Part 1: Chern-Simons matter theories

Karthik Inbasekar

tifr

IMSc, Chennai

25 Jan 2016

Plan of the talk

Chern-Simons theory

- Physics in 2+1 dimensions has many interesting features and intriguing surprises.

- There exist a new type of gauge theory completely different from the usual Maxwell theory called Chern-Simons theory.

- Originally noticed by S.S.Chern and J.Simons that the Pontryagin density in 3+1 dimensions could be written as a total derivative

$$\epsilon^{\mu\nu\rho\sigma} Tr(F_{\mu\nu}F_{\rho\sigma}) = 4\partial_\sigma\left(\epsilon^{\mu\nu\rho\sigma} Tr(A_\mu\partial_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho)\right)$$

- The boundary term has the same form as the Chern-Simons Lagrangian.

- Chern-Simons theories are theoretically novel and have practical application in planar condensed matter phenomena.

Abelian Chern-Simons theory

$$L_{CS} = \frac{\kappa}{2}\epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - A_\mu J^\mu$$

- Involves gauge field instead of manifestly gauge invariant field strength.

- Changes by a total derivative on a gauge transf.

$$\delta L_{CS} = \frac{\kappa}{2}\partial_\mu(\epsilon^{\mu\nu\rho}\partial_\nu A_\rho)$$

- Classical equations of motion

$$\frac{\kappa}{2}\epsilon^{\mu\nu\rho} F_{\nu\rho} = J^\mu$$

- Bianchi identity is compatible with current conservation

$$\epsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho} = 0\ ,\ \ \partial_\mu J^\mu = 0$$

Abelian Chern-Simons theory

- First order in space-time derivatives, source free equation of motion $F_{\mu\nu} = 0$.

- Solutions are pure gauge, in contrast source free Maxwell equations have plane wave solutions.

- Chern-Simons theory is interesting when coupled to matter (charged bosons/fermions) ($J^\mu = (\rho, \vec{J})$)

- Equations of motion in component form when theory is coupled to matter current

$$\rho = \kappa B \ , \ J^i = \kappa \epsilon^{ij} E_j$$

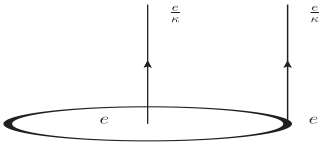
- Electric charge density $\propto$ Magnetic field.

Chern-Simons+matter: Anyons

- Chern-Simons interaction ties magnetic flux to electric charge.

- Charge-flux relation is preserved under time evolution.

- Charge flux coupling leads to Aharonov-Bohm type interactions.



- Adiabatic excursion of one particle around the other: the wave function acquires Aharonov-bohm phase $e^{ie\oint_C A.dx} = e^{\frac{ie^2}{\kappa}}$.

- Point-particle explanation of anyonic statistics

$$\phi(1,2) = e^{i\nu\delta\varphi}\phi(2,1)$$

Non-abelian Chern-Simons theory: κ is quantized

- Non-abelian Chern-Simons action

$$S_{CS} = \kappa \epsilon^{\mu\nu\rho\sigma} Tr(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho)$$

- Under the gauge transformation

$$A_\mu \to g^{-1} A_\mu g + g^{-1} \partial_\mu g$$

- changes by a boundary term

$$S_{CS} \to S_{CS} - 8\pi^2 \kappa w(g) \; ; \; w(g) \in \mathbb{Z}$$

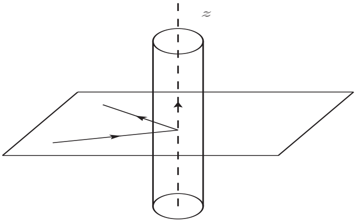
- Gauge invariance of the quantum amplitude $e^{iS_{CS}}$ requires

$$\kappa = \frac{\text{Integer}}{4\pi}$$

Non-relativistic scattering of Anyons

Aharonov-Bohm scattering

- Scattering of a unit charged non-relativistic particle of mass m off a flux tube.



- flux tube oriented in the z direction, located at origin of transverse 2d space.
- study states that preserve translational invariance in z direction - problem effectively in 2 spatial dimensions.
- Assume integral of flux of tube is $2\pi\nu$, anyonic phase of the particle: $2\pi i\nu$.

