Solutions to the Ray Optics Problem Set

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Contents

1 Introduction

In this document, we have answered every question in the Ray Optics problem sheet in comprehensive and rigorous detail, using external applications like Excel or MATLAB to code our solutions where required.

2 Ray Optics Question 1

2.1 (i) Calculating Critical Angles

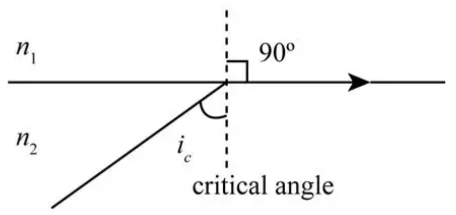

Figure 1: Diagram illustrating light exiting a denser medium at the critical angle and producing an angle of refraction of 90 degrees [2].

We must use Snell's Law of Refraction to find critical angles:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{2.1}$$

For glass to air, we rearrange for θ_1:

$$\theta_1 = \arcsin\left(\frac{n_2 \sin\theta_2}{n_1}\right) \tag{2.2}$$

Subbing the values $n_1 = 1.50, n_2 = 1.00$ and $\theta_2 = 90°$ gives:

$$\theta_1 = \arcsin(\frac{1.00}{1.50}) \tag{2.3}$$

So $\theta_1 = 41.8°$.

For glass to water, apply the same equation, but instead subbing the value $n_2 = 1.34$ since the air is now replaced with water, giving:

$$\theta_1 = \arcsin(\frac{1.34}{1.50}) \tag{2.4}$$

So $\theta_1 = 63.3°$

2.2 (ii) Calculating Number of Wavelengths

When light enters a different medium, the wave speed is calculated by c/n, where the wavelength changes while the frequency stays the same. Thus we have:

$$\frac{c}{n} = \lambda_2 f \tag{2.5}$$

where λ_2 is the wavelength in the new medium, so:

$$\lambda_2 = \frac{c}{fn} \tag{2.6}$$

but since $c/f = \lambda_1$, we have:

$$\lambda_2 = \frac{\lambda_1}{n} \tag{2.7}$$

so the number of wavelengths can be calculated as:

$$\frac{d}{\lambda_2} = \frac{dn}{\lambda_1} \tag{2.8}$$

Subbing values for each variable gives $\frac{0.05 \times 1.31}{520 \times 10^{-9}} = 1.26 \times 10^5$ wavelengths.

2.3 (iii) Ideal Converging Biconvex Lens

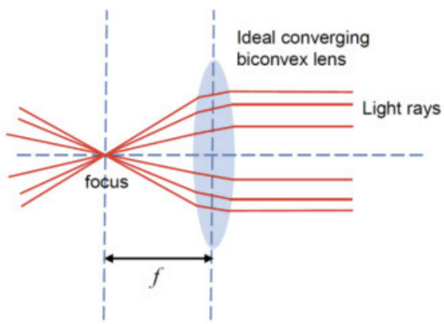

Figure 2: Diagram showing all horizontal lines converging at the focal distance from the lens [3].

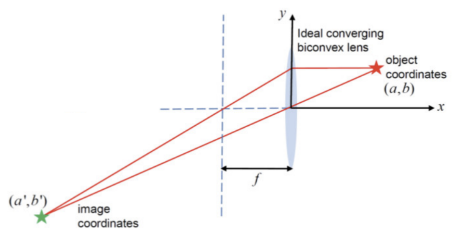

Figure 3: Diagram showing an object at displacement (a, b) from the centre of the lens converging at (a', b') where $a' = -\left(\frac{1}{f} - \frac{1}{a}\right)^{-1}$, $b' = -\frac{b}{a}\left(\frac{1}{f} - \frac{1}{a}\right)^{-1}$ and $a > f$ [3].

To find a' and b' (as per Figure 3) in terms of a, b and f, we find the equations of the lines of the light rays and see where they intersect. If we use the centre of the lens as the origin, then the light ray through the centre of the lens has equation:

$$y_1 = \frac{b}{a}x \tag{2.9}$$

while the light ray entering the lens horizontally has equation:

$$y_2 = \frac{b}{f}x + b \tag{2.10}$$

Setting $y_1 = y_2$ gives:

$$\frac{b}{a}x = \frac{b}{f}x + b \tag{2.11}$$

which can be rearranged to make x the subject:

$$x = a' = -\left(\frac{1}{f} - \frac{1}{a}\right)^{-1} \tag{2.12}$$

and as $a > f$, we know that a' is negative. As $y = \frac{b}{a}x$:

$$y = b' = -\frac{b}{a}\left(\frac{1}{f} - \frac{1}{a}\right)^{-1} \tag{2.13}$$

which is also negative. As the lines intersect at negative coordinates, we get a real, inverted image.

For the projector, we have $f = 20.0$mm, $|b'| = 1.6$m and $|a'| = 5.0$m. Rearranging gives:

$$a = \left(\frac{1}{a'} + \frac{1}{f}\right)^{-1} \tag{2.14}$$

so $a = \left(-\frac{1}{5.0} + \frac{1}{20\times10^{-3}}\right)^{-1} = 20.08$mm, and:

$$b = -ab'\left(\frac{1}{f} - \frac{1}{a}\right) \tag{2.15}$$

so $b = -20.08 \times (-1.6)\left(\frac{1}{20\times10^{-3}} - \frac{1}{2.01\times10^{-2}}\right) = 6.43$mm.

2.4 (iv) Ideal Converging Biconvex Lens 2

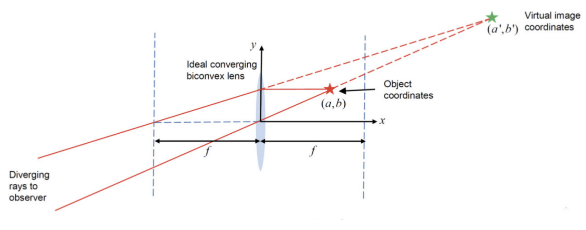

Figure 4: Diagram showing an object at displacement $(a.b)$ from the centre of the lens where $a < f$. A virtual image is seen at coordinates (a', b') where $a < f$ [3].

