

# Discrete Mathematics, Logic, and Reasoning - PMT 2

## Sets and Relations

Hand in October 20, 2025, 19pm - Electronic

**Question 5** (EX. 3.2) Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{a, b, c, d\}$ ,  $C = \{\alpha, \beta, \gamma\}$ , and let

$$R = \{\langle 1, a \rangle, \langle 2, d \rangle, \langle 3, a \rangle, \langle 3, b \rangle, \langle 3, d \rangle\}$$

$$S = \{\langle b, \alpha \rangle, \langle b, \gamma \rangle, \langle c, \beta \rangle, \langle d, \gamma \rangle\}$$

- i) Give the diagram and matrix representations of  $R$  and  $S$ .
- ii) For the following relations, either list their elements and give their types, or explain why they are not well defined: i)  $R^{-1}$  ii)  $\bar{S}$  iii)  $R \cup S$  iv)  $R \circ S$

**Question 6** (EX. 3.3) **PMT** Give examples of relations on  $\{1, 2, 3, 4\}$  having the following properties:

- i) reflexive, symmetric, not transitive.
- ii) reflexive, not symmetric, not transitive.
- iii) not reflexive, not symmetric, and transitive.
- iv) symmetric, transitive, not reflexive.

**Question 7** (EX. 3.8) Let  $A = \{1, 2, 3, 4\}$  and let  $R$  be a binary relation on  $A$ .

- i) If  $R$  is reflexive, what pairs must belong to  $R$ ?
- ii) If  $R$  is symmetric, what pairs must belong to  $R$ ?
- iii) If  $\langle 1, 2 \rangle, \langle 3, 1 \rangle \in R$  and  $R$  is symmetric, then what other pairs must belong to  $R$ ?
- iv) Is the relation  $\{\langle 1, 4 \rangle\}$  transitive?

**Question 8** (EX. 3.13) **PMT** Let  $R, S$  be binary relations on a set  $A$ .

- i) Show that  $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$ .
- ii) Use (i) to deduce that  $R \cup R^{-1}$  is symmetric.
- iii) Show that  $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$ .
- iv) Now suppose that  $R \subseteq S$  and  $S$  are symmetric. Show that  $R \cup R^{-1} \subseteq S$ .
- v) Use part (iii) to show that  $R$  symmetric implies  $R \circ R$  is symmetric.