Actividad 2 Taller de Modelación)

Nombre Social: Karla Romina Juárez Torres
Nombre Legal : Carlos Alberto Juárez Torres
Número de cuenta : 318013712
Profesor : Pascual Di Bella Nava

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1 Calcular las ecuaciones de movimiento, el tiempo donde la restricción deja de existir así como las trayectorias, velocidades y aceleraciones para la trayectoria donde la restricción se anula y donde ésta continua. La restricción es: $4x^2 + 20y^2 = 45$

Podemos definir el lagrangiano como:

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy + \lambda(4x^2 + 20y^2 - 45)$$

$$\Rightarrow$$

$$\frac{\partial L}{\partial x} = 8\lambda x$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}$$

$$\frac{\partial L}{\partial y} = 40\lambda y - mgy$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = m\ddot{y}$$

Derivando la restricción respecto a t tenemos:

$$\frac{d}{dt}\left(\frac{d}{dt}(4x^2+20y^2-45)\right)=\frac{d}{dt}(8x\dot{x}+40y\dot{y})=0$$

Además:

$$m\ddot{x}x=8\lambda x^2\ ,\ 5m\ddot{y}y=200\lambda y^2-5mgy$$
$$\Rightarrow m(\ddot{x}x+5\ddot{y}y)=200\lambda y^2+8yx^2-5mgy$$
$$m(\ddot{x}x+5\ddot{y}y+5gy)=200\lambda y^2+8\lambda x^2$$
$$m(-\dot{x}^2-5\dot{y}^2+5gy)=8\lambda(25y^2+x^2)$$
$$8\lambda=\frac{m(-\dot{x}^2-5\dot{y}^2+5gy)}{25y^2+x^2}$$
$$8\lambda=\frac{m(5gy-(\dot{x}^2+5\dot{y}^2))}{25y^2+x^2}$$

Sustituyendo tenemos:

$$m\ddot{x}=\frac{mx(5gy-(\dot{x}^2+5\dot{y}^2))}{25y^2+x^2}$$
$$\ddot{x}=\frac{x(5gy-(\dot{x}^2+5\dot{y}^2))}{25y^2+x^2}$$

y tambien podemos sustituir de est forma:

$$m\ddot{y}=40\lambda y-mg=5(8\lambda)y-mg$$

$$m\ddot{y}=\frac{5my(5gy-(\dot{x}^2+5\dot{y}^2))}{25y^2+x^2}-mg$$
$$\ddot{y}=\frac{5y(5gy-(\dot{x}^2+5\dot{y}^2))}{25y^2+x^2}-mg$$

Finalmente si $\lambda=0$ entonces tenemos:

$$\Longleftrightarrow 5gy-\dot{x}^2-5\dot{y}^2=0$$

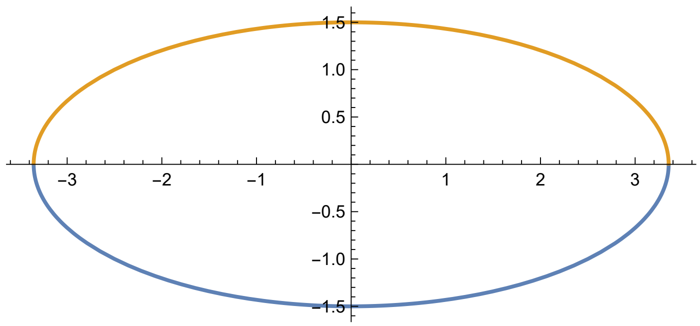
$$\Longleftrightarrow 5gy=\dot{x}^2+5\dot{y}^2$$

Así podemos sustituir en el código de Matemática:

