40018 42

PMT 2: Sets and Relations

Submitters

kw1425

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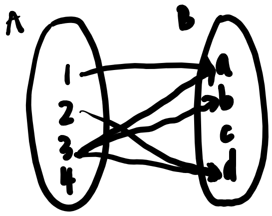
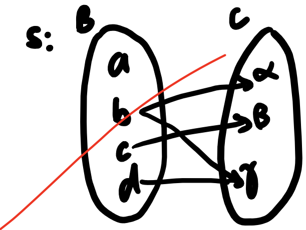
15 October 2025 11:29

5)(i) R:



S:



$$\begin{pmatrix} T & F & F & F \\ F & F & F & T \\ T & T & F & T \\ F & F & F & F \end{pmatrix} \qquad \begin{pmatrix} F & F & F \\ T & F & T \\ F & T & F \\ F & F & T \end{pmatrix}$$

(ii) 1. $R^{-1} = \{\langle a,1\rangle, \langle d,2\rangle, \langle a,3\rangle, \langle b,3\rangle, \langle d,3\rangle\}$
$R^{-1} \subseteq B \times A$

2. $\breve{S} = \{\langle a,\alpha\rangle, \langle a,\beta\rangle, \langle a,\gamma\rangle, \langle b,\beta\rangle, \langle c,\alpha\rangle, \langle c,\gamma\rangle, \langle d,\alpha\rangle, \langle d,\beta\rangle\}$
$\breve{S} \subseteq B \times C$

3. $R \cup S$ not well defined, as R is type $A \times B$ and S is type $B \times C$, so they have different types

4. $R \circ S = \{\langle 2,\gamma\rangle, \langle 3,\alpha\rangle, \langle 3,\gamma\rangle\}$
$R \circ S \subseteq A \times C$

6) (i) $\{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 4,1\rangle, \langle 1,4\rangle, \langle 4,2\rangle, \langle 2,4\rangle\}$

(ii) $\{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 1,2\rangle, \langle 2,3\rangle\}$

(iii) $\{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$ R! S! T

(iv) $\{\langle 1,2\rangle, \langle 2,1\rangle, \langle 1,1\rangle\}$ is this transitive...?

7) (i) $\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle$

(ii) there is no specific pairs that must belong to R.

true, but state rule for a symmetric relation, then come up w/ an explanation

(iii) $\langle 2,1\rangle, \langle 1,3\rangle \in R$

(iv) Yes, as by definition, an intermediate element y cannot be found, so since transitivity uses implication, it is transitive.

8) (i) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$ using equality

show: $(R \cup S)^{-1} \subseteq R^{-1} \cup S^{-1}$ and $(R \cup S)^{-1} \supseteq R^{-1} \cup S^{-1}$

8) (i) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$

to show: $(R \cup S)^{-1} \subseteq R^{-1} \cup S^{-1}$ and $(R \cup S)^{-1} \supseteq R^{-1} \cup S^{-1}$ using equality

should state $a, b \in A$

1. show 'for all $x \in (R \cup S)^{-1}$, $x \in R^{-1} \cup S^{-1}$'

take $p \in (R \cup S)^{-1}$, then $\langle a, b \rangle \in (R \cup S)^{-1}$, then $a (R \cup S)^{-1} b$

by definition of inverse, $b (R \cup S) a$.

by definition of $\cup$, $b R a$ or $b S a$.

by definition of inverse, $a R^{-1} b$ or $a S^{-1} b$.

so, $\langle a, b \rangle \in R^{-1} \cup S^{-1}$ by def of $\cup$

so, $p \in R^{-1} \cup S^{-1}$

so $(R \cup S)^{-1} \subseteq R^{-1} \cup S^{-1}$

2. for $(R \cup S)^{-1} \supseteq R^{-1} \cup S^{-1}$, read the above in reverse

write out! You will find you must show by cases...

(ii) to show: 'for all $x, y \in A$, $x (R \cup R^{-1}) y \Rightarrow y (R \cup R^{-1}) x$'

take $p \in R$, then $\langle a, b \rangle \in R$, then 'by definition of $\cup$', $\langle a, b \rangle \in R \subseteq R \cup R^{-1}$

by definition of inverse, $b R^{-1} a$, so $\langle b, a \rangle \in R^{-1}$

then by definition of $\cup$, $\langle b, a \rangle \in R^{-1} \subseteq R \cup R^{-1}$

, so $a (R \cup R^{-1}) b \Rightarrow b (R \cup R^{-1}) a$

, so $R \cup R^{-1}$ is symmetric.

you can also use symmetric : $R = R^{-1}$ def.

(iii) $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

to show: $(R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1}$ and $(R \circ S)^{-1} \supseteq S^{-1} \circ R^{-1}$ by definition of equality

1. $(R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1}$

take $p \in (R \circ S)^{-1}$, then $\langle a, b \rangle \in (R \circ S)^{-1}$, then $a (R \circ S)^{-1} b$.

by definition of inverse, $b (R \circ S) a$.

then there exists $c \in A$, such that $b R c$ and $c S a$.

by definition of inverse, $c R^{-1} b$ and $a S^{-1} c$.

by definition of composition, $a (S^{-1} \circ R^{-1}) b$

be a little more detailed here, state def. of composition

, so $\langle a, b \rangle \in S^{-1} \circ R^{-1}$, so $p \in S^{-1} \circ R^{-1}$

, so $(R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1}$

2. for $(R \circ S)^{-1} \supseteq S^{-1} \circ R^{-1}$, read the above in reverse.

(iv) 'If $R \subseteq S$ and S are symmetric, $R \cup R^{-1} \subseteq S$'

Assume $R \subseteq S$ and S are symmetric.

Take $p \in R$, then $\langle a,b \rangle \in R$ ~~$\subseteq S$~~

By definition of symmetry, $aRb \Rightarrow bRa$

so, $\langle b,a \rangle \in R$ ~~$\subseteq S$~~

by definition of inverse, $\langle b,a \rangle \in R^{-1}$, so $R^{-1} \subseteq S$

since $R \subseteq S$ and $R^{-1} \subseteq S$, $R \cup R^{-1} \subseteq S$

we know S is symmetric, but can we say so for R?

Try again, maybe use $S = S^{-1}$ identity instead...

(v) show 'R symmetric $\Rightarrow$ $R \circ R$ symmetric.'

Assume R is symmetric.

Using part (iii), $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

set $S = R$, then $(R \circ R)^{-1} = R^{-1} \circ R^{-1}$

R is symmetric by assumption

so, by definition of symmetry, $R = R^{-1}$

so, $(R \circ R)^{-1} = R \circ R$.

so, 'if R is symmetric, $R \circ R$ is symmetric'.

great!

Great, proofs structured nicely! Just need to elaborate a bit more.