Stability of Bianchi attractors in gauged supergravity

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Plan of the talk

Introduction

- In gauge/gravity correspondence, black branes are holographic duals to field theories at finite temperature.
- Extremal branes exhibit vanishing entropy density at zero temperature and describe ground states of the field theory.
- Bianchi attractors: Classification of homogeneous anisotropic extremal black brane horizons in $d = 5$.

  Iizuka-Kachru-Kundu-Narayan-Sircar-Trivedi
- Lifshitz, Schrödinger geometries belong to Bianchi Type I in the classification.
- Appear as generalised attractor solutions in extensions of attractor mechanism to gauged supergravity.

  Cacciatori-Klemm , Kachru-Kallosh-Shmakova

Generalised attractors

- Generalised attractors: solutions to equations of motion that reduce to algebraic equations when all the fields, curvature tensors are constants in tangent space.
- Gauge field, and Einstein equations reduce to algebraic equations at the attractor point.
- Scalar field equations reduce to a minimisation condition on an attractor potential.
- Generalised attractor geometries are characterised by constant anholonomy coefficients.

$$[e_a, e_b] = c_{ab}{}^{c} e_c \ ; \quad e_a \equiv e_a^{\mu}\partial_\mu$$

$$c_{ab}{}^{c} = e_a^{\mu} e_b^{\nu}(\partial_\mu e_\nu^c - \partial_\nu e_\mu^c)$$

- Bianchi attractors have constant anholonomy by construction.

Bianchi Attractors in gauged supergravity

- Bianchi type metrics can be easily realised in simple Einstein-Maxwell systems with massive gauge fields.

- Typical scalar kinetic term of gauged supergravities,

$$g_{xy}\mathcal{D}_\mu\phi^x\mathcal{D}^\mu\phi^y; \quad \mathcal{D}_\mu\phi^x \equiv \partial_\mu\phi^x + gA^I_\mu K^x_I(\phi).$$

- At attractor points scalars are constant, terms like

$$g^2 g_{xy}K^x_I K^y_J A^I_\mu A^{J\mu}$$

act as effective mass term for the gauge field.

- Several Bianchi attractors were embedded in gauged supergravity as generalised attractors. Inbasekar-Tripathy

Motivation: Stability of Bianchi metrics

- Instabilities due to scalar fluctuations exist in large class of such metrics. Donos-Gauntlett-Pantelidou, Cremonini-Sinkovics, Andrade-Ross, Keeler
- Studying such instabilities help in understanding how the geometry might get corrected in the IR.
- A common recipe to study the stability of Bianchi type metrics will be useful.
- Embedding Bianchi type metrics as generalised attractors in gauged supergravity provided an ideal platform for this study.

Motivation: Stability of generalised attractors

- Generalised attractor analysis does not involve susy, relies on extremisation of an attractor potential.
- Solutions were found at critical points, not at absolute minima of attractor potential.
- Preliminary susy analysis of existing solutions using KSI indicated broken susy.
- Non-susy attractors can be unstable to scalar fluctuations about critical value.

Overview

- Analyse the stability of Bianchi attractors in gauged supergravity under scalar fluctuations about the attractor value.
- Examine the field equations at the linearised level and demand that fluctuations vanish near the horizon.
- Determine conditions of stability.
- Identify the class of Bianchi attractors which satisfy the condition.

Bianchi Attractors

Biachi Attractors: Symmetries

- Bianchi Attractors: Five dimensional extremal black brane horizons with homogeneous symmetries in spatial directions. Iizuka-Kachru-Kundu-Narayan-Sircar-Trivedi

- Homogeneous symmetries: invariant basis $\tilde{e}_i, i = 1, 2, 3$ that commutes with Killing vectors.

$$[\xi_j, \tilde{e}_i] = 0, \quad [\tilde{e}_i, \tilde{e}_j] = c_{ij}{}^k \tilde{e}_k$$

- Invariant vectors close to form a Lie algebra - isomorphic to Bianchi classification (I-IX) of 3d real Lie algebras Bianchi.

