$2 \to 2$ scattering in supersymmetric matter Chern-Simons theories at large N

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The fourth Indo-Israel conference on string theory
23 Dec 2015

Based on

- K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh, S.Yokoyama: Arxiv 1505.06571, *JHEP* **1510** (2015) 176

- S.Jain, M.Mandlik, S.Minwalla, T.Takimi, S.Wadia, S.Yokoyama: Arxiv 1404.6373, *JHEP* **1504** (2015) 129

CS matter theories: Preliminaries

- Lots of activity in $U(N)$ Chern-Simons theories with fundamental matter.
- Motivations: AdS/CFT, Vasiliev duality, limit of ABJ theory, solvable in large N.
- 2+1 d bosonization duality: (Ofer and Shiroman's talk)

  $U(N)$ CS+fundamental boson at Wilson Fisher limit

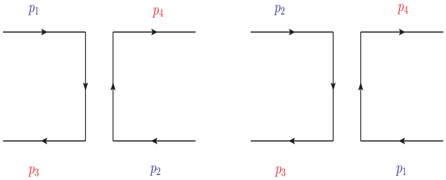
  $\Leftarrow$ dual $\Rightarrow$

  $U(N)$ CS+fundamental fermion
- Supersymmetric theories are conjectured to be self dual.
- Evidence for duality: Matching of spectrum of single trace operators, three point functions, thermal partition functions, $2 \to 2$ S matrices.. [Aharony, Bardeen, Benini, Chang, Frishman, Giombi, Giveon, Gur-Ari, Gurucharan, Kutasov, Jain, Maldacena, Mandlik, Minwalla, Moshe, Sharma, Prakash, Takimi, Trivedi, Umesh, Sonnenschein, Yacoby, Yin, Yokoyama, Wadia, Zhiboedov]

Channels of scattering

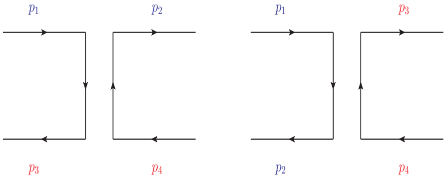
- Particle-Particle scattering: $p_1 + p_2 \to p_3 + p_4$

  Fundamental$\otimes$Fundamental $\to$ Symmetric(U_d)$\oplus$Anti-symmetric(U_e)



- Particle-Antiparticle scattering

  Fundamental $\otimes$ Antifundamental $\to$ Adjoint(T) $\oplus$ Singlet(S)



Scattering in CS matter theories: Peculiarities

- Scattering results consistent with duality.
- In singlet channel (particle-Antiparticle) S matrices obtained from naive crossing symmetry rules are inconsistent with unitarity and have incorrect non-relativistic limit.
- Consistency with unitarity requires
  - Delta function term at forward scattering.
  - Modified crossing symmetry rules.
- Conjecture: [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama]
  Singlet channel S matrices have the form

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$

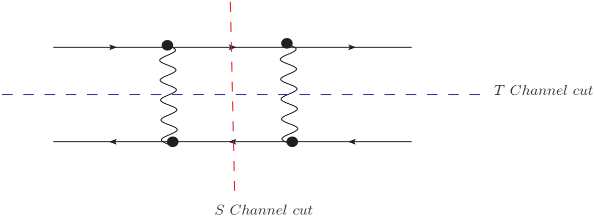
- $\mathcal{T}^{S;\text{naive}}$ is the matrix obtained from naive analytic continuation of particle-particle scattering.

Unitarity and anyonic behavior in Singlet Channel

- Unitarity $i(T^\dagger - T) = TT^\dagger$: non-trivial only for singlet channel in the large N limit .

$$T_{Asym} \sim T_{Sym} \sim T_{Adj} \sim O\left(\frac{1}{N}\right) \ , \ T_{sing} \sim O(1)$$

- One loop cut structure (generalizes to all loops)



- Singlet channel is effectively anyonic - usual crossing rules fail unitarity. Anyonic phase operator $\nu_m = \frac{C_2(R_1)+C_2(R_2)-C_2(R_m)}{2\kappa}$,

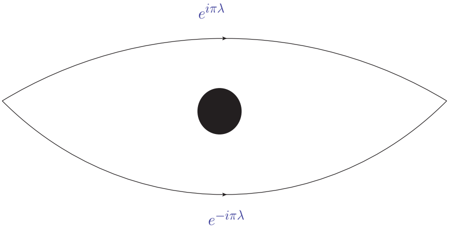
$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right) , \nu_{Sing} \sim O(\lambda)$$

- Remedy: delta function and modified crossing rules.