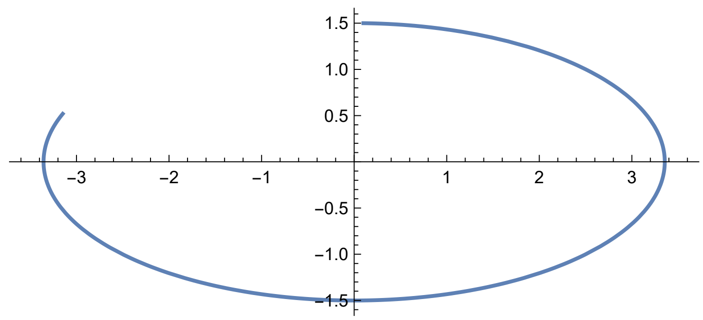
rt = Solve[20 * y^2 + 4 * x^2 − 45==0, y]

$$\left\{\left\{y\to-\frac{\sqrt{45-4x^2}}{2\sqrt{5}}\right\},\left\{y\to\frac{\sqrt{45-4x^2}}{2\sqrt{5}}\right\}\right\}$$

```
Plot[{rt[[1, 1, 2]], rt[[2, 1, 2]]}, {x, −3.5, 3.5},
AspectRatio->All]
```



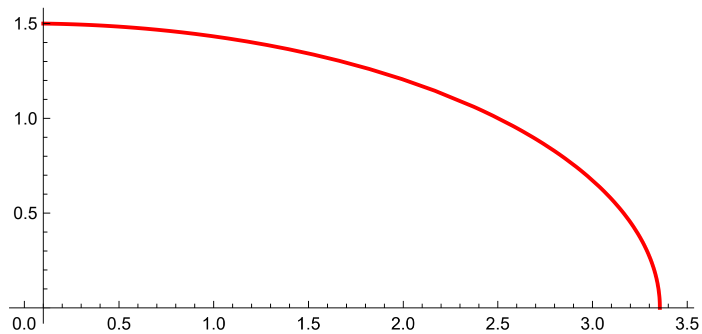

```
sol = With[{g = 9.81},
NDSolve[
{x”[t]==
(x[t] ∗ ((5 ∗ g ∗ y[t]) − (x′[t]^2 + 5 ∗ y′[t]^2)))/
(x[t]^2 + 25 ∗ y[t]^2),
y”[t]==
((5 ∗ y[t]((5 ∗ gy[t]) − (x′[t]^2 + 5 ∗ y′[t]^2)))/
(x[t]^2 + 25 ∗ y[t]^2)) − g, x′[0]==0,
y′[0]==0, x[0]==0.1,
y[0]==Sqrt[(45 − 4 ∗ 0.001^2)/20]}, {x, y},
{t, 0, 5}]];

ParametricPlot[{x[t], y[t]}/.sol, {t, 0, 5},
PlotRange->All]
```




```
{sol1, pt1} =
With[{g = 9.8},
Reap@
NDSolve[
{x”[t]==
(x[t] * ((5 * g * y[t]) − (x′[t]^2 + 5 * y′[t]^2)))/
(x[t]^2 + 25 * y[t]^2),
y”[t]==
((5 * y[t]((5 * gy[t]) − (x′[t]^2 + 5 * y′[t]^2)))/
(x[t]^2 + 25 * y[t]^2)) − g, x′[0]==0,
y′[0]==0, x[0]==0.1, y[0]==Sqrt[45/20],
WhenEvent[y[t]==0, Sow[{t, x[t]}, 1]]}, {x, y},
{t, 0, 5}]];
pt1
```

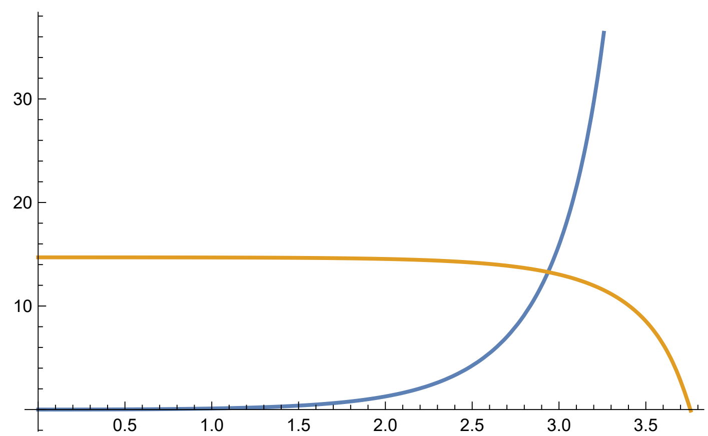
$\{\{\{3.75843, 3.35559\}, \{4.88567, -3.35559\}\}\}$

```
a = ParametricPlot[{x[t], y[t]}/.sol1, {t, 0, pt1[[1, 1, 1]]},
PlotStyle->{Red, Thick}, PlotRange->All]
```



