Exercise set 2 Solutions

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1. Given the following result of a partial digest experiment, find all solutions:

2, 3, 7, 8, 9, 9, 10, 11, 12, 17, 18, 19, 21, 26, 29.

Solution:

D(x) = {2, 3, 7, 8, 9, 9, 10, 11, 12, 17, 18, 19, 21, 26, 29}

L = D(x) and first point is $x_1 = 0$: x = {0}

- $29 \in L$

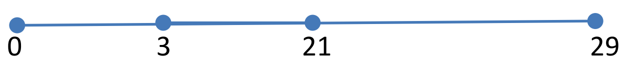
  The second point is $x_2 = 29$.
  x = {0,29}.
  Hence, L = {2, 3, 7, 8, 9, 9, 10, 11, 12, 17, 18, 19, 21, 26}.

- $26 \in L$

  There are two possibilities: either $3 \in x$ or $26 \in x$ (considered in another possible solution).
  Let's Choose $3 \in x$.
  x = {0, 3, 29}.
  Take distances |0-3| = 3 and |29-3| = 26 out of L.
  Hence, L = {2, 7, 8, 9, 9, 10, 11, 12, 17, 18, 19, 21}.

- $21 \in L$

  There are two possibilities: either $21 \in x$ or $8 \in x$.
  Let's Choose $21 \in x$. $8 \in x$ gives contradiction since |8-3| = $5 \notin L$.
  x = {0, 3,21, 29}.
  Computing the distances between 21 and all other points in x. Take them out of L.
  So in this case taking 21,18, and 8 out of L.
  Hence, L = {2, 7, 9, 9, 10, 11, 12, 17, 19}.

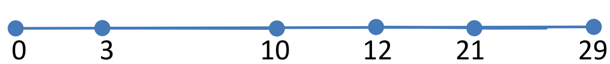


- $19 \in L$

  There are two possibilities: either $19 \in x$ or $10 \in x$.
  $19 \in x$ gives contradiction since |19-3| = $16 \notin L$.
  Let's Choose $10 \in x$ and x = {0, 3,10,21, 29}.
  Take 10,7,11, and 19 out of L.
  Hence, L = {2,9, 9, 12, 17}.

- $17 \in L$

  There are two possibilities: either $17 \in x$ or $12 \in x$.
  $17 \in x$ gives contradiction since the differences $14, 4 \notin L$.
  Let's Choose $12 \in x$ and x = {0, 3,10,12,21, 29}.
  Take the differences 12,9,2,9, and 17 out of L.
  Hence, L = {}.



Hence one solution is x ={0,3,10,12,21,29}

Another possible solution:

D(x) = {2, 3, 7, 8, 9, 9, 10, 11, 12, 17, 18, 19, 21, 26, 29} , L = D(x) and first point is x_1 = 0: x = {0}

- 29 ϵ L

  The second point is x_2 = 29.
  x = {0,29}.
  Hence, L = {2, 3, 7, 8, 9, 9, 10, 11, 12, 17, 18, 19, 21, 26}.

- 26 ϵ L

  There are two possibilities: either 26 ϵ x or 3 ϵ x (Already considered in previous solution).
  Let's Choose 26 ϵ x.
  x = {0, 26, 29}.
  Take the distances |0-26| = 26 and |29-26| = 3 out of L.
  Hence, L = {2, 7, 8, 9, 9, 10, 11, 12, 17, 18, 19, 21}.

- 21 ϵ L

  There are two possibilities: either 21 ϵ x or 8 ϵ x. 21 ϵ x gives contradiction since the distance 5∉ L.
  Hence, 8 ϵ x.
  x = {0, 8,26, 29}.
  Take the distances 8,18,21 out of L.
  Hence, L = {2, 7, 9, 9, 10, 11, 12, 17, 19}.

- 19 ϵ L

  There are two possibilities: either 19 ϵ x or 10 ϵ x. 10 ϵ x gives contradiction since the distance 16∉ L.
  Hence, 19 ϵ x.
  x = {0, 8,19,26, 29}.
  Take the distances 19,11,7,10 out of L.
  Hence, L = {2, 9, 9, 12, 17}.

- 17 ϵ L

  There are two possibilities: either 17 ϵ x or 12 ϵ x. 12 ϵ x gives contradiction since the distance 4,14∉ L.
  Hence, 17 ϵ x.
  x = {0, 8, 17,19,26, 29}.
  Take the distances 17,9,2,9,12 out of L.
  Hence, L = {}.
  Another possible solution is x ={0, 8, 17,19,26, 29}

2. Given the following result of a partial digest experiment, find one solution: 1, 1, 2, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 7, 7, 7, 8, 9, 10, 11, 12.