- Metric written in terms of invariant one forms ω^i dual to $\tilde{e}_i$ displays manifest homogeneous symmetries.

$$d\omega^k = \frac{1}{2} c_{ij}{}^k \omega^i \wedge \omega^j$$

Bianchi Attractors: Symmetries

- Additional symmetries: scale invariance, time translation invariance

$$\hat{r} \to \lambda \hat{r} \ , \quad \hat{t} \to \lambda^{-u_0} \hat{t} \ , \quad \omega^i \to \lambda^{-u_i} \omega^i$$

- Fix the form of the metric completely.

$$ds^2 = L^2 \left[-\hat{r}^{2u_0} d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + \hat{r}^{2(u_i+u_j)} \eta_{ij} \omega^i \otimes \omega^j \right]$$

- Have constant anholonomy coefficients by construction.

Example: Bianchi II

- One forms, invariant vectors, structure constants,

$$
\begin{aligned}
&{c_{23}}^1 = 1 = -{c_{32}}^1, \\
&\xi_1 = \partial_{\hat{y}}, && \tilde{e}_1 = \partial_{\hat{y}}, && \omega^1 = d\hat{y} - \hat{x}d\hat{z}, && d\omega^1 = \omega^2 \wedge \omega^3, \\
&\xi_2 = \partial_{\hat{z}}, && \tilde{e}_2 = \hat{x}\partial_{\hat{y}} + \partial_{\hat{z}}, && \omega^2 = d\hat{z}, && d\omega^2 = 0, \\
&\xi_3 = \partial_{\hat{x}} + \hat{z}\partial_{\hat{y}}, && \tilde{e}_3 = \partial_{\hat{x}}, && \omega^3 = d\hat{x}, && d\omega^3 = 0
\end{aligned}
$$

- scaling in coordinates,

$$(\hat{x}, \hat{y}, \hat{z}) \to (\lambda^{-u_1}\hat{x}, \lambda^{-(u_1+u_3)}\hat{y}, \lambda^{-u_3}\hat{z})$$

- scaling in one forms,

$$(\omega^1, \omega^2, \omega^3) \to (\lambda^{-(u_1+u_3)}\omega^1, \lambda^{-u_3}\omega^2, \lambda^{-u_1}\omega^3)$$

- metric

$$ds^2 = L^2\left[-\hat{r}^{2u_0}d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + \hat{r}^{2(u_1+u_3)}(\omega^1)^2 + \hat{r}^{2u_3}(\omega^2)^2 + \hat{r}^{2u_1}(\omega^3)^2\right]$$

Example: Bianchi IX

- Invariant one forms,

$$
\begin{aligned}
\omega^1 &= -\sin(\hat{z})d\hat{x} + \sin(\hat{x})\cos(\hat{z})d\hat{y}, & d\omega^1 &= \omega^2 \wedge \omega^3 \ , \\
\omega^2 &= \cos(\hat{z})d\hat{x} + \sin(\hat{x})\sin(\hat{z})d\hat{y}, & d\omega^2 &= \omega^3 \wedge \omega^1 \ , \\
\omega^3 &= \cos(\hat{x})d\hat{y} + d\hat{z}, & d\omega^3 &= \omega^1 \wedge \omega^2
\end{aligned}
$$

- no scaling symmetry in $(\hat{x}, \hat{y}, \hat{z})$ coordinates and one forms

$$ds^2 = L^2\left[-\hat{r}^{2u_0}d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + (\omega^1)^2 + (\omega^2)^2 + (\omega^3)^2 \right]$$

- Metric can be rewritten in direct product form $AdS_2 \times M_{IX}$

$$ds^2 = L_1^2\left(-\tilde{r}^2 d\tilde{t}^2 + \frac{d\tilde{r}^2}{\tilde{r}^2} \right) + L_2^2\left((\omega^1)^2 + (\omega^2)^2 + (\omega^3)^2 \right)$$

Stability of Bianchi attractors in gauged sugra

Gauged Sugra: Symmetries

- The most general $\mathcal{N}=2, d=5$ gauged sugra has gravity coupled to vector, tensor and hypermultiplets.

