

Specific Heat: $Q = mc\Delta T$

Heat Capacity: $Q = C\Delta T$

Energy in Capacitor: $Q = CV$

Switching Energy: $E = \frac{1}{2}CV^2$ (per charge/discharge)

Switching Energy: $E = CV^2$ (full cycle)

Power per Cycle: $P = E \times f$

Power per Cycle w/ Activity Factor: $P = \alpha CV^2 f$

Resistance: $R = \rho \frac{L}{A} = \rho \frac{L}{w \times h}$

Wire Thickness: $t = \frac{\rho}{R_s}$

Sheet Resistance: $R_s = \frac{\rho}{h} \parallel R = R_s \frac{L}{W}$

Capacitance per Unit Area: $C = C_A WL$

Elmore Delay: $\delta_E = \frac{1}{2}rcn(n+1) \parallel \delta_E = c_1 r_1 + c_2(r_1 + r_2) \dots$

$\delta_E = \frac{1}{2}R_s \frac{L}{W} \times C_A WL \times \frac{L_T^2}{L^2} = \frac{1}{2}R_s C_A L_T^2 \parallel n = \frac{L_T}{L}$

Capacitance Scaling: $C_g = \frac{c_g}{x}$

Drive Voltage Scaling: $V_{dd} = \frac{V_{dd}}{x}$

Current Scaling: $I = \frac{I}{x}$

Switching Time Scaling: $\tau = RC = \frac{\tau}{x}$

Switching Energy Scaling: $E = \frac{E}{x^3}$

Power Scaling: $P = \frac{P}{x^2}$

Power Density Scaling: not changed!

Delay Scaling: unchanged for local interconnect, scales with L^2 for global interconnect

Local	Global
$R' = R \cdot x$	$R' \propto L$
$C' = C/x$	$C' \propto L$
Delay not changed	Delay $\propto L^2$

Zeroth Law: If bodies A and B are each in thermal equilibrium with a third body T, then A and B are in thermal equilibrium with each other.

Heat: the transferred energy between a system and its environment because of a temperature difference that exists between them.

Energy Transfer Rate: $q = \frac{Q}{t}$

Fourier's Law of Heat Conduction: $q = -k \frac{\Delta T}{\Delta x} (T_H - T_C)$

Thermal Resistance: $q = \frac{\Delta T}{R_T} = \frac{T_1 - T_2}{L/kA} \parallel R_T = \frac{L}{kA} = \frac{\Delta T}{q}$

Newton's Law of Cooling: $q_c = hA(T_s - T_a)$

- h = heat transfer coefficient, T_a = environment temp.

Fins: $q_c = NhA_{fin} \frac{3}{4}(T_{base/s} - T_a)$

- $A_{fin} = W_{chip} \times L_{fin} \times 2$
- $N = \frac{W_{chip} + gap_{fin}}{t_{fin} + gap_{fin}}$
- $T_s = T_j - q(R_{ic} + R_{el} + R_{hs})$

First Law: The internal energy E_{int} of a system tends to increase if energy is added as heat Q and tends to decrease if energy is lost as work W done by the system ($\Delta E_{int} = Q - W$)

Second Law: If a process occurs in a closed system, the entropy of the system increases for irreversible processes and remains constant for reversible processes.

Adiabatic Process: occurs so rapidly or system is so well insulated that no transfer of energy as heat occurs between the system and its environment, i.e. $Q = 0$.

Constant-Volume Process: If the volume of a system is held constant, that system cannot do work, i.e. $W = 0$.

Cyclical Process: After certain interchanges of heat and work, the system is restored to its initial state, i.e. $Q = W$.

Free Expansion: Special adiabatic processes in which no transfer of energy as heat occurs between the system and its environment and no work is done by the system, i.e. $Q = W = 0$.

