The Basic New Keynesian Example Model used in Herbst and Schorfheide (2015): Bayesian Estimation of DSGE Models

Setup and Derivation of the Example Model
Katrin Rabitsch

Last change: 2018

1 The Model

The example model is a standard New Keynesian model: there is a representative household, that consumes, supplies labor and saves with a one-period bond. Monopolistically competitive good firms produce output using labor as a production input, and facing nominal (price) rigities a la Calvo. The central bank follows an interest rate (Taylor) rule with feedback to inflation, the output gap and with interest rate smoothing. There are three sources of exogenous disturbances: a shock to the TFP growth rate, a government expenditure shock and a monetary (interest rate) shock. The model is estimated on three macroeconomic time series: output growth, inflation, and the nominal interest rate. Below is a summary of setup, derivations, and necessary stationarizations to code up the model.

1.1 Households

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(\frac{C_t}{A_t}\right)^{1-\tau}}{1-\tau} - \xi_t \frac{N_t^{1+\varphi}}{1+\varphi} \right\} \tag{1}$$

subject to

$$P_t C_t + B_t \leq B_{t-1} R_{t-1} + W_t N_t + T_t \tag{2}$$

optimization problem:

$$L_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(\frac{C_t}{A_t}\right)^{1-\tau}}{1-\tau} - \xi_t \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t \left[B_{t-1} R_{t-1} + W_t N_t + T_t - P_t C_t - B_t \right] \right\}$$

w.r.t. C_t:

$$C_t^{-\tau} A_t^{-(1-\tau)} = \lambda_t P_t \tag{3}$$

w.r.t. N_t:

$$\xi_t N_t^{\varphi} = \lambda_t W_t \tag{4}$$

w.r.t. B_t:

$$\lambda_t = \beta E_t \lambda_{t+1} R_t \tag{5}$$

w.r.t. λ_t:

$$P_t C_t + B_t = B_{t-1} R_{t-1} + W_t N_t + T_t \tag{6}$$

combining:

$$\begin{aligned}
\xi_t N_t^{\varphi} C_t^{\tau} A_t^{(1-\tau)} &= \frac{W_t}{P_t} &(7)\\
C_t^{\tau} A_t^{(1-\tau)} &= \beta E_t C_{t+1}^{\tau} A_{t+1}^{(1-\tau)} \frac{P_t}{P_{t+1}} R_t &(8)\\
P_t C_t + B_t &= B_{t-1} R_{t-1} + W_t N_t + T_t &(9)
\end{aligned}$$

detrending and deflating, that is, rewrite conditions in terms of stationary allocations and real (relative) prices, denoted by lowercase variables, i.e.: $c_t = \frac{C_t}{A_t}$, $y_t = \frac{Y_t}{A_t}$, $\pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t A_t}$, $b_t = \frac{B_t}{P_t A_t}$, $t_t = \frac{T_t}{P_t A_t}$

$$\begin{aligned}
\xi_t N_t^{\varphi} c_t^{\tau} &= w_t &(10)\\
c_t^{\tau} &= \beta E_t c_{t+1}^{\tau} \frac{R_t}{\pi_{t+1} dA_{t+1}} &(11)\\
c_t + b_t &= \frac{b_{t-1}}{dA_t \pi_t} R_{t-1} + w_t N_t + t_t &(12)
\end{aligned}$$

1.2 Final good firms

buy $Y_t(i)$ at prices $P_t(i)$ and sell Y_t at P_t production technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{13}$$

optimization problem:

$$\max_{Y_t(i)} \left[P_t Y_t - P_t(i) Y_t(i)\right] = \max_{Y_t(i)} \left[P_t \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - P_t(i) Y_t(i)\right]$$

w.r.t. $Y_t(i)$:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t \tag{14}$$

with

$$P_t = \left[\int_0^1 P_t\left(i\right)^{1-\varepsilon} di \right]^{1-\varepsilon} \tag{15}$$

