Amplitudes and hidden symmetries in N=2 Chern-Simons Matter theory

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ISRAEL PHYSICAL SOCIETY

Based on

K.I, S.Jain, P.Nayak, V.Umesh, **arXiv :1710.04227** **(BCFW)**

K.I, S.Jain, S.Majumdar, P.Nayak, T.Neogi, T.Sharma, R.Sinha, V.Umesh, **arXiv : 1711.02672** **(Dual Superconformal symmetry)**

K.I, S.Jain, P.Nayak, T.Sharma, V.Umesh, **arXiv : 1801.nnppq** **(Yangian)**

References:

K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh, S.Yokoyama, **arXiv: 1505.06571**, JHEP 1510 (2015) 176.

S.Jain, M.Mandlik, S.Minwalla, T.Takimi, S.Wadia, S.Yokoyama, **arXiv: 1404.6373,** JHEP 1504 (2015) 129.

Part I

Introduction

Introduction

- When we say that a theory is **integrable**, what do we really mean?
- Classically $\text{symmetries} = \text{degrees of freedom}$
- Several examples: n - dimensional harmonic oscillator, Central force motion, Heisenberg spin chain system, Sine-Gordon equation.
- At **quantum** level, there is **no universal notion of integrability**, often presence of infinite dimensional symmetry structures.
- Some of the best known quantum field theories that arise in the low energy limits of string theory are **conjectured to be integrable**.

$$d = 4,\ \mathcal{N} = 4 \text{ Super Yang Mills}$$
$$d = 3,\ \mathcal{N} = 6 \text{ ABJM}$$

- In both cases, in the planar limit, all the **tree level superamplitudes possess an infinite dimensional symmetry known as the Yangian**.
- It will be interesting to ask if such symmetry structures arise in **theories with less or no supersymmetry**.

Introduction

- $d = 3$, $\mathcal{N} = 2$ superconformal Chern-Simons theory coupled to matter in fundamental representation of U(N)

$$\mathcal{S}^L_{\mathcal{N}=2} = \int d^3x \Big[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right) + \bar{\psi} i \not{\mathcal{D}} \psi - \mathcal{D}^\mu\bar{\phi}\mathcal{D}_\mu\phi + \frac{4\pi^2}{\kappa^2}(\bar{\phi}\phi)^3 + \frac{4\pi}{\kappa}(\bar{\phi}\phi)(\bar{\psi}\psi) + \frac{2\pi}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi)\Big]$$

- The theory exhibits a **strong-weak self duality** under the duality map

$$\kappa' = -\kappa \ , \ \ N' = |\kappa| - N + 1 \ , \ \ \lambda' = \lambda - \mathrm{Sgn}(\lambda)$$

- **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama :** $2 \to 2$ scattering amplitudes to all orders in the ’t Hooft coupling. (summing planar diagrams)

- In the symmetric, anti-symmetric and adjoint channels of scattering the **amplitude is tree-level exact to all orders in λ**.

- In the singlet channel the coupling dependence is **extremely simple**.

2→2 scattering amplitude to all orders in λ

- Tree level super amplitude

$$T_{\text{tree}} = \frac{4\pi}{\kappa}\frac{\langle 12\rangle}{\langle 23\rangle}\delta(\sum_{i=1}^{4} p_i)\delta^2\left(\mathcal{Q}\right)$$

$$\delta^2(Q) = \sum_{i<j=1}^{n} \langle ij\rangle \eta_i\eta_j$$

- **All loop** super amplitude

$$T_{symm}^{all\ loop} = T_{Asymm}^{all\ loop} = T_{Adj}^{all\ loop} = \ T_{tree}$$

$$T_{singlet}^{all\ loop} = N\frac{\sin\pi\lambda}{\pi\lambda}T_{tree}$$

$$S_{Sy/Asy/Ad} = I + T_{Sy/Asy/Ad}^{all\ loop}$$

$$S_{Sing} = \cos(\pi\lambda)I + T_{Sing}^{all\ loop}$$

- Passes all consistency checks: **Unitarity and Duality**

Motivation

- Why is the $2 \to 2$ particle scattering in the Sym/Asym/Adj channels **tree level exact**? and why does it have a very **simple coupling dependence in singlet channel**?

- Maybe some **powerful symmetry** that protects the amplitude from renormalization.

