Amplitudes and hidden symmetries in N=2 SUSY CS matter theory

References: 1505.06571, 1710.04227, 1711.02672, work in progress

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Motivation: Bosonization duality

- 2+1 d bosonization duality in U(N) Chern-Simons matter theories at Large N

$U(N)$ CS+fundamental boson at Wilson Fisher limit **dual to** $U(N)$ CS+fundamental fermion

Aharony, Benini, Giombi, Giveon, Gur-Ari, Gurucharan, Kutasov, Jain, Maldacena, Minwalla, Prakash, Yacoby, Yin, Yokoyama, Wadia, Zhiboedov

- Plenty of tests in the large N, large κ limit : Spectrum of single trace primaries, Three point functions, Thermal partition functions, 2 → 2 S matrices match under duality.
- At finite N, there is a conjectured duality but less is known. Aharony
- The bulk description corresponds to Vasiliev higher spin gravity theories in AdS_4 that are "expected" to arise as tensionless limit of string theories.
- It is possible that some of the integrability features of the underlying string theory survive the tensionless limit! Integrability may help understand the duality beyond planar limit.
- One possible indicator of integrability: infinite dimensional symmetry structures. eg amplitudes in N=4 SYM, N=6 ABJM.
- In Chern-Simons matter theories, planar amplitudes can be computed exactly to all orders in the coupling $\lambda = \frac{N}{\kappa}$

Candidate: N=2 Supersymmetric Chern-SImons matter theory

- Renormalizable $\mathcal{N}=2$ Chern-Simons coupled to fundamental matter in U(N)

$$\mathcal{S}_{\mathcal{N}=2} = \int d^3x \Big[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right) + i\bar\psi\gamma^\mu\mathcal{D}_\mu\psi - \mathcal{D}^\mu\bar\phi\mathcal{D}_\mu\phi$$
$$-\frac{4\pi^2}{\kappa^2}(\bar\phi\phi)^3 + \frac{4\pi}{\kappa}(\bar\phi\phi)(\bar\psi\psi) + \frac{2\pi}{\kappa}(\bar\psi\phi)(\bar\phi\psi)\Big]$$

- In the symmetric, anti-symmetric and adjoint channels of scattering, the 2 → 2 S matrices are tree level exact to all orders in λ in the planar limit.

$$T^{all\ loop}_{symm} = T^{all\ loop}_{Asymm} = T^{all\ loop}_{Adj} = T_{Tree} \qquad T_{Tree} = \frac{4\pi}{\kappa}\frac{\langle 12\rangle}{\langle 23\rangle}\delta(\sum_{i=1}^{4}p_i)\delta^2(\mathcal{Q})$$

- The singlet channel S matrix continues to be simple, but not tree level exact.

$$T^{all\ loop}_{Singlet} = N\frac{\sin\pi\lambda}{\pi\lambda}T_{Tree}$$ K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama

All loop exact Dual superconformal symmetry of 2 → 2 amplitude

- Why is the 2 → 2 amplitude in the Symm/Anti-symm/Adjoint channels tree level exact? and why does it have a very simple coupling dependence in Singlet channel?
- Maybe some powerful symmetry protects the amplitude.
- Amplitude in Dual space

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\,\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

