Generalised Attractors in Five Dimensional Gauged Supergravity

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Plan of the talk

Introduction:

- Attractor mechanism plays a crucial role in understanding the origin of black hole entropy in supergravity theories. [Ferrara-Kallosh-Strominger]

- Moduli fields in black hole background are attracted to specific charge dependent values on the horizon.

- Attractor values are determined by solving sets of algebraic equations.

- Macroscopic entropy is determined in terms of charges - independent of asymptotic values of moduli.

- Agrees with microscopic results in string theory.

Introduction:

- Attractor mechanism is a consequence of near horizon geometry rather than supersymmetry.

  [Ferrara-Gibbons-Kallosh]

- Extends to non-supersymmetric cases.

  [Goldstein-Iizuka-Jena-Trivedi]

- Recently, Attractor mechanism generalised for $\mathcal{N} = 2, d = 4$ gauged supergravity.

  [Cacciatori-Klemm , Kachru-Kallosh-Shmakova]

- Generalised attractors: solutions to equations of motion that reduce to algebraic equations when all the fields, curvature tensors are constants in tangent space.

Introduction:

- Lifshitz, Schrödinger geometries are some examples of generalised attractors.
- Such geometries are near horizon geometries of extremal black branes.

  [Goldstein-Kachru-Prakash-Trivedi]
- Bianchi attractors: Classification of homogeneous anisotropic extremal black brane horizons in $d = 5$.

  [Iizuka-Kachru-Kundu-Narayan-Sircar-Trivedi]
- Lifshitz, Schrödinger geometries belong to Bianchi Type I in the classification.

Motivation: Why Bianchi type metrics?

- In gauge/gravity correspondence, black branes are holographic duals to field theories at finite temperature.
- Extremal branes exhibit vanishing entropy density at zero temperature and describe ground states of the field theory.
- Bianchi type metrics are homogeneous, and have been studied recently in the context of AdS/CFT.

  [Domokos-Harvey, Nakamura-Ooguri-Park,Donos-Hartnoll]
- Embedding these metrics in supergravity will be useful to study their supersymmetry properties and pave way for string embedding.

Motivation: Why gauged sugra?

- Bianchi attractors arise in 5d Einstein-Maxwell systems with massive gauge fields.

- Explicit mass terms break SUSY and are not allowed in Sugra.

- Typical scalar kinetic term of Gauged supergravities,

$$g_{\tilde{x}\tilde{y}}\mathcal{D}_\mu\phi^{\tilde{x}}\mathcal{D}^\mu\phi^{\tilde{y}}; \quad \mathcal{D}_\mu\phi^{\tilde{x}} \equiv \partial_\mu\phi^{\tilde{x}} + gA^I_\mu K^{\tilde{x}}_I(\phi).$$

- At attractor points scalars are constant, terms like

$$g^2 g_{\tilde{x}\tilde{y}} K^{\tilde{x}}_I K^{\tilde{y}}_J A^I_\mu A^{J\mu}$$

  act as effective mass term for the gauge field.

- Gauged sugras (not all) : from string theory via flux compactifications.

Our Work

- We extend the work of [Kachru-Kallosh-Shmakova] to $\mathcal{N} = 2, d = 5$ gauged supergravity.
- We show that homogeneous anisotropic extremal black brane horizons are generalised attractor solutions of gauged supergravity.
- Examples: Using a simple gauged sugra model we realise Bianchi Type I ($z = 3$ Lifshitz), Bianchi Type II and Bianchi Type VI solutions as generalised attractors.

Gauged Sugra and Generalised Attractors

Gauged Sugra: Symmetries

- The most general $\mathcal{N} = 2, d = 5$ gauged sugra has gravity coupled to vector, tensor and hypermultiplets.