- Is it possible to compute **all loop $m \to n$ scattering amplitudes** in the N=2 theory at least in the planar limit?

- Does the **non-renormalization** results of the $2 \to 2$ scattering continue to persist for arbitrary higher point amplitudes?

- These computations would also **test the duality** in regions un-probed by large N perturbation theory yet.

Our results

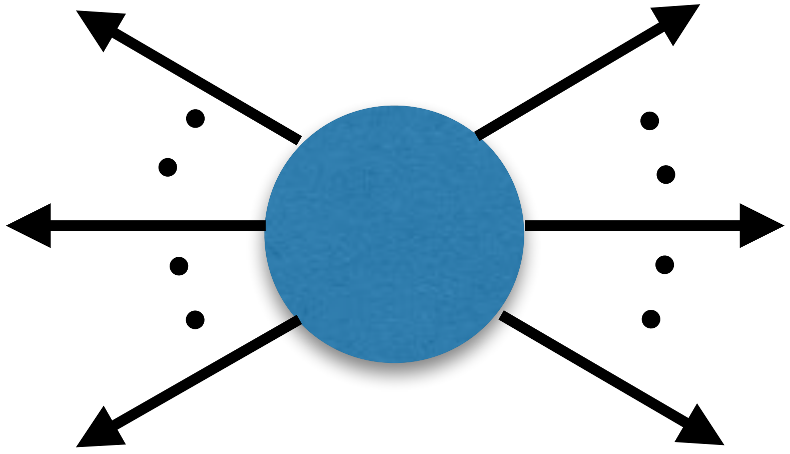
- As a first step towards the all loop m→n scattering, is it possible to write down **arbitrary m → n tree level amplitudes** ?
- For the particle only scattering, we are able to achieve this via **BCFW recursions.** **K.I, Jain, Nayak, Umesh**
- As a first step towards thinking about higher point loop amplitudes we identify a **hidden symmetry** in the 2 → 2 amplitude computed earlier that might explain the non-renormalization.
- This symmetry is known as **dual superconformal symmetry**.

  K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh
- The superconformal symmetry and dual superconformal symmetry together generate an infinite dimensional symmetry known as the **Yangian**.

  K.I, Jain, Nayak, Sharma, Umesh, to appear
- If all tree level amplitudes possess this symmetry then the theory may be **integrable!**

Part II

All tree level amplitudes



- K.I, S.Jain, P.Nayak, V.Umesh, **arXiv :1710.04227**

BCFW recursions in 2+1 dimensions

- Recursion relations enable to construct **n point tree level scattering amplitudes from lower point tree level amplitudes.**

  Britto, Cachazo, Feng, Witten

- Central idea: **Dixon**
  - Tree level amplitudes are **continuously deformable** analytic functions of momenta.
  - Only type of singularities that can appear at tree level are **simple poles.**
  - One can **reconstruct amplitudes** for generic scattering kinematics knowing its behavior in **singular kinematics**.
  - In these singular regions **amplitudes factorize** into causally disconnected amplitudes with fewer legs, connected by an i**ntermediate onshell state**.

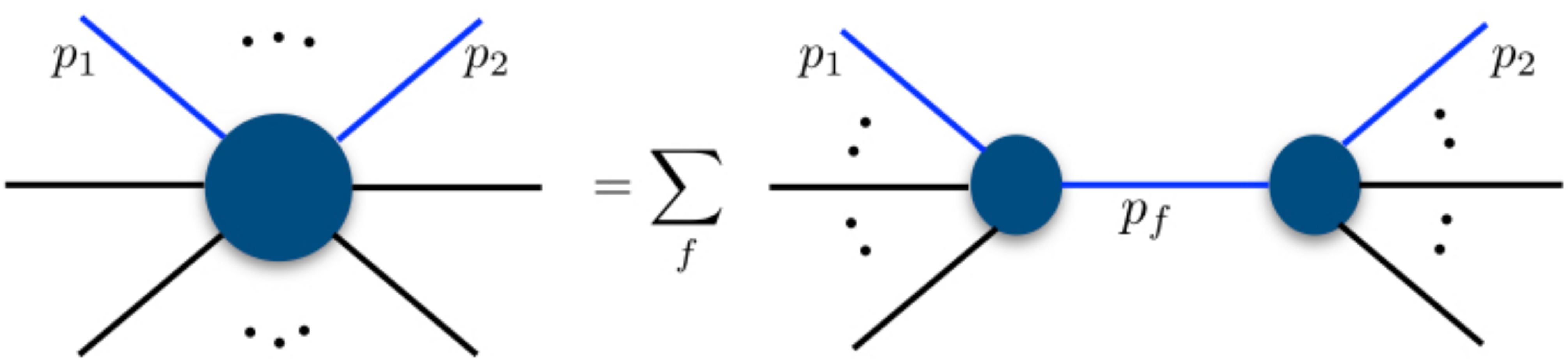
- We will focus on situation where the external particles are massless.