Nature of the conjecture: Delta function

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$

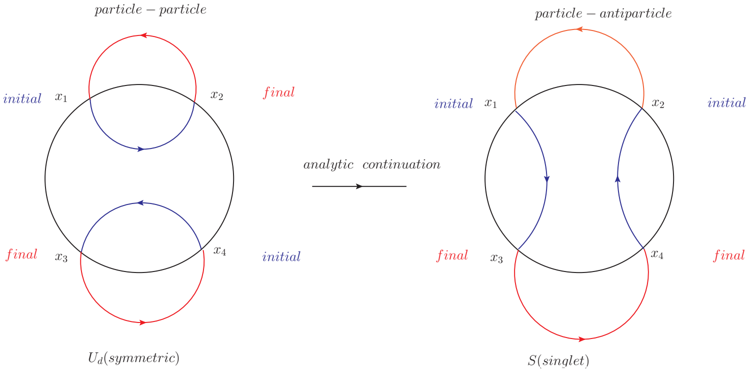
- The conjectured S matrix has a non-analytic $\delta(\theta)$ piece.
- delta function is already known to be necessary to unitarize non-relativistic Aharanov-Bohm scattering [Ruijsenaars; Bak,Jackiw,Pi].



- $\cos(\pi\lambda)$ is due to the interference of the Aharonov-Bohm phases of the wave packets.

Modified crossing rules: A heuristic explanation

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$



$$T_{U_d} W_{U_d} \to T_S W_S$$

$$\frac{W_{U_d}}{W_S} = \frac{\oint \text{with 2 circular Wilson lines}}{\oint \text{with 1 circular Wilson line}} = \frac{\sin(\pi\lambda)}{\pi\lambda} \quad \text{[Witten]}$$

Universality and tests

- delta function and modified crossing rules conjectured in [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama] - appear to be universal
- Tests:
  - Unitarity of the S matrix
  - Bose-Fermi duality
  - Non-relativistic limit gives Aharanov-Bohm
- All the tests have been explicitly verified for
  - $U(N)$ Chern-Simons coupled to fundamental bosons
  - $U(N)$ Chern-Simons coupled to fundamental fermions
- We test the conjecture in $\mathcal{N} = 1, 2$ Supersymmetric $U(N)$ Chern-Simons matter theories.

Our work

- Test the conjecture in the most general renormalizable supersymmetric $\mathcal{N} = 1$ Chern-Simons matter theory.

- Superspace - manifest supersymmetry

- Use supersymmetric light cone gauge - no gauge superfield self interactions.

- Work in large N - only planar diagrams .

- Compute off-shell four point correlator,take on-shell limit and extract the S matrix.

- Provide evidence for duality and subject the conjecture to stringent unitarity test.

S matrix in onshell superspace

- S matrix solution (in-states: p_1, p_2 ,out-states p_3, p_4) is determined in terms of two functions $\mathcal{S}_B$ and $\mathcal{S}_F$ of momenta,couplings and mass.

