Duality in supersymmetric Chern-Simons matter theories

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Work done in Collaboration with
Jain, Minwalla, Mazumdar, Umesh, Yokoyama

Plan of the talk

Introduction

- Physics in 2+1 dimensions has many interesting features and intriguing surprises.
- There exist a new type of gauge theory completely different from the usual Maxwell theory called Chern-Simons theory.
- Originally noticed by S.S.Chern and J.Simons that the Pontryagin density in 3+1 dimensions could be written as a total derivative

$$\epsilon^{\mu\nu\rho\sigma} Tr(F_{\mu\nu}F_{\rho\sigma}) = 4\partial_\sigma\left(\epsilon^{\mu\nu\rho\sigma} Tr(A_\mu\partial_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho)\right)$$

- The boundary term has the same form as the Chern-Simons Lagrangian.
- Chern-Simons theories are theoretically novel and have practical application in planar condensed matter phenomena.

Pure Chern-Simons theory

$$L_{CS} = \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - A_\mu J^\mu$$

features:

- Involves gauge field instead of manifestly gauge invariant field strength.
- Changes by a total derivative on a gauge transformation.
- First order in space-time derivatives, source free equation of motion $F_{\mu\nu} = 0$.
- Solutions are pure gauge, in contrast source free Maxwell equations have plane wave solutions.

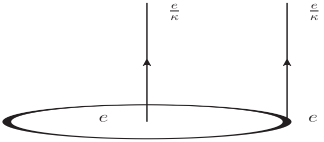
Chern-Simons+matter: Anyons

- Chern-Simons theory is interesting when coupled to matter (charged bosons/fermions)

- Equations of motion in component form when theory is coupled to matter current

$$\rho = \kappa B \ , \ J^i = \kappa \epsilon^{ij} E_j$$

- Chern-Simons interaction ties magnetic flux to electric charge (anyons). Leads to Aharonov-Bohm interactions.



- Adiabatic interaction of anyonic particles: at quantum level non relativistic wave function acquires Aharonov-bohm phase $e^{ie\oint_C A.dx} = e^{\frac{ie^2}{\kappa}}$.

κ is quantized

- Under the gauge transformation

$$A_\mu \to g^{-1} A_\mu g + g^{-1} \partial_\mu g$$

- non-abelian Chern-Simons action

$$S_{CS} = \kappa \epsilon^{\mu\nu\rho\sigma} Tr(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho)$$

- changes by a boundary term

$$S_{CS} \to S_{CS} - 8\pi^2 \kappa w(g) \ ; \ w(g) \in \mathbb{Z}$$

- Gauge invariance of the quantum amplitude $e^{iS_{CS}}$ requires

$$\kappa = \frac{\text{Integer}}{4\pi}$$

Level rank duality in CS matter theory

- $U(N_B)$ Chern-Simons theory coupled to fundamental boson [Giombi, Minwalla, Prakash, Trivedi, Wadia]

$$S = \int d^3x \Big(i\epsilon^{\mu\nu\rho} \frac{\kappa_B}{4\pi} Tr(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho)$$
$$D_\mu \bar{\phi} D^\mu \phi + \sigma \bar{\phi}\phi + N_B \frac{m_B^2}{b_4}\sigma - N_B \frac{\sigma^2}{2b_4} \Big)$$

- Wilson-Fisher limit

$$b_4 \to \infty\ ,\ m_B \to \infty\ ,\ 4\pi \frac{m_B^2}{b_4} = fixed$$

- $U(N_F)$ Chern-Simons theory coupled to fundamental fermion

$$S = \int d^3x \Big(i\epsilon^{\mu\nu\rho} \frac{\kappa_F}{4\pi} Tr(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho)$$
$$+ \bar{\psi}\gamma^\mu D_\mu \psi + m_F \bar{\psi}\psi \Big)$$

Level rank duality in CS matter theory

- Statement of duality [Jain, Minwalla, Yokoyama]

  $U(N_B)$ CS+fundamental boson at Wilson Fisher limit

  $\Leftarrow$ dual $\Rightarrow$

  $U(N_F)$ CS+fundamental fermion

- under the duality map

$$
\begin{aligned}
\kappa_F &= -\kappa_B \\
N_F &= |\kappa_B| - N_B \\
\lambda_B &= \lambda_F - sgn(\lambda_F) \\
m_F &= -m_B^{cri}\lambda_B
\end{aligned}
$$