  [Ceresole-Dall'Agata]

- The scalars in the theory parametrise a manifold that factorises into a direct product of a very special and quaternionic manifold,

$$\mathcal{M}_{scalar} = \mathcal{S}(n_v + n_T) \otimes \mathcal{Q}(n_H).$$

- The R symmetry group is $SU(2)_R$.

Gauged Sugra: Gauging the Symmetries

- Gauging: Suitable subgroup K of the isometry group G of the full scalar manifold $\mathcal{M}_{scalar}$, and the $SU(2)_R$ symmetry group.

- Ordinary derivatives on scalar and fermions are replaced with K-covariant derivatives.

$$\begin{aligned}
\partial_\mu \phi^{\tilde{x}} &\rightarrow \mathcal{D}_\mu \phi^{\tilde{x}} \equiv \partial_\mu \phi^{\tilde{x}} + g A^I_\mu K^{\tilde{x}}_I(\phi) \\
\partial_\mu q^X &\rightarrow \mathcal{D}_\mu q^X \equiv \partial_\mu q^X + g A^I_\mu K^X_I(q) \\
\nabla_\mu B^M_{\nu\rho} &\rightarrow \mathcal{D}_\mu B^M_{\nu\rho} \equiv \nabla_\mu B^M_{\nu\rho} + g A^I_\mu \Lambda^M_{IN} B^N_{\nu\rho},
\end{aligned}$$

- Gauging the $SU(2)_R$ Symmetry:

$$\nabla_\mu \psi_{\nu i} \rightarrow \nabla_\mu \psi_{\nu i} + g_R A^I_\mu P_{Ii}{}^j(q) \psi_{\nu j}.$$

Gauged Sugra: Lagrangian

The bosonic part of the five dimensional $\mathcal{N}=2$ gauged supergravity:

$$\begin{aligned}\hat{e}^{-1}\mathcal{L}^{\mathcal{N}=2}_{Bosonic} =& -\frac{1}{2}R-\frac{1}{4}a_{\tilde{I}\tilde{J}}\mathcal{H}^{\tilde{I}}_{\mu\nu}\mathcal{H}^{\tilde{J}\mu\nu}-\frac{1}{2}g_{XY}\mathcal{D}_{\mu}q^{X}\mathcal{D}^{\mu}q^{Y}\\ &-\frac{1}{2}g_{\tilde{x}\tilde{y}}\mathcal{D}_{\mu}\phi^{\tilde{x}}\mathcal{D}^{\mu}\phi^{\tilde{y}}+\frac{\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\tau}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}A^{K}_{\tau}\\ &+\frac{\hat{e}^{-1}}{4g}\epsilon^{\mu\nu\rho\sigma\tau}\Omega_{MN}B^{M}_{\mu\nu}\mathcal{D}_{\rho}B^{N}_{\sigma\tau}-\mathcal{V}(\phi,q).\end{aligned}$$

$$\begin{aligned}&\mathcal{H}^{\tilde{I}}_{\mu\nu}=(F^{I}_{\mu\nu},B^{M}_{\mu\nu}), && \mu=0,\ldots,4\\ &M=1,\ldots,n_T, && I=0,1,\ldots,n_V\\ &\tilde{x}=0,1,\ldots n_v+n_T, && X=1,2,\ldots 4n_H.\end{aligned}$$

Gauged Sugra: Potential and fermionic shifts

$$\mathcal{V}(\phi, q) = 2g^2 W^{\tilde{a}} W^{\tilde{a}} - g_R^2 [2P_{ij}P^{ij} - P^{\tilde{a}}_{ij} P^{\tilde{a}ij}] + 2g^2 \mathcal{N}_{iA}\mathcal{N}^{iA}$$

$$P_{ij} \equiv h^I P_{Iij}, \qquad P^{\tilde{a}}_{ij} \equiv h^{\tilde{a}I} P_{Iij}$$

$$W^{\tilde{a}} \equiv \frac{\sqrt{6}}{4} h^I K^{\tilde{x}}_I f^{\tilde{a}}_{\tilde{x}}, \qquad \mathcal{N}^{iA} \equiv \frac{\sqrt{6}}{4} h^I K^X_I f^{Ai}_X.$$