BCFW recursions in 2+1 dimensions

- Promote the amplitude into a one complex parameter family of amplitudes

$$A(z) = A(p_1, \ldots p_i(z), p_{i+1}, \ldots p_l(z), \ldots, p_{2n})$$

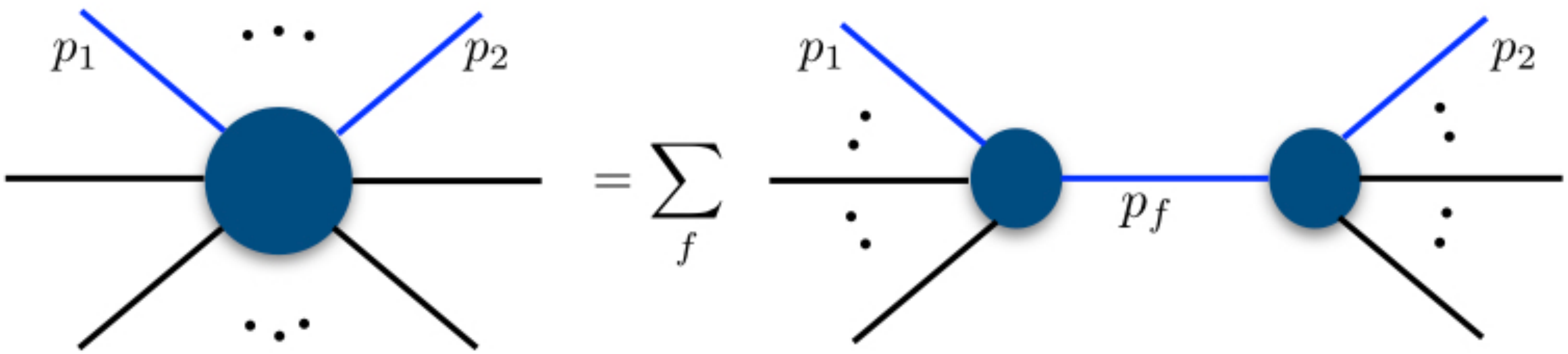
- The necessary and sufficient conditions are:
    - **The momentum deformation should preserve on-shell conditions and momentum conservation.**
    - **The amplitude should be asymptotically well behaved under the deformation.**



- A higher point amplitude factorizes into lower point amplitudes!

The recursion formula for arbitrary 2n point superamplitude

$$A_{2n}(z=1) = \sum_f \int \frac{d\theta}{p_f^2} \left(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} A_L(z_{a;f}, \theta) A_R(z_{a;f}, i\theta) + (z_{a;f} \leftrightarrow z_{b;f}) \right)$$