Aharonov-Bohm scattering

- Seek scattering state solutions at energy $E = \frac{p^2}{2m}$.
- Time dependent Schrodinger equation of the system

$$\left(-\frac{1}{2m}(\nabla + 2\pi i \nu G)^2 - \frac{k^2}{2m}\right)\psi = 0$$

$$G_i = \frac{\epsilon_{ij}}{2\pi}\partial_j \ln r$$

- Aharanov-Bohm boundary conditions: regularity of wave function at origin.
- Most general solution:

$$\psi(r,\theta) = \sum_{n>0} a_n e^{in\theta} J_{n+\nu}(kr) + \sum_{n>0} a_{-n} e^{-in\theta} J_{n-\nu}(kr) + a_0 J_{|\nu|}(kr)$$

- unknown coefficients are fixed by demanding that at large r, the ingoing piece ($\propto e^{-ikr}$) reduces to that of an incoming wave.

Aharonov-Bohm scattering

- Bessel function expansion of plane wave in the large r limit

$$\sum_n i^n J_n(kr)e^{in\theta} \sim \frac{\sqrt{2\pi}}{\sqrt{kr}}\left(e^{\frac{-i\pi}{4}}e^{ikr}\delta(\theta) + e^{\frac{i\pi}{4}}e^{-ikr}\delta(\theta-\pi)\right)$$

- describes an incoming wave at negative x axis ($\theta = -\pi$) and outgoing wave at positive x axis ($\theta = 0$).

- solution $\psi(r,\theta)$ (ingoing part is identical to plane wave)

$$\psi(r,\theta) = \sum_{n=1}^{\infty} i^n e^{-\frac{i\pi\nu}{2}} J_{n+\nu}(kr)e^{in\theta} + \sum_{n=1}^{\infty} i^n e^{\frac{i\pi\nu}{2}} J_{n-\nu}(kr)e^{-in\theta} + e^{-\frac{i\pi|\nu|}{2}} J_{|\nu|}(kr)$$

- large r limit

$$\psi(r,\theta) = \frac{1}{\sqrt{2\pi kr}}\left(2\pi e^{\frac{i\pi}{4}}\delta(\theta-\pi)e^{-ikr} + H(\theta)e^{-\frac{i\pi}{4}}e^{ikr}\right)$$

$$H(\theta) = 2\pi\cos(\pi\nu)\delta(\theta) + \sin(\pi\nu)\left(Pv\cot\left(\frac{\theta}{2}\right) - i\mathrm{Sgn}(\nu)\right)$$

Aharonov-Bohm scattering

- The scattering amplitude is read off by writing wave function as plane wave+scattered part

$$\psi(r,\theta) = e^{-ikx} + \frac{h(\theta)e^{-\frac{i\pi}{4}}e^{ikr}}{\sqrt{2\pi kr}}$$

- using the large r expansion for the incoming wave above

$$h(\theta) = H(\theta) - 2\pi\delta(\theta)$$

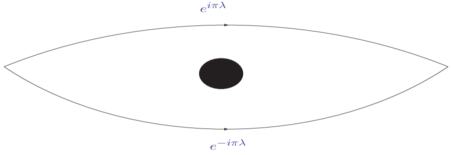
- The final non-relatvistic scattering amplitude

$$h(\theta) = 2\pi(\cos(\pi\nu) - 1)\delta(\theta) + \sin(\pi\nu)\left(Pv\cot\left(\frac{\theta}{2}\right) - i\mathrm{Sgn}(\nu)\right)$$

Aharanov-Bohm scattering - Interpretation

$$h(\theta) = 2\pi(\cos(\pi\nu) - 1)\delta(\theta) + \sin(\pi\nu)\left(Pv\cot\left(\frac{\theta}{2}\right) - i\mathrm{Sgn}(\nu)\right)$$

- $2\pi\delta(\theta)$ is the usual term that appears in a traditional scattering problem.
- $2\pi\cos(\pi\nu)\delta(\theta)$ specific to Aharanov-Bohm scattering.
- $\cos(\pi\nu)$ is due to the interference of the Aharanov-Bohm phases of the wave packets.