Solution:

D(x) = {1, 1, 2, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 7, 7, 7, 8, 9, 10, 11, 12} , L = D(x) and first point is x_1 = 0: x = {0}

- 12 ϵ L

The second point is $x_2 = 12$.
x = {0,12}.
Hence, L = {1, 1, 2, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 7, 7, 7, 8, 9, 10, 11}.

- 11 ϵ L

    There are two possibilities, either 11 ϵ x or 1 ϵ x.

    Lets choose 1 ϵ x. x = {0,1,12}

    Taking the differences/distances 1, 11 out of L.

    Hence, L = {1 , 2, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 7, 7, 7, 8, 9, 10 }.

- 10 ϵ L

    There are two possibilities, either 10 ϵ x or 2 ϵ x.
    Lets choose 10 ϵ x. x = {0,1,10,12}
    Taking the differences/distances 10, 9,2 out of L.
    L = {1 , 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 7, 7, 7, 8}

- 8ϵ L

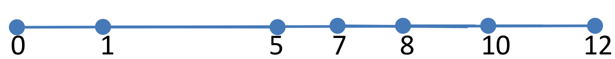
    There are two possibilities, either 8ϵ x or 4 ϵ x.
    For 4 ϵ x gives contradiction since the computed distances are 4,3,6,6 and 4,3,6 ϵ L but one more 6∉ L.
    So we consider 8 ϵ x. x = {0,1,8,10,12}
    Taking the differences/distances 8, 7,2,4 out of L.
    L = {1 ,2, 3, 3, 4, 5, 5, 5, 6, 7, 7}.

- 7ϵ L

    There are two possibilities, either 7ϵ x or 5 ϵ x.
    Choosing 7ϵ x.
    x = {0,1,7,8,10,12}.
    Taking the distances 7,6,1,3,5 out of L.
    L = {2, 3, 4, 5, 5, 7}.

- 5ϵ L

    There are two possibilities, either 5ϵ x or 7 ϵ x.
    Choosing 5ϵ x.
    x = {0,1,5,7,8,10,12}.
    Taking the distances 5,4,2,3,5,7 out of L.
    L = { }.

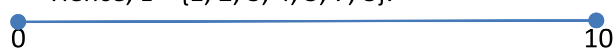
3. Given the following result of a partial digest experiment, find a solution: 1, 2, 3, 4, 5, 7, 8, 10. Same problem knowing that some of the numbers in the previous list may occur several times although they are only listed once.

Solution:

D(x) = {1, 2, 3, 4, 5, 7, 8, 10} , L = D(x) and first point is $x_1 = 0$: x = {0}

- 10 ϵ L

  The second point is $x_2 = 10$.
  x = {0,10}.
  Hence, L = {1, 2, 3, 4, 5, 7, 8}.



- 8ϵ L

  There are two possibilities, either 8ϵ x or 2 ϵ x.
  Choosing 8ϵ x. Taking the distances 8,2 out of L.
  x = {0,8,10}, L = {1,3, 4, 5, 7}.

- 7ϵ L

  There are two possibilities, either 7ϵ x or 3 ϵ x.
  Choosing 3 ϵ x. Taking the distances 3,5,7 out of L.
  x = {0,3,8,10}, L = {1, 4 }.

- 4ϵ L

  There are two possibilities, either 4ϵ x or 6 ϵ x.

  For 4ϵ x the computed distances are 4,1,4,6 and it gives contradictions since distances 4,1 ϵ L but ,4,6∉ L.

  For 6ϵ x the computed distances are 6,3,2,4 and it gives contradictions since distances 4ϵ L but 6,3,2∉ L.

So here go back to option 7ϵ L. x = {0,8,10}, L = {1,3, 4, 5, 7}.

  Choosing 7ϵ x. Taking the distances 7,1,3 out of L.

  x = {0,7,8,10}, L = { 4, 5}

- 5ϵ L

  There are two possibilities, either 5ϵ x or 5 ϵ x.
  5 ϵ x. Gives contradiction since the distances 5 ϵ L but 5,2,3∉ L .

So here go back to option 8ϵ L. x = {0,10} and L = {1, 2, 3, 4, 5, 7, 8}.