1.3 Intermediate goods firms

Cost minimization

$\min_{N_t(j)} \left\{ W_t N_t\left(j\right) + MC_t\left(j\right)\left[Y_t\left(j\right) - A_t N_t\left(j\right)\right]\right\}$

w.r.t. $N_t\left(j\right)$:

$$MC_t\left(j\right) = MC_t = \frac{W_t}{A_t}$$

w.r.t. $MC_t\left(j\right)$:

$$Y_t\left(j\right) = A_t N_t\left(j\right)$$

Profit maximization

$\max_{P_t(j)} E_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left\{\left[P_t\left(j\right) - MC_t\right] Y_t\left(j\right)\right\} = \max_{P_t(j)} E_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left\{\left[P_t\left(j\right) - MC_t\right] \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t\right\}$

decompose infinite sum to derive first order condition...:

$$\left[\begin{array}{c} \max_{P_t(j)} \left\{ \left[P_t\left(j\right) - MC_t\right] \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t + \right\} + \\ + \left(\beta\theta\right) \frac{u_{Ct+1}}{u_{Ct}} \frac{P_t}{P_{t+1}} \left\{ \left[P_t\left(j\right) - MC_{t+1}\right] \left[\left(\frac{P_t(j)}{P_{t+1}}\right)^{-\varepsilon} Y_{t+1} \right] \right\} + \\ + \left(\beta\xi_P\right)^2 \frac{u_{t+2}}{u_{Ct}} \frac{P_t}{P_{t+2}} \left\{ \left[P_t\left(j\right) - MC_{t+2}\right] \left[\left(\frac{P_t(j)}{P_{t+2}}\right)^{-\varepsilon} Y_{t+2} \right] \right\} + ... \end{array} \right]$$

w.r.t. $P_t\left(j\right)$:

$$\left\{ \begin{array}{c} 0 = \left[\begin{array}{c} \left[\left(\frac{P_t^*(j)}{P_t}\right)^{-\varepsilon} Y_t \right] + P_t^*\left(j\right)\left(-\varepsilon\right) \left[\left(\frac{P_t^*(j)}{P_t}\right)^{-\varepsilon} Y_t \right] \frac{1}{P_t^*(j)} \\ -MC_t\left(-\varepsilon\right) \left[\left(\frac{P_t^*(j)}{P_t}\right)^{-\varepsilon} Y_t \right] \frac{1}{P_t^*(j)} \end{array} + \right] \\ \left[\left(\beta\theta\right) E_t \left\{ \frac{u_{Ct+1}}{u_{Ct}} \frac{P_t}{P_{t+1}} \left[\begin{array}{c} \left[\left(\frac{P_t^*(j)}{P_{t+1}}\right)^{-\varepsilon} Y_{t+1} \right] + P_t^*\left(j\right)\left(-\varepsilon\right) \left[\left(\frac{P_t^*(j)}{P_{t+1}}\right)^{-\varepsilon} Y_{t+1} \right] \frac{1}{P_t^*(j)} \\ -MC_{t+1}\left(-\varepsilon\right) \left[\left(\frac{P_t^*(j)}{P_{t+1}}\right)^{-\varepsilon} Y_{t+1} \right] \frac{1}{P_t^*(j)} \end{array} \right] \right\} + \right] \\ \left[\left(\beta\theta\right)^2 E_t \left\{ \frac{u_{Ct+2}}{u_{Ct}} \frac{P_t}{P_{t+2}} \left[\begin{array}{c} \left[\left(\frac{P_t^*(j)}{P_{t+2}}\right)^{-\varepsilon} Y_{t+2} \right] + P_t^*\left(j\right)\left(-\varepsilon\right) \left[\left(\frac{P_t^*(j)}{P_{t+2}}\right)^{-\varepsilon} Y_{t+2} \right] \frac{1}{P_t^*(j)} \\ -MC_{t+2}\left(-\varepsilon\right) \left[\left(\frac{P_t^*(j)}{P_{t+2}}\right)^{-\varepsilon} Y_{t+2} \right] \frac{1}{P_t^*(j)} \end{array} \right] \right\} + ... \right] \end{array} \right\}$$