Plot[{(4x'[t]^2 + 20y'[t]^2)/.sol1, 9.8y[t]/.sol},

{t, 0, pt1[[1, 1, 1]]}]



{sol1, pt2} =

With[{g = 9.8},

Reap@

NDSolve[

{x"[t]==

(x[t] * ((5 * g * y[t]) − (x'[t]^2 + 5 * y'[t]^2)))/

(x[t]^2 + 25 * y[t]^2),

y"[t]==

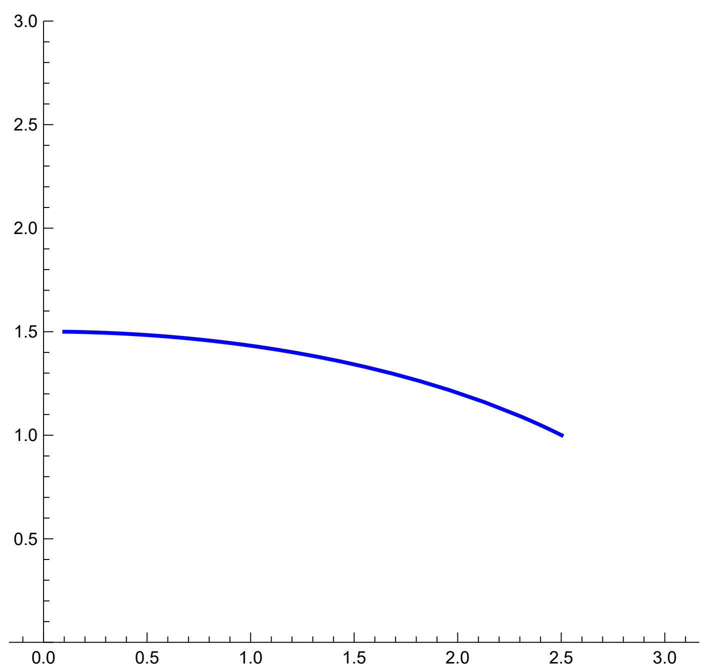
((5 * y[t]((5 * gy[t]) − (x'[t]^2 + 5 * y'[t]^2)))/

(x[t]^2 + 25 * y[t]^2)) − g, x'[0]==0,

```
y'[0]==0, x[0]==0.1,
y[0]==Sqrt[(45 - 4 * 0.001^2)/20],
WhenEvent[g * y[t]==x'[t]^2 + y'[t]^2,
Sow[{t, x[t], y[t], x'[t], y'[t]}, 1]]},
{x, y}, {t, 0, pt1[[1, 1, 1]]}]];
pt2
```

{{{3.42229, 2.502, 1., 2.79955, −1.4009}}}

```
b = ParametricPlot[{x[t], y[t]}/.sol1, {t, 0, pt2[[1, 1, 1]]},
PlotStyle->{Blue, Thick}, PlotRange->{0, 3}]
```

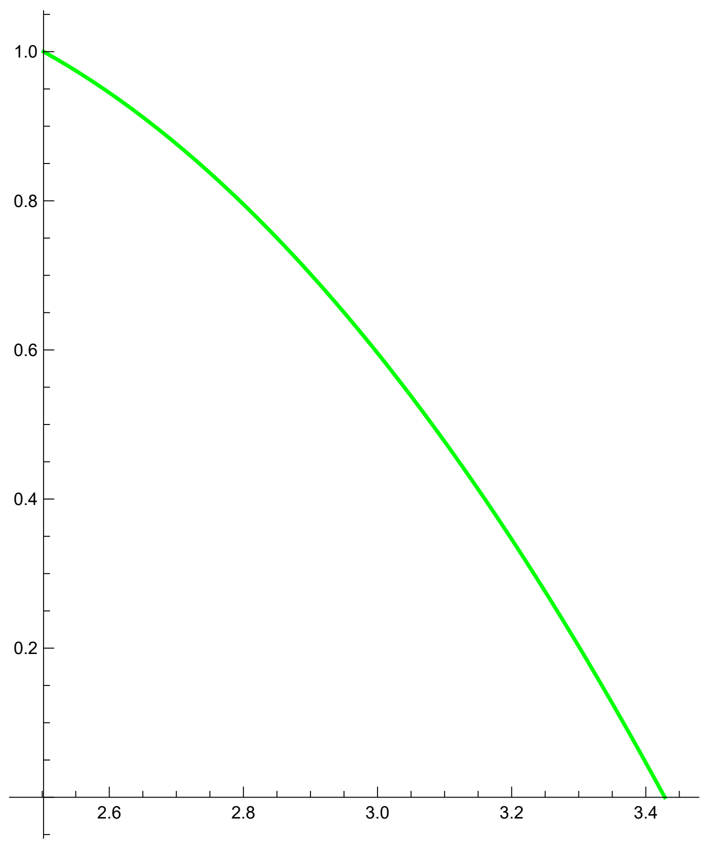



```
{sol3, pt3} =
With[{g = 9.8},
Reap@NDSolve[{x''[t]==0, y''[t]== - g,
x'[pt2[[1, 1, 1]]]==pt2[[1, 1, 4]],
```