- $\Delta S = nR \ln \frac{V_f}{V_i}$

Isothermal Process: Temperature is constant while volume/pressure changes, $\Delta T = 0$. ($pV = nRT = \text{constant}$)

Ideal Gas: $pV = nRT$ ($R = 8.314 \text{ Jmol}^{-1} \text{ K}^{-1}$)

Isothermal Work: $W = nRT \ln \frac{V_f}{V_i}$ (done by ideal gas)

- If positive, gas does work on environment

Average Translational Kinetic Energy: $K_{avg} = \frac{3}{2}kT$

- At a given temperature T, all ideas gases (no matter their mass) have the same average translational KE

Internal Energy: $E_{int} = \frac{3}{2}nRT$

Molar Specific Heat at Const. Vol:

- Monatomic: $C_v = \frac{3}{2}R$
- Diatomic: $C_v = \frac{5}{2}R$
- Polyatomic: $C_v = 3R$
- $Q = nC_v \Delta T$

Molar Specific Heat at Constant Pressure: $C_p = C_v + R$

- $Q = nC_p \Delta T$

Adiabatic Process: $pV^\gamma = \text{constant}$, $TV^{\gamma-1} = \text{constant}$

- $\gamma = C_p/C_v$
- $p_B V_B^\gamma = p_C V_C^\gamma$
- $T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$

Adiabatic Work: $W = \frac{1}{1-\gamma}(P_2 V_2 - P_1 V_1)$

Fridge: Remove heat from low-temp reservoir, move it to high-temp reservoir

- Efficiency $= K_R = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L}$

- $W = Q_H - Q_L$
- You do 1 J of work to remove K_R J of heat
- $W = \frac{W}{\% \text{ Carnot limit}}$
- Q_L = heat removed/added to low-temp per cycle
- Q_H = heat removed/added to high-temp per cycle

Heat Pump:

- Efficiency $= K_P = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} = \frac{T_H}{T_H - T_L}$

Entropy Equality: $\frac{Q_L}{T_L} = \frac{Q_H}{T_H}$

Entropy: $\Delta S = \frac{Q}{T}$

Entropy Change: $\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_v \ln \frac{T_f}{T_i}$

- For one molecule: $\Delta S = k_B \ln \frac{V_f}{V_i}$
- $\Delta S = k_B N \ln \frac{V_f}{V_i}$

Constant Volume Process: $\Delta S = nC_v \ln \frac{T_f}{T_i}$

Constant Pressure Process: $\Delta S = nR \ln \frac{V_f}{V_i}$

Maxwell Speed Distribution: $P(v) = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}$

- This is for 3D

Average Molecule Speed: $v_{avg} = \sqrt{\frac{8RT}{\pi M}}$

- M: molar mass [g/mol]

RMS Molecule Speed: $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{(v^2)_{avg}}$

Most Probable Speed: $v_p = \sqrt{\frac{2RT}{M}}$

- The speed at which $P(v)$ is maximum

Magnitudes: $v_p < v_{avg} < v_{rms}$

Degrees of Freedom: the number of independent ways in which a molecule can store energy; each DoF has an energy of $\frac{1}{2}kT$ associated with it on average.

- $E_{int} = (f/2)nRT$
- $C_v = (f/2)R$
- Translational, rotational, and oscillatory motions (diatomic and polyatomic only)

Equipartition Theorem: Energy is shared equally amongst all energetically accessible degrees of freedom of a system, and each share is $\frac{1}{2}kT$.

Boltzmann Entropy: $S = k_B \ln W$

- $k_B = 1.3806 \times 10^{-23}$

Information Content: $Q = -k \log P$

Average Uncertainty: $H = -k \sum p_i \log(p_i)$

- Typically $k = 1$

Maximum Entropy: $H_{MAX} = \log N$

Algorithmic Entropy: the shortest code that produces a specified string as output.