As before:

$$a' = \left(\frac{1}{a} - \frac{1}{f}\right)^{-1} \tag{2.16}$$

and:

$$b' = \frac{b}{a}\left(\frac{1}{f} - \frac{1}{a}\right)^{-1} \tag{2.17}$$

however, now that $f > a$, a' and b' are now positive. As $M = b'/b$, we have:

$$M = \frac{b'}{b} = \frac{1}{a}\left(\frac{1}{a} - \frac{1}{f}\right)^{-1} = \frac{f}{f-a} \tag{2.18}$$

So for Sherlock Holmes's magnifying glass, $M = 5.0$ and $a = 8.0$cm. So $\frac{f}{f-0.08} = 5$, giving $f = 10$cm.

2.5 (v) Solar Eclipse Calculations

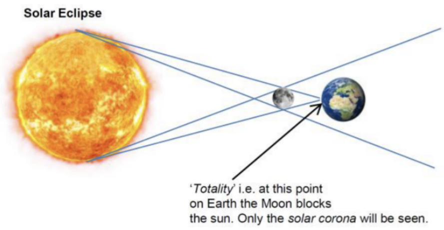

Figure 5: Diagram showing a total solar eclipse occurring at only one point on Earth [3].

For a solar eclipse to only occur at a single point on Earth, we have a right triangle of height as the radius of the Sun (R_S), and base as one AU minus Earth's radius ($a - R_E$). This triangle must be similar to another triangle of height as the lunar radius (R_M) and base as the Earth-moon distance minus Earth's radius($x - R_E$). This gives the equation:

$$\frac{R_M}{x - R_E} = \frac{R_S}{a - R_E} \tag{2.19}$$

and rearranging gives:

$$x = \frac{R_M}{R_S}(a - R_E) + R_E \tag{2.20}$$

so $x = \frac{1737.1}{696340}(1.496 \times 10^8 - 6371) + 6371 = 379,550$km.

To find how many years until there will be no more solar eclipses, we take our previously calculated value of 379,550km and subtract the current Earth-moon distance between centres as 356,500km, then divide by the distance increase in x per year as 3.8cm. So we have:

$$t = \frac{1000(379550 - 356500)}{3.8 \times 10^{-2}} \tag{2.21}$$

which gives $t = 607$ million years.

2.6 (vi) Lunar Eclipse Calculations

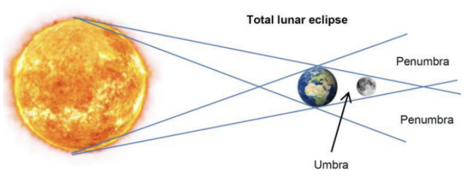

Figure 6: Diagram showing a total lunar eclipse, where the moon is at the maximum distance from the Earth where it is still entirely within the umbra region so that no light from the Sun reaches the moon [3].

To find the maximum distance of the moon from the Earth, we can use a similar approach of using similar triangles. Labeling the distance of the moon to the end of the umbra as f, and the distance between the Earth and the moon as α, as well as labeling the Earth-sun distance as a, we get that:

$$\frac{R_S - R_E}{a} = \frac{R_E - R_M}{\alpha} = \frac{R_E}{x+f} = \frac{R_S}{a+x+f} \tag{2.22}$$

Comparing expressions 1 and 3, we get:

$$x = \frac{a(R_E - R_M)}{R_S - R_E} \tag{2.23}$$

so $x = \frac{1.496\times10^8\times(6371-1737.1)}{696340-6371} = 1,004,727\text{km}$. So you will still get total lunar eclipses long after there are no more total solar eclipses.

2.7 (vii) Lunar Eclipse Calculations 2

The situation of Cassini with Saturn is analagous to a total lunar eclipse, so we can use the same method as in the question prior, substituting values for Saturn and Cassini instead of Earth and the moon. Using similar triangles, we find that:

$$\frac{r + x + R_{Sa}}{R_S} = \frac{R_{Sa} + x}{R_{Sa}} \tag{2.24}$$

Rearranging gives:

$$x = \left(\frac{r + R_{Sa}}{R_S} - 1\right)\left(\frac{1}{R_{Sa}} - \frac{1}{R_S}\right)^{-1} \tag{2.25}$$

So $x = \left(\frac{9.957\times1.496\times10^8+58232}{690340} - 1\right)\left(\frac{1}{58232} - \frac{1}{696340}\right)^{-1}$ giving $x = 1,245,486\text{km}$.

2.8 (viii) Optic Fibre Cables

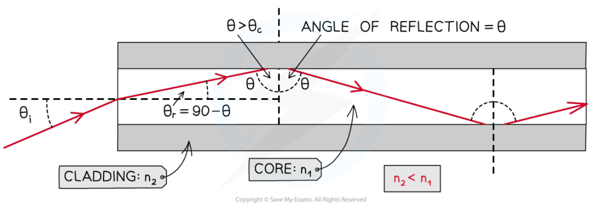

Figure 7: Diagram showing light propagating through an optic fibre cable where the angle of incidence is greater than the critical angle between the core and cladding, so total internal reflection occurs and no light is lost [4].

To find the critical angle between the core and cladding, we again use Snell's Law, substituting $\sin\theta_2 = 1$, giving:

$$\theta_c = \arcsin\left(\frac{n_{clad}}{n_{core}}\right) \tag{2.26}$$

so $\theta_c = \arcsin\left(\frac{1.4440}{1.4475}\right) = 86.0°$.

