

## # Assignment - (04).

1). Rank = ?

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \\ R_4 \rightarrow R_4 - 6R_1}} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$\xrightarrow{\substack{R_3 \rightarrow R_3 - R_4 \\ R_2 \leftrightarrow R_4}} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

 $R_4 \rightarrow R_4 + R_3$ 

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

# Rank = No of non-zero rows

$$\boxed{\neq \text{Rank} = 3}$$

③  $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Eigen values & E vectors of  $A^{-1}$  &  $A+4I$

char eq  $(A) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix}$

Eigen values of  $A = 1, 3$   
 $\rightarrow$  Eigen values of  $A^{-1} = 1, \frac{1}{3}$  Property

$$(2-\lambda)^2 - 1 = 0$$

$$\lambda^2 + 4 - 4\lambda - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda^2 - 3\lambda - \lambda + 3 = 0$$

$$\lambda(\lambda-3) - 1(\lambda-3) = 0$$

$$(\lambda-1)(\lambda-3) = 0$$

$$\boxed{\lambda = 1, \lambda = 3}$$

for  $\lambda = 1$ ,

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{cases} x - y = 0 \\ -x + y = 0 \end{cases} \rightarrow \begin{bmatrix} k \\ k \end{bmatrix}$$

$\rightarrow$  If  $A$  is invertible,

Eigen vector of  $A^{-1} =$  Eigen vector of  $A$

$\Rightarrow$  Eigen vector of  $(A+4I) =$  Eigen vector of  $A$

$\lambda = 3$ ,

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$-x - y = 0$$

$$-x - y = 0$$

$$\begin{bmatrix} k \\ -k \end{bmatrix}$$

④  $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

$\rightarrow z = \frac{71.4 - 0.3x - 0.2y}{10}$

$y = \frac{-19.3 - 0.1x + 0.3z}{7}$

$x = \frac{7.85 + 0.1y + 0.2z}{3}$

for first iter<sup>n</sup>,

$x = \frac{7.85}{3} = \boxed{2.6167}$

$y = \frac{-19.3 - 0.1(2.6167)}{7} = \boxed{19.5617} \Rightarrow z = \frac{71.4 - 0.3(2.6167) - 0.2(19.5617)}{10}$

$\boxed{z = 6.67}$

$\frac{71.4 - 0.785 - 3.912}{10}$



for second iteration,

base case  $(x, y, z) = (2.6167, 19.5617, 6.6703)$

$$x = \frac{7.85 + 0.1(19.5617) + 0.2(6.6703)}{3} = 7.716$$

$$y = \frac{-19.3 + 0.1(7.716) + 0.3(6.6703)}{7} = -2.36$$

$$z = \frac{71.4 - 0.3(7.716) - 0.2(-2.36)}{10} = 6.956$$

for third iteration,

base case  $(x, y, z) = \{7.716, -2.36, 6.956\}$

$$x = \frac{7.85 + 0.1(-2.36) + 0.2(6.956)}{3} \Rightarrow x = 3.002$$

$$y = \frac{-19.3 + 0.1(3.002) + 0.3(6.956)}{7} \Rightarrow y = -2.455$$

$$z = \frac{71.4 - 0.3(3.002) - 0.2(-2.455)}{10} \Rightarrow z = 7.099$$

- ⑤. Consistent equations  $\rightarrow$  Equations which have solutions.  
Inconsistent equation  $\rightarrow$  Equations which don't have solutions.

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$  /  $R_3 \rightarrow R_3 - 3R_1$   
 $R_4 \rightarrow R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_4 \rightarrow R_4 + R_3$   
 $R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x + 3y + 2z = 0 \\ 2x - y + 3z = 0 \\ 3x - 5y + 4z = 0 \\ x + 17y + 4z = 0 \end{cases}$$

$S(A) = S(AB) \neq n$

Inconsistent solution