PMT 1

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1) (i) Assume P. then P holds, so P implies P. by 'showing implies'

(ii) Assume P and Q. Then P holds, so P or Q holds, so '(P and Q) implies (P or Q)' by 'using and' etc...

(iii) Assume 'for every $x \in A$, we have $P(x)$', then
'not (there exists $x \in A$, such that not $(P(x))$'
Then, assume 'there exists $x \in A$, such that not $P(x)$'.
Let $a \in A$, such that not $P(a)$ holds.
Contradiction with initial assumption.
Reject 2nd assumption and conclude 'not (there exists $x \in A$, such that not $P(x)$)'. Yes!

(iv) Assume 'P implies Q'
By cases...
1) Assume P. then Q holds by assumption.
then '(not P) or Q' holds
2) Assume not P. then '(not P) or Q' holds

Both cases lead to '(not P) or Q', so '(P implies Q) implies ((not P) or Q)'
Yes!

2) 1. $\{x\} \subseteq \{x\}$
'for all $a \in \{x\}$, $a \in \{x\}$' holds, so true.

2. $\{x\} \in \{x\}$
false, as $\{x\}$ is not an element in $\{x\}$

3. $\{x\} \in \{x, \{x\}\}$
true, as $\{x\}$ is an element of $\{x, \{x\}\}$.

4. $\{x\} \subseteq \{x, \{x\}\}$
true, as for $a \in \{x\}, a \in \{x, \{x\}\}$

5. $\emptyset \in \emptyset$
false, as an empty set contains no elements.

6. $\emptyset \subseteq \emptyset$
true, as there are no elements in $\emptyset$

7. $\emptyset \subseteq \{\emptyset\}$
true, as there are no elements in $\emptyset$

3) $A \cap B = \{\{a\}\}$
$A \cup B = \{\{a\}, \{b\}, a, b\}$
$\wp B = \{\{a\}, \{b\}, \{\{a\}\}, \emptyset, \{a, b\}, \{a, \{a\}\}, \{b, \{a\}\}, \{a, b, \{a\}\}\}$
$A \cap \wp B = \{\{a\}, \{b\}\}$ A
$A \times B = \{\langle \{a\}, a \rangle, \langle \{a\}, b \rangle, \langle \{a\} \{a\} \rangle, \langle \{b\}, a \rangle, \langle \{b\}, b \rangle, \langle \{b\}, \{a\} \rangle\}$
$(A \times B) \cap (B \times A) = \{\langle \{a\}, \{a\} \rangle\}$
$A \Delta B = \{a, b, \{b\}\}$

4) (i) $A \cup (B \cap C) = (A \cap B) \cup C$

False. $A = \{0,1\}$ $B = \{1,2\}$ $C = \{2,3\}$

great! just be more specific, explain counter example fully.

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\subseteq$) take $x \in A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$.

then $x \in B$ or $x \in C$.

By cases.

1. $x \in B$, so $x \in A \cap B$
2. $x \in C$, so $x \in A \cap C$

so, $x \in (A \cap B) \cup (A \cap C)$

great!

2) take $x \in (A \cap B) \cup (A \cap C)$

then $x \in A \cap B$ or $x \in A \cap C$

by cases.

1. $x \in A \cap B$, then '$x \in A$ and $x \in B$'
   so '$x \in B$ or $x \in C$ and $x \in A$'
2. $x \in A \cap C$, then '$x \in A$ and $x \in C$'
   so '$x \in B$ or $x \in C$ and $x \in A$'

so, we conclude $x \in A \cap (B \cup C)$.

great understanding!

(iii) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

False. $A = \{0\}$ $B = \{1\}$ $C = \{2\}$

(iv) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

$\subseteq$) take $x \in A \setminus (B \cup C)$, then $x \in A$ and $x \notin B \cup C$

then $x \in A$ and $x \notin B$ and $x \notin C$.

then $x \in A$ and $x \notin B$, so $x \in A \setminus B$.

then $x \in A$ and $x \notin C$, so $x \in A \setminus C$.

so $x \in A \setminus B$ and $x \in A \setminus C$.