dividing the above f.o.c. by $P_t^*(j)^{-\varepsilon}$ multiplying by $P_t^*(j)$

$$\left[\begin{array}{c} 0 = P_t^*(j)\left[\left(\frac{1}{P_t}\right)^{-\varepsilon} Y_t\right] + P_t^*(j)(-\varepsilon)\left[\left(\frac{1}{P_t}\right)^{-\varepsilon} Y_t\right] + \varepsilon MC_t\left[\left(\frac{1}{P_t}\right)^{-\varepsilon} Y_t\right] \\ +P_t^*(j)(\beta\theta)E_t\frac{u_{Ct+1}}{u_{Ct}}\frac{P_t}{P_{t+1}}\left\{\begin{array}{c} \left[\left(\frac{1}{P_{t+1}}\right)^{-\varepsilon} Y_{t+1}\right] + P_t^*(j)(-\varepsilon)\left[\left(\frac{1}{P_{t+1}}\right)^{-\varepsilon} Y_{t+1}\right] \\ +\varepsilon MC_{t+1}\left[\left(\frac{1}{P_{t+1}}\right)^{-\varepsilon} Y_{t+1}\right] \end{array}\right\} \\ +P_t^*(j)(\beta\theta)^2 E_t\frac{u_{Ct+2}}{u_{Ct}}\frac{P_t}{P_{t+2}}\left\{\begin{array}{c} \left[\left(\frac{1}{P_{t+2}}\right)^{-\varepsilon} Y_{t+2}\right] + P_t^*(j)(-\varepsilon)\left[\left(\frac{1}{P_{t+2}}\right)^{-\varepsilon} Y_{t+2}\right] \\ +\varepsilon MC_{t+2}\left[\left(\frac{1}{P_{t+2}}\right)^{-\varepsilon} Y_{t+2}\right] \end{array}\right\} \\ +... \end{array}\right]$$

writing more compactly:

$$\left[\begin{array}{c} P_t^*(j)(\varepsilon-1)E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{u_{Ct+k}}{u_{Ct}}\frac{P_t}{P_{t+k}}\left[\left(\frac{1}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}\right] \\ =\varepsilon E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{u_{Ct+k}}{u_{Ct}}\frac{P_t}{P_{t+k}}MC_{t+k}\left[\left(\frac{1}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}\right] \end{array}\right]$$

or:

$$\begin{aligned} P_t^*(j) &= \frac{\varepsilon}{(\varepsilon-1)}\frac{E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{u_{Ct+k}}{u_{Ct}}\frac{P_t}{P_{t+k}}MC_{t+k}\left[\left(\frac{1}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}\right]}{E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{u_{Ct+k}}{u_{Ct}}\frac{P_t}{P_{t+k}}\left[\left(\frac{1}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}\right]} \\ \frac{P_t^*(j)}{P_t} &= \frac{\varepsilon}{(\varepsilon-1)}\frac{E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{u_{Ct+k}}{u_{Ct}}\frac{P_t}{P_{t+k}}\frac{MC_{t+k}}{P_{t+k}}\left[\left(\frac{1}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}\right]\frac{P_{t+k}}{P_t}P_t^{-\varepsilon}}{E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{u_{Ct+k}}{u_{Ct}}\frac{P_t}{P_{t+k}}\left[\left(\frac{1}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}\right]P_t^{-\varepsilon}} \\ \frac{P_t^*(j)}{P_t} &= \frac{\varepsilon}{(\varepsilon-1)}\frac{E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{u_{Ct+k}}{u_{Ct}}\frac{MC_{t+k}}{P_{t+k}}\left[\left(\frac{P_t}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}\right]}{E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{u_{Ct+k}}{u_{Ct}}\left[\left(\frac{P_t}{P_{t+k}}\right)^{1-\varepsilon} Y_{t+k}\right]} \end{aligned}$$