Ceresole-Dall'Agata

- The scalars in the theory parametrise a manifold that factorises into a direct product of a very special and quaternionic manifold,

$$\mathcal{M}_{scalar} = \mathcal{S}(n_v + n_T) \otimes \mathcal{Q}(n_H).$$

$$C_{IJK} h^I h^J h^K = 1, \quad h^I \equiv h^I(\phi).$$

- The R symmetry group is $SU(2)_R$.

- We consider the truncated theory with vector multiplets, abelian gauging of the isometries of the scalar manifold and gauging of $U(1)_R$ symmetry.

Gauged Sugra: Gauging the Symmetries

- Gauging: Suitable subgroup K of the isometry group G of the full scalar manifold $\mathcal{M}_{scalar}$, and the $U(1)_R$ symmetry group.

- Ordinary derivatives on scalar and fermions are replaced with K-covariant derivatives.

$$\partial_\mu \phi^x \rightarrow \mathcal{D}_\mu \phi^x \equiv \partial_\mu \phi^x + g A^I_\mu K^x_I(\phi)$$

- Gauging the $U(1)_R$ Symmetry:

$$\nabla_\mu \psi_{\nu i} \rightarrow \nabla_\mu \psi_{\nu i} + \imath g_R A^I_\mu V_I \psi_{\nu i}.$$

- Gauging leads to scalar potentials in the theory - possibility of AdS vacuum.

Lagrangian

- The bosonic part of the Lagrangian:

$$\hat{e}^{-1}\mathcal{L}^{\mathcal{N}=2}_{Bosonic} = -\frac{1}{2}R - \frac{1}{4}a_{IJ}F^{I}_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}\mathcal{D}_{\mu}\phi^{x}\mathcal{D}^{\mu}\phi^{y} - \mathcal{V}(\phi) + \frac{\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\tau}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}A^{K}_{\tau}$$

- The potential in this case comes only from the R symmetry gauging,

$$\mathcal{V}(\phi) = -g_R^2[2P_{ij}P^{ij} - P^{x}_{ij}P^{xij}],$$

$$P_i{}^k \equiv h^I V_I \delta_i{}^k, \quad P^x_i{}^k \equiv h^{xI} V_I \delta_i{}^k.$$

Fluctuations about attractor value

- Fluctuations

$$\phi^x_c + \epsilon\delta\phi^x,$$
$$A^I_\mu + \epsilon\delta A^I_\mu,$$
$$g_{\mu\nu} + \epsilon\gamma_{\mu\nu},$$

- Gauge field equations,

$$\begin{aligned} a_{IJ}|_{\phi_c}\nabla_\mu F_\delta^{I\mu\nu} - g^2 K_{IJ}|_{\phi_c}\delta A^{\nu J} = \\ -\left(\left.\frac{\partial a_{IJ}}{\partial\phi^z}\right|_{\phi_c}\nabla_\mu(F^{I\mu\nu}\delta\phi^z) - g^2\left.\frac{\partial K_{IJ}}{\partial\phi^z}\right|_{\phi_c}\delta\phi^z A^{\nu J}\right) \\ + gK_{Iy}|_{\phi_c}\partial^\nu\delta\phi^y \end{aligned}$$

- Regularity of the gauge fields requires well behaved scalar fluctuations near the horizon.

Linearised Einstein equations

- Linearised Einstein equation,

$$\nabla^{\alpha}\nabla_{\alpha}\bar{\gamma}_{\mu\nu} + 2R_{(\mu\ \ \nu)}^{\ \ \alpha\ \ \beta}\bar{\gamma}_{\beta\alpha} - 2R_{(\mu}^{\ \ \beta}\bar{\gamma}_{\nu)\beta} + g_{\mu\nu}(R_{\alpha\beta}\bar{\gamma}^{\alpha\beta} + \frac{2}{2-D}R\bar{\gamma}) + R\bar{\gamma}_{\mu\nu}$$
$$+2\dot{T}^{attr}_{\mu\nu}(g_{\alpha\beta} + \epsilon\gamma_{\alpha\beta})|_{\epsilon=0} + 2\dot{T}_{\mu\nu}(\phi_c + \epsilon\delta\phi)|_{\epsilon=0} = 0$$

