Technical Appendix to Stockman and Tesar (1995): Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements, *American Economic Review*, Vol. 85, No. 1 (Mar., 1995), p. 168-185

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1 Social Planner Solution

1.1 Setup of the Model

Social Planner problem:

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \omega u \left(c_{1t}, c_{2t}, d_t, L_t \right) + (1-\omega) u \left(c_{1t}^*, c_{2t}^*, d_t^*, L_t^* \right) \right\} \tag{1}$$

where

$$u \left(c_{1t}, c_{2t}, d_t, L_t \right) = \frac{1}{1-\sigma} \left\{ \left[\left(c_{1t}^{\theta} c_{2t}^{1-\theta} \right)^{-\mu} + d_t^{-\mu} \right]^{-\frac{1}{\mu}} \right\}^{1-\sigma} L_t^a \tag{2}$$

subject to:

the production functions:

$$Y_t^i = A_t^i K_t^{i\alpha_i} N_t^{i(1-\alpha_i)} \tag{3}$$

law of motion of capital stocks:

$$I_t^i = \gamma K_{t+1}^i - (1-\delta) K_t^i \tag{4}$$

for $i = T, T^*, NT, NT^*$.

labor constraints:

$$\begin{aligned} N_t^T + N_t^{NT} + L_t &= 1 \\ N_t^{*T} + N_t^{*NT} + L_t^* &= 1 \end{aligned} \tag{5}$$

resource constraints:

$$Y_t^T = c_{1t}^T + c_{1t}^{T*} + I_t^T \tag{6}$$

$$Y_t^{T*} = c_{2t}^T + c_{2t}^{T*} + I_t^{T*} \tag{7}$$

$$Y_t^{NT} = d_t + I_t^{NT} \tag{8}$$

$$Y_t^{NT*} = d_t^* + I_t^{NT*} \tag{9}$$

1.2 Set up the Lagrangian

$$L = E_0 \sum_{t=0}^{\infty} \beta^t$$
$$\left\{ \begin{array}{c} \omega \frac{1}{1-\sigma} \left\{ \left[\left(c_{1t}^{\theta} c_{2t}^{1-\theta} \right)^{-\mu} + d_t^{-\mu} \right]^{-\frac{1}{\mu}} \right\}^{1-\sigma} \left(1 - N_t^T - N_t^{NT}\right)^a + \\ (1-\omega) \frac{1}{1-\sigma} \left\{ \left[\left(c_{1t}^{*\theta^*} c_{2t}^{*1-\theta^*} \right)^{-\mu^*} + d_t^{*-\mu^*} \right]^{-\frac{1}{\mu^*}} \right\}^{1-\sigma} \left(1 - N_t^{T*} - N_t^{NT*}\right)^{a^*} - \\ \lambda_t^T \left[c_{1t} + c_{1t}^* + \gamma K_{t+1}^T - (1-\delta) K_t^T - A_t^T K_t^{T\alpha_T} N_t^{T(1-\alpha_T)} \right] - \\ \lambda_t^{T*} \left[c_{2t} + c_{2t}^* + \gamma K_{t+1}^{T*} - (1-\delta) K_t^{T*} - A_t^{T*} K_t^{T*\alpha_T^*} N_t^{T*(1-\alpha_T^*)} \right] - \\ \lambda_t^{NT} \left[d_t + \gamma K_{t+1}^{NT} - (1-\delta) K_t^{NT} - A_t^{NT} K_t^{NT\alpha_T} N_t^{NT(1-\alpha_{NT})} \right] - \\ \lambda_t^{NT*} \left[d_t^* + \gamma K_{t+1}^{NT*} - (1-\delta) K_t^{NT*} - A_t^{NT*} K_t^{NT*\alpha_{NT}^*} N_t^{NT*(1-\alpha_{NT}^*)} \right] \end{array} \right\}$$

where we have substituted in for constraints (3)-(5).

1.3 First Order Conditions

$\boxed{\text{w.r.t. } c_{1t}:}$

$$\omega C_t^{1-\sigma+\mu} L_t^a \left(c_{1t}^{\theta} c_{2t}^{1-\theta} \right)^{-\mu} \theta c_{1t}^{-1} = \lambda_t^T \tag{10}$$

$\boxed{\text{w.r.t. } c_{2t}:}$

$$\omega C_t^{1-\sigma+\mu} L_t^a \left(c_{1t}^{\theta} c_{2t}^{1-\theta} \right)^{-\mu} (1-\theta) c_{2t}^{-1} = \lambda_t^{T*} \tag{11}$$

$\boxed{\text{w.r.t. } c_{1t}^*:}$

$$(1-\omega)\, C_t^{*1-\sigma+\mu^*} L_t^{*a^*} \left(c_{1t}^{*\theta^*} c_{2t}^{*1-\theta^*}\right)^{-\mu^*} \theta^* c_{1t}^{*-1} = \lambda_t^T \tag{12}$$