If a light ray travels distance $2r$ between reflections and travels cable length $2x$ during this, then we can draw a right triangle with hypoteneuse $2r$, height r and base $2x$. Then if θ is the angle of internal reflection, then $\sin\theta = \frac{2x}{2r} = \frac{x}{r}$, so $r = x/\sin\theta$. So if the cable length is equal to the circumference of the Earth which is $2\pi R_E$, then light travels distance:

$$r = \frac{2\pi R_E}{\sin\theta} \tag{2.27}$$

so the time taken to travel the distance is:

$$t = \frac{2\pi R_E}{\sin\theta}(c/n)^{-1} \tag{2.28}$$

so $t = \frac{2\pi\times6371\times10^3\times1.4475}{2.998\times10^8\times\sin 86.0°}$ so $t = 0.194\text{s}$.

So for the time delay in an email from London to Sydney, we take our previously calculated time of $t = 0.194\text{s}$ and multiply it by the factor $\frac{16983\times10^3}{2\pi R_E}$, which gives $t = 0.082\text{s}$.

2.9 (ix) Polarisation of Light

For S-polarized light, we are given the equation:

$$|r_\perp|^2 = \left(\frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}\right)^2 \tag{2.29}$$

where $|r_\perp|^2$ is the fraction of light power reflected. Subbing the values $\theta_i = 42°$, $n_1 = 1.00$ and $n_2 = 1.50$ into the equation gives:

$$|r_\perp|^2 = \left|\frac{1.00\cos 42° - 1.50\cos\theta_t}{1.00\cos 42° + 1.50\cos\theta_t}\right|^2 \tag{2.30}$$

where $\theta_t = \sin^{-1}\left(\frac{\sin 42°\times1.00}{1.50}\right)$, so $|r_\perp|^2 = 0.083$ and $|t_\perp|^2 = 1 - |r_\perp|^2$, so $|t_\perp|^2 = 0.917 = 91.7\%$.

3 Ray Optics Question 2

3.1 (i) Ideal Diverging Biconcave Lens

To find the coordinates of (a', b'), we use a similar approach to the converging lens. The equations of the two light rays through the diverging lens are given by:

$$y_1 = \frac{b}{a}x \tag{3.1}$$

$$y_2 = -\frac{b}{f}x + b \tag{3.2}$$

and setting $y_1 = y_2$ gives:

$$a' = \left(\frac{1}{a} + \frac{1}{f}\right)^{-1} \tag{3.3}$$

$$b' = \frac{b}{a}a' \tag{3.4}$$

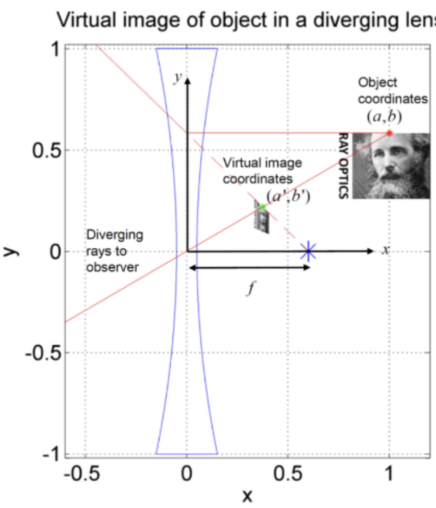

Figure 8: Diagram showing an object at displacement (a, b) from the centre of the lens converging at (a', b') where $a' = -\left(\frac{1}{f} - \frac{1}{a}\right)^{-1}$, $b' = -\frac{b}{a}\left(\frac{1}{f} - \frac{1}{a}\right)^{-1}$ [3].

3.2 (ii) (De)magnification Factor of a Diverging Lens

The (de)magnification factor is given by $M = b'/b$, which gives $M = \frac{f}{f+a}$. Since $f, a > 0$, $M < 1$ and b' has the same sign as b, giving an upright, demagnified virtual image.

4 Ray Optics Question 3

4.1 (i) Pepper's Ghost

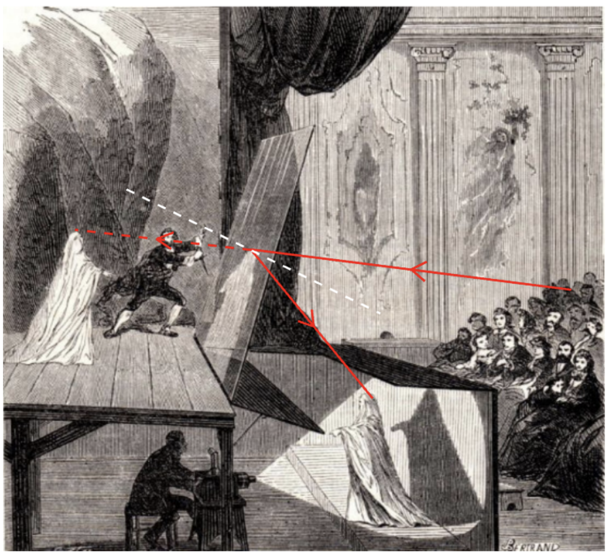

Figure 9: Diagram illustrating how how the effect is produced and made realistic [3].

We can see from Figure 9 that the 'ghost' seen on stage is the virtual image of the actor below the stage, or the apparent source of rays reaching the audience. It may not be immediately apparent that there is a glass sheet between the audience and stage because the glass is angled, so they are unlikely to observe any reflected light originating from the audience.