- 8ϵ L

  Choosing 2ϵ x. Taking the distances 2,8 out of L.

  x = {0,2,10} and L = {1, 3, 4, 5, 7}

- 7ϵ L

  There are two possibilities, either 7ϵ x or 3 ϵ x.

  Choosing 7ϵ x, take out the distances 7,5,3 out of L.

  x = {0,2,7,10} and L = {1, 4}

Backtracking does not help to find a solution.

Knowing that some numbers might occur several times although they are listed once in {1,2,3,4,5,7,8,10}. We assume 3 and 7 appear 2 times. Given problem still have solutions.

{0,3,7,8,10} and {0,2,3,7,10} both digest in to {1,2,3,3,4,5,7,7,8,10}.

	0	3	7	8	10
0		3	7	8	10
3			4	5	7
7				1	3
8					2
10					

	0	2	3	7	10
0		2	3	7	10
2			1	5	8
3				4	7
7					3
10					

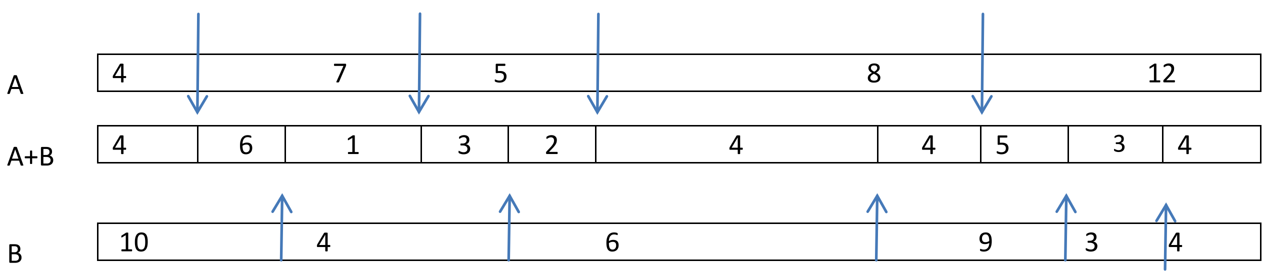
4. Given the following results of a double digest experiment, find a solution:

Enzyme A: 4, 5, 7, 8, 12.

Enzyme B: 3, 4, 4, 6, 9, 10.

Enzymes A+B: 1, 2, 3, 3, 4, 4, 4, 4, 5, 6

<u>Solution:</u>



5.Check if the matrix below has the C1P property and if it has it, determine all permutations of the columns that leave the 1s consecutive in each row.

	1	2	3	4	5	6	7	8	9	10
1	1	1	0	1	0	1	1	1	1	1
2	1	0	0	0	0	0	0	1	0	0
3	0	1	0	0	0	1	0	0	1	0
4	0	0	0	1	0	1	1	0	1	0
5	1	0	1	0	1	0	0	0	0	0
6	0	1	0	0	0	0	0	0	1	0

Solution:

Making sets of columns. For S_i the set of columns k with $M_{ik} = 1$.

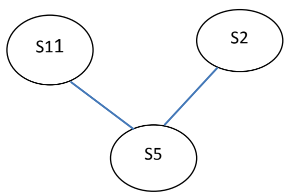
S1 = {1,2,4,6,7,8,9,10}	S1 ∩ S5 = {1}
S2 = {1,8}	S2 ⊆ S1
S3 = {2,6,9}	S3 ⊆ S1, S3 ∩ S4 = {6,9}(6,9 are consecutive in S3,

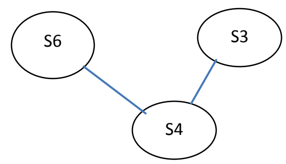
	component and matrix have C1P property)
S4 = {4,6,7,9}	S4 ⊆ S1 , S4 ∩ S6 = {9}
S5 = {1,3,5}	S5 ∩ S2 = {1}
S6 = {2,9}	S6 ⊆ S1

Creating a graph on the basis of above information. Each S_i is a node.

Considering 3 cases:

- If $S_i \cap S_j = \emptyset$ then rows i & j have to be dealt separated. S_i and Sj are not connected.
- If $S_i \subseteq S_j$ Or $S_i \subseteq S_j$ then we deal with rows i and j separately. S_i and Sj are not connected.
- If $S_i \cap S_j \neq \emptyset$ then they have to treated simultaneously. S_i and Sj are connected.