$\boxed{\text{w.r.t. } c_{2t}^* :}$

$$(1-\omega)\, C_t^{*1-\sigma+\mu^*} L_t^{*a^*} \left(c_{1t}^{*\theta^*} c_{2t}^{*1-\theta^*}\right)^{-\mu^*} (1-\theta^*)\, c_{2t}^{*-1} = \lambda_t^{T*} \tag{13}$$

$\boxed{\text{w.r.t. } d_t :}$

$$\omega C_t^{1-\sigma+\mu} L_t^a d_t^{-\mu-1} = \lambda_t^{NT} \tag{14}$$

$\boxed{\text{w.r.t. } d_t^* :}$

$$(1-\omega)\, C_t^{*1-\sigma+\mu^*} L_t^{*a^*} d_t^{*-\mu^*-1} = \lambda_t^{NT*} \tag{15}$$

$\boxed{\text{w.r.t. } N_t^T :}$

$$\omega \frac{C_t^{1-\sigma}}{1-\sigma} a L_t^{a-1} = \lambda_t^T A_t^T K_t^{T\alpha_T} (1-\alpha_T)\, N_t^{T(-\alpha_T)} \tag{16}$$

$\boxed{\text{w.r.t. } N_t^{NT} :}$

$$\omega \frac{C_t^{1-\sigma}}{1-\sigma} a L_t^{a-1} = \lambda_t^{NT} A_t^{NT} K_t^{NT\alpha_{NT}} (1-\alpha_{NT})\, N_t^{NT(-\alpha_{NT})} \tag{17}$$

$\boxed{\text{w.r.t. } N_t^{T*} :}$

$$(1-\omega)\, \frac{C_t^{*1-\sigma}}{1-\sigma} a^* L_t^{*a^*-1} = \lambda_t^{T*} A_t^{T*} K_t^{T*\alpha_T^*} (1-\alpha_T^*)\, N_t^{T*(-\alpha_T^*)} \tag{18}$$

$\boxed{\text{w.r.t. } N_t^{NT*} :}$

$$(1-\omega)\, \frac{C_t^{*1-\sigma}}{1-\sigma} a^* L_t^{a^*-1} = \lambda_t^{NT*} A_t^{NT*} K_t^{NT*\alpha_{NT}^*} (1-\alpha_{NT}^*)\, N_t^{NT*(-\alpha_{NT}^*)} \tag{19}$$

$\boxed{\text{w.r.t. } K_{t+1}^{T}:}$

$$\lambda_t^T \gamma = \beta E_t \lambda_{t+1}^T \left\{ 1 - \delta + \alpha_T A_{t+1}^T K_{t+1}^{T\alpha_T - 1} N_{t+1}^{T1-\alpha_T} \right\} \tag{20}$$

$\boxed{\text{w.r.t. } K_{t+1}^{NT}:}$

$$\lambda_t^{NT} \gamma = \beta E_t \lambda_{t+1}^{NT} \left\{ 1 - \delta + \alpha_{NT} A_{t+1}^{NT} K_{t+1}^{NT\alpha_{NT} - 1} N_{t+1}^{NT1-\alpha_{NT}} \right\} \tag{21}$$

$\boxed{\text{w.r.t. } K_{t+1}^{T*}:}$

$$\lambda_t^{T*} \gamma = \beta E_t \lambda_{t+1}^{T*} \left\{ 1 - \delta + \alpha_T^* A_{t+1}^{T*} K_{t+1}^{T*\alpha_T^* - 1} N_{t+1}^{T*1-\alpha_T^*} \right\} \tag{22}$$

$\boxed{\text{w.r.t. } K_{t+1}^{NT*}:}$

$$\lambda_t^{NT*} \gamma = \beta E_t \lambda_{t+1}^{NT*} \left\{ 1 - \delta + \alpha_{NT}^* A_{t+1}^{NT*} K_{t+1}^{NT*\alpha_{NT}^* - 1} N_{t+1}^{NT*1-\alpha_{NT}^*} \right\} \tag{23}$$

$\boxed{\text{w.r.t. } \lambda_t^{T}:}$

$$c_{1t} + c_{1t}^* + \gamma K_{t+1}^T - (1-\delta) K_t^T = A_t^T K_t^{T\alpha_T} N_t^{T(1-\alpha_T)} \tag{24}$$

$\boxed{\text{w.r.t. } \lambda_t^{NT}:}$

$$d_t + \gamma K_{t+1}^{NT} - (1-\delta) K_t^{NT} = A_t^T K_t^{NT\alpha_T} N_t^{NT(1-\alpha_{NT})} \tag{25}$$