$$
\begin{aligned}
&S(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) = \mathcal{S}_B + \mathcal{S}_F\ \theta_1\theta_2\theta_3\theta_4 + \\
&\left(\frac{1}{2}C_{12}\mathcal{S}_B - \frac{1}{2}C^*_{34}\mathcal{S}_F\right)\ \theta_1\theta_2 + \left(\frac{1}{2}C_{13}\mathcal{S}_B - \frac{1}{2}C^*_{24}\mathcal{S}_F\right)\ \theta_1\theta_3 \\
&+ \left(\frac{1}{2}C_{14}\mathcal{S}_B + \frac{1}{2}C^*_{23}\mathcal{S}_F\right)\ \theta_1\theta_4 + \left(\frac{1}{2}C_{23}\mathcal{S}_B + \frac{1}{2}C^*_{14}\mathcal{S}_F\right)\ \theta_2\theta_3 \\
&+ \left(\frac{1}{2}C_{24}\mathcal{S}_B - \frac{1}{2}C^*_{13}\mathcal{S}_F\right)\ \theta_2\theta_4 + \left(\frac{1}{2}C_{34}\mathcal{S}_B - \frac{1}{2}C^*_{12}\mathcal{S}_F\right)\ \theta_3\theta_4
\end{aligned}
$$

- No θ term: four boson scattering, four θ term : four fermion scattering.

- All other processes (two boson to two fermion etc) determined completely in terms of the two independent functions $\mathcal{S}_B$ and $\mathcal{S}_F$.

Dyson-Schwinger equations

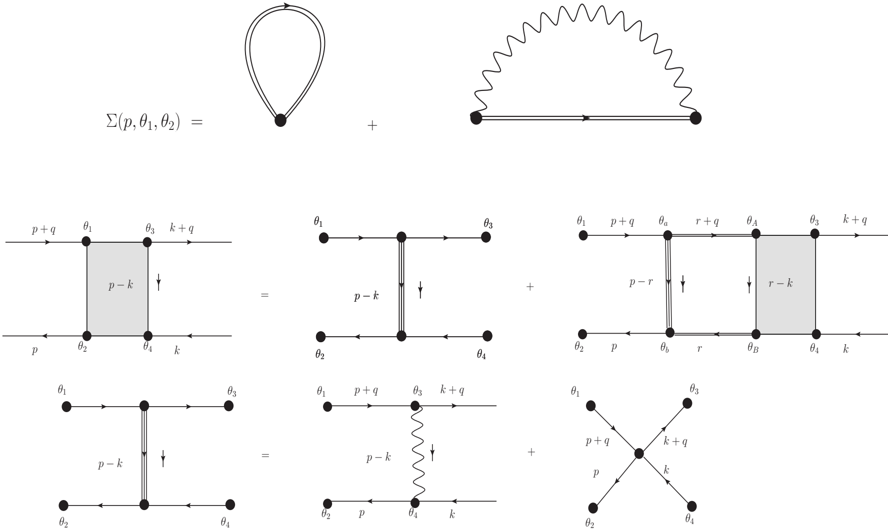


Figure: Dyson-Schwinger equation for exact propagator and exact offshell four point function.

S matrix in T , U_d, U_e channels for $\mathcal{N}=1,2$ theories,duality and unitarity

- We have computed S matrices in the $\mathcal{N}=1$ theory

$$\mathcal{T}_B = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} + J_B(q,\lambda,w,m,\kappa) \ ,$$
$$\mathcal{T}_F = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} + J_F(q,\lambda,w,m,\kappa) \ ,$$

- Remarkable simplification in the $\mathcal{N}=2$ point (w=1).

$$\mathcal{T}_B^{\mathcal{N}=2} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} - \frac{8\pi m}{\kappa} \ ,$$
$$\mathcal{T}_F^{\mathcal{N}=2} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} + \frac{8\pi m}{\kappa}$$

- under duality bosonic and fermionic S matrices map to one another upto overall phase.

- Explicitly verified unitarity equations (linear in these channels).

Conjectured S matrix in S channel $\mathcal{N}=1$ theory

- Following [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama], the conjectured S matrix for the $\mathcal{N}=1$ theory in c.o.m. frame

$$\mathcal{S}_B^S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\left(4\pi i\lambda\sqrt{s}\cot(\theta/2) + J_B(\sqrt{s},\lambda)\right) \ ,$$
$$\mathcal{S}_F^S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\left(4\pi i\lambda\sqrt{s}\cot(\theta/2) + J_F(\sqrt{s},\lambda)\right).$$

- conjectured S channel S matrix has simple non-relativistic limit leading to unitary version of known Aharonov-Bohm result.

- Surprisingly this result is also same as the $\mathcal{N}=2$ S channel S matrix.

