Discrete Mathematics, Logic, and Reasoning - PMT 3

Relations

Hand in October 27, 2025, 19pm - Electronic

Question 9 (Ex. 3.16) For the following problems, either give a proof when the statement is correct, or give a clear counter example when it is not.

i) Is $R \circ R^{-1}$ reflexive for all binary relations and sets?

ii) Is a reflexive relation always symmetric?

iii) Is a symmetric relation always reflexive?

iv) **PMT** Is the union of two symmetric relations always symmetric?

v) **PMT** Is the complement of a transitive relation a transitive relation?

vi) Is the complement of a non-symmetric relation a symmetric relation?

Question 10 (Ex. 3.17) **PMT** Let R and S be binary relations on A.

i) Give specific relations R and S, such that R and S are transitive but $R \cup S$ is not.

ii) Prove that if R and S are transitive then $R \cap S$ is transitive; how about the converse?

iii) Give specific relations R and S, such that R and S are transitive but $R \circ S$ is not.

Question 11 (Ex. 3.18) **PMT** A relation R on a set A is called *circular* when it satisfies:

$$\forall a,c \in A \left(\exists b \left(a \, R \, b \wedge b \, R \, c \right) \Rightarrow c \, R \, a \right)$$

Prove that a relation R is an equivalence relation on A if and only if it is reflexive and circular on A.

Question 12 (Ex. 3.19) *i*) Suppose $R = \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 3,4\rangle, \langle 4,1\rangle\}$. Give R^+. Illustrate both R and its transitive closure R^+ as directed graphs.

ii) Let R be the binary relation on the natural numbers defined by $a \, R \, b \triangleq b = 2x$. Give R^+.

Question 13 (Ex. 3.20) **PMT** Find a binary relation R on a set A with $|A| = n$ such that $R^+ \neq R \cup \ldots \cup R^{n-1}$.