Shannon Coding Theorem: $n = \frac{M}{M_c}$

- Describes efficiency of compression code
- M_c = entropy of compressed string

- M = lower bound Shannon entropy

Joint Entropy: $H(X, Y) = - \sum_{x,y} p(x,y) \log(p(x,y))$

- $p(x,y)$ = probability of x AND y
- $H(X, Y) = H(Y, X)$

Conditional Entropy: $H(X|Y) = - \sum_{x,y} p(x,y) \log(p(x|y))$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(B) \cdot P(A|B) = P(A) \cdot P(B|A)$

Mutual Information:

$I(X; Y) = H(X) - H(X|Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$

- Measures the correlation between two variables
- $I(X; Y) = I(Y; X)$

Relationship 1:

- $I(X; Y) = H(X) + H(Y) - H(X, Y)$
- If X,Y corr: $H(X, Y) < H(X) + H(Y)$
- else: $H(X, Y) = H(X) + H(Y)$

Relationship 2:

- $I(X; Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$
- If X,Y corr: $H(X|Y) < H(X)$; $H(Y|X) < H(Y)$
- else: $H(X|Y) = H(X)$; $H(Y|X) = H(Y)$
 - $I = 0$

Differential Entropy: $h(X) = - \int_0^L f(x) \log_2 f(x) dx$

Uniform DE: $h(X) = - \int_0^L \frac{1}{L} \log_2 \frac{1}{L} dx = \log_2 L$

Quantized Entropy: $H(X^\Delta) = (- \sum \Delta f(x_i) \log(f(x_i))) - \log \Delta$

- $H(X^\Delta) = h(X) - \log \Delta = \log_2 (L/\Delta)$
- L/Δ = number of microstates

1D Momentum: $f(v_x) = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-mv_x^2/2k_B T}$

3D Momentum: $f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2k_B T}$

Uncertainty Principle: you cannot determine the position and momentum of a molecule at the same time with the same accuracy.

- $\sigma_x \sigma_y \geq \frac{h}{4\pi}$
- $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$

Entropy Increase: As N increases, the probability of the system being in state n^* (the most probable macrostate) actually decreases. Although this decreases, we notice the “peak” is less sharp and broader, i.e. the probability of the system occupying states close to n^* increases (close to 100%).

- Probability drives entropy to increase, and also drives systems to equilibrium state

Fuel Value: the amount of energy that can be extracted from a tape.

- Random tape has zero fuel value
- $FV = (N - I)k_B T \ln 2$ (for N bits)
- Zero uncertainty/information = max fuel value

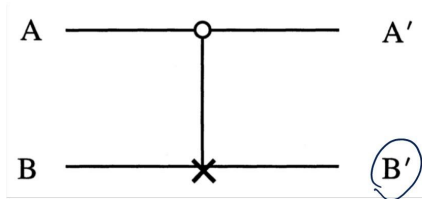
Landauer's Principle: the energy cost associated with erasing/resetting memory is $k_B T \ln 2$ per bit.

- Information is proportional to the amount of energy we need to spend if we want to reset memory

Minimum Energy for Reversible Computer: $E_{min} = Nk_B T \ln 2$

- For N-bit output, theoretical minimum that is being chased
- T in Kelvin!

Controlled NOT Gate:



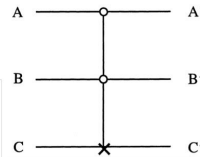
A	B	A'	B'
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

- If input to O-wire is 1, then input to X-wire (NOT gate) is inverted
- Otherwise, both pass through unchanged
- O-wire input is never changed

CCN Gate:

Controlled Controlled NOT (CCN) gate

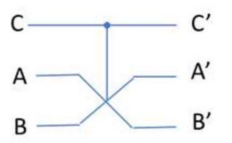
A	B	C	A'	B'	C'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0



- X-wire is inverted only if both O-inputs are 1

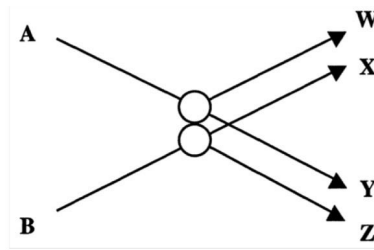
Fredkin Gate:

C	A	B	C'	A'	B'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1



- AND: Set B to 0; $B' = A \text{ AND } C$
- NOT: Set A to 0 and B to 1; $B' = \text{NOT } C$
- OR: Set B to 1; $A' = A \text{ OR } C$

Billiard Balls:



- Outputs X and Y can realize the CN Gate

Feynman Rate r: At any given stage, it is r times more likely to make a forward calculation than a backward one.