- with condition

$$\lambda_F m_F > 0$$

Evidence for duality

- Spectrum of single trace operators and three point functions on both sides match.
  [Giombi, Minwalla, Prakash, Trivedi, Wadia] ,
  [Aharony, Gur-Ari, Yacoby], [Maldacena, Zhiboedov]

- Thermal partition functions on both sides match.
  [Jain, Trivedi, Wadia, Yokoyama] ,
  [Aharony, Giombi, Gur-Ari, Maldacena,Yacoby], [Takimi]

- Duality follows from a deformation of the known Giveon-Kutasov duality in supersymmetric theory.
  [Jain,Minwalla,Yokoyama]

- The S matrices for $2 \to 2$ processes on both sides have been computed to all orders in t'Hooft coupling and map into one another.
  [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama]

Duality and the S matrix

- The statement of duality is actually a statement of bosonization of fermions.
- Bosonic and fermionic S matrices related by duality is equivalent to a bosonization map.
- Such a mapping is possible in 2+1 dimensions: Dirac equation uniquely determines the polarization spinors as a function of the momentum.
- In large N limit, only planar diagrams contribute. Possible to get exact results as a function of λ.
- It has been shown that the S matrices for $2 \to 2$ processes in the CS+bosonic theory map to the CS+fermionic theory under duality.
  [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama]

Duality and the S matrix: Peculiarities

- In usual relativistic QFT the S matrix of particle-particle scattering is related to S matrix for particle-antiparticle scattering by crossing symmetry.
- There is no known example of crossing symmetry violation in 3+1 dimensions.
- For Chern-Simons matter theory usual crossing symmetry rules lead to unitarity violation and incorrect non-relativistic limit for the singlet channel.
- The conjectured S channel S matrix for bosons:

$$S_S^B = \cos(\pi\lambda_B) I(p_1, p_2, p_3, p_4) + i\frac{\sin(\pi\lambda_B)}{\pi\lambda_B} T$$

- It obeys unitarity, has the correct non-relativistic limit and maps to the corresponding fermionic S matrix under duality.

Our work

- Interesting to compute the S matrix in a supersymmetric theory as a test for duality.
- Check for the unusual features of the S matrix, which appear to be a general feature of Chern-Simons matter theories.
- Easy to work with $\mathcal{N} = 1, 2$ theories, Superspace formulation exists and manifest supersymmetry can be maintained.
- Eventually make contact with scattering in maximally supersymmetric Chern-Simons theories (ABJM) in string theory.
- Ongoing work in collaboration with Jain, Mazumdar, Minwalla, Umesh, Yokoyama.

Superspace

Superspace

- Superspace: Coordinate space of supersymmetric theories.
- Consists of usual space time coordinates and anticommuting (Grassmann) coordinates
- Superspace formulation maintains manifest supersymmetry.
- Superfields: Functions in superspace which package component fields.
- eg: Scalar superfield

$$\Phi(x,\theta) = \phi(x) + \theta\psi(x) - \theta^2 F(x)$$

- Superspace formalism contains auxiliary fields and realises supersymmetry offshell.

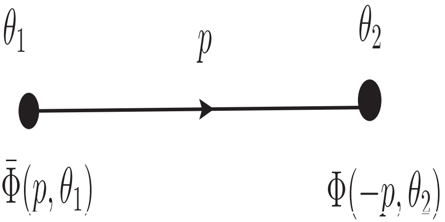
Toy model

- Scalar theory in superspace with a quartic interaction

$$S_E = -\int d^3x d^2\theta \left(-\frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi + m\bar{\Phi}\Phi + \frac{\eta}{4}(\bar{\Phi}\Phi)^2 \right)$$

- Superfield propagator

$$\langle \bar{\Phi}(\theta_1, p)\Phi(\theta_2, -p)\rangle = \frac{D^2_{\theta_1,p} - m}{p^2 + m^2}\delta^2(\theta_1 - \theta_2)$$