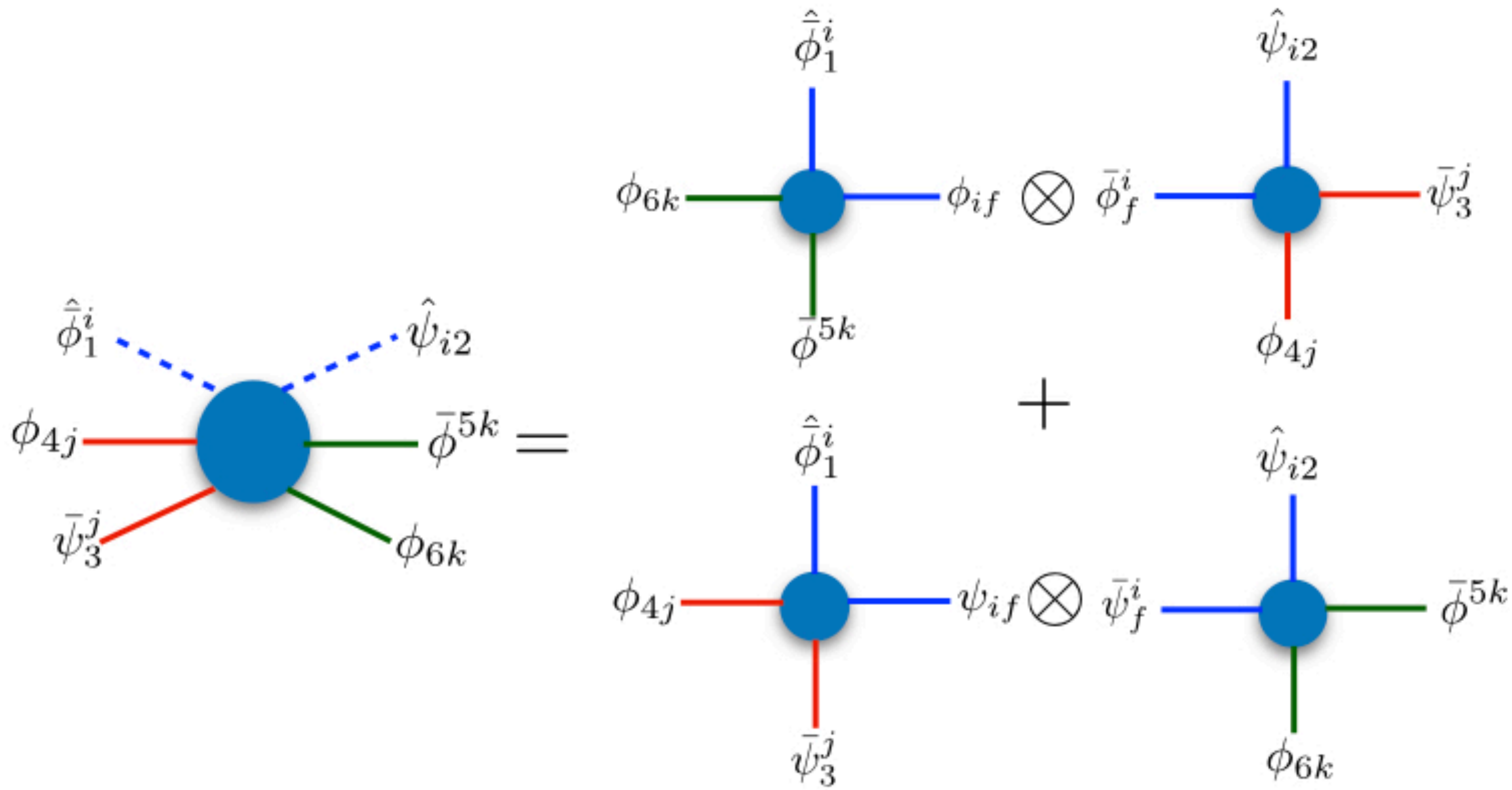
- $z_{a;f}, z_{b;f}$ are zeroes of $p_f^2(z) = 0$
- The formula can be recursively applied to write down any **higher point superamplitude in terms of products of the four point superamplitude**.

Eg: Six point amplitude as product of four point amplitudes

$$\langle\bar{\phi}_1\psi_2\bar{\psi}_3\phi_4\bar{\phi}_5\phi_6\rangle =$$

$$\left(z_{a;f}\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2}\langle\hat{\bar{\phi}}_1\hat{\phi}_f\bar{\phi}_5\phi_6\rangle_{z_{a;f}}\langle\hat{\bar{\phi}}_{(-f)}\hat{\psi}_2\bar{\psi}_3\phi_4\rangle_{z_{a;f}}+(z_{a;f}\leftrightarrow z_{b;f})\right)\frac{i}{p_f^2}\Big|_{p_f=p_{234}}$$

$$+\left(z_{a;f}\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2}\langle\hat{\bar{\phi}}_1\hat{\psi}_f\bar{\psi}_3\phi_4\rangle_{z_{a;f}}\langle\hat{\bar{\psi}}_{(-f)}\hat{\psi}_2\bar{\phi}_5\phi_6\rangle_{z_{a;f}}+(z_{a;f}\leftrightarrow z_{b;f})\right)\frac{i}{p_f^2}\Big|_{p_f=p_{256}}$$



Recursion relations for non-supersymmetric theories!

- BCFW does not apply to the **non-susy CS coupled to fermions/bosons** since the amplitudes **do not have good asymptotic behavior**.