5 Ray Optics Question 4

5.1 (i) Circular Mirror

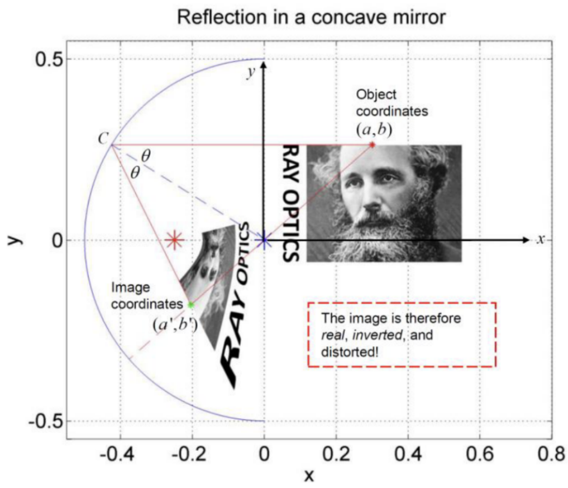

Figure 10: Diagram showing a circular mirror with object coordinates (a, b) and image coordinates (a', b') [3].

In Figure 10, if the radius of the circle is R, then the Cartesian equation of the circle is given by:

$$R^2 = x^2 + y^2 \tag{5.1}$$

where rearranging gives:

$$x = \sqrt{R^2 - y^2} \tag{5.2}$$

so when $y = b$, $x = -\sqrt{R^2 - b^2}$. We can thus draw a right triangle with base $\sqrt{R^2 - b^2}$ and height b, so:

$$\tan\theta = \frac{b}{\sqrt{R^2 - b^2}} \tag{5.3}$$

giving $\theta = \arctan \frac{b}{\sqrt{R^2 - b^2}}$.

5.2 (ii) Circular Mirror 2

We can now find the Cartesian equation of the reflected line, by drawing a right triangle with angle 2θ, coordinates $(-\sqrt{R^2 - b^2}, b)$ at the top and coordinates (a', b') at the bottom of the hypoteneuse. We know that the line is in the form:

$$y = -mx + c \tag{5.4}$$

where $m = -\tan 2\theta$, and subbing:

$$y = b \tag{5.5}$$

$$x = -\sqrt{R^2 - b^2} \tag{5.6}$$

gives:

$$c = b - m\sqrt{R^2 - b^2} \tag{5.7}$$

so:

$$y_1 = -mx + b - m\sqrt{R^2 - b^2} \tag{5.8}$$

and:

$$y_2 = \frac{b}{a}x \tag{5.9}$$

and so when $y_1 = y_2$ we get:

$$\frac{b}{a}x = -m\,x + b - m\sqrt{R^2 - b^2} \tag{5.10}$$

so:

$$x = a' = -\frac{m\sqrt{R^2 - b^2} - b}{m + \frac{b}{a}} \tag{5.11}$$

and using $y_2 = \frac{b}{a}a'$, we get:

$$b' = \frac{b}{a}a' \tag{5.12}$$

6 Ray Optics Question 5

6.1 (i) Angle of Elevation of Rainbows

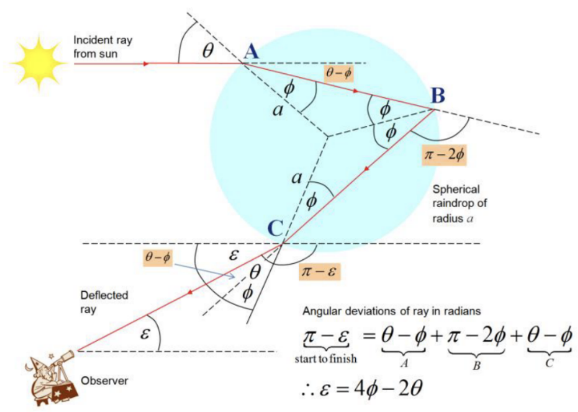

Figure 11: Diagram showing the path of light rays through a rain drop for a primary rainbow [3].

To find the elevation angle in terms of θ and n only, we use Snell's Law at point A in Figure 11. We get that $\sin\theta = n\sin\phi$, so:

$$\phi = \arcsin\left(\frac{\sin\theta}{n}\right) \tag{6.1}$$

As we know from Figure 11 that $\varepsilon = 4\phi - 2\theta$, we simply substitute our previous expression for ϕ to give:

$$\varepsilon = 4\sin^{-1}\left(\frac{\sin\theta}{n}\right) - 2\theta \tag{6.2}$$

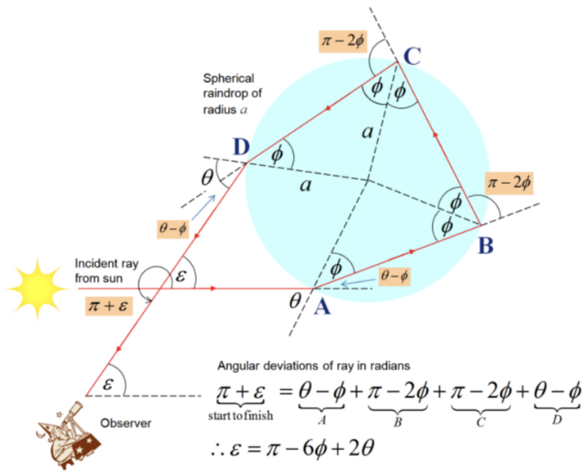

Figure 12: Diagram showing the path of light rays through a rain drop for a secondary rainbow [3].