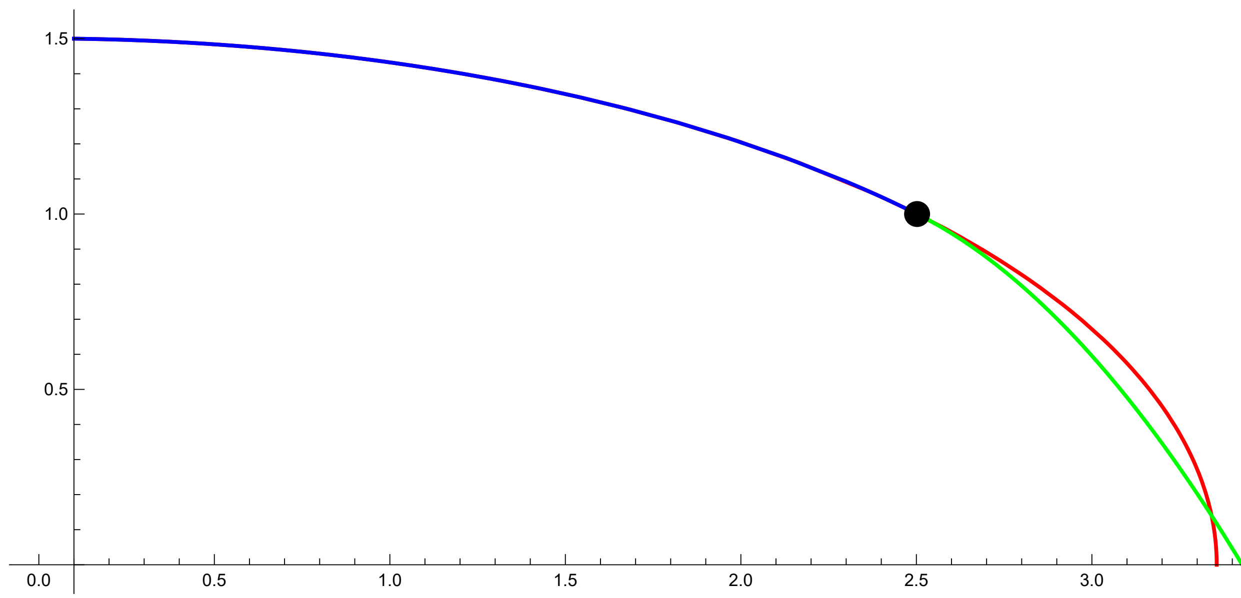
```
y'[pt2[[1, 1, 1]]]==pt2[[1, 1, 5]],
x[pt2[[1, 1, 1]]]==pt2[[1, 1, 2]],
y[pt2[[1, 1, 1]]]==pt2[[1, 1, 3]],
WhenEvent[y[t]==0, Sow[{t, x[t]}, 1]]}, {x, y},
{t, pt2[[1, 1, 1]], 4}]];
pt3
```

{{{3.75317, 3.42832}}}

```
c = ParametricPlot[{x[t], y[t]}/.sol3,
{t, pt2[[1, 1, 1]], pt3[[1, 1, 1]]}, PlotStyle->{Green, Thick},
PlotRange->All, AspectRatio->Full]
```



```
Show[a, b, c, PlotRange->All,
Epilog->
{{Black, PointSize[0.02],
Point[{pt2[[1, 1, 2]], pt2[[1, 1, 3]]}]},
{Black, Dashed,
Line[{{pt2[[1, 1, 2]], 30}, {pt2[[1, 1, 2]], 40}}]}}]
```




```
pt1[[1, 1, 2]]
pt3[[1, 1, 2]]
```

