PMT 1

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1) (i) Assume P. then P holds, so P implies P.

(ii) Assume P and Q. Then P holds, so P or Q holds, so '(P and Q) implies (P or Q)'

(iii) Assume 'for every $x \in A$, we have $P(x)$', then 'not (there exists $x \in A$, such that not $(P(x))$'

Then, assume 'there exists $x \in A$, such that not $P(x)$'.

Let $a \in A$, such that not $P(a)$ holds.

Contradiction with initial assumption.

Reject 2nd assumption and conclude 'not (there exists $x \in A$, such that not $P(x)$)'.

(iv) Assume 'P implies Q'

By cases...

1) Assume P. then Q holds by assumption.
then '(not P) or Q' holds

2) Assume not P. then '(not P) or Q' holds

Both cases lead to '(not P) or Q', so '(P implies Q) implies ((not P) or Q)'

2) 1. $\{x\} \subseteq \{x\}$

'for all $a \in \{x\}$, $a \in \{x\}$' holds, so true.

2. $\{x\} \in \{x\}$

false, as $\{x\}$ is not an element in $\{x\}$

3. $\{x\} \in \{x, \{x\}\}$

true, as $\{x\}$ is an element of $\{x, \{x\}\}$.

4. $\{x\} \subseteq \{x, \{x\}\}$

true, as for $a \in \{x\}$, $a \in \{x, \{x\}\}$

5. $\emptyset \in \emptyset$

false, as an empty set contains no elements.

6. $\emptyset \subseteq \emptyset$

true, as there are no elements in $\emptyset$

7. $\emptyset \subseteq \{\emptyset\}$

true, as there are no elements in $\emptyset$

3) $A \cap B = \{\{a\}\}$

$A \cup B = \{\{a\}, \{b\}, a, b\}$

$\wp B = \{\{a\}, \{b\}, \{\{a\}\}, \emptyset, \{a, b\}, \{a, \{a\}\}, \{b, \{a\}\}, \{a, b, \{a\}\}\}$

$A \cap \wp B = \{\{a\}, \{b\}\}$

$A \times B = \{\langle \{a\}, a \rangle, \langle \{a\}, b \rangle, \langle \{a\}, \{a\} \rangle, \langle \{b\}, a \rangle, \langle \{b\}, b \rangle, \langle \{b\}, \{a\} \rangle\}$

$(A \times B) \cap (B \times A) = \{\langle \{a\}, \{a\} \rangle\}$

$A \Delta B = \{a, b, \{b\}\}$

4) (i) $A \cup (B \cap C) = (A \cap B) \cup C$

False. $A = \{0,1\}$ $B = \{1,2\}$ $C = \{2,3\}$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\subseteq$) take $x \in A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$.

then $x \in B$ or $x \in C$.

By cases.

1. $x \in B$, so $x \in A \cap B$
2. $x \in C$, so $x \in A \cap C$

so, $x \in (A \cap B) \cup (A \cap C)$

$\supseteq$) take $x \in (A \cap B) \cup (A \cap C)$

then $x \in A \cap B$ or $x \in A \cap C$

by cases.

1. $x \in A \cap B$, then '$x \in A$ and $x \in B$'

so '$x \in B$ or $x \in C$ and $x \in A$'

2. $x \in A \cap C$, then '$x \in A$ and $x \in C$'

so '$x \in B$ or $x \in C$ and $x \in A$'

so, we conclude $x \in A \cap (B \cup C)$.

(iii) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

False. $A = \{0\}$ $B = \{1\}$ $C = \{2\}$

(iv) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

$\subseteq$) take $x \in A \setminus (B \cup C)$, then $x \in A$ and $x \notin B \cup C$

then $x \in A$ and $x \notin B$ and $x \notin C$.

then $x \in A$ and $x \notin B$, so $x \in A \setminus B$.

then $x \in A$ and $x \notin C$, so $x \in A \setminus C$.

so $x \in A \setminus B$ and $x \in A \setminus C$.

$\supseteq$) take $x \in (A \setminus B) \cap (A \setminus C)$, then

$x \in A \setminus B$ and $x \in A \setminus C$

then $x \in A$ and $x \notin B$ and $x \in A$ and $x \notin C$

so $x \in A$ and $x \notin B$ and $x \notin C$

so $x \in A$ and $x \notin B \cup C$

so $x \in A \setminus (B \cup C)$.

(v) $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$

False. $A = \{0,1\}$, $B = \{1\}$, $C = \{2\}$

(vi) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

$\subseteq$) take $x \in A \cap (B \setminus C)$, then $x \in A$ and $x \in B \setminus C$.

then $x \in A$ and $x \in B$ and $x \notin C$

so, $x \in A \cap B$

if $x \notin C$, then $x \notin C$ or $x \notin A$

so $x \notin A \cap C$.

so $x \in (A \cap B) \setminus (A \cap C)$

$\supseteq$) take $x \in (A \cap B) \setminus (A \cap C)$, then $x \in A \cap B$ and $x \notin A \cap C$.

then $x \in A$ and $x \in B$ and ($x \notin A$ or $x \notin C$).

by cases ...

1. $x \notin A$. $\lightning$ so $x \in A$
2. $x \notin C$, so $x \in A$ and $x \in B$ and $x \notin C$.

so, $x \in A \cap (B \setminus C)$

(vii) $A \triangle (B \cap C) = (A \triangle B) \cap (A \triangle C)$

False. $A = \{0, 1\}$ $B = \{1, 2\}$ $C = \{2, 3\}$

(viii) $A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$

$\subseteq$) take $x \in A \cap (B \triangle C)$, then $x \in A$ and $x \in B \triangle C$

then $x \in A$ and $x \in (B \setminus C) \cup (C \setminus B)$

then $x \in A$ and $x \in B \setminus C$ or $x \in C \setminus B$

by cases...

1. $x \in B \setminus C$ then $x \in B$ and $x \notin C$
   so $x \in A \cap B$ and $x \notin A \cap C$
   so $x \in (A \cap B) \setminus (A \cap C)$
2. $x \in C \setminus B$ then $x \in C$ and $x \notin B$
   then $x \in A \cap C$ and $x \notin A \cap B$
   so $x \in (A \cap C) \setminus (A \cap B)$

since $x \in (A \cap B) \setminus (A \cap C) \cup (A \cap C) \setminus (A \cap B)$

then $x \in (A \cap B) \triangle (A \cap C)$

2) take $x \in (A \cap B) \Delta (A \cap C)$

then $x \in (A \cap B) \setminus (A \cap C) \cup (A \cap C) \setminus (A \cap B)$

then $x \in A \cap B$ and $x \notin A \cap C$ or $x \in A \cap C$ and $x \notin A \cap B$

by cases,

1. $x \in A \cap B$ and $x \notin A \cap C$, then $x \in A$ and $x \in B$ and $x \notin A$ or $x \notin C$.

   If $x \notin A$, $\lightning$

   If $x \notin C$, $x \in A$ and $x \in B$ and $x \notin C$

   So $x \in A$ and $x \in B \setminus C$

2. $x \in A \cap C$ and $x \notin A \cap B$, then $x \in A$ and $x \in C$ and $x \notin A$ or $x \notin B$.

   If $x \notin A$, $\lightning$

   If $x \notin B$, $x \in A$ and $x \notin B$ and $x \in C$

   so, $x \in A$ and $x \in C \setminus B$

so $x \in A$ and $x \in C \setminus B$ or $x \in B \setminus C$

so $x \in A$ and $x \in B \Delta C$

so $x \in A \cap (B \Delta C)$