Unitarity of Aharanov-Bohm - importance of delta function

- This delta function piece was originally missed by Aharanov-Bohm.
- As we will see, it is necessary for unitarity of the amplitude [Ruijsenaars; Bak,Jackiw,Pi].
- Consider an S matrix (in center of mass frame) of the form

$$S(s,\theta) = I + iT(s,\theta)$$

- The unitarity condition is

$$-i(T(s,\theta) - T^*(s,-\theta)) = \frac{1}{8\pi\sqrt{s}} \int d\alpha \, T(s,\alpha) T^*(s,-(\alpha-\theta))$$

Unitarity of Aharanov-Bohm - importance of delta function

- Consider the general structure ($x(\theta) = i\cot(\theta/2)$)

$$T(\sqrt{s},\theta) = H(\sqrt{s})\ x(\theta) + W_1(\sqrt{s}) - iW_2(\sqrt{s})\delta(\theta)$$

- unitarity equation can be written as

$$H - H^* = \frac{1}{8\pi\sqrt{s}}(W_2 H^* - H W_2^*)\ ,$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H H^*)\ ,$$

$$W_1 - W_1^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_1^* - W_2^* W_1) - \frac{i}{4\sqrt{s}}(H H^* - W_1 W_1^*)$$

- For Aharanov-Bohm scattering

$$H = 4\sqrt{s}\sin(\pi\nu)\ ,\ W_1 = -4\sqrt{s}\sin(\pi\nu)\mathrm{Sgn}(\nu)\ ,\ W_2 = 8\pi\sqrt{s}(\cos(\pi\nu) - 1)$$

- First and third equations trivially obeyed. Second equation is obeyed due to

$$2(1-\cos(\pi\nu)) = (1-\cos(\pi\nu))^2 + \sin^2(\pi\nu)$$

Unitarity of Aharanov-Bohm - lessons

- Important: Delta function piece is absolutely necessary for unitarity.
- evident from second equation!
- One should expect (and we will show later) that the delta function is necessary for unitarity of the S matrix for the relativistic theory as well.
- natural to expect, since non-relativistic limit should give Aharanov-Bohm.
- hint of universality!
- more details on lecture 3.

Aharanov-Bohm scattering - nonabelian case

- Consider $2 \to 2$ scattering of particles in representation R_1 and R_2 of $U(N)$.

$$R_1 \times R_2 = \sum_m R_m$$

- The S matrix takes the schematic form

$$S = \sum_m P_m S_m.$$

P_m : projector in m^{th} rep, S_m is scattering in m^{th} channel.

- Aharonov-Bohm phase of particle (R_1) as it circles around R_2 is $2\pi\nu_m$

$$\nu_m = \frac{4\pi}{\kappa} T_1^a T_2^a = \frac{2\pi}{\kappa} \left(C_2(R_1) + C_2(R_2) - C_2(R_m) \right)$$

- Non-relativistic scattering amplitude in the R_m exchange channel = Aharanov-Bohm scattering of a unit charge $U(1)$ particle off a flux tube of flux $2\pi\nu_m$.

Aharanov-Bohm scattering - nonabelian case

- Channels of scattering

$$\text{Fundamental} \otimes \text{Fundamental} \to \text{Symm}(U_d) \oplus \text{Asymm}(U_e)$$
$$\text{Fundamental} \otimes \text{Antifundamental} \to \text{Adjoint}(T) \oplus \text{Singlet}(S)$$

- The quadratic Casimirs

$$C_2(F) = C_2(A) = \frac{N^2-1}{2N}\ ,\ \ C_2(Sym) = \frac{N^2+N-2}{N}$$
$$C_2(ASym) = \frac{N^2-N-2}{N}\ ,\ \ C_2(Adj) = N\ ,\ \ C_2(Sing) = 0$$

- anyonic phase

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa}\ ,\ \ \nu_{Asym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}$$
$$\nu_{Adj} = \frac{1}{N\kappa}\ ,\ \ \nu_{Sin} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

2+1 d bosonization duality

Level rank duality in pure CS theory

- $SU(N)$ Chern-Simons theory at level k on S^3

$$S_{CS} = \frac{k}{4\pi} \int_{S^3} Tr(A \wedge dA + \frac{2}{3} A \wedge A \wedge A)$$

- The partition function on S^3 [Witten;Camperi, Levstein, Zemba; Naculich et.al]

$$\tilde{Z}(N,k) = \sqrt{N} Z(N,k) = (k+N)^{-\frac{(N-1)}{2}} \prod_{j=1}^{N-1} \left(2 \sin\left(\frac{\pi j}{N+k} \right) \right)^{N-j}$$

- The partition function posseses a symmetry under the exchange of N and k.

$$\tilde{Z}(N,k) = \tilde{Z}(k,N)$$

- This symmetry is an illustration of what is known as the "level-rank duality" in pure Chern-Simons theory.