detrending and deflating: define $p_t^* = \frac{P_t^*(j)}{P_t}$, $mc_t = \frac{MC_t}{P_t}$, $y_t = \frac{Y_t}{A_t}$

$$\frac{P_t(j)}{P_t} = \frac{\varepsilon}{(\varepsilon-1)} \frac{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{u_{Ct+k}}{u_{Ct}} \frac{MC_{t+k}}{P_{t+k}} \left[\left(\frac{P_t}{P_{t+k}} \right)^{-\varepsilon} \frac{Y_{t+k}}{A_{t+k}} \frac{A_{t+k}}{A_t} \right]}{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{u_{Ct+k}}{u_{Ct}} \left[\left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} \frac{Y_{t+k}}{A_{t+k}} \frac{A_{t+k}}{A_t} \right]}$$

$$p_t^* = \frac{\varepsilon}{(\varepsilon-1)} \frac{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{u_{Ct+k}}{u_{Ct}} mc_{t+k} \left[\left(\frac{P_t}{P_{t+k}} \right)^{-\varepsilon} y_{t+k} \frac{A_{t+k}}{A_t} \right]}{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{u_{Ct+k}}{u_{Ct}} \left[\left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} y_{t+k} \frac{A_{t+k}}{A_t} \right]}$$

$$p_t^* = \frac{\varepsilon}{(\varepsilon-1)} \frac{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{C_{t+k}^{-\tau} A_{t+k}^{-(1-\tau)}}{C_t^{-\tau} A_t^{-(1-\tau)}} mc_{t+k} \left[\left(\frac{P_t}{P_{t+k}} \right)^{-\varepsilon} y_{t+k} \frac{A_{t+k}}{A_t} \right]}{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{C_{t+k}^{-\tau} A_{t+k}^{-(1-\tau)}}{C_t^{-\tau} A_t^{-(1-\tau)}} \left[\left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} y_{t+k} \frac{A_{t+k}}{A_t} \right]}$$

$$p_t^* = \frac{\varepsilon}{(\varepsilon-1)} \frac{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{c_{t+k}^{-\tau}}{c_t^{-\tau}} mc_{t+k} \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} \left(\frac{P_t}{P_t} \right)^{-\varepsilon} \left[\left(\frac{P_t}{P_{t+k}} \right)^{-\varepsilon} y_{t+k} \right]}{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{c_{t+k}^{-\tau}}{c_t^{-\tau}} \left[\left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} y_{t+k} \right]}$$

$$p_t^* = \frac{\varepsilon}{(\varepsilon-1)} \frac{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{c_{t+k}^{-\tau}}{c_t^{-\tau}} mc_{t+k} \left(\frac{P_t}{P_{t+k}} \right)^{-\varepsilon} y_{t+k} \frac{A_{t+k}}{A_t}}{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{c_{t+k}^{-\tau}}{c_t^{-\tau}} \left[\left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} y_{t+k} \frac{A_{t+k}}{A_t} \right]}$$

finally:

$$p_t^* = \frac{\varepsilon}{(\varepsilon-1)} \frac{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{c_{t+k}^{-\tau}}{c_t^{-\tau}} mc_{t+k} \left(\frac{P_t}{P_{t+k}} \right)^{-\varepsilon} y_{t+k}}{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{c_{t+k}^{-\tau}}{c_t^{-\tau}} \left(\frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} y_{t+k}}$$