2) take $x \in (A \setminus B) \cap (A \setminus C)$, then

$x \in A \setminus B$ and $x \in A \setminus C$

then $x \in A$ and $x \notin B$ and $x \in A$ and $x \notin C$

so $x \in A$ and $x \notin B$ and $x \notin C$

so $x \in A$ and $x \notin B \cup C$

so $x \in A \setminus (B \cup C)$.

(v) $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$

False. $A = \{0,1\}$, $B = \{1\}$, $C = \{2\}$

(vi) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

$\subseteq$) take $x \in A \cap (B \setminus C)$, then $x \in A$ and $x \in B \setminus C$.
then $x \in A$ and $x \in B$ and $x \notin C$
so, $x \in A \cap B$
if $x \notin C$, ~~then $x \notin C$ or $x \notin A$~~
so $x \notin A \cap C$.

so $x \in (A \cap B) \setminus (A \cap C)$

$\supseteq$) take $x \in (A \cap B) \setminus (A \cap C)$, then $x \in A \cap B$
and $x \notin A \cap C$.
then $x \in A$ and $x \in B$ and ($x \notin A$ or $x \notin C$). demorgans

by cases...
1. $x \notin A$. ↯ so $x \in A$
2. $x \notin C$, so $x \in A$ and $x \in B$ and $x \notin C$.

so, $x \in A \cap (B \setminus C)$

(vii) $A \Delta (B \cap C) = (A \Delta B) \cap (A \Delta C)$
False. $A = \{0,1\}$ $B = \{1,2\}$ $C = \{2,3\}$

(viii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

$\subseteq$) take $x \in A \cap (B \Delta C)$, then $x \in A$ and $x \in B \Delta C$
then $x \in A$ and $x \in (B \setminus C) \cup (C \setminus B)$
then $x \in A$ and $x \in B \setminus C$ or $x \in C \setminus B$
by cases...
1. $x \in B \setminus C$ then $x \in B$ and $x \notin C$
so $x \in A \cap B$ and $x \notin A \cap C$
so $x \in (A \cap B) \setminus (A \cap C)$
2. $x \in C \setminus B$ then $x \in C$ and $x \notin B$
then $x \in A \cap C$ and $x \notin A \cap B$
so $x \in (A \cap C) \setminus (A \cap B)$

since $x \in (A \cap B) \setminus (A \cap C) \cup (A \cap C) \setminus (A \cap B)$
then $x \in (A \cap B) \Delta (A \cap C)$

2) take $x \in (A \cap B) \Delta (A \cap C)$

then $x \in (A \cap B) \setminus (A \cap C) \cup (A \cap C) \setminus (A \cap B)$

then $x \in A \cap B$ and $x \notin A \cap C$ or $x \in A \cap C$ and $x \notin A \cap B$

by cases,

1. $x \in A \cap B$ and $x \notin A \cap C$, then $x \in A$ and $x \in B$ and $x \notin A$ or $x \notin C$.

   If $x \notin A$, ↯

   If $x \notin C$, $x \in A$ and $x \in B$ and $x \notin C$

   so $x \in A$ and $x \in B \setminus C$

2. $x \in A \cap C$ and $x \notin A \cap B$, then $x \in A$ and $x \in C$ and $x \notin A$ or $x \notin B$.

   If $x \notin A$, ↯

   If $x \notin B$, $x \in A$ and $x \notin B$ and $x \in C$

   so, $x \in A$ and $x \in C \setminus B$

so $x \in A$ and $x \in C \setminus B$ or $x \in B \setminus C$

so $x \in A$ and $x \in B \Delta C$

so $x \in A \cap (B \Delta C)$

perfect! MS uses other parts of tutorial to prove this... can you see how?

Great work!

Really good Q1 proofs – try use 'using and', 'showing implies' labels to make proof clearer

Q4 really good! You have a great understanding. A little more indepth for counter examples though