Toy model

- unpackage in component form and eliminate auxiliary fields

$$S_E = \int d^3x \Big(\partial\bar{\phi}\partial\phi + m^2\bar{\phi}\phi - \bar{\psi}(i\not{\partial} + m)\psi + \eta m(\bar{\phi}\phi)^2 + \frac{\eta^2}{4}(\bar{\phi}\phi)^3 - \frac{\eta}{2}(\bar{\phi}\phi)(\bar{\psi}\psi) - \frac{\eta}{4}(\bar{\phi}\psi + \bar{\psi}\phi)^2 \Big)$$

- component propagators

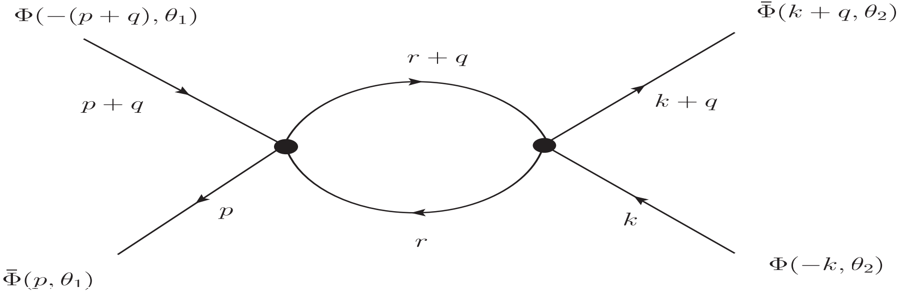
$$\langle\bar{\phi}\phi\rangle = \frac{1}{p^2+m^2} \quad , \langle\bar{\psi}_\alpha\psi_\beta\rangle = \frac{p_{\alpha\beta} + mC_{\beta\alpha}}{p^2+m^2}$$

- Dynamics in superspace encapsulates all dynamics of components while maintaining manifest supersymmetry.

- The Feynman rules for Superspace are analogous to the usual QFT rules.

One loop four point function

The one loop effective action in superspace



$$\frac{\eta^2}{8}\int d^2\theta_1 d^2\theta_2 \frac{d^3r}{(2\pi)^3} P(\theta_1,\theta_2,r+q)P(\theta_2,\theta_1,r)(\bar{\Phi}\Phi)(\theta_1)(\bar{\Phi}\Phi)(\theta_2)$$

One loop four point function

- Do the grassmann integrals and get component action

$$\frac{\eta^2}{8} H(q) \left((\bar{\phi}\phi)^2 (12m^2 - q^2) - 8m(\bar{\phi}\phi)(\bar{\psi}\psi) + (\bar{\psi}\psi)^2 + \dots \right)$$

- From tree component action, one loop effective action can be computed to match the above result.

- Coefficients of relevant terms in effective action correspond to the corresponding four point function.

Gauge theory with interactions

Gauge theory with interactions

- The non-abelian $\mathcal{N}=1$ Chern-Simons action in superspace [Avdeev, Grigoriev, Kazakov], [Ivanov].

$$S = \int d^3x d^2\theta \Big[-\frac{\kappa}{8\pi} Tr(D_\alpha \Gamma^\beta D_\beta \Gamma^\alpha) - \frac{1}{6} g f^{abc} (D^\alpha \Gamma^\beta_a) \Gamma^b_\alpha \Gamma^c_\beta \\ - \frac{1}{24} g^2 f^{abc} f^{ade} \Gamma^\alpha_b \Gamma^\beta_c \Gamma^d_\alpha \Gamma^e_\beta \\ - \frac{1}{2} (D^\alpha \bar{\Phi} + ig \bar{\Phi} \Gamma^\alpha)(D_\alpha \Phi - ig \Gamma_\alpha \Phi) + m_0 \bar{\Phi}\Phi + \frac{\eta}{4} (\overline{\Phi}\Phi)^2 \Big]$$

- Scalar superfield : fundamental rep of $U(N)$

$$\Phi = \phi + \theta\psi - \theta^2 F$$

- Gauge superfield : adjoint rep of $U(N)$

$$\Gamma^\alpha = \chi^\alpha - \theta^\alpha B + i\theta^\beta A_\beta{}^\alpha - \theta^2 (2\lambda^\alpha - i\partial^{\alpha\beta}\chi_\beta)$$

Supersymmetric Light cone gauge

- Self interacting gauge fields very complicated for dynamics.