Similarly, we know that $\varepsilon = \pi - 6\phi + 2\theta$ for the secondary rainbow as per Figure 12, so making the same substitution for ϕ gives:

$$\varepsilon = \pi - 6\sin^{-1}\left(\frac{\sin\theta}{n}\right) + 2\theta \tag{6.3}$$

6.2 (ii) Angle of Elevation of Rainbows 2

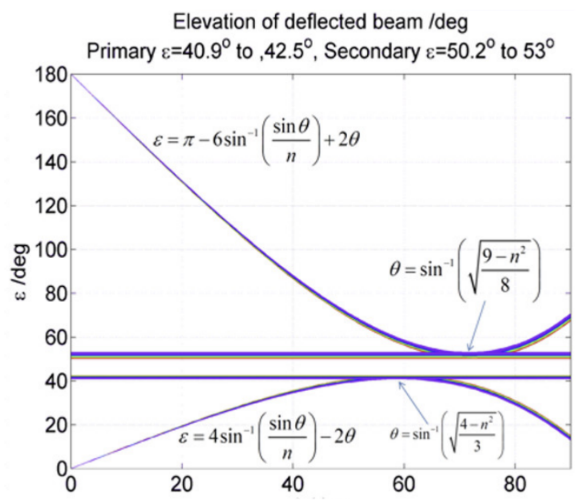

Figure 13: Graph of elevation (ε) against angle of incidence (θ) where $\frac{d\varepsilon}{d\theta} = 0$ when $\varepsilon = 40.9°$ to $42.5°$ for the primary rainbow and $\varepsilon = 50.2°$ to $53°$ for the secondary rainbow [1].

As seen in Figure 13, when $\frac{d\varepsilon}{d\theta} = 0$, there is a maximum in light intensity because there is a single value of ε for a range of angles of incidence θ, causing a focussing of light. These are coincidentally the angles of elevations of the primary and secondary rainbows observed in the sky.

6.3 (iii) Angle of Elevation of Rainbows 3

Using Snell's Law at point A again, we have:

$$\sin\phi = \frac{\sin\theta}{n} \tag{6.4}$$

and differentiating with respect to θ gives:

$$\cos\phi\frac{d\theta}{d\phi} = \frac{1}{n}\cos\theta \tag{6.5}$$

Squaring both sides, using trigonometric identities and performing the substitution (10.4) gives:

$$\left(n^2 - \sin^2\theta\right)\left(\frac{d\phi}{d\theta}\right)^2 = 1 - \sin^2\theta \tag{6.6}$$

and thus we achieve:

$$\left(\frac{d\phi}{d\theta}\right)^2 = \frac{1-\sin^2\theta}{n^2-\sin^2\theta} \tag{6.7}$$

For a primary rainbow, from before, $\varepsilon = 4\phi - 2\theta$ so:

$$\frac{d\varepsilon}{d\theta} = 4\frac{d\phi}{d\theta} - 2 \tag{6.8}$$

setting $\frac{d\varepsilon}{d\theta} = 0$ gives the maximum intensity, and so $\left(\frac{d\phi}{d\theta}\right)^2 = \frac{1}{4}$ so:

$$\frac{1-\sin^2\theta}{n^2-\sin^2\theta} = \frac{1}{4} \tag{6.9}$$

and rearranging to make θ the subject gives:

$$\theta = \arcsin\left(\sqrt{\frac{4-n^2}{3}}\right) \tag{6.10}$$

For the secondary rainbow, we take a similar approach, knowing that $\varepsilon = \pi - 6\phi + 2\theta$, so:

$$\frac{d\varepsilon}{d\theta} = -6\frac{d\phi}{d\theta} + 2 \tag{6.11}$$

and setting $\frac{d\varepsilon}{d\theta} = 0$ gives:

$$\frac{1-\sin^2\theta}{n^2-\sin^2\theta} = \frac{1}{9} \tag{6.12}$$

and rearranging gives:

$$\theta = \arcsin\left(\sqrt{\frac{9-n^2}{8}}\right) \tag{6.13}$$

6.4 (iv) Angle of Elevation of Rainbows 4

I have used MATLAB to code the graph as shown in Figure 14, where the code can be found here: `https://github.com/kqtiiv/optics/blob/main/task_11a.m`.

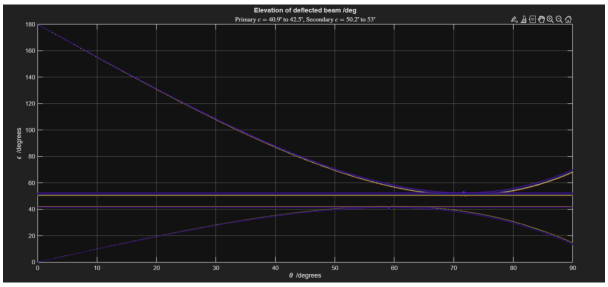

Figure 14: Graph of elevation (ε) against angle of incidence (θ) coded in MATLAB.

n_1	2.00	Critical angle (degrees)	30.00
n_2	1.00	Brewster angle (degre	26.57

Angle of incidence (degrees)	Angle of incidence (radians)	Angle of refraction (degrees)	Angle of refraction (radians)	Reflected (S-polarized)	Transmitted (S-polarized)	Reflected (P-polarized)	Transmitted (P-polarized)
0.00	0.00	0.00	0.00	0.111	0.889	0.111	0.889
0.05	0.00	0.10	0.00	0.111	0.889	0.111	0.889
0.10	0.00	0.20	0.00	0.111	0.889	0.111	0.889
0.15	0.00	0.30	0.01	0.111	0.889	0.111	0.889
0.20	0.00	0.40	0.01	0.111	0.889	0.111	0.889
0.25	0.00	0.50	0.01	0.111	0.889	0.111	0.889
0.30	0.01	0.60	0.01	0.111	0.889	0.111	0.889
0.35	0.01	0.70	0.01	0.111	0.889	0.111	0.889
0.40	0.01	0.80	0.01	0.111	0.889	0.111	0.889
0.45	0.01	0.90	0.02	0.111	0.889	0.111	0.889
0.50	0.01	1.00	0.02	0.111	0.889	0.111	0.889
0.55	0.01	1.10	0.02	0.111	0.889	0.111	0.889
0.60	0.01	1.20	0.02	0.111	0.889	0.111	0.889
0.65	0.01	1.30	0.02	0.111	0.889	0.111	0.889
0.70	0.01	1.40	0.02	0.111	0.889	0.111	0.889
0.75	0.01	1.50	0.03	0.111	0.889	0.111	0.889

Figure 15: Table in Excel of $|r_\parallel|^2$, $|t_\parallel|^2$, $|r_\perp|^2$ and $|t_\perp|^2$ against θ_i over the range $0 \le \theta_i \le 90°$ for $n_2 = 2$ and $n_1 = 1$.