- It is possible to extract the recursion relations for non-susy fermionic/ bosonic CS matter theories from the N=2 results!! Eg:

    - At **tree level,** the Feynman diagrams for an **all fermion amplitude are same** for susy/non-susy theory.

    - **Susy ward identity**: The four point super amplitude is completely specified by one function, choose it to be the four fermion amplitude.

    - Use this information recursively in the BCFW formula!

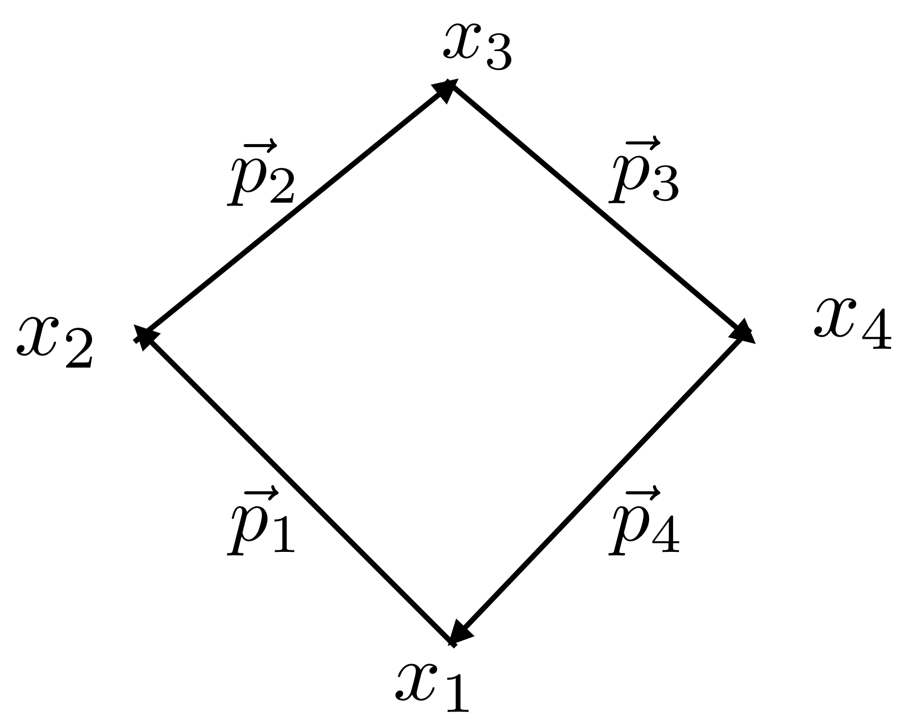
- An arbitrary higher point tree level amplitude in the fermionic CS matter theory can be entirely written in terms of **4 fermion amplitude**.

Recursion relations for non-supersymmetric theories!

$$
\begin{aligned}
&\langle \bar\psi_1 \psi_2 \bar\psi_3 \psi_4 \bar\psi_5 \psi_6 \rangle = \\
&\Bigg(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} \left[-\frac{z_{a;f}^2 + 1}{2 z_{a;f}} + i \, \frac{z_{a;f}^2 - 1}{2 z_{a;f}} \, \frac{\langle \hat{1} 4 \rangle}{i \langle \hat{f} 4 \rangle} \, \frac{\langle \hat{f} 6 \rangle}{\langle \hat{2} 6 \rangle} \right] \\
&\qquad\qquad\qquad \times \langle \hat{\bar\psi}_1 \hat\psi_f \bar\psi_3 \psi_4 \rangle \langle \hat{\bar\psi}_{(-f)} \hat\psi_2 \bar\psi_5 \psi_6 \rangle_{z_{a;f}} \\
&+ (z_{a;f} \leftrightarrow z_{b;f}) \Bigg) \frac{i}{p_f^2} \bigg|_{p_f = p_{234}} \\
&- \Bigg(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} \left[-\frac{z_{a;f}^2 + 1}{2 z_{a;f}} + i \, \frac{z_{a;f}^2 - 1}{2 z_{a;f}} \, \frac{\langle \hat{1} 6 \rangle}{i \langle \hat{f} 6 \rangle} \, \frac{\langle \hat{f} 4 \rangle}{\langle \hat{2} 4 \rangle} \right] \\
&\qquad\qquad\qquad \times \langle \hat{\bar\psi}_1 \hat\psi_f \bar\psi_5 \psi_6 \rangle \langle \hat{\bar\psi}_{(-f)} \hat\psi_2 \bar\psi_3 \psi_4 \rangle_{z_{a;f}} \\
&+ (z_{a;f} \leftrightarrow z_{b;f}) \Bigg) \frac{i}{p_f^2} \bigg|_{p_f = p_{256}}
\end{aligned}
$$