$\boxed{\text{w.r.t. } \lambda_t^{T*}:}$

$$c_{2t} + c_{2t}^* + \gamma K_{t+1}^{T*} - (1-\delta) K_t^{T*} = A_t^{T*} K_t^{T*\alpha_T^*} N_t^{T*(1-\alpha_T^*)} \tag{26}$$

$\boxed{\text{w.r.t. } \lambda_t^{NT*}:}$

$$d_t^* + \gamma K_{t+1}^{NT*} - (1-\delta) K_t^{NT*} = A_t^{NT*} K_t^{NT*\alpha_{NT}^*} N_t^{NT*(1-\alpha_{NT}^*)} \tag{27}$$

the above system of optimality conditions (10)-(27) provide 18 equations for 18 variables: c_1, c_2 , c_1^*, c_2^* , d, d^*, N^T, N^{NT}, N^{T*}, N^{NT*}, K^T, K^{NT}, K^{T*}, K^{NT*}, λ^T, λ^{NT}, λ^{T*}, λ^{NT*}, where we have also used the following definitions:

$$C_t = \left[\left(c_{1t}^{\theta} c_{2t}^{1-\theta}\right)^{-\mu} + d_t^{-\mu}\right]^{-\frac{1}{\mu}} \tag{28}$$

$$C_t^* = \left[\left(c_{1t}^{*\theta^*} c_{2t}^{*1-\theta^*}\right)^{-\mu^*} + d_t^{*-\mu^*}\right]^{-\frac{1}{\mu^*}} \tag{29}$$

$$L_t = 1 - N_t^T - N_t^{NT} \tag{30}$$

$$L_t^* = 1 - N_t^{T*} - N_t^{NT*} \tag{31}$$

Finally, the disturbances to technology are assumed to follow an AR(1) process:

$$\mathbf{A}_{t+1} = \Omega \mathbf{A}_t + \boldsymbol{\varepsilon}_{t+1} \tag{32}$$

where $\mathbf{A}$ is the vector $\left[A^T, A^{NT}, A^{T*}, A^{NT*}\right]$ and Ω is a 4x4 matrix describing the autoregressive component of the disturbance. The innovation to $\mathbf{A}$ is $\left[\varepsilon^T, \varepsilon^{NT}, \varepsilon^{T*}, \varepsilon^{NT*}\right]$.

The above equations can directly be coded up (see the Matlab file *ST95_planner_model.m*).

2 Decentralized Economy

2.1 Consumers

2.1.1 Intratemporal Consumption Allocation

Consumers' aggregate consumption index:

$$C_t = \left[c_t^{-\mu} + d_t^{-\mu}\right]^{-\frac{1}{\mu}}, \ C_t^* = \left[c_t^{*-\mu^*} + d_t^{*-\mu^*}\right]^{-\frac{1}{\mu^*}} \tag{33}$$

where c (c^*) is an index of tradable consumption:

$$c_t = \left(c_{1t}^{\theta} c_{2t}^{1-\theta}\right) \ , \ c_t^* = \left(c_{1t}^{*\theta^*} c_{2t}^{*1-\theta^*}\right)$$

$max P_t C_t - P_{T,t} c_t - P_{NT,t} d_t$

$\boxed{\text{w.r.t. } c_t :}$

$$c_t = \left(\frac{P_{T,t}}{P_t}\right)^{-\frac{1}{1+\mu}} C_t \ , \ c_t^* = \left(\frac{P_{T,t}^*}{P_t^*}\right)^{-\frac{1}{1+\mu^*}} C_t^* \tag{34}$$

$\boxed{\text{w.r.t. } d_t :}$

$$d_t = \left(\frac{P_{N,t}}{P_t}\right)^{-\frac{1}{1+\mu}} C_t \ , \ d_t^* = \left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\frac{1}{1+\mu^*}} C_t^* \tag{35}$$

Price index:

$$P_t = \left[P_{T,t}^{\frac{\mu}{1+\mu}} + P_{N,t}^{\frac{\mu}{1+\mu}}\right]^{\frac{1+\mu}{\mu}} \ , \ P_t = \left[P_{T,t}^{*\frac{\mu^*}{1+\mu^*}} + P_{N,t}^{*\frac{\mu^*}{1+\mu^*}}\right]^{\frac{1+\mu^*}{\mu^*}} \tag{36}$$

$max P_{T,t} c_t - p_{1,t} c_{1,t} - p_{2,t} c_{2,t}$

$\boxed{\text{w.r.t. } c_{1,t} :}$

$$c_{1,t} = \theta \left(\frac{P_{1,t}}{P_{T,t}}\right)^{-1} c_t \ , \ c_{1,t}^* = \theta^* \left(\frac{P_{1,t}^*}{P_{T,t}^*}\right)^{-1} c_t^* \tag{37}$$