- Non-abelian gauge superfield transforms as

$$\delta\Gamma_\alpha^a = D_\alpha K^a + g f^{abc}\Gamma_\alpha^b K^c$$

- There exists a supersymmetric light cone gauge Siegel

$$\Gamma_- = 0 \implies A_- = 0$$

- Gauge superfield self interactions vanish in this gauge, action abelianizes!

Action in Light cone gauge

- Action simplifies in supersymmetric light cone gauge

$$S_E = -\int d^3x d^2\theta \left[\frac{\kappa}{8\pi} Tr(\Gamma^- i\partial_- \Gamma^-) - \frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi \right.$$
$$\left. - \frac{i}{2} \Gamma^- (\bar{\Phi} D_- \Phi - D_- \bar{\Phi} \Phi) + m_0 \bar{\Phi} \Phi + \frac{\eta}{4} (\bar{\Phi} \Phi)^2 \right]$$

- Gauge superfield propagator

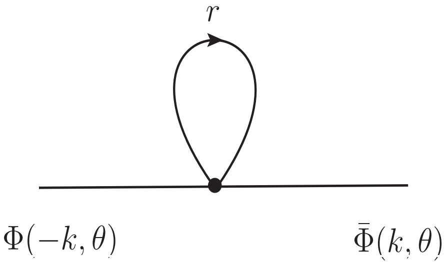
$$\langle \Gamma^-(\theta_1, p) \Gamma^-(\theta_2, -p) \rangle = \frac{8\pi}{\kappa} \frac{\delta^2(\theta_1 - \theta_2)}{p_-}$$

- Component propagators

$$\langle A_+(p) A_3(-p) \rangle = \frac{4\pi i}{\kappa} \frac{1}{p_-} \ , \ \langle A_3(p) A_+(-p) \rangle = -\frac{4\pi i}{\kappa} \frac{1}{p_-}$$

Two point function

- Two point function: Self interactions of scalar



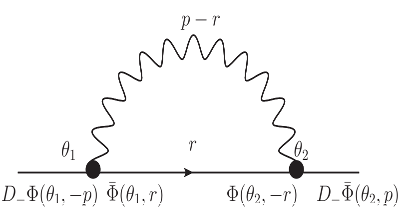
- Effective action for self interactions

$$S_1 = -\frac{\eta}{8\pi}|m| \int d^2\theta_1 d^2\theta_2 \delta^2(\theta_1 - \theta_2)\bar{\Phi}(\theta_1, p)\Phi(\theta_2, -p) \ .$$

- The self energy correction is just a shift in the mass.

Two point function

- Two point function: Gauge superfield exchange (Rainbow diagrams)



- Effective action for gauge superfield exchange

$$S_2 = -\frac{1}{4}\int d^2\theta_1 d^2\theta_2 \frac{d^3 r}{(2\pi)^3}\Big(\Gamma^-(\theta_1, p-r)J_-(\theta_1 - p, r)$$
$$\Gamma^-(\theta_2, r-p)J_-(\theta_2, -r, p)\Big)$$

Duality invariance of Pole mass

- The bare mass is corrected by self interactions and gauge superfield interaction

$$m = \frac{2m_0}{2 + (-1 + w)\lambda \ \mathrm{Sgn(m)}}$$

- Under duality map Jain, Minwalla, Yokoyama

$$\lambda \to \lambda - \mathrm{Sgn}(\lambda) \ , \ \ w \to \frac{3 - w}{1 + w} \ , m_0 \to \frac{-2m_0}{1 + w}$$

- the pole mass transforms as

$$m' = \frac{2m_0}{-1 - w + (-1 + w)\mathrm{Sgn}(m')(\lambda - \mathrm{Sgn}(\lambda))}$$

- $m' = -m$ under duality and provided we satisfy the condition

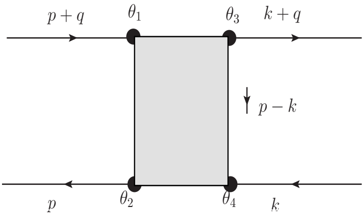
$$\mathrm{Sgn}(\lambda)\mathrm{Sgn}(m) = 1$$

- the pole mass is invariant.

