

# Discrete Mathematics, Logic, and Reasoning - PMT 1

## Mathematical Reasoning and Sets

Hand in October 13, 2025, 19pm - Electronic

**Question 1** **PMT** Prove the following statements.

- i) ' $P$  implies  $P$ ';
- ii) ' $(P \text{ and } Q) \text{ implies } (P \text{ or } Q)$ ';
- iii) ' $(\text{for every } x \in A \text{ we have } P(x)) \text{ implies } (\text{not } (\text{there exists } y \in A \text{ such that } (\text{not } P(y))))$ ';
- iv) ' $(P \text{ implies } Q) \text{ implies } ((\text{not } P) \text{ or } Q)$ ';

**Question 2** (EX. 2.6) Determine whether the following statements are true or false; explain your answer. 1)  $\{x\} \subseteq \{x\}$ ; 2)  $\{x\} \in \{x\}$ ; 3)  $\{x\} \in \{x, \{x\}\}$ ; 4)  $\{x\} \subseteq \{x, \{x\}\}$ ; 5)  $\emptyset \in \emptyset$ ; 6)  $\emptyset \subseteq \emptyset$ ; 7)  $\emptyset \subseteq \{\emptyset\}$ .

**Question 3** (EX. 2.5) **PMT** Let  $A = \{\{a\}, \{b\}\}$  and  $B = \{a, b, \{a\}\}$ . Determine  $A \cap B$ ,  $A \cup B$ ,  $\wp B$ ,  $A \cap \wp B$ ,  $A \times B$ ,  $(A \times B) \cap (B \times A)$  and  $A \triangle B$ .

**Question 4** (EX. 2.9) Let  $A$ ,  $B$  and  $C$  be any sets. Decide whether each of the following statements is true or false. You can use earlier results, or de Morgan's law ('not ( $A$  or  $B$ ) if and only if ((not  $A$ ) and (not  $B$ ))'), or ' $(A \text{ and } (\text{not } A \text{ or } B)) \text{ implies } B$ ' whenever convenient. Justify your answer either with a formal proof or a counter-example.

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|---|---|
| i) <b>PMT</b> $A \cup (B \cap C) = (A \cap B) \cup C$               | v) <b>PMT</b> $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$ |
| ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$                | vi) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$                |
| iii) <b>PMT</b> $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$    | vii) $A \triangle (B \cap C) = (A \triangle B) \cap (A \triangle C)$          |
| iv) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ | viii) $A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$              |