3.35559

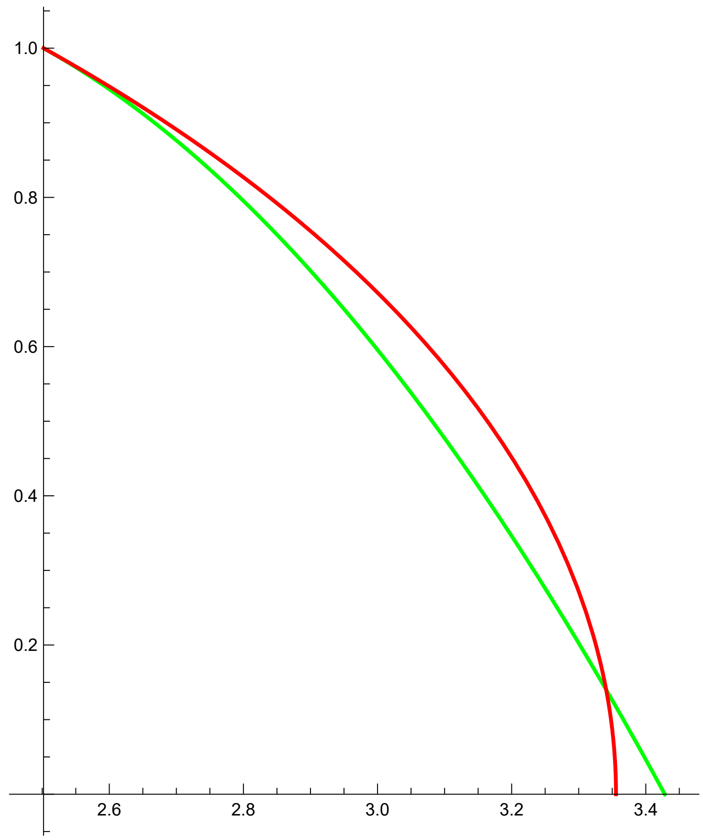
3.42832

```
pt1[[1, 1, 1]]
pt3[[1, 1, 1]]
```