Level rank duality in pure CS theory

- Let us make the statement of duality precise.
- The level k of the $SU(N)/U(N)$ Chern-Simons theory depends on the regularization procedure.
- In regulation by dimensional reduction the renormalized coupling is [Chen, Semenoff, Wu]

$$\kappa = k + N\text{Sgn}(k)$$

- Shift due to one loop diagram for a gluon loop, no shift for the $U(1)$ gauge group.
- Statement of duality is [Aharony]

$$U(N)_\kappa \leftrightarrow U(|\kappa| - N)_{-\kappa}$$

- It is valid at any finite value of N and k.

Level rank duality in CS matter theory

- Two bosonic CFT's in $2+1d$
  - "$O(N)$ vector model - N real massless free scalars φ^i.
  - "Critical $O(N)$ model" - Wilson-Fisher fixed point.

- Two fermionic CFT's $2+1d$
  - N massless fermions ψ^a.
  - Gross-Neveu model.

- The free field theories have higher spin symmetries, in the interacting theories the symmetries are weakly broken $\partial.J \sim \frac{1}{N}$.

- All the theories have a good $1/N$ expansion - could have classical gravity duals in AdS_4.

- Gravity duals must have massless higher spin fields, to match CFT spectrum.

- Conjecture: bosonic theories are dual to Vasiliev A higher-spin gravity, fermionic theories are dual to Vasilieve B higher spin gravity.[Kelbanov, Polyakov; Sezgin,Sundell; Sundborg; Witten]

Level rank duality in CS matter theory

- Vasiliev type A and Vasiliev type B have same spectrum but different interactions.
- A one parameter (θ) family of higher spin gravity theories that interpolates between type A and type B theories.
- The bosonic theories arise when $\theta = 0$ and fermionic theories arise at $\theta = \pi/2$,
- Hint that there is a family of field theories in the CFT that interpolate between bosonic and fermionic theories.
- [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin] conjectured that the family of field theories arise by coupling bosonic/fermionic theories to CS at level κ.
- In large N, large κ limit , $\lambda = \frac{N}{\kappa}$ is a continuous parameter that allows the interpolation.

Level rank duality in CS matter theory

- $U(N_B)$ Chern-Simons theory coupled to fundamental boson [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin]

$$S = \int d^3x \Big(i\epsilon^{\mu\nu\rho} \frac{\kappa_B}{4\pi} Tr(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho)$$
$$D_\mu \bar{\phi} D^\mu \phi + \sigma \bar{\phi}\phi + N_B \frac{m_B^2}{b_4}\sigma - N_B \frac{\sigma^2}{2b_4} \Big)$$

- Wilson-Fisher limit

$$b_4 \to \infty \ , \ m_B \to \infty \ , \ 4\pi \frac{m_B^2}{b_4} = fixed$$

- $U(N_F)$ Chern-Simons theory coupled to fundamental fermion

$$S = \int d^3x \Big(i\epsilon^{\mu\nu\rho} \frac{\kappa_F}{4\pi} Tr(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho)$$
$$+ \bar{\psi}\gamma^\mu D_\mu \psi + m_F \bar{\psi}\psi \Big)$$

Level rank duality in CS matter theory

- Statement of duality

  $U(N_B)$ CS+fundamental boson at Wilson Fisher limit

  $\Leftarrow$ dual $\Rightarrow$

  $U(N_F)$ CS+fundamental fermion

- under the duality map

$$
\begin{aligned}
\kappa_F =& -\kappa_B \\
N_F =& |\kappa_B| - N_B \\
\lambda_B =& \lambda_F - sgn(\lambda_F) \\
m_F =& -m_B^{cri}\lambda_B
\end{aligned}
$$

- with condition

$$\lambda_F m_F > 0$$

Toy evidence from anyonic phases

- recollect anyonic phase

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa}\ ,\ \nu_{Asym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}\ ,\ \nu_{Adj} = \frac{1}{N\kappa}\ ,\ \nu_{Sin} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

- in large N limit

$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right), \nu_{Sing} \sim O(\lambda)$$

- Aharonov-Bohm phase of bosons scattering off flux tube

$$e^{-i\pi\lambda_B}$$

- Aharonov-Bohm phase of fermions scattering off flux tube

$$(-1)e^{-i\pi\lambda_F} = e^{-i\pi(\lambda_F - \mathrm{Sgn}(\lambda_F))}$$

- The phases are identical when

$$\lambda_B = \lambda_F - \mathrm{Sgn}(\lambda_F)$$

- Precisely the map induced by level-rank duality! (not a derivation!)