$\boxed{\text{w.r.t. } c_{2,t} ::}$

$$c_{2,t} = (1-\theta)\left(\frac{P_{2,t}}{P_{T,t}}\right)^{-1} c_t \ , \ \ c_{2,t}^{*} = (1-\theta^{*})\left(\frac{P_{2,t}^{*}}{P_{T,t}^{*}}\right)^{-1} c_t^{*} \tag{38}$$

Price index:

$$P_{T,t} = \frac{\left[P_{1,t}^{\theta} P_{2,t}^{1-\theta}\right]}{\theta^{\theta}(1-\theta)^{(1-\theta)}} \ , \ \ P_{T,t}^{*} = \frac{\left[P_{1,t}^{*\theta^{*}} P_{2,t}^{*1-\theta^{*}}\right]}{\theta^{*\theta^{*}}(1-\theta^{*})^{(1-\theta^{*})}} \tag{39}$$

2.1.2 Intertemporal Consumer's Problem

maximizes expected lifetime consumption $E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} C_t^{1-\sigma} L_t^a$ subject to the budget constraint:

$$\begin{array}{c} \gamma \left[P_{1t} K_{t+1}^{T} + P_{NTt} K_{t+1}^{NT} + \sum\limits_{s_{t+1}} Q\left(s_{t+1}|s_t\right) B_{t+1}\left(j, s_{t+1}\right) \right] \\ = B_t\left(j, s_t\right) + (1-\delta)\left[P_{1t} K_t^{T} + P_{TNt} K_t^{NT}\right] \\ -P_t C_t + W_t^{T} N_t^{T} + W_t^{NT} N_t^{NT} + R_t^{T} K_t^{T} + R_t^{NT} K_t^{NT} \end{array} \tag{40}$$

Set up the Lagrangian of the Home representative consumer:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{1-\sigma} C_t^{1-\sigma}\left(1 - N_t^{T} - N_t^{NT}\right)^a \\ +\lambda_t \left[\begin{array}{c} B_t\left(j, s_t\right) + (1-\delta)\left[P_{1t} K_t^{T} + P_{NTt} K_t^{NT}\right] \\ +W_t^{T} N_t^{T} + W_t^{NT} N_t^{NT} + R_t^{T} K_t^{T} + R_t^{NT} K_t^{NT} \\ -P_t C_t - \gamma \left[P_{1t} K_{t+1}^{T} + P_{NTt} K_{t+1}^{NT} + \sum\limits_{s_{t+1}} Q\left(s_{t+1}|s_t\right) B_{t+1}\left(j, s_{t+1}\right) \right] \end{array} \right] \end{array} \right\}$$

$\boxed{\text{w.r.t. } C_t :}$

$$\lambda_t = P_t^{-1} C_t^{-\sigma} L_t^a \tag{41}$$

$\boxed{\text{w.r.t. } K_{t+1}^{T} :}$

$$\gamma \lambda_t P_{1t} = \beta E_t \lambda_{t+1}\left[(1-\delta) P_{1t+1} + R_{t+1}^{T}\right] \tag{42}$$

$\boxed{\text{w.r.t. } K_{t+1}^{NT} :}$

$$\gamma\lambda_t P_{NTt} = \beta E_t \lambda_{t+1} \left[(1-\delta) P_{NTt+1} + R_{t+1}^{NT} \right] \tag{43}$$

$\boxed{\text{w.r.t. } N_t^{T} :}$

$$\frac{C_t^{1-\sigma}}{1-\sigma} a L_t^{a-1} = \lambda_t W_t^T \tag{44}$$

$\boxed{\text{w.r.t. } N_t^{NT} :}$

$$\frac{C_t^{1-\sigma}}{1-\sigma} a L_t^{a-1} = \lambda_t W_t^{NT} \tag{45}$$

$\boxed{\text{w.r.t. } B_{t+1} :}$

$$\gamma\lambda_t Q_t = \beta E_t \lambda_{t+1} \tag{46}$$

$\boxed{\text{w.r.t. } \lambda_t :}$

$$\begin{gathered} \gamma \left[P_{1t} K_{t+1}^T + P_{NTt} K_{t+1}^{NT} + \sum_{s_{t+1}} Q\left(s_{t+1}|s_t\right) B_{t+1}\left(j, s_{t+1}\right) \right] \\ = B_t\left(j, s_t\right) + (1-\delta)\left[P_{1t} K_t^T + P_{TNt} K_t^{NT} \right] \\ -P_t C_t + W_t^T N_t^T + W_t^{NT} N_t^{NT} + R_t^T K_t^T + R_t^{NT} K_t^{NT} \end{gathered} \tag{47}$$