Bosonic part of supersymmetry transformations:

$$\delta_\epsilon \psi_{\mu i} = \sqrt{6}\nabla_\mu \epsilon_i + \frac{i}{4} h_{\tilde{I}} (\gamma_{\mu\nu\rho}\epsilon_i - 4 g_{\mu\nu}\gamma_\rho \epsilon_i)\mathcal{H}^{\nu\rho\tilde{I}} + i g_R P_{ij}\gamma_\mu \epsilon^j$$

$$\delta_\epsilon \lambda^{\tilde{a}}_i = -\frac{i}{2} f^{\tilde{a}}_{\tilde{x}} \gamma^\mu \epsilon_i \mathcal{D}_\mu \phi^{\tilde{x}} + \frac{1}{4} h^{\tilde{a}}_{\tilde{I}} \gamma^{\mu\nu}\epsilon_i \mathcal{H}^{\tilde{I}}_{\mu\nu} + g_R P^{\tilde{a}}_{ij}\epsilon^j + g W^{\tilde{a}}\epsilon_i$$

$$\delta_\epsilon \zeta^A = -\frac{i}{2} f^A_{iX}\gamma^\mu \epsilon^i \mathcal{D}_\mu q^X + g\mathcal{N}^A_i \epsilon^i.$$

The potential can be written as squares of fermionic shifts.

Generalised Attractors: Definition

Ansatz:

- In tangent space, all the bosonic fields in the theory take constant values at the attractor points.

$$\phi^{\tilde{z}} = \mathrm{const}\ ; q^{Z} = \mathrm{const}\ ; A^{I}_{a} = \mathrm{const}\ ;$$
$$B^{M}_{ab} = \mathrm{const}\ ; {c_{bc}}^{a} = \mathrm{const}.$$

- The attractor geometries are characterised by constant anholonomy coefficients.

$$[e_a, e_b] = {c_{ab}}^{c} e_c\ ; \quad e_a \equiv e^{\mu}_{a}\partial_{\mu}$$
$${c_{ab}}^{c} = e^{\mu}_{a} e^{\nu}_{b}(\partial_{\mu} e^{c}_{\nu} - \partial_{\nu} e^{c}_{\mu})$$

Generalised Attractors: Features

- Gauge field, Tensor field and Einstein equations reduce to algebraic equations at the attractor point.
- Scalar field equations reduce to a minimisation condition on an attractor potential.
- The attractor potential is also independently constructed from squares of fermionic shifts.
- Constant anholonomy $\Rightarrow$ regular geometries.

Attractor Potential

Using attractor ansatz,

- Equation of motion for $\phi^{\tilde{x}}$ reduces to,

$$\frac{\partial}{\partial \phi^{\tilde{z}}}\left[\mathcal{V}(\phi, q)+\frac{1}{2}g^2\, g_{\tilde{x}\tilde{y}} K_I^{\tilde{x}} K_J^{\tilde{y}} A^{Ia} A_a^J+\frac{1}{4} a_{\tilde{I}\tilde{J}} \mathcal{H}^{\tilde{I}}_{ab} \mathcal{H}^{\tilde{J}ab}\right]=0.$$

- Equation of motion for q^Z reduces to,

$$\frac{\partial}{\partial q^Z}\left[\mathcal{V}(\phi, q)+\frac{1}{2}g^2 g_{XY} K_I^X K_J^Y A^{aI} A_a^J\right]=0.$$

Attractor Potential

- Scalar field equations reduce to an extremisation condition on an attractor potential.

$$\mathcal{V}_{attr}(\phi, q) = \Big[\mathcal{V}(\phi, q) + \frac{1}{4} a_{\tilde{I}\tilde{J}} \mathcal{H}^{\tilde{I}}_{ab} \mathcal{H}^{\tilde{J}ab} + \frac{1}{2} g^2 [\, g_{\tilde{x}\tilde{y}} K^{\tilde{x}}_I K^{\tilde{y}}_J + g_{XY} K^X_I K^Y_J] A^{Ia} A^J_a \Big]$$

- The attractor potential gives rise to the attractor values of the scalars upon extremisation.

