classmate

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Practice Assignment - 01.

(a) Test for consistency & solve -

i) $2x-3y+7z=5$ & $3x+y-3z=13$ $2x+19y-47z=32$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - \frac{3}{2}R_1} \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1/3 & -\frac{23}{3} & \frac{29}{3} \\ 0 & 22 & -54 & 27 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2 \cdot 66} \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1/3 & -\frac{23}{3} & \frac{29}{3} \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$\rho(A) \neq \rho(AB)$

$\hookrightarrow$ Inconsistent.

ii) $2x-y+3z=8$ $-x+2y+z=4$ $3x+y-4z=0$

$$\begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - \frac{3R_1}{2}]{R_2 \to R_2 + \frac{R_1}{2}} \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 5/2 & 8 \\ 0 & 5/2 & -\frac{17}{2} & -12 \end{bmatrix} \xrightarrow{R_3 \to R_3 - \frac{5}{3}R_2} \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 5/2 & 8 \\ 0 & 0 & -10/6 & \frac{-76}{3} \end{bmatrix}$$

$\rho(A) = \rho(AB) = n = 3$ $\therefore$ unique sol^n

iii) $4x-y=12$ $-x+5y-2z=0$ $-2x+4z=-8$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2R_2]{R_2 \to R_2 + R_1/4} \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 19/4 & -2 & 3 \\ 0 & -10 & 8 & -8 \end{bmatrix} \xrightarrow{R_3 \to R_3 + \frac{40}{19}R_2} \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 19/4 & -2 & 3 \\ 0 & 0 & \frac{72}{19} & \end{bmatrix}$$

$\rho(A) = \rho(AB) = n = 3$ $\therefore$ unique sol^n

(b) $x+3y-2z=0$ $2x-y+4z=0$, $x-11y+14z=0$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rightarrow \rho(A) = \rho(AB) \neq n$

$\Downarrow$

∞ sol^n

$x+3y-2z=0$

$-7y+8z=0$

$\boxed{z=k}$

$\boxed{y=\frac{8}{7}k}$

$\boxed{x=\frac{-10k}{7}}$

(b). $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right] \xrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - R_2} \left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

i) no $sol^n \Rightarrow \rho(A) \neq \rho(AB)$

$\rightarrow \boxed{\lambda = 3,\ \mu \neq 10}$

ii) uniq. $sol^n \Rightarrow \rho(A) = \rho(AB) = n = 3$

$\rightarrow \boxed{\lambda \neq 3}$

iii) ∞ $sol^n \Rightarrow \rho(A) = \rho(AB) \neq 3$

$\boxed{\lambda = 3}\ \boxed{\mu = 10}$

(c) $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right] \xrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - R_2} \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 2 & 6 & \lambda^2-\lambda \end{array}\right]$$ have a sol^n

$R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{array}\right]$$

have a sol^n

$\underbrace{\rho(A)}_{2} = \underbrace{\rho(AB)}_{2} \neq n \Rightarrow \infty$ sol^n

$\lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda^2 - 2\lambda - \lambda + 2 = 0$

$= \lambda(\lambda-2) - 1(\lambda-2) = 0$

$\boxed{\lambda = 1}\ \boxed{\lambda = 2}$

for $\lambda = 1$

$x+y+z=1$

$y+3z=0$

$\boxed{z = k}$

$\boxed{y = -3k}$

$\boxed{x = 1+2k}$

for $\lambda = 2$,

$x+y+z=1$

$y+3z=1$

$\boxed{z = k}$

$\boxed{y = 1-3k}$

$\boxed{x = 2k}$

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ⓒ. $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$

$$\begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 4 & -2 & -3 & 0 \\ 2\lambda & 4 & \lambda & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - \frac{2\lambda R_1}{3}]{R_2 \to R_2 - \frac{4R_1}{3}} \begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 0 & -10/3 & \frac{-9-4\lambda}{3} & 0 \\ 0 & \frac{12-2\lambda}{3} & \frac{3\lambda-2\lambda^2}{3} & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - \frac{(12-2\lambda)}{10}} \begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 0 & -10/3 & \frac{-9-4\lambda}{3} & 0 \\ 0 & 0 & x & 0 \end{bmatrix}$$

for non-trivial solⁿ

$\rho(A) = \rho(AB) \neq n$

$\therefore \quad x = 0$

$-14\lambda^2 + 15\lambda + 69 = 0 \Rightarrow 14\lambda^2 - 15\lambda - 69 = 0$

$\lambda = \frac{15 \pm \sqrt{225-(-3864)}}{28} = \frac{15 \pm 64}{28} \Rightarrow 2.82, -1.75$

$\boxed{\lambda = \frac{79}{28}}$, $\boxed{\lambda = -\frac{49}{28}}$