 \leq t \leq 1$

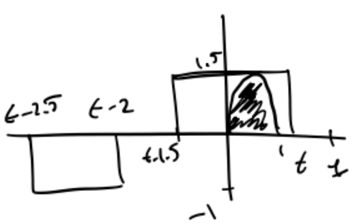


$$y(t) = \int_0^t 1.5^2 \sin \pi\tau \, d\tau$$

$$= -2.25 \frac{\cos(\pi\tau)}{\pi} \Big|_0^t$$

$$y(t) = \frac{2.25}{\pi}(1 - \cos \pi t)$$

3) $1 \leq t \leq 1.5$

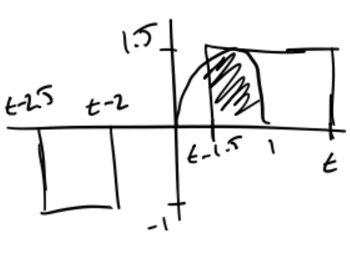


$$y(t) = \int_0^1 1.5^2 \sin(\pi\tau)\, d\tau$$

$$y(t) = -\frac{2.25}{\pi} \cos \pi\tau \Big|_0^1 = \frac{2.25}{\pi} + \frac{2.25}{\pi}$$

$$= \frac{4.5}{\pi} = 1.432$$

4) $1.5 \leq t \leq 2$

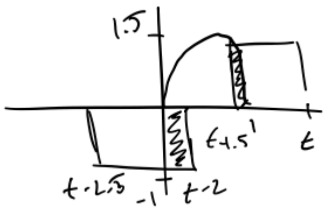


$$y(t) = \int_{t-1.5}^{1} 1.5^2 \sin \pi\tau \, d\tau$$

$$= -\frac{1.5^2}{\pi} (\cos(\pi\tau)) \Big|_{t-1.5}^{1}$$

$$y(t) = \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right)$$

5) $2 \le t \le 2.5$

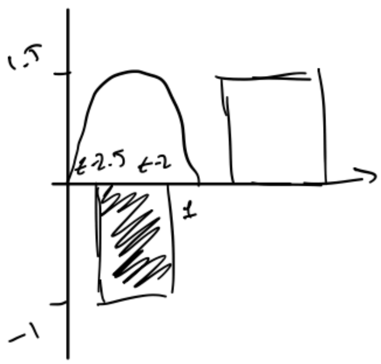


$$y(t) = \int_0^{t-2} -1.5 \sin \pi\tau \, d\tau + \int_{t-1.5}^{1} 1.5^2 \sin \pi\tau \, d\tau$$

$$= \frac{1.5}{\pi} \cos \pi\tau \Big|_0^{t-2} + \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right)$$

$$y(t) = \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - 1\right) + \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right)$$

6) $2.5 \le t \le 3$

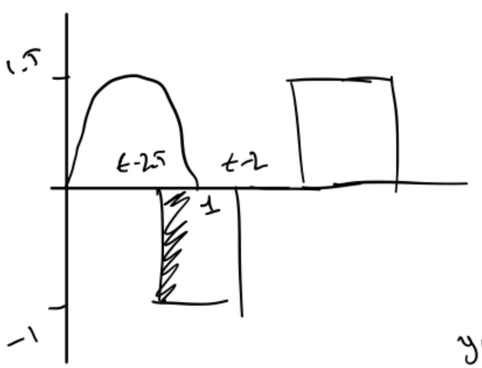


$$y(t) = \int_{t-2.5}^{t-2} -1.5 \sin(\pi\tau)\, d\tau$$

$$= \frac{1.5}{\pi} \cos(\pi\tau) \Big|_{t-2.5}^{t-2}$$

$$= \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right)$$

7) $3 \le t \le 3.5$

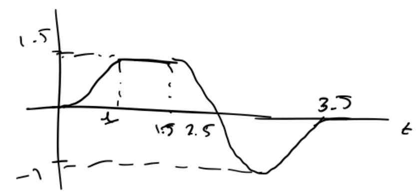


$$y(t) = \int_{t-2.5}^{1} -1.5 \sin \pi\tau \, d\tau$$

$$= \frac{1.5}{\pi} \cos \pi\tau \Big|_{t-2.5}^{1}$$

$$y(t) = -\frac{1.5}{\pi}\left(\cos(\pi(t-2.5)) + 1\right)$$

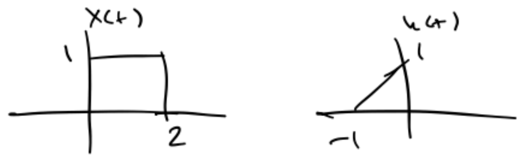
$$\left.\begin{array}{ll}
0 & t \leq 0 \\
\frac{2.25}{\pi}(1-\cos(\pi t)) & 0 \leq t \leq 1 \\
\frac{4.5}{\pi} & 1 \leq t \leq 1.5 \\
\frac{2.25}{\pi}(\cos(\pi(t-1.5))+1) & 1.5 \leq t \leq 2 \\
\frac{1.5}{\pi}(\cos(\pi(t-2))-1)+\frac{2.25}{\pi}(\cos\pi(t-1.5)+1) & 2 \leq t \leq 2.5 \\
\frac{1.5}{\pi}(\cos(\pi(t-2))-\cos(\pi(t-2.5))) & 2.5 \leq t \leq 3 \\
-\frac{1.5}{\pi}(\cos\pi(t-2.5)+1) & 3 \leq t \leq 3.5 \\
0 & t \geq 3.5
\end{array}\right\} y(t) \Rightarrow$$