- $E = kT \log r$
- Smaller r means less energy is required

Brownian Computer:

- Two "wells" and a barrier
- Wells have energy E_1 and E_2 , $E_1 > E_2$. To go forward, a molecule in well E_1 needs less energy to overcome the barrier and so forward computation is favoured.
- Rate is proportional to $\exp[\frac{E_1 - E_2}{kT}]$
- Prob. of moving forward is proportional to $\exp[\frac{-(A - E_1)}{kT}]$
- Prob. of moving backward is prop. to $\exp[\frac{-(A - E_2)}{kT}]$
- If $r > 1$, then forward; if $0 < r < 1$, then backward

State-Based Rate: $r = n_2/n_1$

- n_2 states in forward direction, n_1 in backward

Energy Loss per Step:

$$kT \log r = kT (\log n_2 - \log n_1) = (S_2 - S_1)T$$

- Energy loss per step is equal to entropy generated in that step

Reversible Computing:

- Process can go forward or backward with equal probability (not desirable, $r = 1$)
- All inputs can be deduced from outputs
- Zero energy cost for computations
- Energy cost comes from memory erasing/resetting

Adiabatic Computing: we can approach zero energy as speed is reduced.

- Each switching operation minimizes irreversible energy dissipation
- Conventional computer architectures/circuits can be used (can lower operating frequency to reduce energy loss)
- Could be far more efficient than reversible principles
- A balance between speed and cost
- Energy: $E = I^2 R T = C V^2 \times R C / T$

At Landauer's Limit: $k_B T \ln 2 = \frac{1}{2} C V_{DD}^2$

Johnson Noise (Capacitor): $V_N = \sqrt{\frac{k_B T}{C}} \parallel V_{DD} = N \cdot V_N$

Johnson Noise (Resistor):

- $i_{n\Delta f} = \frac{4k_B T \Delta f}{R}$
- $V_N^2 = 4k_B T R \Delta f$

Minimum Switching Energy: $E_{min} = \frac{1}{2} C V_{DD}^2 = \frac{1}{2} N^2 k_B T$

- For full switching cycle: $E_{min} = N^2 k_B T$
- For adiabatic: $k_B T \ln 2 = C V_{DD}^2 \times RC/T$

Emissivity:

- $J = \epsilon \sigma T^4$ (T in Kelvin)
- J = radiance, $\sigma = 5.67 \times 10^{-8}$
- Heat flux = Radiance \times Area

Prefix		Base 1000	Base 10
Name	Symbol		
yotta	Y	1000^8	10^{24}
zetta	Z	1000^7	10^{21}
exa	E	1000^6	10^{18}
peta	P	1000^5	10^{15}
tera	T	1000^4	10^{12}
giga	G	1000^3	10^9
mega	M	1000^2	10^6
kilo	k	1000^1	10^3
hecto	h	$1000^{2/3}$	10^2
deca	da	$1000^{1/3}$	10^1
		1000^0	10^0
deci	d	$1000^{-1/3}$	10^{-1}
centi	c	$1000^{-2/3}$	10^{-2}
milli	m	1000^{-1}	10^{-3}
micro	μ	1000^{-2}	10^{-6}
nano	n	1000^{-3}	10^{-9}
pico	p	1000^{-4}	10^{-12}
femto	f	1000^{-5}	10^{-15}
atto	a	1000^{-6}	10^{-18}
zepto	z	1000^{-7}	10^{-21}
yocto	y	1000^{-8}	10^{-24}

$$h_{si} = 130 \text{ W/m} \cdot ^\circ\text{C}; h_{el} = 0.65 \text{ W/m} \cdot ^\circ\text{C}; h_{hs} = 202 \text{ W/m} \cdot ^\circ\text{C}$$