3.75843

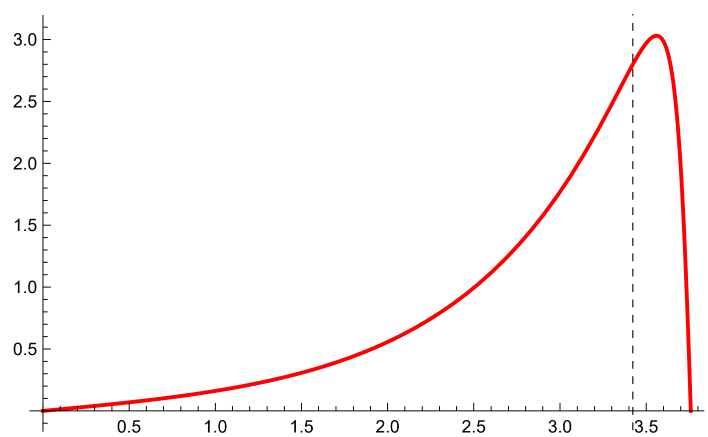
3.75317

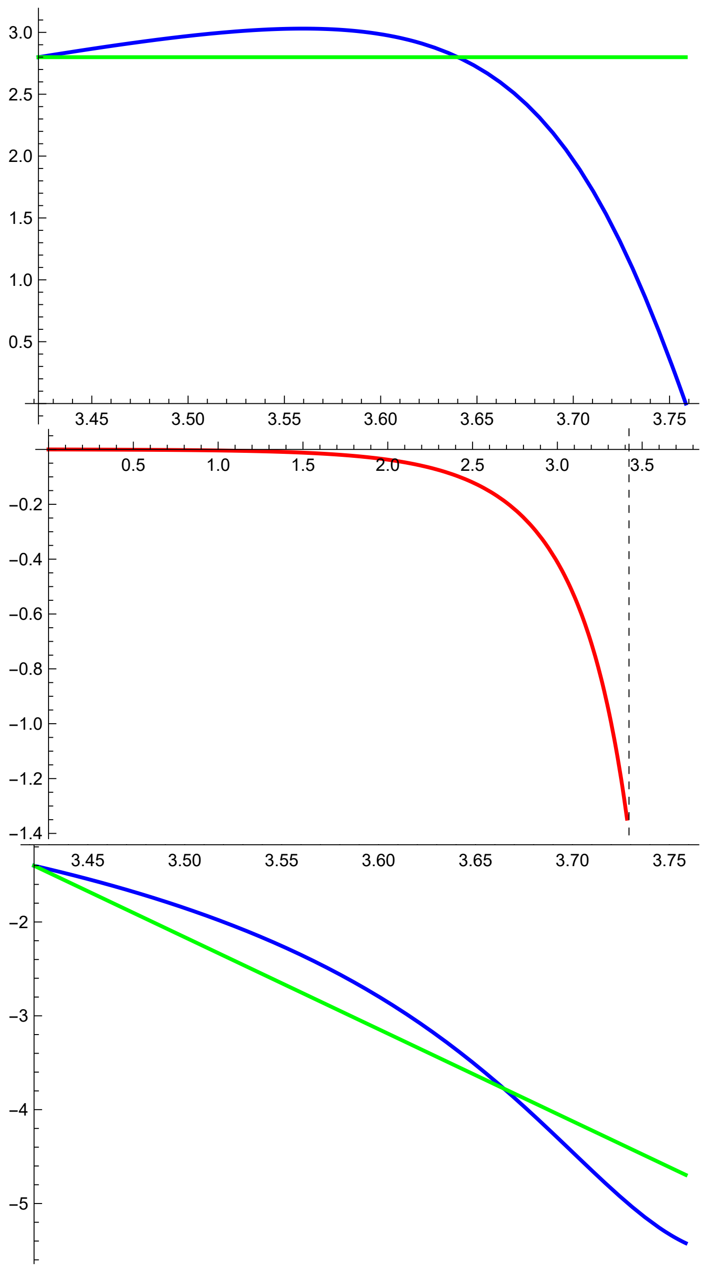
```
d = ParametricPlot[{x[t], y[t]}/.sol1,
{t, pt2[[1, 1, 1]], pt1[[1, 1, 1]]}, PlotStyle->{Red, Thick},
PlotRange->{0, Automatic}];
Show[c, d]
```




```
vx1 = Plot[{x'[t]/.sol1}, {t, 0, pt1[[1, 1, 1]]},
PlotStyle->{Red, Thick},
```

```
Epilog->
{{Black, Dashed,
Line[{{pt2[[1, 1, 1]], −30}, {pt2[[1, 1, 1]], 30}}]}}]
vx2 = Plot[{x'[t]/.sol1, x'[t]/.sol3},
{t, pt2[[1, 1, 1]], pt1[[1, 1, 1]]},
PlotStyle->{{Blue, Thick}, {Green, Thick}},
PlotRange->All]
vy1 = Plot[{y'[t]/.sol1}, {t, 0, pt1[[1, 1, 1]]},
PlotStyle->{Red, Thick},
Epilog->
{{Black, Dashed,
Line[{{pt2[[1, 1, 1]], −30}, {pt2[[1, 1, 1]], 30}}]}}]
vy2 = Plot[{y'[t]/.sol1, y'[t]/.sol3},
{t, pt2[[1, 1, 1]], pt1[[1, 1, 1]]},
PlotStyle->{{Blue, Thick}, {Green, Thick}}]
```