Foreign country's Lagrangian:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{1-\sigma} C_t^{*1-\sigma} \left(1 - N_t^{*T} - N_t^{*NT}\right)^{a^*} \\ +\lambda_t^* \left[\begin{array}{c} \varepsilon_t B_t^*\left(j, s_t\right) + (1-\delta)\left[P_{2t}^* K_t^{T*} + P_{NTt}^* K_t^{NT*} \right] \\ +W_t^{T*} N_t^{T*} + W_t^{NT*} N_t^{NT*} + R_t^{T*} K_t^{T*} + R_t^{NT*} K_t^{NT*} \\ -P_t^* C_t^* - \gamma \left[P_{2t}^* K_{t+1}^{T*} + P_{NTt}^* K_{t+1}^{NT*} + \varepsilon_t \sum_{s_{t+1}} Q\left(s_{t+1}|s_t\right) B_{t+1}^*\left(j, s_{t+1}\right) \right] \end{array} \right] \end{array} \right\}$$

$\boxed{\text{w.r.t. } C_t^{*} :}$

$$\lambda_t^* = P_t^{*-1} C_t^{*-\sigma} L_t^{*a^*} \tag{48}$$

$\boxed{\text{w.r.t. } K_{t+1}^{T*}:}$

$$\gamma \lambda_t^* P_{2t}^* = \beta E_t \lambda_{t+1}^* \left[(1-\delta) P_{2t+1}^* + R_{t+1}^{T*} \right] \tag{49}$$

$\boxed{\text{w.r.t. } K_{t+1}^{NT*}:}$

$$\gamma \lambda_t P_{NTt}^* = \beta E_t \lambda_{t+1}^* \left[(1-\delta) P_{NTt+1}^* + R_{t+1}^{NT*} \right] \tag{50}$$

$\boxed{\text{w.r.t. } N_t^{T*}:}$

$$\frac{C_t^{*1-\sigma}}{1-\sigma} a^* L_t^{*a^*-1} = \lambda_t^* W_t^{T*} \tag{51}$$

$\boxed{\text{w.r.t. } N_t^{NT*}:}$

$$\frac{C_t^{*1-\sigma}}{1-\sigma} a^* L_t^{*a^*-1} = \lambda_t^* W_t^{NT*} \tag{52}$$

$\boxed{\text{w.r.t. } B_{t+1}^*:}$

$$\gamma \lambda_t^* Q_t \varepsilon_t = \beta E_t \lambda_{t+1}^* \varepsilon_{t+1} \tag{53}$$

Combining the first order condition for state-contingent securities in both countries (equation (46) and equation (53)) and iterating backwards gives the risk sharing condition:

$$\varepsilon_t = \frac{\lambda_t^*}{\lambda_t} \tag{54}$$

2.2 Firms

tradable firms maximize $P_{1t}Y_t^T - W_t^T N_t^T - R_t^T K_l^T$ where production function is:

$$Y_t^T = A_t^T K_t^{T\alpha_T} N_t^{T(1-\alpha_T)} \tag{55}$$

FOCs:

$\boxed{\text{w.r.t. } N_t^T :}$

$$W_t^T = P_{1t}\left(1-\alpha_T\right) A_t^T K_t^{T\alpha_T} N_t^{T(-\alpha_T)} \tag{56}$$

$\boxed{\text{w.r.t. } K_t^T :}$

$$R_t^T = P_{1t}\alpha_T A_t^T K_t^{T(\alpha_T-1)} N_t^{T(1-\alpha_T)} \tag{57}$$

nontradable firms maximize $P_{NTt}y_t^{NT} - W_t^{NT} N_t^{NT} - R_t^{NT} K_l^{NT}$ where production function is:

$$Y_t^{NT} = A_t^{NT} K_t^{NT\alpha_{NT}} N_t^{NT(1-\alpha_{NT})} \tag{58}$$

FOCs:

$\boxed{\text{w.r.t. } N_t^{NT} :}$

$$W_t^{NT} = P_{NT,t}\left(1-\alpha_{NT}\right) A_t^{NT} K_t^{NT\alpha_{NT}} N_t^{NT(-\alpha_{NT})} \tag{59}$$

$\boxed{\text{w.r.t. } K_t^{NT} :}$

$$R_t^{NT} = P_{NT,t}\alpha_{NT} A_t^{NT} K_t^{NT(\alpha_{NT}-1)} N_t^{NT(1-\alpha_{NT})} \tag{60}$$

In addition, the law of one price holds:

$$P_{1t} = \varepsilon_t P_{1t}^* \tag{61}$$

$$P_{2t} = \varepsilon_t P_{2t}^* \tag{62}$$

2.3 Optimality Conditions in Nominal Terms:

Home Households' Consumption Allocation:

$$
\begin{aligned}
C_t &= \left[c_t^{-\mu} + d_t^{-\mu}\right]^{-\frac{1}{\mu}} \\
c_t &= \left(\frac{P_{T,t}}{P_t}\right)^{-\frac{1}{1+\mu}} C_t \\
d_t &= \left(\frac{P_{N,t}}{P_t}\right)^{-\frac{1}{1+\mu}} C_t \\
c_{1,t} &= \theta \left(\frac{P_{1,t}}{P_{T,t}}\right)^{-1} c_t \\
c_{2,t} &= (1-\theta) \left(\frac{P_{2,t}}{P_{T,t}}\right)^{-1} c_t \\
c_t &= c_{1,t}^{\theta} c_{2,t}^{1-\theta}
\end{aligned}
$$