Constraints from supersymmetry

Four point function

- The four point function in superspace



$$V(\theta_1,\theta_2,\theta_3,\theta_4,p,k,q)=\langle\bar{\Phi}((p+q),\theta_1)\Phi(-(k+q),\theta_3)\bar{\Phi}(k,\theta_4)\Phi(-p,\theta_2)\rangle$$

- Supersymmetric invariance of the four point function implies

$$(Q_{\theta_1,p+q}+Q_{\theta_2,-p}+Q_{\theta_3,k+q}+Q_{\theta_4,k})V(\theta_1,\theta_2,\theta_3,\theta_4,p,k,q)=0$$

Four point function

- Supersymmetry determines the θ structure of V upto a shift invariant function.

$$V = \exp\left(\frac{1}{4}X.(p.X_{12}+q.X_{13}+k.X_{43})\right)F(X_{12},X_{13},X_{43},p,q,k)$$

$$X = \sum_{i=1}^{4}\theta_i\ ,\ X_{ij} = \theta_i - \theta_j$$

- The general form of F is

$$\begin{aligned} F(X_{12},X_{13},X_{43},p,q,k) = X_{12}^{+}X_{43}^{+}\Big(& A(p,k,q)X_{12}^{-}X_{43}^{-}X_{13}^{+}X_{13}^{-} \\ & + B(p,k,q)X_{12}^{-}X_{43}^{-} \\ & + C(p,k,q)X_{12}^{-}X_{13}^{+} + D(p,k,q)X_{13}^{+}X_{43}^{-}\Big) \end{aligned}$$

Four point function

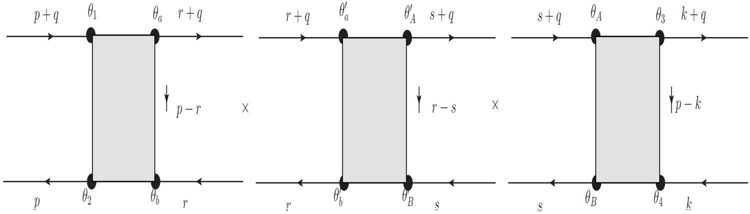
Properties of Supersymmetric four point function

- Symmetry

$$p \to k+q, k \to p+q, q \to -q \ ,$$
$$\theta_1 \to \theta_4, \theta_2 \to \theta_3, \theta_3 \to \theta_2, \theta_4 \to \theta_1$$

- Closure

- Associativity



S matrix

- S matrix for $2 \to 2$ process:

$$\begin{pmatrix} \Phi(\theta_1, p_1) \\ \bar{\Phi}(\theta_3, p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\Phi}(\theta_2, p_2) \\ \Phi(\theta_4, p_4) \end{pmatrix}$$

- Four point function from superspace is offshell.

- Onshell limit gives the S matrix.

- Onshell solutions are obtained from the superfield equation of motion

$$(D^2 + m)\Phi = 0$$

S matrix

- The S matrix in Superspace encodes the following component proceses

$$F_0 : \begin{pmatrix} \phi(p_1) \\ \bar{\phi}(p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\phi}(p_2) \\ \phi(p_4) \end{pmatrix} \ , \ F_7 : \begin{pmatrix} \psi(p_1) \\ \bar{\psi}(p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\psi}(p_2) \\ \psi(p_4) \end{pmatrix}$$

$$F_1 : \begin{pmatrix} \phi(p_1) \\ \bar{\phi}(p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\psi}(p_2) \\ \psi(p_4) \end{pmatrix} \ , \ F_2 : \begin{pmatrix} \psi(p_1) \\ \bar{\psi}(p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\phi}(p_2) \\ \phi(p_4) \end{pmatrix}$$

$$F_3 : \begin{pmatrix} \phi(p_1) \\ \bar{\psi}(p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\phi}(p_2) \\ \psi(p_4) \end{pmatrix} \ , \ F_4 : \begin{pmatrix} \psi(p_1) \\ \bar{\phi}(p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\psi}(p_2) \\ \phi(p_4) \end{pmatrix}$$

$$F_5 : \begin{pmatrix} \phi(p_1) \\ \bar{\psi}(p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\psi}(p_2) \\ \phi(p_4) \end{pmatrix} \ , \ F_6 : \begin{pmatrix} \psi(p_1) \\ \bar{\phi}(p_3) \end{pmatrix} \to \begin{pmatrix} \bar{\phi}(p_2) \\ \psi(p_4) \end{pmatrix}$$

S matrix

- Not all the processes are independent!
- Supersymmetric invariance of the S matrix