7 Ray Optics Question 6

7.1 (i) Fresnel Equations

Using the provided equations:

$$|r_\perp|^2 = \left(\frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}\right)^2 \tag{7.1}$$

$$|t_\perp|^2 = 1 - |r_\perp|^2 \tag{7.2}$$

and:

$$|r_\parallel|^2 = \left(\frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t}\right)^2 \tag{7.3}$$

$$|t_\parallel|^2 = 1 - |r_\parallel|^2 \tag{7.4}$$

while making the substitutions $n_1 = 1$ and $n_2 = 2$, I tabulated the values of $|r_\parallel|^2$, $|t_\parallel|^2$, $|r_\perp|^2$ and $|t_\perp|^2$ at 0.05 increments of θ_i over the range $0 \le \theta_i \le 90°$ and plotted the results in Excel as shown in Figures 15 and 16.

7.2 (ii) Fresnel Equations 2

Simply making the substitution $n_1 = 2$ and $n_2 = 1$ in Excel yields the result of Figure 17, where a zero can be seen in $|r_\parallel|^2$ about 63.4°. When this occurs,

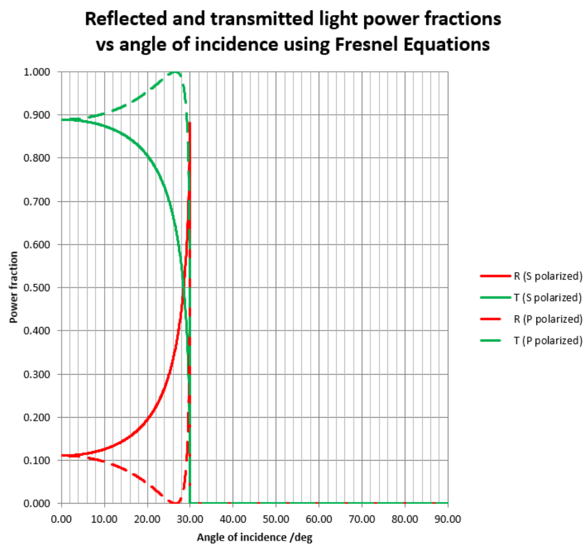

Figure 16: Graph in Excel of $|r_{\parallel}|^2$, $|t_{\parallel}|^2$, $|r_{\perp}|^2$ and $|t_{\perp}|^2$ against θ_i over the range $0 \leq \theta_i \leq 90°$ for $n_2 = 2$ and $n_1 = 1$.

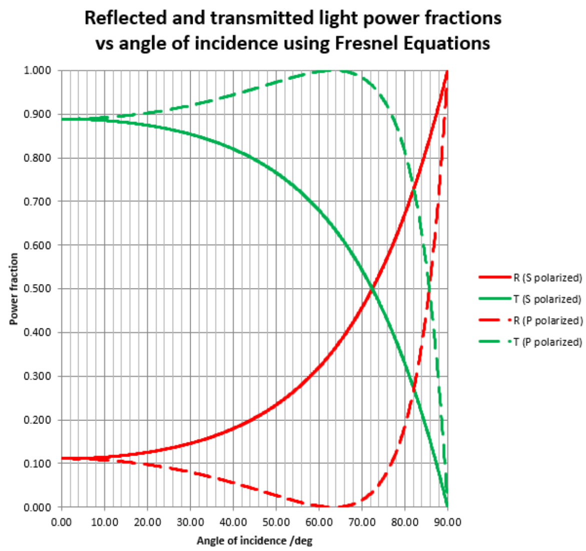

Figure 17: Graph in Excel of $|r_{\parallel}|^2$, $|t_{\parallel}|^2$, $|r_{\perp}|^2$ and $|t_{\perp}|^2$ against θ_i over the range $0 \leq \theta_i \leq 90°$ for $n_2 = 1$ and $n_1 = 2$.

only S-polarized light is reflected, which has many practical applications such as sunglasses which reduce glare in high reflection environments.

7.3 (iii) Fresnel Equations 3

The angle whereby $|r_{\parallel}|^2 = 0$ is called the Brewster angle, denoted by:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) \tag{7.5}$$

To confirm this, we remind ourselves of the equation for $|r_{\parallel}|^2$:

$$|r_{\parallel}|^2 = \left(\frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t}\right)^2 = 0 \tag{7.6}$$

so:

$$n_1\cos\theta_t = n_2\cos\theta_i \tag{7.7}$$

and:

$$n_1^2\left(1-\sin^2\theta_t\right) = n_2^2\left(1-\sin^2\theta_i\right) \tag{7.8}$$

Using Snell's Law, we have:

$$n_1\sin\theta_i = n_2\sin\theta_t \tag{7.9}$$

and algebraic manipulation gives:

$$\sin^2\theta_t = \left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i \tag{7.10}$$

Subbing into our original equation and rearranging to make $\sin^2\theta_i$ the subject gives:

$$\sin^2\theta_i = \frac{\frac{n_2^2}{n_1^2} - 1}{\frac{n_2^2}{n_1^2} - \frac{n_1^2}{n_2^2}} \tag{7.11}$$

and using the identity:

$$1 + \frac{1}{\tan\theta_i} = \frac{1}{\sin\theta_i} \tag{7.12}$$

we get:

$$\tan^2\theta_i = \frac{\frac{n_2^2}{n_1^2}\left(1-\frac{n_1^2}{n_2^2}\right)}{1-\frac{n_1^2}{n_2^2}} \tag{7.13}$$

so $\tan\theta_i = \frac{n_2}{n_1}$ so:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) \tag{7.14}$$