$$P = 40 \text{ W}$$

$$A_{chip} = 625 \text{ mm}^2$$

$$L_{chip} = 0.5 \text{ mm}; L_{el} = 20 \text{ } \mu\text{m}; L_{hs} = 2 \text{ mm}$$

$$L_{fin} = 8 \text{ mm}; t_{fin} = 0.5 \text{ mm}; g_{fin} = 0.5 \text{ mm}$$

$$T_a = 20^\circ\text{C}; T_{J,max} = 85^\circ\text{C}$$

$$W_{chip} = \sqrt{A_{chip}} = N t_{tin} + (N + 1) g_{fin} \Rightarrow N = \frac{W_{chip} + g_{fin}}{t_{fin} + g_{ap}} = 25.5 \rightarrow 25 \text{ fins}$$

$$q = N A_{fin} h \frac{3}{4} (T_s - T_a) = \frac{\Delta T}{R_{total}} = 40 \text{ W}$$

$$A_{fin} = W_{chip} * L_{fin} * 2 = 400 \text{ } \mu\text{m}$$

$$T_s = T_J - q(R_J + R_{el} + R_{hs})$$

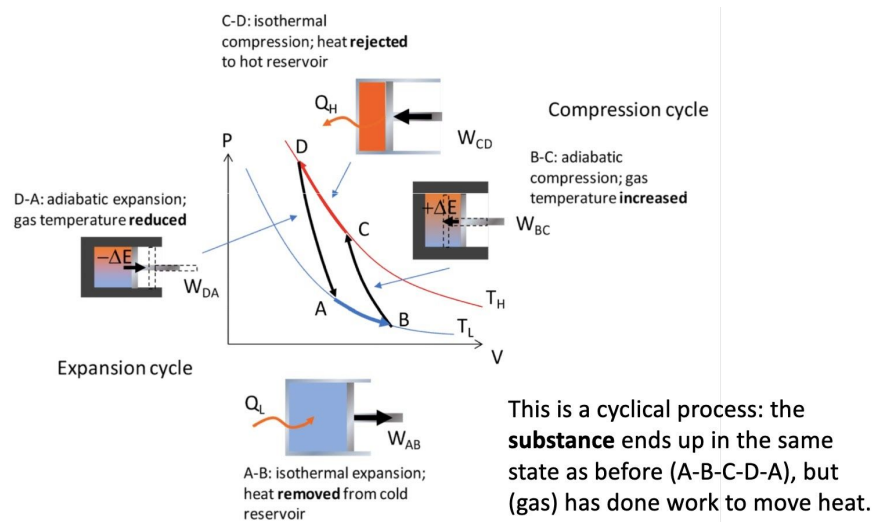
$$R_J = \frac{L_{chip}}{h_{si} A_{chip}} = 0.0062 \text{ }^\circ\text{C/W}$$

$$R_{el} = \frac{L_{el}}{h_{el} A_{chip}} = 0.0492 \text{ }^\circ\text{C/W}$$

$$R_{hs} = \frac{L_{hs}}{h_{hs} A_{chip}} = 0.0158 \text{ }^\circ\text{C/W}$$

$$T_s = T_J - q(R_J + R_{el} + R_{hs}) = 82.15^\circ\text{C}$$

$$q = N A_{fin} h \frac{3}{4} (T_s - T_a) \Rightarrow h = 85.8 \text{ W/m}^2 \cdot \text{K} \Rightarrow \text{Forced convection}$$



“If you know a ball’s suit, how much information do you gain and how much uncertainty is left about its color and vice versa?”

- How much information do you gain:
 $H(S) \equiv \text{Entropy of ball's suit}$
- How much uncertainty is left about its color:
 $H(C|S) \equiv \text{Entropy of color given suit}$
- And vice-versa: $H(C)$ and $H(S|C)$

“How much information is shared between these two features?”

- Mutual information: $I(S; C)$