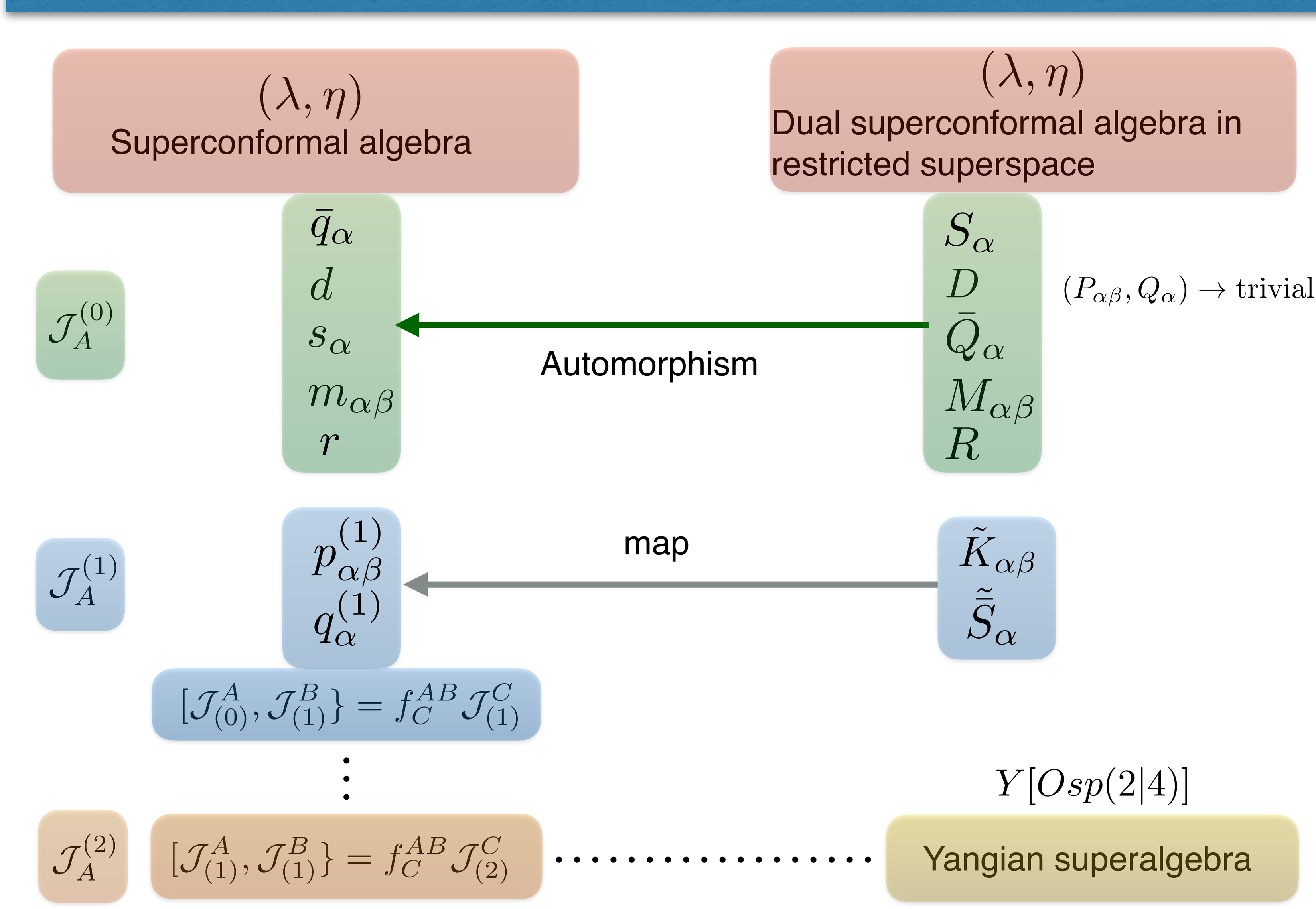
- Action of Dual generators: Manifestly invariant under super translations, rotations, transforms as weight +2 under R symmetry and weight +4 under Dilatations.
- The amplitude is covariant under the action of the special conformal and superconformal supersymmetry with weights $\Delta_i = \{4-1, 1, -1, 1\}$

$$\left(K^{\alpha\beta} + \frac{1}{2}\sum_{j=1}^{4}\Delta_j x_j^{\alpha\beta}\right)\mathcal{A}_4 = 0 \qquad \left(\bar S_\alpha + \frac{1}{2}\sum_{j=1}^{4}\Delta_j\theta_{j\alpha}\right)\mathcal{A}_4 = 0 \qquad T_{tree} = \frac{4\pi}{\kappa}\mathcal{A}_4$$

- The tree level superamplitude is dual superconformal invariant. From the results of four point amplitude, It follows that dual superconformal symmetry is all loop exact.

K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh

All loop exact Yangian symmetry of 2 → 2 amplitude



Level one Yangian generators in N=2 theory

- The general form of the level 1 Yangian generators.

$$\mathcal{J}^A_{(1)} = \frac{1}{2}f^A_{BC}\sum_{j<k}\mathcal{J}^C_{j,(0)}\mathcal{J}^B_{k,(0)} + \sum_k v^l\mathcal{J}^A_{l,(0)}$$

- The level 1 generators of the $\mathcal{N}=2$ theory

$$p^{(1)}_{\alpha\beta} \equiv \tilde K_{\alpha\beta} = \frac{1}{4}\sum_{i>j}\left[p_{(i\alpha}{}^{\gamma}d_{j\beta)\gamma} + q_{i(\alpha}\bar q_{j\beta)} - (i\leftrightarrow j)\right] - \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{i-1}(\Delta_i - 1)p_{i\alpha\beta}$$

$$q^{(1)}_\alpha \equiv \tilde{\bar S}_\alpha = -\frac{1}{4}\sum_{i>j}\left[\bar q_j^{\beta}(m_{i\alpha\beta} + \epsilon_{\alpha\beta}d_i) - \bar q_{j\alpha}r_i + p_{j\alpha}{}^{\beta}s_{i\beta} - (i\leftrightarrow j)\right] - \frac{1}{2}\sum_{i>j}^{4}(\Delta_i - 1)q_{j\alpha}$$

- Remaining level one generators can be obtained from adjoint condition

$$[\mathcal{J}^A_{(0)}, \mathcal{J}^B_{(1)}\} = f_C^{AB}\mathcal{J}^C_{(1)}$$

- Superconformal and dual superconformal symmetries generate Yangian symmetry!

$$\mathcal{J}^A_{(0)}\mathcal{A}_4 = 0\ ,\ \mathcal{J}^A_{(1)}\mathcal{A}_4 = 0\ ,\ \Longrightarrow\ \mathcal{Y}\mathcal{A}_4 = 0,$$

K.I,Jain, Nayak, Sharma (to appear)