Part III

Hidden symmetry: Dual superconformal invariance



- K.I, S.Jain, S.Majumdar, P.Nayak, T.Neogi, T.Sharma, R.Sinha, V.Umesh, **arXiv : 1711.02672**

Dual variables

- The dual variables realize momentum conservation linearly in the x variables

$$x_{i,i+1}^{\alpha\beta} = x_i^{\alpha\beta} - x_{i+1}^{\alpha\beta} = p_i^{\alpha\beta} = \lambda_i^{\alpha}\lambda_i^{\beta}$$
$$\theta_{i,i+1}^{\alpha} = \theta_i^{\alpha} - \theta_{i+1}^{\alpha} = q_i^{\alpha} = \lambda_i^{\alpha}\eta_i$$

- momentum and supermomentum conservation imply

$$P^{\alpha\beta} = \sum_i p_i^{\alpha\beta} = x_{n+1}^{\alpha\beta} - x_1^{\alpha\beta} = 0,$$
$$\mathcal{Q}^{\alpha} = \sum_i q_i^{\alpha} = \theta_{n+1}^{\alpha} - \theta_1^{\alpha} = 0.$$

- The **four point super amplitude in dual space**

$$\mathcal{A}_4 = \frac{\langle 12\rangle}{\langle 23\rangle}\delta(\sum_{i=1}^{4} p_i)\delta^2(\mathcal{Q}) \xrightarrow[\text{dual space}]{} \mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- Goal is to show that this is **invariant under the superconformal symmetry in the dual variables**.

Superconformal algebra in dual space

- The N=2 superconformal algebra in **dual space** is generated by

$$\{P_{\alpha\beta},\ M_{\alpha\beta},\ D,\ K_{\alpha\beta},\ R,\ Q_\alpha,\ \bar{Q}_\alpha,\ S_\alpha,\ \bar{S}_\alpha\}$$

$$P_{\alpha\beta} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i^{\alpha\beta}}, \quad D = -\sum_{i=1}^{n} \left(x_i^{\alpha\beta} \frac{\partial}{\partial x_i^{\alpha\beta}} + \frac{1}{2} \theta_i^\alpha \frac{\partial}{\partial \theta_i^\alpha} \right),$$

$$Q_\alpha = \sum_{i=1}^{n} \frac{\partial}{\partial \theta_i^\alpha}, \quad \bar{Q}_\alpha = \sum_{i=1}^{n} \theta_i^\beta \frac{\partial}{\partial x_i^{\beta\alpha}},$$

$$M_{\alpha\beta} = \sum_{i=1}^{n} \left(x_{i\alpha}{}^{\gamma} \frac{\partial}{\partial x_i^{\gamma\beta}} + \frac{1}{2} \theta_{i\alpha} \frac{\partial}{\partial \theta_i^\beta} \right), \quad R = \sum_{i=1}^{n} \theta_i^\alpha \frac{\partial}{\partial \theta_i^\alpha}$$

- The remaining generators can be expressed using the inversion operator

$$I\left[x_i^{\alpha\beta}\right] = \frac{x_i^{\alpha\beta}}{x_i^2}, \quad I\left[\theta_i^\alpha\right] = \frac{x_i^{\alpha\beta}\theta_{i\beta}}{x_i^2}$$

$$K_{\alpha\beta} = I P_{\alpha\beta} I, \quad S_\alpha = I Q_\alpha I, \quad \bar{S}_\alpha = I \bar{Q}_\alpha I.$$

Dual superconformal invariance of the superamplitude

- The four point amplitude in the N=2 theory is

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- In this form the **translation, Lorentz invariance and supersymmetry invariance of the amplitude is manifest.**

- The invariance under Dilatations and R symmetry is also simple.