8 Ray Optics Problem 7

8.1 (i) Light Through a Glass Prism

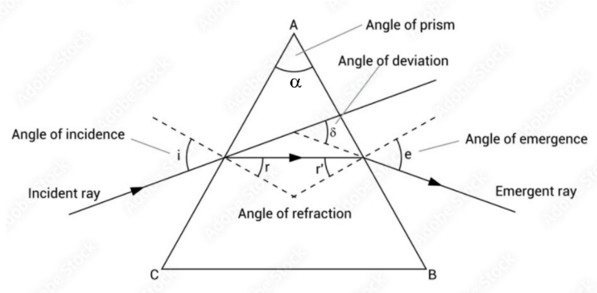

Figure 18: Graph of light going through a prism with apex angle α.

From Figure 18, we can deduce that:

$$\delta = \theta_i - r + \theta_t - r' \tag{8.1}$$

so:

$$\delta = \theta_i + \theta_t - (r + r') \tag{8.2}$$

but since we know that $\alpha = r + r'$, the deflection angle is therefore:

$$\delta = \theta_i + \theta_t - \alpha \tag{8.3}$$

8.2 (ii) Light Through a Glass Prism 2

Using Snell's Law at both boundaries gives:

$$\sin\theta_t = n \sin r' \tag{8.4}$$

$$n \sin r = \sin\theta_i \tag{8.5}$$

and as we know that $r' = \alpha - r$, we can make use of the identities:

$$\sin^2 x + \cos^2 x = 1 \tag{8.6}$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y \tag{8.7}$$

$$\cos x = \sqrt{1 - \sin^2 x} \tag{8.8}$$

to get:

$$\sin\theta_t = n \left(\sin\left(\sqrt{1 - \frac{\sin^2\theta_i}{n_g^2}} \right) - \frac{\cos\alpha \sin\theta_i}{n} \right) \tag{8.9}$$

which simplifies to:

$$\sin\theta_t = \sqrt{n^2 - \sin^2\theta_i}\, \sin\alpha - \sin\theta_i \cos\alpha \tag{8.10}$$

as required.

8.3 (iii) Light Through a Glass Prism 3

For crown glass, the refractive index is calculated by the Sellmeier equation:

$$n = \sqrt{1 + \sum_k \frac{a_k \lambda^2}{\lambda^2 - b_k}} \tag{8.11}$$

where a_k and b_k are the Sellmeier coefficients and k represents the resonances in the material. Here, we are using three terms to model the dispersion:

$$a_1 = 1.03961212, \quad b_1 = 0.00600069867, \tag{8.12}$$

$$a_2 = 0.231792344, \quad b_2 = 0.0200179144, \tag{8.13}$$

$$a_3 = 1.01146945, \quad b_3 = 103.560653 \tag{8.14}$$

For our implementation, we used TypeScript to create the simulation shown in Figure 19, which has two sliders: one for the apex angle of the prism and one for the angle of incidence of the light to see what happens to the light in real time as it is split into its colour components. To view the raw code, click here: `https://github.com/kqtiiv/optics/blob/main/task_12b1.m`.

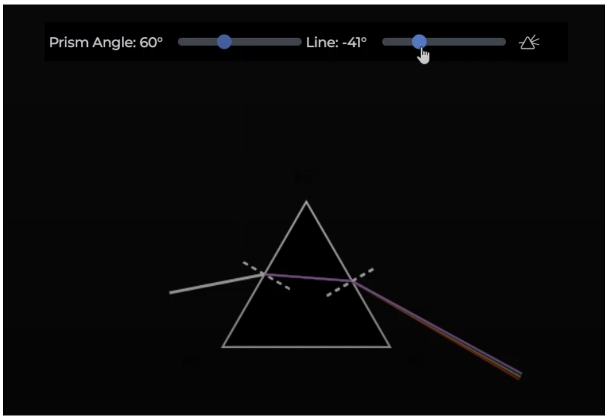

Figure 19: Simulation of light passing through a prism with two sliders, created using TypeScript.

9 Ray Optics Problem 8

9.1 (i) Proof of the Law of Reflection

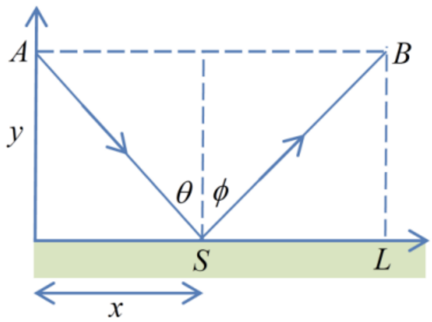

Figure 20: Variables x, θ and ϕ are denoted, with constants y and L also denoted [1].

Using basic trigonometry, we find:

$$x = y \tan\theta \tag{9.1}$$

$$L - x = y \tan\phi \tag{9.2}$$

The total distance travelled by the light ray can be calculated using Pythagoras's theorem as:

$$d = \sqrt{x^2 + y^2} + \sqrt{(L - x)^2 + y^2} \tag{9.3}$$

So the total time taken is:

$$t = \frac{\sqrt{x^2 + y^2}}{c/n} + \frac{\sqrt{(L - x)^2 + y^2}}{c/n} \tag{9.4}$$

Taking the partial derivative with respect to x gives:

$$\frac{\partial t}{\partial x} = \frac{n}{c} (\sin\theta - \sin\phi)\,. \tag{9.5}$$

And setting this to 0 gives the minimum time path, meaning:

$$\sin\theta = \sin\phi \tag{9.6}$$

So $\theta = \phi$, proving the law of reflection.