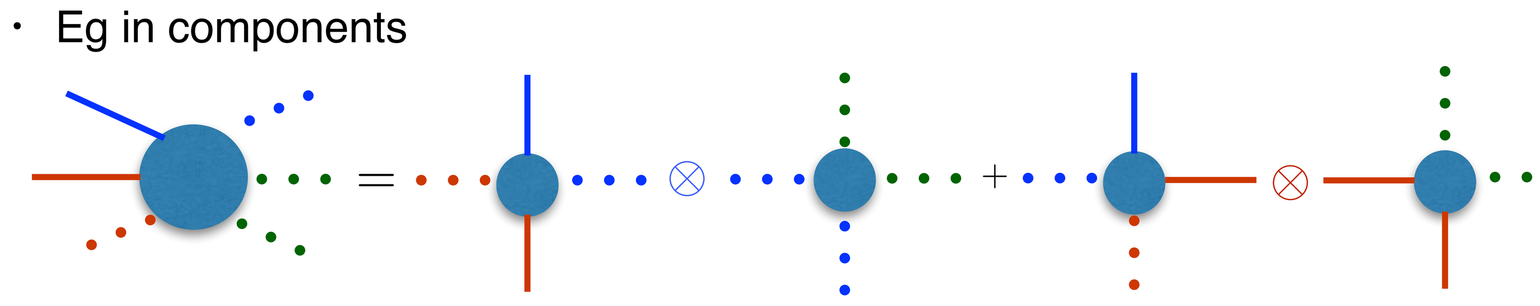
Higher point tree amplitudes: BCFW recursions

Recursion for six point amplitude and factorization channels

- The recursion formula for the six point amplitude for eg takes the form

$$A_6(\Lambda_1\ldots,\Lambda_6) = \sum_f\sum_{z_f = z_{f_i}}\int d\eta\frac{A_4^L(\Lambda_1,\ldots,\Lambda_f,z_f)A_4^R(\Lambda_f,\ldots,\Lambda_6,z_f)}{p_f^2}$$

- Eg in components



Recursion formula for the 2n point superamplitude

- Schematically the recursion relation for a general 2n point amplitude is a factorization in terms of 4 point amplitudes

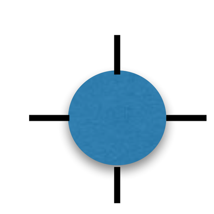
$$A_{2n}(\Lambda_1\ldots,\Lambda_{2n}) = \sum_{f_1}\cdots\sum_{f_{n-2}}\sum_{z_{f_1}=z^i_{f_1}}\cdots\sum_{z_{f_{n-2}}=z^i_{f_{n-2}}}\int d\eta_1\ldots d\eta_{n-2}$$ K.I, Jain, Nayak, Umesh

$$\frac{A_4(\Lambda_1,\ldots,\Lambda_{f_1},z_{f_1},\eta_1)A_4(\Lambda_{f_1},\ldots,\Lambda_{f_2},z_{f_1},z_{f_2},\eta_2)\ldots A_4(\Lambda_{f_{n-2}},\ldots,\Lambda_{2n},z_{f_1}\ldots,z_{f_{n-2}})}{p^2_{f_1}\cdots p^2_{f_{n-2}}}$$

- Dual superconformal symmetry is expected for all n point tree level amplitudes.

K.I,Jain, Nayak, Sharma (in progress)

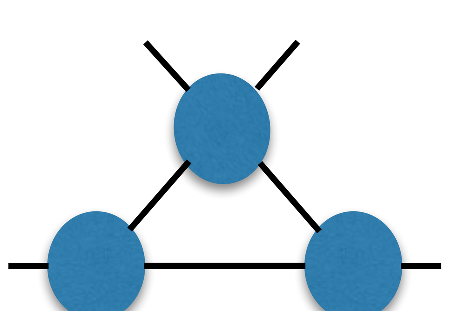
Higher point loop amplitudes



- All loop four point off-shell amplitude in superspace is known.
- Computed in special kinematics using Dyson-Schwinger methods.

K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama

$\langle\bar\Phi(\theta_1,p_1)\Phi(\theta_2,p_2)\bar\Phi(\theta_3,p_3)\Phi(\theta_4,p_4)\rangle$



- Stitch four point functions to form higher point functions.
- Very cumbersome, eg Six point function.
- Possible to extract component amplitudes like six boson/ six fermion, preliminary results indicate vanishing loop corrections!
- If true, then dual superconformal symmetry could be an exact symmetry for higher point amplitudes as well!

K.I,Jain, Nayak, Sharma (in progress)

Things to do

	$\mathcal{N}=4$ SYM	ABJM	$\mathcal{N}=2$ CSM
BCFW recursions for all tree level amplitudes	✔	✔	✔
Dual superconformal symmetry for tree level amplitudes	✔	✔	$2\to 2$
Superconformal x Dual Superconformal symmetry = Yangian	✔	✔	$2\to 2$
Orthogonal Grassmanian	✔	✔	??
Exact Symmetries at loop level	✘	✘	$2\to 2$
Amplitude-Wilson loop duality	✔	$2\to 2$	??
Integrability	✔	✔	