Attractor Potential from fermion shifts

- Susy transformations at attractor points:

$$\delta\psi_{ai} = \sqrt{6} D_a \epsilon_i + (\Sigma_{i|j})^{bc}(\gamma_{abc} - 4\eta_{ab}\gamma_c)\epsilon^j + \gamma_a S_{ij}\epsilon^j$$
$$\delta\lambda_i^{\tilde{a}} = \Sigma^{\tilde{a}}{}_{i|j}\epsilon^j + (\Sigma^{\tilde{a}}{}_{i|j})^a\gamma_a\epsilon^j + (\Sigma^{\tilde{a}}{}_{i|j})^{ab}\gamma_{ab}\epsilon^j$$
$$\delta\zeta^A = (\Sigma^A{}_{|j})\epsilon^j + (\Sigma^A{}_{|j})^a\gamma_a\epsilon^j$$

- Generalised Fermion shifts:

$$\Sigma^{\tilde{a}}{}_{i|j} = g_R P^{\tilde{a}}{}_{ij} - g W^{\tilde{a}}\epsilon_{ij} \quad ; \quad (\Sigma^A{}_{|j}) = g\mathcal{N}_j{}^A$$
$$(\Sigma^{\tilde{a}}{}_{i|j})^a = \frac{i}{2} g f^{\tilde{a}}_{\tilde{x}} K_I^{\tilde{x}} A^{Ia}\epsilon_{ij} \quad ; \quad (\Sigma^A{}_{|j})^a = -\frac{i}{2} g f^A_{jX} K_I^X A^{aI}$$
$$(\Sigma^{\tilde{a}}{}_{i|j})^{ab} = -\frac{1}{4} h^{\tilde{a}}_{\tilde{I}} \mathcal{H}^{\tilde{I}ab}\epsilon_{ij} \quad ; \quad (\Sigma_{i|j})^{bc} = -\frac{i}{4} h_{\tilde{I}} \mathcal{H}^{bc\tilde{I}}\epsilon_{ij}$$
$$S_{ij} = i g_R P_{ij}$$

Attractor Potential from fermion shifts

- The attractor potential can be constructed independently from squares of fermionic shifts

$$-\mathcal{V}_{attr}\frac{\epsilon^{l}{}_{k}}{4} = \bar{S}^{i}{}_{k}S_{i}{}^{l} - \epsilon^{lj}\Big\{\big[(\overline{\Sigma^{A}{}_{|k}})(\Sigma_{A|j}) + \frac{1}{2}(\overline{\Sigma^{\tilde{a}i}{}_{|k}})(\Sigma^{\tilde{a}}{}_{i|j})\big]$$
$$+\big[(\overline{\Sigma^{A}{}_{|k}})_{a}(\Sigma_{A|j})^{a} + \frac{1}{2}(\overline{\Sigma^{\tilde{a}i}{}_{|k}})_{a}(\Sigma^{\tilde{a}}{}_{i|j})^{a}\big]$$
$$+\big[(\overline{\Sigma^{i}{}_{|k}})_{ab}(\Sigma_{i|j})^{ab} + (\overline{\Sigma^{\tilde{a}i}{}_{|k}})_{ab}(\Sigma^{\tilde{a}}{}_{i|j})^{ab}\big]\Big\},$$

which can be shown to reproduce,

$$\mathcal{V}_{attr}(\phi, q) = \Big[\mathcal{V}(\phi, q) + \frac{1}{4}a_{\tilde{I}\tilde{J}}\mathcal{H}^{\tilde{I}}_{ab}\mathcal{H}^{\tilde{J}ab}$$
$$+\frac{1}{2}g^{2}[\,g_{\tilde{x}\tilde{y}}K^{\tilde{x}}_{I}K^{\tilde{y}}_{J} + g_{XY}K^{X}_{I}K^{Y}_{J}]A^{Ia}A^{J}_{a}\Big]$$

Examples

Bianchi Attractors in a simple gauged sugra model

- For illustration, take a gauged supergravity model with one vector and two tensor multiplets.