9.2 (ii) Proof of Snell's Law of Refraction

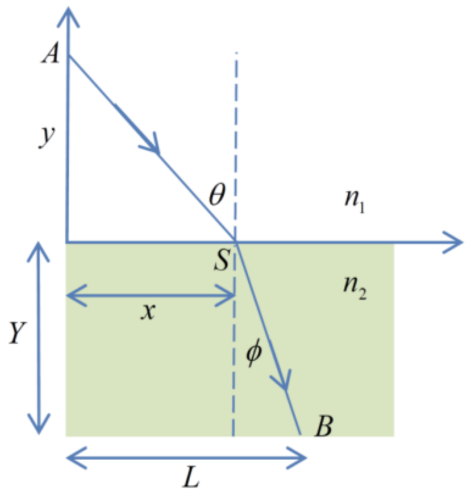

Figure 21: Variables x, θ and ϕ are denoted, with constants y, Y, n_1, n_2 and L also denoted [1].

We can again deduce via trigonometry the following relations:

$$x = y \tan\theta \tag{9.7}$$

$$L - x = Y \tan\phi \tag{9.8}$$

Again, the total time taken is give by:

$$t = \frac{\sqrt{x^2+y^2}}{c/n_1} + \frac{\sqrt{(L-x)^2+Y^2}}{c/n_2}. \tag{9.9}$$

Taking the partial derivative with respect to x gives:

$$\frac{\partial t}{\partial x} = \frac{1}{c}\left(n_1\sin\theta - n_2\sin\phi\right). \tag{9.10}$$

which is equal to 0 when $n_1 \sin\theta = n_2 \sin\phi$, thus proving Snell's Law of Refraction.

10 Ray Optics Question 9

10.1 (i) Deriving the Lensmaker's Equation

Using Snell's Law, we have:

$$\sin\theta_1 = n\sin\theta_2 \tag{10.1}$$

$$\sin\theta_4 = n\sin\theta_3 \tag{10.2}$$

and using small angle approximations, we therefore get:

$$\theta_1 \approx n\theta_2 \tag{10.3}$$

$$\theta_4 \approx n\theta_3 \tag{10.4}$$

Using trigonometry:

$$h_2 \approx (f - d/2)\tan\beta \approx f\beta \tag{10.5}$$

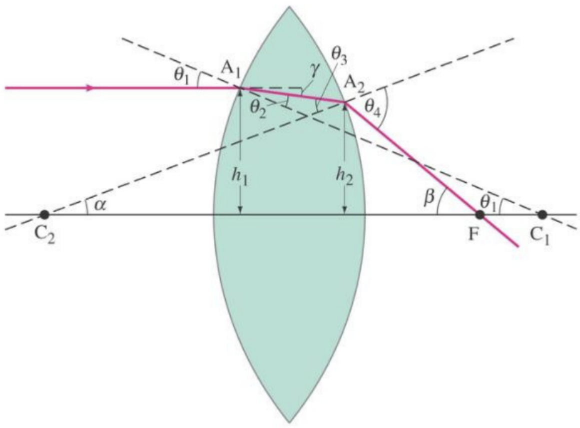

Figure 22: Diagram of a lens [5].

$$h_2 \approx R_2\sin\alpha \approx R_2\alpha \tag{10.6}$$

$$h_1 \approx R_1\sin\theta_1 \approx R_1\theta_1 \tag{10.7}$$

then if $h_1 \approx h_2$:

$$\alpha \approx \frac{R_1}{R_2}\theta_1 \tag{10.8}$$

We also know from basic geometry that $\theta_2 + \theta_3 = \alpha + \theta_1$ and $\alpha + \beta = \theta_4$ so:

$$\frac{\theta_1 + \alpha + \beta}{n} = \theta_1\left(\frac{R_1}{R_2} + 1\right) \tag{10.9}$$

and using $f\beta = R_2\alpha$ we get:

$$\beta = \frac{R_1\theta_1}{f} \tag{10.10}$$

Subbing back into our previous equation, we thus get:

$$\frac{\theta_1}{n} + \frac{R_1}{R_2}\frac{\theta_1}{n} + \frac{R_1\theta_1}{nf} = \frac{R_1}{R_2}\theta_1 + \theta_1 \tag{10.11}$$

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{f} = \frac{n}{R_2} + \frac{n}{R_1} \tag{10.12}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{10.13}$$

10.2 (ii) Deriving a More Accurate Lensmaker's Equation

Now, instead of assuming $h_1 \approx h_2$, we can instead say $h_1 - h_2 \approx d\tan(\theta - \phi) \approx d(\theta_1 - \theta_1/n)$. Then using previously derived results:

$$h_1 = R_1\theta_1 \tag{10.14}$$

$$h_2 = \beta(f - d/2) \tag{10.15}$$

taking $h_1 - h_2$ and rearranging gives:

$$\beta = \frac{R_1 - \frac{d(n-1)}{n}}{f - d/2}\theta_1 \tag{10.16}$$

Now from our previous expression of:

$$\frac{\theta_1 + \alpha + \beta}{n} = \alpha + \theta_1 \tag{10.17}$$

we get:

$$\alpha = \frac{\theta_1 \left(R_1 - d\left(\frac{n-1}{n}\right)\right)}{R_2} \tag{10.18}$$

Now subbing these values for α and β again into the equation, we get:

$$\frac{\theta_1}{n} + \frac{\theta_1}{n}\left(R_1 - \frac{d(n-1)}{n}\right)\left(\frac{1}{R_2} + \frac{1}{f - \frac{d}{2}}\right) = \theta_1\left(\frac{R_1 - \frac{d(n-1)}{n}}{R_2} + 1\right) \tag{10.19}$$

which simplifies to:

$$\frac{1}{f} \approx (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{d(n-1)}{R_1 R_2 n}\right) \tag{10.20}$$

the more accurate Lensmaker's Equation.

11 References

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