

This puzzle actually has a direct answer to it (for both parts) but it is funnier to write a crab function (that moves along a function with a minimum).

That being said, it is still interesting to see why it does have a direct answer, and doing so seeing how the crab function works and find a global optimum (otherwise the crab function may find “wrong one” or even loop indefinitely because there are no optimum).

## 1 General form of the problem

The configuration of the problem is a set of (one-dimensional) points  $(P_i)_{0 \leq i \leq n}$ , representing the abscissa of the crabs. The goal is to find the point  $X$  that minimizes some cost function (the quantity of fuel needed by the crab submarine at position  $P_i$  to reach position  $X$ ).

The cost for crab  $i$  to go from its position to  $X$  is denoted  $c(P_i, X)$  (it is the same for every crab). The total cost is thus:

$$\mathcal{C}(X) = \sum_{i=0}^n c(P_i, X) \quad (1)$$

and thus the general form of the problem is:

$$\min_X \mathcal{C}(X) = \min_X \sum_{i=0}^n c(P_i, X) \quad (2)$$

In the following, we will consider that the positions are ordered (this does not change the solution of the problem and makes everything very much easier):

$$P_0 \leq P_1 \leq \dots \leq P_{n-1} \leq P_n$$

## 2 First case

In the first case, the cost function is:

$$\forall i, c(P_i, X) = |P_i - X| \quad (3)$$

so the total cost function is:

$$\mathcal{C}(X) = \sum_{i=0}^n |P_i - X| \quad (4)$$

Let us try to find an optimum to this function. For that, we note that  $\mathcal{C}$  is continuous, and piecewise differentiable (on intervals of the form  $[P_i, P_{i+1}]$ ).

Let us calculate the derivative of each cost function:

$$\frac{dc(P_i, X)}{dX} = \begin{cases} -1 & \text{if } P_i > X \\ 1 & \text{if } P_i < X \\ 0 & \text{else} \end{cases} \quad (5)$$