- Stress energy dependence on $\gamma_{\mu\nu}$ and $\delta\phi^z$

$$\begin{aligned}\dot{T}^{attr}_{\mu\nu}(g_{\alpha\beta} + \epsilon\gamma_{\alpha\beta})|_{\epsilon=0} =& \mathcal{V}_{attr}(\phi_c)(\bar{\gamma}_{\mu\nu} + \frac{2\bar{\gamma}}{2-D}g_{\mu\nu}) \\ & - (\bar{\gamma}_{\lambda\sigma} + \frac{\bar{\gamma}}{2-D}g_{\lambda\sigma})(\frac{1}{2}T^{\lambda\sigma}_{attr}g_{\mu\nu} + a_{IJ}|_{\phi_c}F^{I\ \lambda}_{\mu}F^{J\ \sigma}_{\nu})\end{aligned}$$

$$\begin{aligned}\dot{T}_{\mu\nu}(\phi_c + \delta\phi)|_{\epsilon=0} =& T^{attr}_{\mu\nu}|_{\phi_c} \\ & + gK_{yI}|_{\phi_c}\Big(A^{\lambda I}\partial_{\lambda}(\delta\phi^y)g_{\mu\nu} - A^I_{\mu}\partial_{\nu}(\delta\phi^y) - A^I_{\nu}\partial_{\mu}(\delta\phi^y)\Big) \\ & - \left[\frac{\partial a_{IJ}}{\partial\phi^z}\bigg|_{\phi_c}F^I_{\mu\lambda}F^{J\ \lambda}_{\nu} + g^2\frac{\partial K_{IJ}}{\partial\phi^z}\bigg|_{\phi_c}A^I_{\mu}A^J_{\nu}\right]\delta\phi^z\end{aligned}$$

Stress energy tensor: Backreaction at first order

- For Gauged sugra with generic gauging, trace of Einstein equation,

$$R(g_{\mu\nu},\gamma_{\mu\nu})\frac{(2-D)}{2} = T_{\mu}^{attr\,\mu}|_{\phi_c} + (D-2)gK_{yI}|_{\phi_c}A^{\lambda I}\partial_{\lambda}(\delta\phi^{y}) + g^{2}\frac{\partial K_{IJ}}{\partial\phi^{z}}\bigg|_{\phi_c}A^{I}_{\mu}A^{J\mu}\delta\phi^{z}$$

$$T_{\mu}^{\mu attr}|_{\phi_c} = \mathcal{V}_{attr}(\phi_c)D - \left[a_{IJ}|_{\phi_c}F^{I}_{\mu\nu}F^{\mu\nu J} + g^{2}K_{IJ}|_{\phi_c}A^{I}_{\mu}A^{\mu J}\right]$$

$$K_{IJ} = g_{xy}K^{x}_{I}K^{y}_{J}$$

- Scalar fluctuation terms indicate backreaction even at first order perturbation.

- Relevant boundary conditions for scalars should be such that they are well behaved near the horizon.

- For $U(1)_R$ gauging, $g = 0$ and back reaction is absent.

Scalar fluctuation equations

- Scalar fluctuation equation for arbitrary gauged sugra,

$$\nabla_\mu \nabla^\mu \delta\phi^x - g^{zx} \frac{\partial^2 \mathcal{V}_{attr}}{\partial\phi^z \partial\phi^y}\bigg|_{\phi_c} \delta\phi^y + 2g\, (g^{zx} \tilde{\nabla}_y K_{Iz})|_{\phi_c} A^{\mu I} \nabla_\mu \delta\phi^y = 0$$

  $\tilde{\nabla}$ - covariant derivative w.r.t g_{xy}.
  ∇ - covariant derivative w.r.t near horizon metric.

- higher order metric/gauge field fluctuations can be ignored for solving the above equation at lowest order.