Evidence for duality

- Spectrum of single trace operators and three point functions on both sides match.
  [Giombi, Minwalla, Prakash, Trivedi, Wadia] ,
  [Aharony, Gur-Ari, Yacoby], [Maldacena, Zhiboedov]

- Thermal partition functions on both sides match.
  [Jain, Trivedi, Wadia, Yokoyama] ,
  [Aharony, Giombi, Gur-Ari, Maldacena,Yacoby], [Takimi]

- Duality follows from a deformation of Giveon-Kutasov duality in supersymmetric theory.
  [Jain,Minwalla,Yokoyama], [Gur-Ari, Yacoby]

- Most recent: $2 \to 2$ S matrices in C.S+bosonic and C.S+fermionic theories map to one another.

Evidence for duality - mapping of current correlators

- N massless scalars in 2+1 d, coupled to $U(N_B)$ Chern-Simons theory at the Wilson-Fisher fixed point.

- In large N limit, spectrum of operators include a single primary operator[Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin]

$$J^{(s)}_{\mu_1\mu_2\ldots\mu_s} = \phi^\dagger_i D_{\mu_1}\ldots D_{\mu_s}\phi^i + \ldots$$

  for spin $s \geq 0$, and conformal dimension $\Delta = s + 1 + O(1/N)$.

- $J^{(s)}$ are symmetric and traceless.

- eg

$$J^{(0)} = \phi^\dagger\phi$$
$$J_\mu = i\phi^\dagger(\overleftarrow{D}_\mu - \overrightarrow{D}_\mu)\phi$$

- Other primaries are products of these single trace operators.

Evidence for duality - mapping of current correlators

- N massless fermions in 2+1 d, coupled to $U(N_F)$ Chern-Simons theory

$$S_{F+CS} = -\frac{i\kappa_F}{4\pi}\int d^3x\left(Tr(A\wedge dA + \frac{2}{3}A\wedge A\wedge A) + \bar{\psi}\gamma^\mu D_\mu\psi\right)$$

- In large N limit, spectrum of operators [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin],
  - single primary operator $J^{(s)}$ for spin $s \geq 1$, conformal dimension $\Delta = s + 1 + O(1/N)$
  - scalar operator $J^{(0)}$ of dimension $\Delta = 2 + O(1/N)$
- eg

$$J^{(0)} = \bar{\psi}^i\psi_i$$
$$J^{(1)}_\mu = i\bar{\psi}\gamma_\mu\psi_i$$

- Other primaries are products of these single trace operators.

Evidence for duality - mapping of current correlators

- The gauge invariant correlation functions computed in the critical bosonic and fermionic theories map to each other under the duality map.
- As an example we will show that the duality map can be derived by matching the two point functions in the regular fermion and the critical boson theory.
- The computation relies on planarity in large N and the bootstrap equations (Dyson-Schwinger equations)
- compute exact propagator in the two theories.
- compute the exact vertex for the operator of interest.
- Glue the vertices with exact propagator to get your favourite n point function of currents. (gets very difficult beyond 3, so we will present 2 point functions here :P)

Correlators in Bosonic theory

- Work in light cone gauge (gauge self interaction vanishes) [Aharony, Gur-Ari, Yacoby]

$$\langle A_+(-p)A_3(q)\rangle = \frac{4\pi i}{\kappa}\frac{1}{p_+}(2\pi)^3\delta^3(q-p)$$

- compute exact propagator for bosons (only interaction $\phi^\dagger A_3 A_3 \phi$, others vanish in large N limit.)

$$\langle \phi_i^\dagger(p)\phi^j(q)\rangle = \frac{\delta_i^j}{p^2-\Sigma(p,\lambda)}(2\pi)^3\delta^3(q-p)$$

- The three types of seagull diagrams conspire to give a divergence, which is subtracted out by counter terms.

- As a result exact propagator is same as bare propagator!

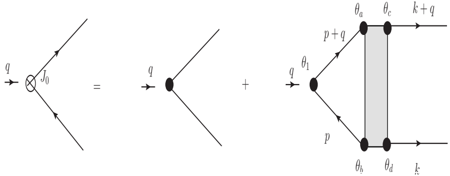
- Next compute the exact four point function $\langle \phi^{i_1}(p+q)\phi\,{\dagger}_{i_2}\,(-p)\phi\,{\dagger}_{j_1}\,(-k-q)\phi^{j_2}(k)\rangle$.