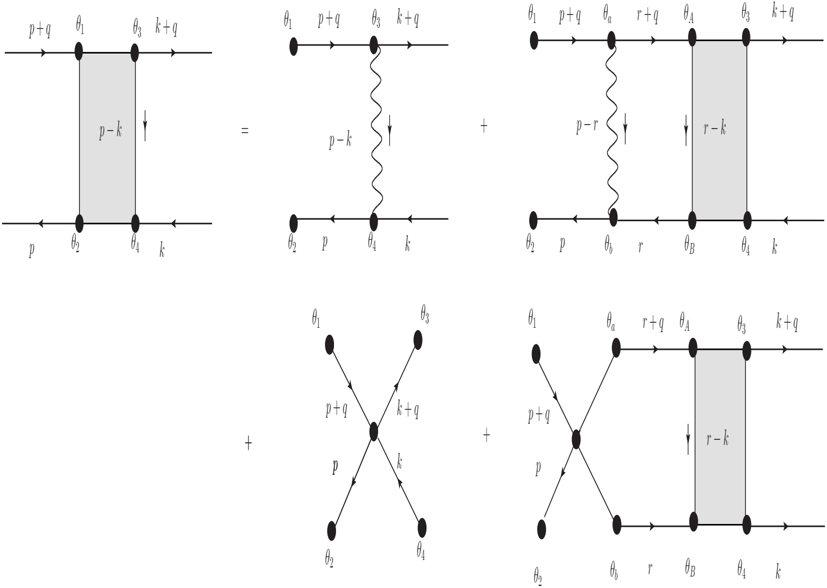
$$QS(\theta_1, \theta_2, \theta_3, \theta_4, p_1, p_2, p_3, p_4) = 0$$

- Only F_0 and F_7 are independent.

$$F_i = a_i F_0 + b_i F_7 \ , \forall i = 1, \ldots, 6$$

Four point function

Schwinger-Dyson equation



Schwinger-Dyson equation

- Schematically the integral equations are of the form

$$V(\theta_i, p_i) = V_0(\theta_i, p_i) + \int \frac{d^3 r}{(2\pi)^3} d^2\theta'_j V_0(\theta_i, \theta'_j, p_i, r) \\ P(\theta'_j, p_i + r) P(\theta'_j, r) V(\theta'_j, \theta_i, p_i)$$

- The integral equation generates the geometric series that sums over the contributing feynman diagrams.

- In the large N limit only the ladder and candy diagrams contribute.

- We have solved the integral equations exactly for arbitrary values of the t'Hooft coupling λ and determined the offshell four point function.

- S matrix is obtained by plugging onshell solutions into offshell four point function.

Duality in T channel

- T channel is particle-antiparticle scattering in the adjoint sector

$$P_i(p_1) + A^j(p_2) \to P_i(p_3) + A^j(p_4)$$

- The S matrix for the boson and fermion have the form

$$T_B = \frac{4i\pi q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} + \frac{4\pi q_3}{\kappa} J_1(\lambda, w, m; q_3)$$
$$T_F = \frac{4i\pi q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} + \frac{4\pi q_3}{\kappa} J_2(\lambda, w, m; q_3)$$

- They map to each other under the duality transformation upto overall sign

$$\lambda \to \lambda - \text{sign}(\lambda)\ ,\ \ w \to \frac{3-w}{1+w}\ ,\ \ m \to -m\ ,\ \ \kappa \to -\kappa$$

- Overall phases in S matrix are unobservable and not physical.

Duality in T channel

$$J_1(\lambda, w, m; q_3) = \frac{n_1 + n_2 + n_3}{d_1 + d_2 + d_3} \ , \ J_2(\lambda, w, m; q_3) = \frac{-n_1 + n_2 + n_3}{d_1 + d_2 + d_3}$$