- Under the action of $K_{\alpha\beta}, S_\alpha, \bar{S}_\alpha$

$$\tilde{K}^{\alpha\beta}\mathcal{A}^{(4)} = \left(K^{\alpha\beta} + \sum_{j=1}^{4}\Delta_j x_j^{\alpha\beta}\right)\mathcal{A}^{(4)} = 0$$

$$\tilde{\bar{S}}^{\alpha\beta}\mathcal{A}^{(4)} = \left(\bar{S}^{\alpha\beta} + \sum_{j=1}^{4}\Delta_j x_j^{\alpha\beta}\right)\mathcal{A}^{(4)} = 0$$

$$S_\alpha\mathcal{A}^{(4)} = 0$$

$$\Delta_i = \frac{1}{2}\{-1, 1 - 1, 1\}$$

Dual superconformal invariance at all loops

- We showed that the function A4 is dual superconformal invariant!

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- The **tree level superamplitude is dual superconformal invariant**.

$$T_{tree} = \frac{4\pi}{\kappa}\mathcal{A}_4$$

- The **all loop results** computed in **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** are **also dual superconformal invariant.**

$$T_{sym}^{all\ loop} = T_{Asym}^{all\ loop} = T_{Adj}^{all\ loop} = T_{tree}$$

$$T_{sing}^{all\ loop} = N\frac{\sin \pi\lambda}{\pi\lambda}T_{tree}$$

- Thus the dual superconformal symmetry is **all loop exact at the planar level**. Now that the symmetry exists, can we invert the argument to bootstrap the amplitude?

4 point amplitude as a free field correlator in dual space

- The **four point amplitude in momentum space** can be interpreted as a **four point correlator in dual space**, then dual conformal invariance fixes

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$
$$= \frac{1}{x_{12}^{\Delta_1+\Delta_2} x_{34}^{\Delta_3+\Delta_4}} \left(\frac{x_{24}}{x_{14}}\right)^{\Delta_1-\Delta_2} \left(\frac{x_{14}}{x_{13}}\right)^{\Delta_3-\Delta_4} f(u,v,\kappa,\lambda)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}.$$

- Since $x_{ij}^2 = p_i^2 = 0$, the correlator is understood in the limit

$$\left.\frac{u}{v}\right|_{onshell} = \left.\frac{x_{12}^2 x_{34}^2}{x_{14}^2 x_{23}^2}\right|_{onshell} = \left.\frac{p_1^2 p_3^2}{p_2^2 p_4^2}\right|_{onshell} = constant$$

- **If dual superconformal symmetry is exact it fixes the momentum (x) dependence completely***

$$f(u,v,\kappa,\lambda) = g(\kappa,\lambda)$$

4 point amplitude as a free field correlator in dual space

- With the identification of the operator dimensions

$$\Delta_{O_1} = \Delta_{O_3} = -\frac{1}{2}$$
$$\Delta_{O_2} = \Delta_{O_4} = \frac{1}{2}$$

- The four point correlator in dual space gets fixed to (cancellations in limiting sense)

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = g(\kappa,\lambda)\sqrt{\frac{x_{13}^2}{x_{24}^2}}$$

- This is exactly same as the amplitude without the delta functions!

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- Thus if dual conformal symmetry is exact to all loops, it fixes the momentum dependence completely! (planar)

Part III
Summary

Summary

- We started with a goal of computing **arbitrary m→n tree level scattering amplitudes** in U(N) $\mathcal{N} = 2$ Chern-Simons matter theories with fundamental matter.
- We achieved this via **BCFW recursion relations**, this enabled us to express arbitrary n point amplitudes as products of four point amplitudes!
- We showed that the **all loop 2→2 scattering amplitude** computed in **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** is **dual superconformal invariant**.
- Thus **dual superconformal symmetry is all loop exact**, at least for the 4 point amplitude.
- The presence of the **superconformal and dual superconformal symmetries indicate a Yangian symmetry** of the four point amplitude.

K.I, Jain, Nayak, Sharma, Umesh, to appear

	$\mathcal{N}=4$ SYM	ABJM	$\mathcal{N}=2$ CSM
BCFW recursions for all tree level amplitudes	✓	✓	✓
Dual superconformal symmetry	✓	✓	$2\to 2$
Superconformal x Dual Superconformal symmetry = Yangian	✓	✓	$2\to 2$
Manifestly Yangian invariant representation (Orthogonal Grassmanian)	✓	✓	🔍
Symmetries at loop level	✗	✗	$2\to 2$
Yangian symmetry of the classical action	🔍	🔍	🔍

תודה רבה!