- S matrix for the $\mathcal{N}=2$ (w=1) theory in c.o.m. frame

$$\mathcal{S}_B^{S;\mathcal{N}=2}(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\sin(\pi\lambda)\left(4i\sqrt{s}\cot(\theta/2) - 8m\right) \ ,$$
$$\mathcal{S}_F^{S;\mathcal{N}=2}(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\sin(\pi\lambda)\left(4i\sqrt{s}\cot(\theta/2) + 8m\right) \ .$$

Unitarity equations in the S channel

- Consider the general structure ($T(\theta) = i\cot(\theta/2)$)

$$\mathcal{T}_B^S = H_B T(\theta) + W_B - iW_2\delta(\theta)\ ,\ \mathcal{T}_F^S = H_F T(\theta) + W_F - iW_2\delta(\theta)\ ,$$

- unitarity equation for four boson scattering

$$H_B - H_B^* = \frac{1}{8\pi\sqrt{s}}(W_2 H_B^* - H_B W_2^*)\ ,$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H_B H_B^*)\ ,$$

$$W_B - W_B^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_B^* - W_2^* W_B) - \frac{i}{4\sqrt{s}}(H_B H_B^* - W_B W_B^*) - \frac{iY}{4\sqrt{s}}(W_B - W_F)(W_B^* - W_F^*)$$

- unitarity equation for four fermion scattering

$$H_F - H_F^* = \frac{1}{8\pi\sqrt{s}}(W_2 H_F^* - H_F W_2^*)\ ,$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H_F H_F^*)\ ,$$

$$W_F - W_F^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_F^* - W_2^* W_F) - \frac{i}{4\sqrt{s}}(H_F H_F^* - W_F W_F^*) - \frac{iY}{4\sqrt{s}}(W_B - W_F)(W_B^* - W_F^*)$$

Unitarity equation in the S channel

- Unitarity equations are verified using

$$H_B = H_F = 4\sqrt{s}\sin(\pi\lambda),\ W_2 = 8\pi\sqrt{s}(\cos(\pi\lambda)-1),\ T(\theta) = i\cot(\theta/2)$$

$$W_B = J_B(\sqrt{s},\lambda)\frac{\sin(\pi\lambda)}{\pi\lambda}\ ,$$
$$W_F = J_F(\sqrt{s},\lambda)\frac{\sin(\pi\lambda)}{\pi\lambda}\ .$$

- Algebraic-miracle: Non-linear unitarity equations obeyed by very complicated functions.

- Important: requires delta function at forward scattering and crossing symmetry rules have to be modified exactly as conjectured [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama], else the unitarity test fails.

- unitarity is an extremely sensitive test [1] .

[1] Tag-line: one sign is doom

Summary of results

- Results in perfect agreement with duality.

- Unitarity requires the delta function at forward scattering and crossing symmetry rules modified in exactly the same way as conjectured in
  [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama]

- Substantial evidence for universality of the conjecture.

- Results of $\mathcal{N} = 2$ theory obtained at special value of quartic scalar coupling.

- Non-renormalization of pole mass and vertex for $\mathcal{N} = 2$ theory - good things happen with more susy .

- $\mathcal{N} = 1$ S matrix has interesting pole structure, with vanishing pole mass on a self-dual codimension one surface in the space of couplings.

Future outlook

- $\mathcal{N}=2$ S matrices are tree level exact in non-anyonic channels and depend on λ very simple way in the anyonic channel - can it reproduced from general principles and $\mathcal{N}=2$ supersymmetry?

- Generalization to higher supersymmetry - mass deformed $\mathcal{N}=3,4,5$, and mass deformed $\mathcal{N}=6$ ABJ theory - in progress [K.I, S.Jain, S.Minwalla, S. Yokoyama]

- Effective field theory for the massless bound states of the S matrix.

- Four point correlator: useful in computation of 2,3,4 point functions of gauge invariant currents - explicit computation in $\mathcal{N}=2$ theory?, possible $\mathcal{N}=2$ generalization of Maldacena-Zhiboedov solutions - in progress [K.I, S.Jain, P.Nayak]

Thank You!

