Homework 2

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October 10, 2019

1 Let $w_T(y_T) = \sum_{y' \in Y^T} exp(\theta^T f(x, y'))$, where Y^T is the set of all possible POS sqeunces with length T. Given $\mathbf{f(x,y)} = \sum_{i=1}^{T} \mathbf{f}_i(x_i, y_i, y_{i-1}) + f_{T+!}(y_{T+1}, y_T)$, We can rewrite the above equation as following:

$$
\begin{aligned}
w_T(y_T) &= \sum_{y' \in Y^T} exp(\theta^T f(x, y')) \\
&= \sum_{y' \in Y^{T-1}} exp(\theta^T \sum_{i=1}^{T} [\mathbf{f}_i(x_i, y'_i, y'_{i-1}) + f_{T+!}(y_{T+1}, y_T)]) \\
&= \sum_{y' \in Y^{T-1}} exp(\theta^T) \prod_{i=1}^{T} exp(\mathbf{f}_i(x_i, y'_i, y'_{i-1}) + f_{T+!}(y_{T+1}, y_T)) \\
&= \sum_{y_{T-1} \in Y} exp(\theta^T) exp(f_T(x_T, y_T, y_{T-1}) + f_{T+!}(y_{T+1}, y_T)) \cdot \\
&\sum_{y' \in Y^{T-2}} exp(\theta^T) \prod_{i=1}^{T-1} exp(\mathbf{f}_i(x_i, y'_i, y'_{i-1}) + f_{T+!}(y_{T+1}, y_T)) \\
&= \sum_{y_{T-1} \in Y} exp(\theta^T) exp(f_T(x_T, y_T, y_{T-1}) + f_{T+!}(y_{T+1}, y_T)) \; w_{T-1}(y_{T-1})
\end{aligned}
$$

With the above recurrent relationship, we can design the following algorithm:

for $n \in \{0 : K\}$:
  $w_1(y_n) = exp(\theta^T) exp(f_1(x_1, y_n, \square) + f_{T+!}(y_{T+1}, y_T))$
for $m \in \{2 : T\}$:
  for $m \in \{0 : K\}$:
    $w_m(y_m) = \sum_{y_{m-1} \in Y} exp(\theta^T) exp(f_1(x_m, y_n, y_{n-1}) + f_{T+!}(y_{T+1}, y_T))$
return $\sum_{y_T \in Y} w_T(y_T)$

The time complexity of the algorithm is $T * K^2$ because the time complexity of calculating the first for loop is K. This is because we iterate the loop K times, within each loop we have to calculate the sum, which has the time complexity of K. In the second loop, we iterate the outer loop $T-1$ times and iterate the inner loop K times. Within each loop, we have to calculate the sum, which has the time complexity of K. Thus, the total time complexity is $K^2 + (T-1) * K^2 = T * K^2$.

2 (d) With $\alpha = 1$ and $\beta = 1$, I have the dev accuracy as 0.947451.
(e) With $\alpha = 100$ and $\beta = 0.1$, I have the best accuracy (among the parameters I tried so far) as 0.956281.
(f) With $\alpha = 100$ and $\beta = 0.1$ obtained in (e), I have the test accuracy as 0.958287.