- Laplacian for any given 5d Bianchi type metric,

$$\nabla_\mu \nabla^\mu = \frac{1}{L^2}\left[\hat{r}^2 \partial_{\hat{r}}^2 + (m+2)\hat{r}\partial_{\hat{r}} - \frac{1}{\hat{r}^{2u_0}}\partial_{\hat{t}}^2\right]$$

  $m = -1 + \sum_I c_I u_I,\ c_I > 0,\ c_0 = 1.$

Scalar fluctuation equations

- For the specific gauged supergravity model fluctuation equation reduce to ,

$$\left[\hat{r}^2\partial_{\hat{r}}^2 + (m+2)\hat{r}\partial_{\hat{r}} - \frac{1}{\hat{r}^{2u_0}}\partial_{\hat{t}}^2 - \lambda\right]\delta\phi^x = 0$$

  λ - Eigenvalue of double derivative of attractor potential.
  Sign of λ - indicates nature of critical point.

- For ansatz $\delta\phi(\hat{r},\hat{t}) = f(\hat{r})e^{ik\hat{t}}$ (with k real), we get Bessel equation

$$\left[\hat{r}^2\partial_{\hat{r}}^2 + (m+2)\hat{r}\partial_{\hat{r}} + (\frac{k^2}{\hat{r}^{2u_0}} - \lambda)\right]f(\hat{r}) = 0$$

Scalar fluctuations

- Scalar fluctuations

$$f(X) = \left(\frac{X}{2}\right)^{\nu_0}\Big[C_1 H^1_{\nu_\lambda}(X)\big[\Gamma(1-\nu_\lambda)e^{i\nu_\lambda\pi} + \Gamma(1+\nu_\lambda)\big]$$
$$+ C_2 H^2_{\nu_\lambda}(X)\big[\Gamma(1-\nu_\lambda)e^{-i\nu_\lambda\pi} + \Gamma(1+\nu_\lambda)\big]\Big]$$

$$X = \frac{k}{u_0 \hat{r}^{u_0}}, \quad \nu_\lambda = \frac{\sqrt{(1+m)^2+4\lambda}}{2u_0}, \quad \nu_0 = \frac{(1+m)}{2u_0}$$

- Consistency condition for ν_λ real,

$$\nu_\lambda = \frac{\sqrt{(1+m)^2+4\lambda}}{2u_0} = \frac{\sqrt{(\sum_I c_I u_I)^2+4\lambda}}{2u_0} \leq 1$$

- implies $\lambda < 0$,

$$-\frac{(\sum_I c_I u_I)^2}{4} \leq \lambda < 0$$

- Scalar fluctuations - well defined for critical points which are maxima of attractor potential.

Conditions for stability

- In our coordinate system horizon is located at $\hat{r} = 0$, $X \simeq 1/\hat{r}$, consider asymptotic expansion of $f(X)$

$$f(X) \sim \left(\frac{X}{2}\right)^{\nu_0 - \frac{1}{2}} \sqrt{\frac{1}{\pi}} \Big[C_1 e^{i(X - \frac{\pi}{2}(\nu_\lambda + \frac{1}{2}))} \left[\Gamma(1-\nu_\lambda) e^{i\nu_\lambda \pi} + \Gamma(1+\nu_\lambda)\right]$$
$$+ C_2 e^{-i(X - \frac{\pi}{2}(\nu_\lambda + \frac{1}{2}))} \left[\Gamma(1-\nu_\lambda) e^{-i\nu_\lambda \pi} + \Gamma(1+\nu_\lambda)\right] \Big]$$

- Leading divergent term is absent only when,

$$\nu_0 = \frac{(1+m)}{2u_0} = \frac{\sum_l c_l u_l}{2u_0} \leq \frac{1}{2}$$

- since $c_0 = 1$,

$$\sum_{l, l \neq 0} c_l u_l \leq 0$$

- But $u_l \geq 0$ for regular horizon, therefore stability conditions are:

$$u_0 \neq 0, \quad u_l = 0 \ \ \forall \ l \neq 0$$

Stable Bianchi attractors

- Bianchi attractors with scale invariance in all directions,

$$ds^2 = L^2\left[-\hat{r}^{2u_0}d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + \hat{r}^{2(u_i+u_j)}\eta_{ij}\omega^i \otimes \omega^j\right]$$

- Stability condition,

$$u_0 \neq 0, \quad u_I = 0 \ \ \forall \ I \neq 0$$

- Stable Bianchi attractors in gauged supergravity are a subclass with scale invariance only in $\hat{r}$ and $\hat{t}$.