Define auxiliary variables aux_{1t} and aux_{2t} to rewrite the optimal price setting equation recursively

$$p_t^* = \frac{\varepsilon}{(\varepsilon-1)} \frac{aux_{1,t}}{aux_{2,t}}$$

if we were to have modeled that firms get a production subsidy that offsets the distortion from monopolistic competition, the above option price setting equation reads (where $\nu = \frac{1}{\varepsilon}$):

$$p_t^* = (1-\nu) \frac{\varepsilon}{(\varepsilon-1)} \frac{aux_{1,t}}{aux_{2,t}} \tag{16}$$

with

$$
\begin{aligned}
aux1_{p,t} &= E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{c_{t+k}^{-\tau}}{c_t^{-\tau}} mc_{t+k} \left(\frac{P_t}{P_{t+k}}\right)^{-\varepsilon} y_{t+k} \\
&= mc_t y_t + E_t \beta\theta \left(\frac{P_t}{P_{t+1}}\right)^{-\varepsilon} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} \underbrace{\left[E_{t+1} \sum_{k=1}^{\infty} (\beta\theta)^k \frac{c_{t+k+1}^{-\tau}}{c_{t+1}^{-\tau}} mc_{t+k+1} \left(\frac{P_{t+1}}{P_{t+k+1}}\right)^{-\varepsilon} y_{t+k}\right]}_{aux_{1t+1}} \\
aux_{1t} &= mc_t y_t + E_t \beta\theta {\pi_{t+1}}^{\varepsilon} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} aux_{1t+1} \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
aux_{2t} &= E_t \sum_{k=0}^{\infty} (\beta\theta)^k \frac{c_{t+k}^{-\tau}}{c_t^{-\tau}} \left(\frac{P_t}{P_{t+k}}\right)^{1-\varepsilon} y_{t+k} \\
&= y_t + E_t \beta\theta \left(\frac{P_t}{P_{t+1}}\right)^{1-\varepsilon} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} \underbrace{\left[E_{t+1} \sum_{k=1}^{\infty} (\beta\theta)^k \frac{c_{t+k+1}^{-\tau}}{c_{t+1}^{-\tau}} \left(\frac{P_{t+1}}{P_{t+k+1}}\right)^{1-\varepsilon} y_{t+k}\right]}_{aux_{2t+1}} \\
aux_{2t} &= y_t + E_t \beta\theta \pi_{t+1}^{\varepsilon-1} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} aux_{2t+1} \qquad (18)
\end{aligned}
$$

Aggregation

production function

$$Y_t(j) = A_t N_t(j)$$

demand

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t^d$$

where

$$Y_t^d = C_t + G_t$$

plugging in and integrating over j::

$$
\begin{aligned}
\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t^d &= A_t N_t(j) \\
Y_t^d \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj &= A_t \int_0^1 N_t(j)\, dj \\
C_t + G_t &= \frac{A_t N_t}{\Delta_t}
\end{aligned}
$$

where price dispersion is defined by writing recursively:

$$\begin{aligned}
\Delta_t &\equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj \\
&= (1-\theta)\left(\frac{P_t^*(j)}{P_t}\right)^{-\varepsilon} + \theta(1-\theta)\left(\frac{P_{t-1}^*(j)}{P_t}\right)^{-\varepsilon} + \theta^2(1-\theta)\left(\frac{P_{t-2}^*(j)}{P_t}\right)^{-\varepsilon} + \ldots \qquad (19) \\
&= (1-\theta)\left(\frac{P_t^*(j)}{P_t}\right)^{-\varepsilon} + \theta\left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon}\left[(1-\theta)\left(\frac{P_{t-1}^*(j)}{P_{t-1}}\right)^{-\varepsilon} + \theta(1-\theta)\left(\frac{P_{t-2}^*(j)}{P_{t-1}}\right)^{-\varepsilon} + \ldots\right] \\
\Delta_t &\equiv (1-\theta)\left(\frac{P_t^*(j)}{P_t}\right)^{-\varepsilon} + \theta\left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon}\Delta_{t-1} \\
\Delta_t &\equiv (1-\theta)(p_t^*)^{-\varepsilon} + \theta(\pi_t)^{\varepsilon}\Delta_{t-1} \qquad (20)
\end{aligned}$$

let government expenditure be a (possibly time varying) share of output net of the costs of price dispersion, $G_t = \eta_t \frac{A_t N_t}{\Delta_t}$, and denote with $g_t = \frac{1}{1-\eta_t}$. Then, we can rewrite:

$$\begin{aligned}
C_t + G_t &= \frac{A_t N_t}{\Delta_t} \\
C_t + \eta_t \frac{A_t N_t}{\Delta_t} &= \frac{A_t N_t}{\Delta_t} \\
C_t \frac{1}{1-\eta_t} &= \frac{A_t N_t}{\Delta_t} \\
C_t g_t &= \frac{A_t N_t}{\Delta_t}.
\end{aligned}$$

we will then assume a government expenditure shock by specifying an exogenous process for g_t

law of motion for prices:

$$\begin{aligned}
P_t &= \int_0^1 \left[P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \\
P_t^{1-\varepsilon} &= (1-\theta)(P_t^*(j))^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \\
1 &= (1-\theta)\left(\frac{P_t^*(j)}{P_t}\right)^{1-\varepsilon} + \theta\left(\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon} \\
p_t^* &= \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \qquad (21)
\end{aligned}$$

1.4 Flexible price output

labor supply:

$$\xi_t N_t^{\varphi} c_t^{\tau} = w_t$$

labor demand:

$$w_t = (1-\nu)\frac{\varepsilon}{(\varepsilon-1)}$$

resource constraint:

$$c_t g_t = y_t$$

production function:

$$y_t = N_t$$

combining:

$$\begin{aligned}
\xi_t N_t^{\varphi} c_t^{\tau} &= (1-\nu)\frac{\varepsilon}{(\varepsilon-1)} \\
\xi_t y_t^{\varphi}\left(\frac{y_t}{g_t}\right)^{\tau} &= (1-\nu)\frac{\varepsilon}{(\varepsilon-1)} \\
\xi_t y_t^{\varphi+\tau}\left(\frac{1}{g_t}\right)^{\tau} &= (1-\nu)\frac{\varepsilon}{(\varepsilon-1)} \\
y_t^{flex} &= \left[(1-\nu)\frac{\varepsilon}{(\varepsilon-1)}\frac{g_t^{\tau}}{\xi_t}\right]^{\frac{1}{\varphi+\tau}}
\end{aligned}$$

1.5 List of first order and equilibrium conditions

$$\xi_t N_t^{\varphi} c_t^{\tau} = w_t \tag{22}$$

$$c_t^{-\tau} = \beta E_t c_{t+1}^{-\tau}\frac{1}{dA_{t+1}}\frac{R_t}{\pi_{t+1}} \tag{23}$$

$$p_t^* = (1-\nu)\frac{\varepsilon}{(\varepsilon-1)}\frac{aux_{1,t}}{aux_{2,t}} \tag{24}$$

$$aux_{1t} = mc_t y_t + E_t\beta\theta{\pi_{t+1}}^{\varepsilon}\frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}aux_{1t+1} \tag{25}$$

$$aux_{2t} = y_t + E_t\beta\theta\pi_{t+1}^{\varepsilon-1}\frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}aux_{2t+1} \tag{26}$$

$$y_t = N_t \tag{27}$$

$$\Delta_t \equiv (1-\theta)\left(p_t^*\right)^{-\varepsilon} + \theta\left(\pi_t\right)^{\varepsilon}\Delta_{t-1} \tag{28}$$

$$p_t^* = \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \tag{29}$$

$$mc_t = w_t \tag{30}$$

$$c_t g_t = \frac{y_t}{\Delta_t} \tag{31}$$

$$\frac{R_t}{R} = \left(\frac{R_t}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{y_t}{y_t^{flex}}\right)^{\rho_y}\right]^{1-\rho_R} e^{\varepsilon_{R,t}} \tag{32}$$

$$y_t^{flex} = \left[(1-\nu)\frac{\varepsilon}{(\varepsilon-1)}\frac{1}{\xi_t}\right]^{\frac{1}{\varphi+\tau}} \tag{33}$$

$$\log(dA_t) = \rho_A \log(dA_{t-1}) + (1-\rho_A)\, dA + \varepsilon_{dA,t} \tag{34}$$

$$\log(g_t) = \rho_g \log(g_{t-1}) + \left(1-\rho_g\right) g + \varepsilon_{g,t} \tag{35}$$