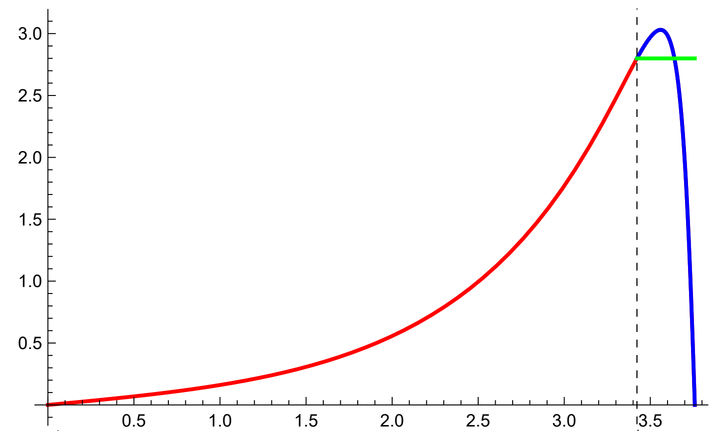


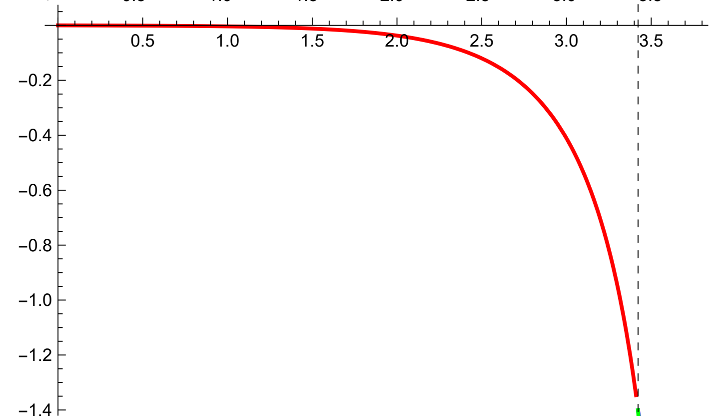
Show[vx1, vx2,

Epilog->

{{Black, Dashed,

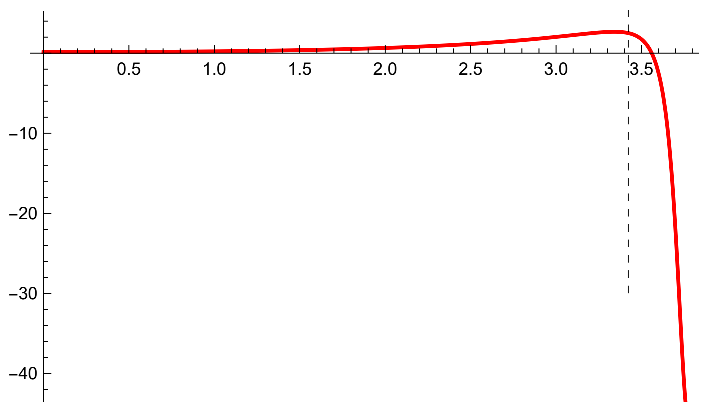
```
Line[{{pt2[[1, 1, 1]], −30}, {pt2[[1, 1, 1]], 30}}]}}]
Show[vy1, vy2,
Epilog->
{{Black, Dashed,
Line[{{pt2[[1, 1, 1]], −30}, {pt2[[1, 1, 1]], 30}}]}}]
```

