9 Sea $f(x) = x^2 - 6$ con $x_0 = 3$ y $x_1 = 2$ hallar x_4 para

a) El metodo de secante $\quad f(x_0) = 9-6 = 3 \quad f(x_1) = 4-6 = -2$

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1) = 2 - \frac{2-3}{(-2)-3}(-2) = 2 - \frac{-1}{-5}(-2) = \frac{12}{5} \qquad f(x_2) = \frac{144}{25} - 6 = -\frac{6}{25}$$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = \frac{12}{5} - \frac{\frac{12}{5} - 2}{-\frac{6}{25} - (-2)}\left(-\frac{6}{25}\right) = \frac{12}{5} - \frac{\frac{2}{5}}{\frac{44}{25}}\left(-\frac{6}{25}\right) = \frac{12}{5} + \frac{\frac{12}{5\cdot 25}}{\frac{44}{25}} = \frac{132}{55} + \frac{3}{55} = \frac{135}{55} = \frac{27}{11}$$

$$f(x_3) = \frac{729}{121} - 6 = \frac{3}{121}$$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3) = \frac{27}{11} - \frac{\frac{27}{11} - \frac{12}{5}}{\frac{3}{121} - \left(-\frac{6}{25}\right)}\left(\frac{3}{121}\right) = \frac{27}{11} - \frac{\frac{135-132}{55}}{\frac{75+726}{25\cdot 121}}\left(\frac{3}{121}\right) = \frac{27}{11} - \frac{\frac{3}{55}\left(\frac{3}{121}\right)}{\frac{801}{25\cdot 121}} = \frac{27}{11} - \frac{3}{55\cdot 121}\left(\frac{5\cdot 121}{801}\right)$$

$$= \frac{27}{11} - \frac{5}{11\cdot 89} = \frac{2403 - 5}{11\cdot 89} = \frac{2398}{11\cdot 89} = \frac{218}{89}$$

b) el metodo de la regla falsa

$f(x_0) = 3 \qquad f(x_1) = -2$

$$x_2 = \frac{f(x_1)x_0 - f(x_0)x_1}{f(x_1) - f(x_0)} = \frac{3\cdot 3 - 3\cdot 2}{-2 - 3} = \frac{12}{5} \qquad f(x_2) = -\frac{6}{25}$$

como $f(x_2) < 0$ y $f(x_1) < 0$ usamos x_2 y x_0 para calcular x_3

$$x_3 = \frac{f(x_2)x_0 - f(x_0)x_2}{f(x_2) - f(x_0)} = \frac{-\frac{6}{25}\cdot 3 - 3\cdot\frac{12}{5}}{-\frac{6}{25} - 3} = \frac{-\frac{18}{25} - \frac{180}{25}}{-\frac{81}{25}} = \frac{198}{81} = \frac{22}{9} \qquad f(x_3) = \left(\frac{22}{9}\right)^2 - 6 = \frac{484}{81} - 6 = -\frac{2}{81}$$

como $f(x_3) < 0$ y $f(x_2) < 0$ usamos x_3 y x_0 para calcular x_4

$$x_4 = \frac{f(x_3)x_0 - f(x_0)x_3}{f(x_3) - f(x_0)} = \frac{-\frac{2}{81}\cdot 3 - 3\cdot\frac{22}{9}}{-\frac{2}{81} - 3} = \frac{\frac{-2-198}{27}}{-\frac{245}{81}} = \frac{200}{27}\left(\frac{81}{245}\right) = \frac{120}{49}$$

c) ¿Cúal de los dos metodos se acerca mas a $\sqrt{6} = 2.4494897\ldots$?

$\frac{218}{89} = 2.4494382 \qquad \left|\frac{218}{89} - \sqrt{6}\right| = 0.0000515$

$\frac{120}{49} = 2.44897959 \qquad \left|\frac{120}{49} - \sqrt{6}\right| = 0.0005101$

$\frac{218}{89}$ se acerca mas a $\sqrt{6}$ que $\frac{120}{49}$

$\therefore$ el método secante se aproxima mas