Foreign Households' Consumption Allocation:

$$
\begin{aligned}
C_t^* &= \left[c_t^{*-\mu^*} + d_t^{*-\mu^*}\right]^{-\frac{1}{\mu^*}} \\
c_t^* &= \left(\frac{P_{T,t}^*}{P_t^*}\right)^{-\frac{1}{1+\mu^*}} C_t^* \\
d_t^* &= \left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\frac{1}{1+\mu^*}} C_t^* \\
c_{1,t}^* &= \theta^* \left(\frac{P_{1,t}^*}{P_{T,t}^*}\right)^{-1} c_t^* \\
c_{2,t}^* &= (1-\theta^*) \left(\frac{P_{2,t}^*}{P_{T,t}^*}\right)^{-1} c_t^* \\
c_t^* &= c_{1,t}^{*\theta^*} c_{2,t}^{*1-\theta^*}
\end{aligned}
$$

Home Households' Intertemporal Problem

$$
\begin{aligned}
\gamma C_t^{-\sigma} L_t^a \frac{P_{1t}}{P_t} &= \beta E_t C_{t+1}^{-\sigma} L_{t+1}^a \frac{1}{P_{t+1}} \left[(1-\delta) P_{1t+1} + R_{t+1}^T\right] \\
\gamma C_t^{-\sigma} L_t^a \frac{P_{NTt}}{P_t} &= \beta E_t C_{t+1}^{-\sigma} L_{t+1}^a \frac{1}{P_{t+1}} \left[(1-\delta) P_{NTt+1} + R_{t+1}^{NT}\right] \\
\frac{1}{1-\sigma} C_t^{1-\sigma} a L_t^{a-1} &= C_t^{-\sigma} L_t^a \frac{W_t^T}{P_t} \\
\frac{1}{1-\sigma} C_t^{1-\sigma} a L_t^{a-1} &= C_t^{-\sigma} L_t^a \frac{W_t^{NT}}{P_t} \\
\frac{\varepsilon_t P_t^*}{P_t} &= \frac{C_t^{*-\sigma} L_t^{*a^*}}{C_t^{-\sigma} L_t^a}
\end{aligned}
$$

Foreign country's Lagrangian:

$$
\begin{aligned}
\gamma C_t^{*-\sigma} L_t^{*a^*} \frac{P_{2t}^*}{P_t^*} &= \beta E_t C_{t+1}^{*-\sigma} L_{t+1}^{*a^*} \frac{1}{P_{t+1}^*} \left[(1-\delta) P_{2t+1}^* + R_{t+1}^{T*}\right] \\
\gamma C_t^{*-\sigma} L_t^{*a^*} \frac{P_{NTt}^*}{P_t^*} &= \beta E_t C_{t+1}^{*-\sigma} L_{t+1}^{*a^*} \frac{1}{P_{t+1}^*} \left[(1-\delta) P_{NTt+1}^* + R_{t+1}^{NT*}\right] \\
\frac{1}{1-\sigma} C_t^{*1-\sigma} a^* L_t^{*a^*-1} &= C_t^{*-\sigma} L_t^{*a^*} \frac{W_t^{T*}}{P_t^*} \\
\frac{1}{1-\sigma} C_t^{*1-\sigma} a^* L_t^{*a^*-1} &= C_t^{*-\sigma} L_t^{*a^*} \frac{W_t^{NT*}}{P_t^*}
\end{aligned}
$$

Home firms' Optimality Conditions:

$$
\begin{aligned}
W_t^T &= P_{1t} (1-\alpha_T) A_t^T K_t^{T\alpha_T} N_t^{T(-\alpha_T)} \\
R_t^T &= P_{1t} \alpha_T A_t^T K_t^{T(\alpha_T-1)} N_t^{T(1-\alpha_T)} \\
W_t^{NT} &= P_{NT,t} (1-\alpha_{NT}) A_t^{NT} K_t^{NT\alpha_{NT}} N_t^{NT(-\alpha_{NT})} \\
R_t^{NT} &= P_{NT,t} \alpha_{NT} A_t^{NT} K_t^{NT(\alpha_{NT}-1)} N_t^{NT(1-\alpha_{NT})} \\
Y_t^T &= A_t^T K_t^{T\alpha_T} N_t^{T(1-\alpha_T)} \\
Y_t^{NT} &= A_t^{NT} K_t^{NT\alpha_{NT}} N_t^{NT(1-\alpha_{NT})}
\end{aligned}
$$