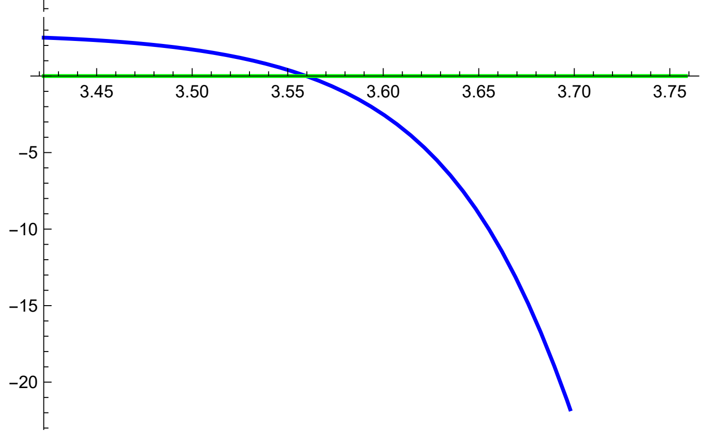


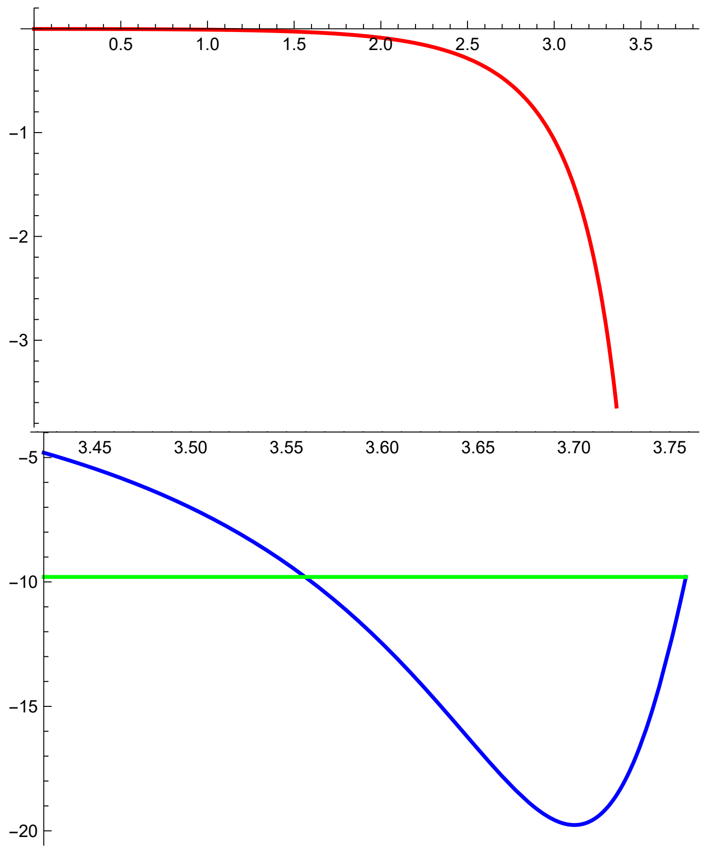



```
ax1 = Plot[{x"[t]/.sol1}, {t, 0, pt1[[1, 1, 1]]},
PlotStyle->{Red, Thick},
Epilog->
{{Black, Dashed,
Line[{{pt2[[1, 1, 1]], −30}, {pt2[[1, 1, 1]], 30}}]}},
PlotRange->All]
```

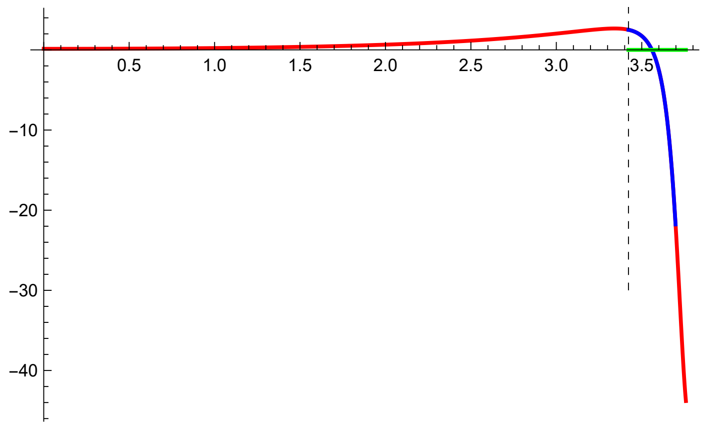
```
ax2 = Plot[{x''[t]/.sol1, x''[t]/.sol3},
{t, pt2[[1, 1, 1]], pt1[[1, 1, 1]]},
PlotStyle->{{Blue, Thick}, {Green, Thick}}]
ay1 = Plot[{y''[t]/.sol1}, {t, 0, pt1[[1, 1, 1]]},
PlotStyle->{Red, Thick},
Epilog->
{{Black, Dashed,
Line[{{0, −9.8}, {pt1[[1, 1, 1]], −9.8}}]}}]
ay2 = Plot[{y''[t]/.sol1, y''[t]/.sol3},
{t, pt2[[1, 1, 1]] + 0.001, pt1[[1, 1, 1]]},
PlotStyle->{{Blue, Thick}, {Green, Thick}}]
```








```
Show[ax1, ax2,
Epilog->
{{Black, Dashed,
Line[{{pt2[[1, 1, 1]], −30}, {pt2[[1, 1, 1]], 30}}]}}]
```




```
Show[ay1, ay2,
Epilog->
{{Black, Dashed,
Line[{{pt2[[1, 1, 1]], −30}, {pt2[[1, 1, 1]], 30}}]}}]
```