Problem B.2:

$x(t) = u(t) - u(t-2)$

$h(t) = (t+1)(u(t+1) - u(t))$



1) $t \leq -1 \Rightarrow y(t) = 0$

2) $-1 \leq t \leq 0$

$$y(t) = \int_{-1}^{t} (t+1)\,dt = \int_{-1}^{t} t\,dt + \int_{-1}^{t} 1\,dt$$

$$= \frac{t^2}{2} - \frac{1}{2} + t - 1$$

$$y(t) = \frac{t^2}{2} + t + \frac{1}{2}$$

3) $0 \leq t \leq 1$

$$y(t) = \int_{-1}^{0} t + \int_{-1}^{0} 1\,dt$$

$$= \frac{t^2}{2} + t \,\Big|$$

$$y(t) = -\frac{1}{2}$$

4) $1 \leq t \leq 2$

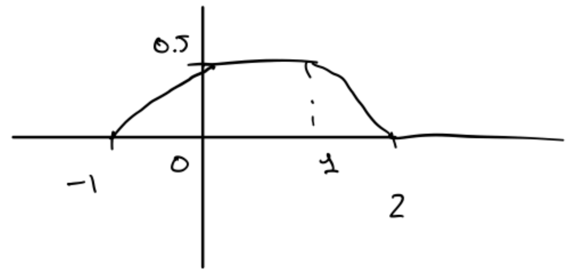
$$y(t) = \int_{t-2}^{0} t + \int_{t-2}^{0} 1\,dt$$

$$= \frac{t^2}{2} + t \,\Big|_{t-2}^{0}$$

$$y(t) = -\left(\frac{(t-2)^2}{2} + (t-2)\right)$$

5) $t \geq 2 \Rightarrow y(t) = 0$

$$
y(t) = \begin{cases} 0 & t \leq -1 \\ \frac{t^2}{2} + t + \frac{1}{2} & -1 \leq t \leq 0 \\ \frac{1}{2} & 0 \leq t \leq 1 \\ -\left(\frac{(t-2)^2}{2} + (t-2)\right) & 1 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases} \Rightarrow
$$



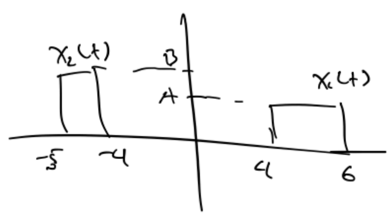
Problem B.3

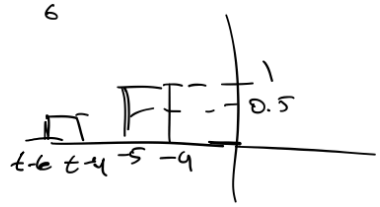
Part A:

Let $A = 0.5$, $B = 1$

$x_1(t) = \frac{1}{2}\left(u(t-4) - u(t-6)\right)$

$x_2(t) = u(t+5) - u(t+4)$





1) $t \leq -1$

$y(t) = 0$

2) $-1 \leq t \leq 0$

$$y(t) = \int_{-5}^{t-4} \frac{1}{2}\, d\tau = \frac{1}{2}t + \frac{1}{2}$$

3) $0 \leq t \leq 1$

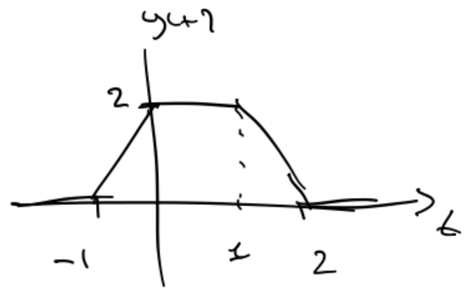
$$y(t) = \int_{-5}^{-4} \frac{1}{2}\, d\tau = \frac{1}{2}$$

4) $1 \leq t \leq 2$

$$y(t) = \int_{t-6}^{-4} \frac{1}{2}\, d\tau = -\frac{1}{2}t + 1$$

5) $t \geq 2$ $y(t) = 0$

$$\left.\begin{array}{ll} 0 & t \le 1 \\ \frac{1}{2}t + \frac{1}{2} & -1 \le t \le 0 \\ \frac{1}{2} & 0 \le t \le 1 \\ -\frac{1}{2}t + 1 & 1 \le t \le 2 \\ 0 & t \geq 2 \end{array}\right\} y(t) \Rightarrow$$