Foreign firms' Optimality Conditions:

$$
\begin{aligned}
W_t^{T*} &= P_{2t}^* \left(1-\alpha_T^*\right) A_t^{T*} K_t^{T*\alpha_T^*} N_t^{T*(-\alpha_T^*)} \\
R_t^{T*} &= P_{2t}^* \alpha_T^* A_t^{T*} K_t^{T*(\alpha_T^*-1)} N_t^{T*(1-\alpha_T^*)} \\
W_t^{NT*} &= P_{NT,t}^* \left(1-\alpha_{NT}^*\right) A_t^{NT*} K_t^{NT*\alpha_{NT}^*} N_t^{NT*(-\alpha_{NT}^*)} \\
R_t^{NT*} &= P_{NT,t}^* \alpha_{NT}^* A_t^{NT*} K_t^{NT*(\alpha_{NT}^*-1)} N_t^{NT*(1-\alpha_{NT}^*)} \\
Y_t^{T*} &= A_t^{T*} K_t^{T*\alpha_T^*} N_t^{T*(1-\alpha_T^*)} \\
Y_t^{NT*} &= A_t^{NT*} K_t^{NT*\alpha_{NT}^*} N_t^{NT*(1-\alpha_{NT}^*)}
\end{aligned}
$$

Resource Constraints:

$$
\begin{aligned}
Y_t^T &= c_{1t} + c_{1t}^* + I_t^T \\
Y_t^{T*} &= c_{2t} + c_{2t}^* + I_t^{T*} \\
Y_t^{NT} &= d_t + I_t^{NT} \\
Y_t^{NT*} &= d_t^* + I_t^{NT*}
\end{aligned}
$$

Law of One Price and Real Exchange Rate:

$$
\begin{aligned}
P_{1t} &= \varepsilon_t P_{1t}^* \\
P_{2t} &= \varepsilon_t P_{2t}^* \\
RER_t &= \frac{\varepsilon_t P_t^*}{P_t}
\end{aligned}
$$

Divide all domestic nominal variables by the domestic CPI, P_t, and all foreign nominal variables by the foreign CPI, P_t^*. Define lower case prices as these real prices deflated by the CPI, e.g. $p_{1,t} = \frac{P_{1,t}}{P_t}$, $p_{T,t} = \frac{P_{T,t}}{P_t}$, $p_{NT,t} = \frac{P_{NT,t}}{P_t}$, $w_{T,t} = \frac{W_t^T}{P_t}$, $w_{T,t}^* = \frac{W_t^{T*}}{P_t^*}$, etc.

2.4 Optimality Conditions in Real Terms/ equations to be coded:

Home Households' Consumption Allocation:

$$C_t = \left[c_t^{-\mu} + d_t^{-\mu}\right]^{-\frac{1}{\mu}} \tag{63}$$

$$c_t = p_{T,t}^{\left(-\frac{1}{1+\mu}\right)} C_t \tag{64}$$

$$d_t = p_{NT,t}^{\left(-\frac{1}{1+\mu}\right)} C_t \tag{65}$$

$$c_{1,t} = \theta \left(\frac{p_{1,t}}{p_{T,t}}\right)^{-1} c_t \tag{66}$$

$$c_{2,t} = (1-\theta) \left(\frac{p_{2,t}}{p_{T,t}}\right)^{-1} c_t \tag{67}$$

$$c_t = c_{1,t}^{\theta} c_{2,t}^{1-\theta} \tag{68}$$

Foreign Households' Consumption Allocation:

$$C_t^* = \left[c_t^{*-\mu^*} + d_t^{*-\mu^*}\right]^{-\frac{1}{\mu^*}} \tag{69}$$

$$c_t^* = p_{T,t}^{*\left(-\frac{1}{1+\mu^*}\right)} C_t^* \tag{70}$$

$$d_t^* = p_{NT,t}^{*\left(-\frac{1}{1+\mu^*}\right)} C_t^* \tag{71}$$

$$c_{1,t}^* = \theta^* \left(\frac{p_{1,t}}{p_{T,t}^* RER_t}\right)^{-1} c_t^* \tag{72}$$

$$c_{2,t}^* = (1-\theta^*) \left(\frac{p_{2,t}}{p_{T,t}^* RER_t}\right)^{-1} c_t^* \tag{73}$$

$$c_t^* = c_{1,t}^{*\theta^*} c_{2,t}^{*1-\theta^*} \tag{74}$$

Home Households' Intertemporal Problem

$$\gamma C_t^{-\sigma} L_t^a p_{1t} = \beta E_t C_{t+1}^{-\sigma} L_{t+1}^a \left[(1-\delta) p_{1t+1} + r_{t+1}^T\right] \tag{75}$$