Unitarity in the S channel - $\mathcal{N} = 2$ case

- The $\mathcal{N} = 2$ T matrix is tree level exact in T, U channels.
- Naive crossing symmetry would imply the same for S channel, unitarity equation $i(T^\dagger - T) = TT^\dagger$ would never be obeyed (LHS would be zero).
- modified crossing rules resolve this puzzle:

$$\mathcal{T}_B^{S;\mathcal{N}=2}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda) - 1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2) - 8m)$$
$$\mathcal{T}_F^{S;\mathcal{N}=2}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda) - 1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2) + 8m)$$

- Non-analytic piece makes $\mathcal{T}_B, \mathcal{T}_F$ not Hermitian, both LHS and RHS are non-zero and non-linear unitarity equation is obeyed.

Pole structure of the singlet channel S matrix

- Both bosonic and fermionic S matrices have a pole at threshold ($s = 4m^2$) for $w \leq -1$. For $w \leq -1 + \epsilon$ the pole is close to threshold.

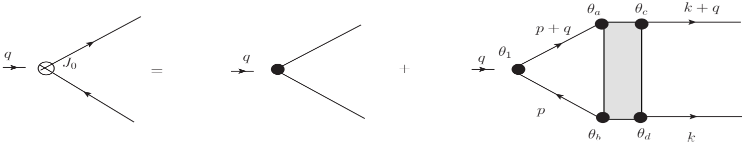
- As w is decreased further and as it hits a critical value $w = w_c$ the pole becomes massless!
$$w = w_c(\lambda) = 1 - \frac{2}{|\lambda|}$$

- As w is further decreased and as $w \to -\infty$ the pole approaches threshold once again.

- To summarize, a one parameter tuning of the superpotential interaction parameter w - sufficient to produce massless bound states in a massive theory.

- w can be scaled to w_c - possible decoupled QFT description of light states.

Current-current correlators in $\mathcal{N} = 2$ theory



- two point J_0 correlator

$$\langle J_0(\theta_1, q) J_0(\theta_2, -q)\rangle = \frac{N}{8\pi|q|\lambda} \exp(-\theta_1^\alpha \theta_2^\beta q_{\alpha\beta}) \Big(\sin(\pi\lambda) + |q|(1-\cos(\pi\lambda))\delta^2(\theta_1 - \theta_2) \Big)$$

- three point J_0 correlator

$$\begin{aligned}\langle J_0(\theta_1, q) J_0(\theta_1', q') J_0(\theta_1'', -q-q')\rangle = &\left(\frac{N}{72 \;\, q_3 q_3'(q_3 + q_3')} \frac{\sin(\pi\lambda)}{\pi\lambda} \right) \Bigg[-9\cos(\pi\lambda) \\ &+ 9i\sin(\pi\lambda) \left(q_3 X^-_{11''} X^+_{11''} + q_3' X^-_{1'1''} X^+_{1'1''} \right) \\ &+ 3\cos(\pi\lambda)\; (q_3' - q_3) \left(X^-_{11''} X^+_{1'1''} - X^-_{1'1''} X^+_{11''} \right) \\ &- \cos(\pi\lambda)\; (q_3^2 + 7 q_3 q_3' + q_3'^2)\;\, X^-_{11''} X^+_{11''} X^-_{1'1''} X^+_{1'1''} \Bigg] \\ &\times e^{\frac{1}{3} X \cdot (q \cdot X_{11''} + q' \cdot X_{1'1''})}\end{aligned}$$

[K.I, S.Jain, P.Nayak]

Open questions

- Rigorous proof of delta function and modified crossing rules, generalization to finite N and κ.
- From perturbative pov modified crossing rules could be related to IR divergences.
- IR divergences can be summed up and exponentiated [Grammer,Yennie; Bern,Dickson,Smirnov]
- Modified crossing factor $\frac{\sin(\pi\lambda)}{\pi\lambda}$ is identical to the expectation value of circular Wilson loop in pure Chern-Simons theory on S^3.
- To explore: Path integral derivation of Witten's result, crossing and fusion rules in RCFT's.