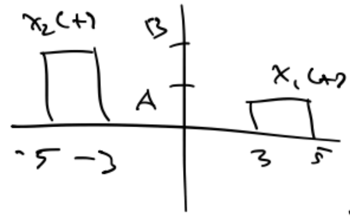


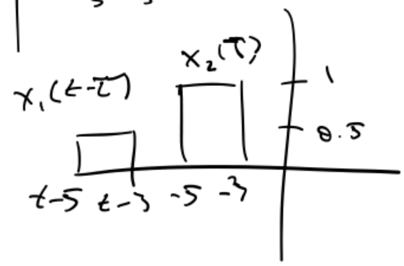
Part B

Let $A = 0.5$, $B = 1$

$x_1(t) = \frac{1}{2}(u(t-3) - u(t-5))$

$x_2(t) = u(t+5) - u(t+3)$





1) $t \le -2$

$y(t) = 0$

2) $-2 \le t \le 0$

$$y(t) = \int_{-5}^{t-3} \frac{1}{2}\, d\tau = \frac{1}{2}t + 1$$

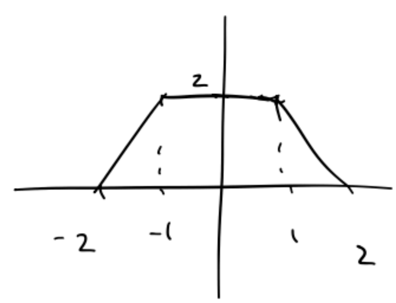
3) $0 \le t \le 2$

$$y(t) = \int_{t-5}^{-3} \frac{1}{2}\, d\tau = -\frac{1}{2}t + 1$$

4) $t \ge 2$

$y(t) = 0$

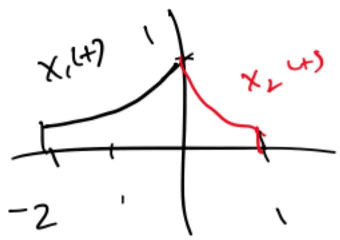
$$y(t) = \begin{cases} 0 & t \le -2 \\ \frac{1}{2}t + 1 & -2 \le t \le 0 \\ -\frac{1}{2}t + 1 & 0 \le t \le 2 \\ 0 & t \ge 2 \end{cases} \Rightarrow$$

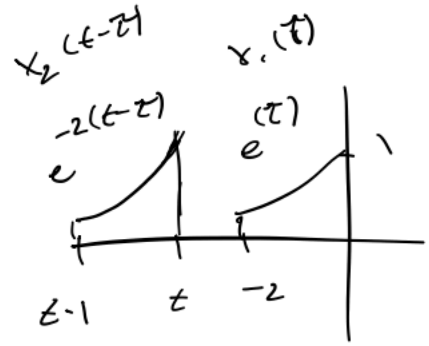


Part H

$$x_1(t) = e^{t}\,(u(t+2) - u(t+1))$$

$$x_2(t) = e^{-2t}\,(u(t) - u(t-1))$$





1) $t \le -2$

$y(t) = 0$

2) $-2 \le t \le -1$

$$y(t) = \int_{-2}^{t} e^{\tau} e^{-2(t-\tau)}\, d\tau = \frac{1}{3}\left(e^{t} - e^{-2(t+3)}\right)$$

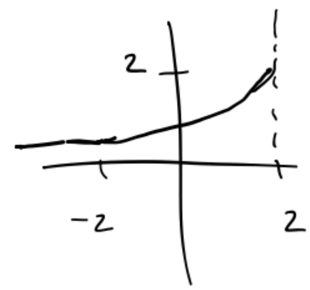
3) $-1 \le t \le 0$

$$y(t) = \int_{t-1}^{t} e^{\tau} e^{-2(t-\tau)}\, d\tau = \frac{1}{3}\left(e^{t} - e^{t-3}\right)$$

4) $0 \le t \le 1$

$$y(t) = \int_{t-1}^{0} e^{\tau} e^{-2(t-\tau)}\, d\tau = \frac{1}{3}\left(e^{-2t} - e^{t-3}\right)$$

$$\left.\begin{array}{ll} 0 & t \le -2 \\ \frac{1}{3}\left(e^{t} - e^{-2(t+3)}\right) & -2 \le t \le -1 \\ \frac{1}{3}\left(e^{t} - e^{t-3}\right) & -1 \le t \le 0 \\ \frac{1}{3}\left(e^{-2t} - e^{t-3}\right) & 0 \le t \le 1 \\ 0 & t \ge 1 \end{array}\right\} y(t) \Rightarrow$$



The results obtained by solving by hand and solving through MATLAB were the same.

<u>Problem D.2:</u> What can you say about the width/duration of the signal resulting from the convolution of two signals?

The width/duration of the convoluted signal is equal to the sum of the widths of each signal x(t) and h(t).