First we can take S_6 = {2,9}, S_4 = {4,6,7,9}, S_3 = {2,6,9}

	...	..	{4}	{6}	{2,9}	{2,9}	{7}	..
1			1	1	1	1	1	
2			0	0	0	0	0	
3			0	1	1	1	0	
4			1	1	1	1	1	
5			0	0	0	0	0	
6			0	0	1	1	0	

Now we can take S_1 = {1,2,4,6,7,8,9,10} , S_2 = {1,8}, S_5 = {1,3,5}.

	{3,5}	{3,5}	{1,8}	{1,8}	{2}	{9}	{6}	{4,7}	{4,7}	{10}
1	0	0	1	1	1	1	1	1	1	1
2	0	0	1	1	0	0	0	0	0	0
3	0	0	0	0	1	1	1	0	0	0
4	0	0	0	0	0	1	1	1	1	0
5	1	1	1	0	0	0	0	0	0	0
6	0	0	0	0	1	1	0	0	0	0

The given matrix has C1P property and above is one of the possible permutations that is found.

Some of the possible permutations are listed as follows.

(3,5,1,8,2,9,6,4,7,10)	(5,3,1,8,2,9,6,4,7,10)
(3,5,1,8,2,9,6,7,4,10)	(5,3,1,8,2,9,6,7,4,10)
(3,5,1,8,10,2,9,6,7,4)	(3,5,1,8,4,7,6,9,2,10)
(4,7,6,9,2,10,8,1,3,5)	(10,2,9,6,4,7,8,1,3,5)

6. Does the following matrix have the C1P property? If not, find a permutation of columns that transforms it into a matrix with a minimal number of gaps in its blocks of consecutive 1s.

	P1	P2	P3	P4
C1	1	0	1	0
C2	0	1	1	0
C3	1	1	0	1
C4	0	1	1	0

<u>Solution:</u>

Consider following submatrix

1	0	1
0	1	1
1	1	0

This submatrix does not have C1P property. Any matrix which contains this as submatrix is not having C1P property. So given matrix does not have C1P property.

Now finding a permutation of columns that transforms it into a matrix with minimum gaps in between the blocks of consecutive 1s.

Making a graph from this matrix.

Reduce the problem in to the traveling salesman problem. Visiting each vertex exactly once. Minimizing the number of gaps in rows is the same to minimizing the weight of the graph cycle.

Cycle weight = No. of Gap transitions + 2*(Number of vertices)

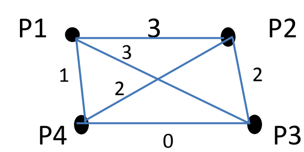
Each gap contributes 2 to weight.

Cycle weight = 2*(Number of gaps) + 2*(Number of vertices)

-Vertex for each column.

-Number of columns where the two columns differ will be the weight of the edge between two vertices.

	P1	P2	P3	P4
C1	1	0	1	0
C2	0	1	1	0
C3	1	1	0	1
C4	0	1	1	0



Path	Gaps	Cycle weight 2*(Number of gaps) + 2*(Number of vertices)
P1 -> P4 -> P3 -> P2	2	12
P1 -> P2 -> P3 -> P4	2	12
P1 -> P4 -> P2 -> P3	1	10
P2 -> P1 -> P3 -> P4	3	14
P2 -> P3 -> P4 -> P1	2	12
P2 -> P4 -> P1 -> P3	2	12
P1 -> P2 -> P4 -> P3	3	14

Permutation that transforms the matrix with minimum gaps is : (P1, P4, P2, P3)

	P1	P4	P2	P3
C1	1	0	0	1
C2	0	0	1	1
C3	1	1	1	0
C4	0	0	1	1

Note: Following is one more example giving an idea about the C1P property.

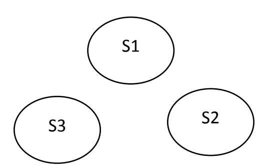
We have following matrix M.

	A	B	C	D	E	F
1	1	0	0	1	0	0
2	0	0	1	0	0	1
3	0	1	0	0	1	0

We calculate all the subsets. The subset S_i is the set of columns k with $M_{ik} = 1$.

S1= {A,D}, S2 = {F,C}, S3 = {B,E}

Next we make a graph. Here no subsets are connected. The given matrix has consecutive 1 property.



	A	D	F	C	B	E
1	1	1	0	0	0	0
2	0	0	1	1	0	0
3	0	0	0	0	1	1