1.6 Steady state

Flexible price output:

$$y^{flex} = \left[(1-\nu)\frac{\varepsilon}{(\varepsilon-1)}\frac{1}{\xi(1-\eta)^\tau}\right]^{\frac{1}{\varphi+\tau}} \tag{36}$$

Taylor rule at stst:

$$\pi = \pi^{target} \tag{37}$$

Euler eq. at stst:

$$R = \frac{dA\pi}{\beta} \tag{38}$$

p^* at stst

$$p^* = \left[\frac{1-\theta\pi^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \tag{39}$$

Price dispersion at stst

$$\Delta \equiv \left[\frac{1}{1-\theta(\pi)^\varepsilon}(1-\theta)(p^*)^{-\varepsilon}\right]^{1-\alpha} \tag{40}$$

Pricing equation and auxiliary variables, to obtain stst mc:

$$p^* = (1-\nu)\frac{\varepsilon}{(\varepsilon-1)}\frac{aux_1}{aux_2}$$

$$
\begin{aligned}
aux_1 &= mc\ y + \beta\theta\pi^{\varepsilon} aux_1 \\
aux_1 &= \left[\frac{1}{1-\beta\theta\pi^{\varepsilon}}\right] mc\ y
\end{aligned}
$$

$$
\begin{aligned}
aux_2 &= y + \beta\theta\pi^{\varepsilon-1} aux_2 \\
aux_2 &= \left[\frac{1}{1-\beta\theta\pi^{\varepsilon-1}}\right] y
\end{aligned}
$$

$$
p^* = (1-\nu)\,\frac{\varepsilon}{(\varepsilon-1)}\,\frac{\left[\frac{1}{1-\beta\theta\pi^{\varepsilon}}\right] mc\ y}{\left[\frac{1}{1-\beta\theta\pi^{\varepsilon-1}}\right] y}
$$

$$
mc = p^*\,\frac{\varepsilon-1}{\varepsilon\,(1-\nu)}\,\frac{\left[\frac{1}{1-\beta\theta\pi^{\varepsilon-1}}\right]}{\left[\frac{1}{1-\beta\theta\pi^{\varepsilon}}\right]} \tag{41}
$$

definition of marginal cost, labor supply, resource constraint, and production function:

$$
mc = w
$$

$$
\xi N^{\varphi} c^{\tau} = w
$$

$$
cg = \frac{y}{\Delta}
$$

$$
y = N
$$

$$
\begin{aligned}
mc &= \xi N^{\varphi} c^{\tau} \\
mc &= \xi\,[y]^{\varphi}\left[\frac{y}{g}\right]^{\tau} \\
mc &= \xi\Delta^{\varphi} y^{\tau+\varphi}\frac{1}{g^{\tau}} = \xi\Delta^{\varphi} y^{\tau+\varphi}\,(1-\eta)^{\tau} \\
y &= \left[\frac{mc}{\xi\Delta^{\varphi}\,(1-\eta)^{\tau}}\right]^{\frac{1}{\tau+\varphi}}
\end{aligned}
$$

$$
y = \left[\frac{(1-\alpha)\,mc}{\xi\Delta^{\varphi}}\right]^{\frac{1}{\tau+\varphi}} \tag{42}
$$

$$
c = \frac{y}{\Delta g} \tag{43}
$$

$$N = y \tag{44}$$

$$w = mc \tag{45}$$

$$aux_1 = \left[\frac{1}{1 - \beta \theta \pi^{\frac{\varepsilon}{1-\alpha}}} \right] mc\ y \tag{46}$$

$$aux_2 = \left[\frac{1}{1 - \beta \theta \pi^{\varepsilon - 1}} \right] y \tag{47}$$