Correlators in Bosonic theory

- The seagull terms as before do not contribute to the four point function, as a result only need to sum ladder diagrams.

- four point function determined from the bootstrap equation (Dyson-Schwinger).

- Now exact vertex can be easily computed



- Gluing two of the above exact vertices will give the two point function for spin zero operator at any order in λ.

- Exact vertex can be constructed for every current J^s.

Correlators in Fermionic theory

- In the fermionic theory the exact propagator is obtained by solving the bootstrap equation [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin].
- The contribution in the large N limit simply comes from rainbow diagrams .

$$\langle \psi_i(p)\bar{\psi}^j(-q)\rangle = \delta_i^j S(p)(2\pi)^3\delta^3(q-p)$$

$$S(p) = \frac{-i\gamma^\mu p_\mu + i\lambda^2\gamma^+ p^- + \lambda(p_1^2+p_2^2)}{p^2}$$

- Exact vertex computed for any $J^{(s)}$ computed in a similar way as in the bosonic case[Gur-Ari, Yacoby].
- Correlators computed by gluing the exact vertices appropriately.

Evidence for duality - mapping of current correlators

- results for critical bosonic theory (eg two point function)

$$\langle J^0(-q)J^0\rangle^{crit} = -\frac{1}{\tan(\frac{\pi\lambda_B}{2})}\langle J^0(-q)J^0\rangle_{bos}$$

$$\langle J^1(-q)J^1\rangle^{crit} = \frac{N_B\sin(\pi\lambda_B)}{\pi\lambda_B}\langle J^1(-q)J^1\rangle_{bos}$$

- results for critical fermionic theory (eg two point function)

$$\langle J^0(-q)J^0\rangle = \tan(\frac{\pi\lambda_F}{2})\langle J^0(-q)J^0\rangle_{ferm}$$

$$\langle J^1(-q)J^1\rangle = \frac{N_F\sin(\pi\lambda_F)}{\pi\lambda_F}\langle J^1(-q)J^1\rangle_{ferm}$$

Evidence for duality - mapping of current correlators

- equating the parameters in J^0 and J^1 [Gur-Ari, Yacoby]

$$\tan(\frac{\pi\lambda_F}{2}) = -\cot(\frac{\pi\lambda_B}{2})$$
$$\kappa_B \sin(\pi\lambda_B) = \kappa_F \sin(\pi\lambda_F)$$

- second equation implies λ_B and λ_F have opposite signs.

- First equation then implies $\kappa_B = -\kappa_F$.
- Together

$$\cos(\frac{\pi|\lambda_F|}{2}) = \sin(\frac{\pi|\lambda_B|}{2})$$

  is solved by

$$|\lambda_B| + |\lambda_F| = 1$$

- Using $\lambda = N/\kappa$, implies $N_B = |\kappa_B| - N_F$ (level rank duality!)

Conjectured Duality for susy matter CS

- Jain, Minwalla, Yokoyama conjectured that $\mathcal{N} = 1, 2$ supersymmetric matter coupled Chern-Simons theories are self dual

$$Theory(\lambda', w', m') \Longleftrightarrow Theory(\lambda, w, m)$$

- under the map

$$\lambda' = \lambda - \mathrm{Sgn}(\lambda) \ , \ w' = \frac{3 - w}{1 + w} \quad m_0' = \frac{-2m_0}{1 + w}$$
$$N' = |\kappa| - N + 1 \ , \ \kappa' = -\kappa$$

- with a pole mass

$$m = \frac{2m_0}{2 + (-1 + w)\lambda \ \mathrm{Sgn(m)}}$$

- $m' = -m$ under duality and $\lambda m(m_0, w) \geq 0$

Evidence for conjectured duality in susy CS

- S matrices computed in the $\mathcal{N} = 2$ Chern-Simons matter theory in the large N limit, to all orders in t'Hooft coupling λ.
- T_B - S matrix for $2 \to 2$ boson scattering, T_F - S matrix for $2 \to 2$ fermion scattering.

$$\mathcal{T}_B^{\mathcal{N}=2} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} - \frac{8\pi m}{\kappa} ,$$
$$\mathcal{T}_F^{\mathcal{N}=2} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} + \frac{8\pi m}{\kappa}$$

- Duality easy to see, $\kappa \to -\kappa$ and $m \to -m$.
- The S matrices map to each other upto an overal unobservable phase.
- More details on lecture 3.

Thank You!