$$\gamma C_t^{-\sigma} L_t^a p_{NT,t} = \beta E_t C_{t+1}^{-\sigma} L_{t+1}^a \left[(1-\delta) p_{NT,t+1} + r_{t+1}^{NT}\right] \tag{76}$$

$$\frac{1}{1-\sigma} C_t^{1-\sigma} a L_t^{a-1} = C_t^{-\sigma} L_t^a w_t^T \tag{77}$$

$$\frac{1}{1-\sigma} C_t^{1-\sigma} a L_t^{a-1} = C_t^{-\sigma} L_t^a w_t^{NT} \tag{78}$$

$$RER_t = \frac{C_t^{*-\sigma} L_t^{*a^*}}{C_t^{-\sigma} L_t^a} \tag{79}$$

Foreign country's Lagrangian:

$$
\begin{aligned}
\gamma C_t^{*-\sigma} L_t^{*a^*} \frac{p_{2t}}{RER_t} &= \beta E_t C_{t+1}^{*-\sigma} L_{t+1}^{*a^*} \left[(1-\delta) \frac{p_{2t}}{RER_{t+1}} + r_{t+1}^{T*} \right] && (80) \\
\gamma C_t^{*-\sigma} L_t^{*a^*} p_{NT,t}^* &= \beta E_t C_{t+1}^{*-\sigma} L_{t+1}^{*a^*} \left[(1-\delta) p_{NT,t+1}^* + r_{t+1}^{NT*} \right] && (81) \\
\frac{1}{1-\sigma} C_t^{*1-\sigma} a^* L_t^{*a^*-1} &= C_t^{*-\sigma} L_t^{*a^*} w_t^{T*} && (82) \\
\frac{1}{1-\sigma} C_t^{*1-\sigma} a^* L_t^{*a^*-1} &= C_t^{*-\sigma} L_t^{*a^*} w_t^{NT*} && (83)
\end{aligned}
$$

Home firms' Optimality Conditions:

$$
\begin{aligned}
\frac{w_t^T}{p_{1t}} &= (1-\alpha_T) A_t^T K_t^{T\alpha_T} N_t^{T(-\alpha_T)} && (84) \\
\frac{r_t^T}{p_{1t}} &= \alpha_T A_t^T K_t^{T(\alpha_T-1)} N_t^{T(1-\alpha_T)} && (85) \\
\frac{w_t^{NT}}{p_{NT,t}} &= (1-\alpha_{NT}) A_t^{NT} K_t^{NT\alpha_{NT}} N_t^{NT(-\alpha_{NT})} && (86) \\
\frac{r_t^{NT}}{p_{NT,t}} &= \alpha_{NT} A_t^{NT} K_t^{NT(\alpha_{NT}-1)} N_t^{NT(1-\alpha_{NT})} && (87) \\
Y_t^T &= A_t^T K_t^{T\alpha_T} N_t^{T(1-\alpha_T)} && (88) \\
Y_t^{NT} &= A_t^{NT} K_t^{NT\alpha_{NT}} N_t^{NT(1-\alpha_{NT})} && (89)
\end{aligned}
$$

Foreign firms' Optimality Conditions:

$$
\begin{aligned}
\frac{w_t^{T*} RER_t}{p_{2t}^*} &= (1-\alpha_T^*) A_t^{T*} K_t^{T*\alpha_T^*} N_t^{T*(-\alpha_T^*)} && (90) \\
\frac{r_t^{T*} RER_t}{p_{2t}^*} &= \alpha_T^* A_t^{T*} K_t^{T*(\alpha_T^*-1)} N_t^{T*(1-\alpha_T^*)} && (91) \\
\frac{w_t^{NT*}}{p_{NT,t}^*} &= (1-\alpha_{NT}^*) A_t^{NT*} K_t^{NT*\alpha_{NT}^*} N_t^{NT*(-\alpha_{NT}^*)} && (92) \\
\frac{r_t^{NT*}}{p_{NT,t}^*} &= \alpha_{NT}^* A_t^{NT*} K_t^{NT*(\alpha_{NT}^*-1)} N_t^{NT*(1-\alpha_{NT}^*)} && (93) \\
Y_t^{T*} &= A_t^{T*} K_t^{T*\alpha_T^*} N_t^{T*(1-\alpha_T^*)} && (94) \\
Y_t^{NT*} &= A_t^{NT*} K_t^{NT*\alpha_{NT}^*} N_t^{NT*(1-\alpha_{NT}^*)} && (95)
\end{aligned}
$$

Resource Constraints:

$$
\begin{aligned}
Y_t^T &= c_{1t} + c_{1t}^* + I_t^T && (96)\\
Y_t^{T*} &= c_{2t} + c_{2t}^* + I_t^{T*} && (97)\\
Y_t^{NT} &= d_t + I_t^{NT} && (98)\\
Y_t^{NT*} &= d_t^* + I_t^{NT*} && (99)
\end{aligned}
$$