$$ds^2 = L^2\left(-\hat{r}^{2u_0}d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2}\right) + L^2\left(\eta_{ij}\omega^i \otimes \omega^j\right)$$

- They are of the direct product form $AdS_2 \times M$.

$$ds^2 = L_1^2\left(-\tilde{r}^2 d\tilde{t}^2 + \frac{d\tilde{r}^2}{\tilde{r}^2}\right) + L_2^2\left(\eta_{ij}\omega^i \otimes \omega^j\right)$$

Stable Bianchi attractors

- Unstable generalised attractors

Geometry	λ	u_0	$u_I, I \neq 0$
Lifshitz	-34	3	1
Bianchi II	$-\frac{22}{3}$	$\sqrt{2}$	$u_1 = u_3 = \frac{1}{2\sqrt{2}}$
Bianchi VI $h < 0$	$-1 + \frac{14h}{3} - h^2$	$\frac{1}{\sqrt{2}}(1-h)$	$u_2 = -\frac{1}{\sqrt{2}}h, u_3 = \frac{1}{\sqrt{2}}$

- Stable generalised attractors in direct product form

Geometry	λ	u_0	$u_I, I \neq 0$
$Lif_{u_0}(2) \times M_I$	$-\frac{5u_0^2}{3}$	any $u_0 > 0$	0
$AdS_2 \times M_I$	$-\frac{5}{3}$	1	0
$Lif_{u_0}(2) \times M_{II}$	$-\frac{61}{6}$	$\sqrt{\frac{11}{2}}$	0
$Lif_{u_0}(2) \times M*$	$\lambda < 0$	any $u_0 > 0$	0

Summary and Future outlook

Summary

- Bianchi attractors are generalised attractor solutions in gauged supergravity.
- Generalised attractor procedure relies on extremisation of an attractor potential rather than susy.
- non-supersymmetric fixed points can be unstable attractors.
- We studied scalar fluctuations about the attractor value and derived stability conditions by demanding regularity near the horizon.

Results

- Stress energy tensor in gauged supergravity depends on scalar fluctuations even at first order.

- Instability - Ill behaved fluctuations near the horizon will backreact strongly $\implies$ significant deviation from the attractor geometry.

- Consistency condition on scalar fluctuations: critical point is a maxima of the attractor potential.

- Regularity of the fluctuations near the horizon require the near horizon geometry to factorise as $AdS_2 \times M$,

$$ds^2 = L_1^2 \left(-\tilde{r}^2 d\tilde{t}^2 + \frac{d\tilde{r}^2}{\tilde{r}^2} \right) + L_2^2 \left(\eta_{ij} \omega^i \otimes \omega^j \right)$$

  $M = M_I, M_{II} \ldots M_{IX}$ - 3d homogeneous subspaces invariant under the Bianchi type symmetries.

Caveats

- Purely radial fluctuations $k = 0$,

$$\left[\hat{r}^2\partial_{\hat{r}}^2 + (m+2)\hat{r}\partial_{\hat{r}} - \lambda\right]\delta\phi^x = 0.$$

$$\delta\phi^x = C_1 r^{\left(\sqrt{4\lambda+(1+m)^2}-(1+m)\right)/2} + C_2 r^{\left(-\sqrt{4\lambda+(1+m)^2}-(1+m)\right)/2}$$

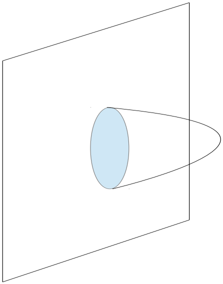
- Fluctuations vanishing as $\hat{r} \to 0$ exist when $\lambda > 0$, $C_2 = 0$.

- However, no bianchi attractors were found for critical points with $\lambda > 0$ - possible model dependent artifacts.

Future outlook

- Based on stability analysis we conclude that Bianchi attractors of the form $AdS_2 \times M$ are stable geometries in the deep IR.
- The factorisation of the near horizon geometry is reminiscent of the situation for extremal black holes.
- It is known that the near horizon geometry of extremal black holes preserve supersymmetry.
- Reasonable to expect $AdS_2 \times M$ geometries to preserve some fraction of the supersymmetry (work in progress).

Thank You!



Minimal Surface