Službeni podsjetnik iz Tehnika upravljanja u mehatronici

Ziegler-Nicholsovi postupci

Postupak ruba stabilnosti:

Tip regulatora	Vrijednosti parametara regulatora		
	K_R	T_I	T_D
P	$0.5K_{Rkr}$	-	-
PI	$0.4K_{Rkr}$	$0.8T_{kr}$	-
PID	$0.6K_{Rkr}$	$0.5T_{kr}$	$0.125T_{kr}$

Postupak prijelazne funkcije:

Tip regulatora	Vrijednosti parametara regulatora		
	K_R	T_I	T_D
P	$1/a$	-	-
PI	$0.9/a$	$3T_T$	-
PID	$1.2/a$	$2T_T$	$0.5T_T$

Osnovni izrazi za amplitudno-frekvencijsku karakteristiku

Prijenosna funkcija:

$$G(s) = \frac{K}{1 + sT}$$

Amplitudna karakteristika:

$$|G(j\omega)| = \frac{K}{\sqrt{1 + (\omega T)^2}}$$

Fazna karakteristika:

$$\phi(\omega) = -\operatorname{atan}(\omega T)$$

Pomoćni izrazi

$$\operatorname{atan}(x) - \operatorname{atan}(y) = \operatorname{atan}\frac{x - y}{1 + xy}.$$

$$\frac{d}{dx}(\operatorname{atan}(x)) = \frac{1}{1 + x^2}$$

Kompenzacijski regulator

Fazno prethođenje:	$G_R(s) = \sqrt{\alpha}\frac{1+Ts}{1+\alpha Ts}, \quad 0 < \alpha < 1$
Fazno kašnjenje:	$G_R(s) = \alpha\frac{1+T_s}{1+\alpha Ts}, \quad \alpha > 1$
Maksimum faze:	$\varphi_m = \arcsin\frac{1-\alpha}{1+\alpha}$
Frekvencija maksimuma:	$\omega_m = \frac{1}{T\sqrt{\alpha}}$

Regulator po varijablama stanja

Ackermannova formula:

$$K = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix} P^{-1}\alpha(A)$$

$$P = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$$

$$\alpha(A) = A^n + \alpha_{n-1}A^{n-1} + \cdots + \alpha_2 A^2 + \alpha_1 A + \alpha_0 I$$

Standardni oblici modelske prijenosne funkcije

Za modelsku prijenosnu funkciju $G_M = \frac{B_M(s)}{A_M(s)}$, standardni polinomi $A_M(s)$ za različite redove n_M glase:

Binomni oblik	$s + \omega_n$ $s^2 + 2\omega_n s + \omega_n^2$ $s^3 + 3\omega_n s^2 + 3\omega_n^2 s + \omega_n^3$ $s^4 + 4\omega_n s^3 + 6\omega_n^2 s^2 + 4\omega_n^3 s + \omega_n^4$ $s^5 + 5\omega_n s^4 + 10\omega_n^2 s^3 + 10\omega_n^3 s^2 + 5\omega_n^4 s + \omega_n^5$
Butterworthov oblik	$s + \omega_n$ $s^2 + 1.414\omega_n s + \omega_n^2$ $s^3 + 2\omega_n s^2 + 2\omega_n^2 s + \omega_n^3$ $s^4 + 2.613\omega_n s^3 + 3.414\omega_n^2 s^2 + 2.613\omega_n^3 s + \omega_n^4$ $s^5 + 3.236\omega_n s^4 + 5.236\omega_n^2 s^3 + 5.236\omega_n^3 s^2 + 3.236\omega_n^4 s + \omega_n^5$

Veza karakterističnih veličina u vremenskom i frekvencijskom području

$$t_m = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, \quad \sigma_m[\%] = 100e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}, \quad \omega_r = \omega_n\sqrt{1-2\zeta^2}$$

$$\omega_c \approx \frac{3}{t_m}, \quad \text{za } 0.3 < \zeta < 0.8$$

$$\gamma\,[^\circ] \approx 70 - \sigma_m[\%], \quad \text{za } 0.3 < \zeta < 0.8$$

Upravljanje s unutarnjim modelom

$$G_R(s) = \frac{G_{IMC}(s)}{1 - G_{IMC}(s)\hat{G}_p(s)}$$

Preporuke za odabir perioda uzorkovanja

$$T = (0.16 \div 1.05)\frac{1}{\omega_b}, \quad T = (0.17 \div 0.34)\frac{1}{\omega_c}, \quad T = (0.08 \div 0.5)t_r, \quad T = \frac{t_u}{4 \div 12}$$

Postupci diskretizacije

Postupak diskretizacije	$G(s) \to G(z)$
ZOH	$G(z) = \left(1 - z^{-1}\right)\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$
Tustin	$s = \frac{2}{T}\frac{z-1}{z+1}$
Unazadni Euler	$s = \frac{z-1}{Tz}$
Unaprijedni Euler	$s = \frac{z-1}{T}$

Tablica $\mathcal{L}$- i $\mathcal{Z}$-transformacija

$f(t)$	$F(s)$	$f(kT)$	$F(z)$
$\delta(t)$	1	$1, k = 0$ $0, k \neq 0$	1
1	$\frac{1}{s}$	1	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
e^{-at}	$\frac{1}{s+a}$	e^{-akT}	$\frac{z}{z-e^{-aT}}$

Juryjev kriterij

Karakteristični polinom:

$$f(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n = 0$$

Juryjeva tablica:

Redak	z^0	z^1	z^2	...	z^{n-k}	...	z^{n-2}	z^{n-1}	z^n
1	a_0	a_1	a_2	...	a_{n-k}	...	a_{n-2}	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}	...	a_k	...	a_2	a_1	a_0
3	b_0	b_1	b_2	...	b_{n-k}	...	b_{n-2}	b_{n-1}	
4	b_{n-1}	b_{n-2}	b_{n-3}	...	b_{k-1}	...	b_1	b_0	
5	c_0	c_1	c_2	...	c_{n-k}	...	c_{n-2}		
6	c_{n-2}	c_{n-3}	c_{n-4}	...	c_{k-2}	...	c_0		
$\vdots$				$\vdots$		$\vdots$			
$2n-5$	p_0	p_1	p_2	p_3					
$2n-4$	p_3	p_2	p_1	p_0					
$2n-3$	q_0	q_1	q_2						

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}$$

$$c_k = \begin{vmatrix} b_0 & b_{n-k-1} \\ b_{n-1} & b_k \end{vmatrix}$$

Uvjeti:

$$f(1) > 0, \quad (-1)^n f(-1) > 0,$$
$$|a_0| < |a_n|, \quad |b_0| > |b_{n-1}|,$$
$$|c_0| > |c_{n-2}|, \ldots,$$
$$|q_0| > |q_2|$$

RST

$$G_{z\omega R} = \frac{B(z)T(z)}{A(z)R(z) + B(z)S(z)} = \frac{B_M(z)A_o(z)}{A_M(z)A_o(z)} = \frac{B_z(z)}{A_z(z)}$$

Uvjeti minimalne izvedivosti bez integratora:

$$deg\,A_o = deg\,R = deg\,S = deg\,T = n - 1$$

Implementacija

$$\frac{\partial p_i}{\partial a_k} = -\frac{p_i^{n-k}}{\prod_{j=1, j\neq i}^{n}(p_i - p_j)}$$

Pretvorba realnog broja u broj u aritmetici s nepomičnim zarezom:

$$X := \text{round}(x \cdot 2^n)$$

Pretvorba broja zapisanog u aritmetici s nepomičnim zarezom u realni broj:

$$x := X \cdot 2^{-n}$$