$$\begin{aligned}
n_1 =& 16 m q_3 (w+1) e^{i\lambda\left(2\tan^{-1}\left|\frac{2m}{q_3}\right| + \pi \mathrm{sgn}(q_3)\right)} \\
n_2 =& (w-1)(q_3 + 2im)(2m(w-1) + iq_3(w+3)) \left(-e^{2i\pi\lambda \mathrm{sgn}(q_3)}\right) \\
n_3 =& (w-1)(2m + iq_3)(q_3(w+3) + 2im(w-1)) e^{4i\lambda \tan^{-1}\left|\frac{2m}{q_3}\right|} \\
d_1 =& (w-1)\left(4m^2(w-1) - 8imq_3 + q_3^2(w+3)\right) e^{4i\lambda \tan^{-1}\left|\frac{2m}{q_3}\right|} \\
d_2 =& (w-1)\left(4m^2(w-1) + 8imq_3 + q_3^2(w+3)\right) e^{2i\pi\lambda \mathrm{sgn}(q_3)} \\
d_3 =& -2\left(4m^2(w-1)^2 + q_3^2(w(w+2)+5)\right) e^{i\lambda\left(2\tan^{-1}\left|\frac{2m}{q_3}\right| + \pi \mathrm{sgn}(q_3)\right)}
\end{aligned}$$

Duality in T channel, $\mathcal{N} = 2$ point

- There is a massive simplification at the $\mathcal{N} = 2$ point, corresponding to $w \to 1$.

- In this limit, the S matrices reduce to tree level answer

$$T_B = \frac{4i\pi q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} - \frac{8\pi m}{\kappa}$$
$$T_F = \frac{4i\pi q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} + \frac{8\pi m}{\kappa}$$

- Under duality transformation $m \to -m$ and the S matrices for boson and fermion map to one another.

- We would like to stress that this is an enormous collapse compared to the complicated answer in the $\mathcal{N} = 1$ case.

- We are exploring the implications of this result, may be large $\mathcal{N}$ is as useful as large N !.

Summary

Summary

- We studied the $2 \to 2$ scattering in supersymmetric Chern-Simons matter theory.
- We computed the offshell four point function in a manifestly supersymmetric formalism.
- We obtained the S matrix for boson and fermion processes by taking appropriate onshell limit.
- Under the level rank duality the S matrices map to one another.
- There is a massive collapse at the $\mathcal{N} = 2$ point, we are investigating its implications.

More interesting stuff to come!

- Duality in U channel.
- Constraints from unitarity of the S matrix.
- Peculiarities of the scattering process in S channel and duality.
- Non relativistic limits.
- Possible direct computation of the S matrix in S channel for $\mathcal{N} = 2$ case.
- $\mathcal{N} = 3, \mathcal{N} = 4, \ldots$ ABJM.

Thank You![1]

[1] One sign is DOOM

Duality in Supersymmetric theory

- Giveon-Kutasov duality is a strong-weak type duality in $U(N)$, $\mathcal{N} = 2$ superconformal Chern-Simons theory with a fundamental chiral multiplet.

$$S_E = \int d^3x \Bigg(i\epsilon^{\mu\nu\rho} \frac{\kappa}{4\pi} Tr(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) + D_\mu \bar{\phi} D^\mu \phi + \bar{\psi}\gamma^\mu D_\mu \psi$$
$$\frac{4\pi}{\kappa}(\bar{\psi}\psi)(\bar{\phi}\phi) + \frac{2\pi}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi) + \frac{4\pi^2}{\kappa^2}(\bar{\phi}\phi)^3 \Bigg)$$

- Class of theories labelled by integers N and κ.

- Duality: $U(N)$ theory at level κ is dual to $U(|\kappa| - N)$ theory with level $-\kappa$.

- Thermal partition functions on both sides match.
  [Jain, Trivedi, Wadia, Yokoyama] ,
  [Aharony, Giombi, Gur-Ari, Maldacena,Yacoby], [Takimi]

Duality in Supersymmetric theory

- The statement of duality between pairs of $\mathcal{N} = 2$ theories has been generalised further by including relevant/marginal deformations in the large N limit.
  Jain, Minwalla, Yokoyama

- relevant deformations: mass terms for scalar and fermion, $(\bar{\phi}\phi)^2$ term.

- marginal deformations: $(\bar{\phi}\phi)^3$ term, Yukawa terms.

- Includes a two parameter set (w, m_0) of $\mathcal{N} = 1$ theories characterised by a superpotential

$$W = -\frac{w}{4\kappa}(\bar{\phi}\phi)^2 - m_0(\bar{\phi}\phi)$$

- The $\mathcal{N} = 1$ theory maps to itself under the transformations

$$w' = \frac{3 - w}{1 + w}\ , m_0 = -\frac{2m_0}{1 + w}$$