[Gunaydin-Zagermann]

- Within this model, we realise a $z = 3$ Lifshitz solution, a Bianchi Type II and a Bianchi Type VI solution as attractors.

- The other Bianchi type metrics can be realised as attractor solutions of more generic gauged supergravity models.

Model dependent data

- Moduli space

$$\mathcal{M}_{scalar} = SO(1,1) \times \frac{SO(2,1)}{SO(2)}.$$

- Metric on moduli space $g_{\tilde{x}\tilde{y}}$, $a_{\tilde{I}\tilde{J}}$.

- Gauging: $SO(2)$ subgroup of G using a single vector A^0 (graviphoton).

- R-Symmetry: $A_\mu[U(1)_R] = A^0_\mu V_0 + A^1_\mu V_1$

Model dependent data

- Potential:

$$\mathcal{V}(\phi) = \frac{g^2}{8}\left[\frac{[(\phi^2)^2 + (\phi^3)^2]}{||\phi||^6}\right] - 2g_R^2\left[2\sqrt{2}\frac{\phi^1}{||\phi||^2}V_0 V_1 + ||\phi||^2 V_1^2\right].$$

- Conditions for $\mathcal{N} = 2$ supersymmetry and AdS vaccum:

$$\phi^2 = 0, \quad \phi^3 = 0, \quad \phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}, \quad V_0 V_1 > 0, \quad 32\frac{g_R^2}{g^2}V_0^2 \leq 1.$$

- potential evaluated at these values gives the AdS cosmological constant $\mathcal{V}_{AdS} = -6g_R^2(\phi_c^1)^2 V_1^2$.

- Bianchi attractors exist for values of scalars for which the theory would also have an *AdS* Vacuum.

Methodology

- Take metric ansatz: Bianchi types,
- gauge field ansatz: time like gauge field

$$A^t = e^t_a A^a = \frac{1}{Lr^u} A^0$$

- Set all tensor fields $B^M_{\mu\nu}$ to zero!
- Use the generalised attractor procedure and solve the algebraic field equations!

Bianchi Type I - Lifshitz

Bianchi Type I specified by gauging parameters g, V_0, V_1.

$$ds^2 = L^2\Big[-r^{2u}dt^2 + \frac{dr^2}{r^2} + r^2(dx^2+dy^2+dz^2)\Big]$$

$$e_2 = \partial_x \qquad e_3 = \partial_y \qquad e_4 = \partial_z$$

$$[e_2, e_3] = 0 \qquad [e_2, e_4] = 0 \qquad [e_3, e_4] = 0$$

$$u = 3; \quad A^t = \frac{1}{Lr^u}\sqrt{\frac{2}{3}}\frac{1}{(\phi_c^1)^2}; \quad L = \sqrt{3}\frac{(\phi_c^1)^4}{g};$$

$$\phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}; \quad V_0 V_1 > 0; \qquad \frac{32}{3(\phi_c^1)^4} \leq 1.$$

Bianchi Type II

Bianchi Type II specified by gauging parameters g, V_0, V_1.

$$ds^2 = L^2\Big[-r^{2u}dt^2 + \frac{dr^2}{r^2} + r^{2w}dx^2 + r^{2(v+w)}dy^2 - 2xr^{2(v+w)}dydx + [r^{2(v+w)}x^2 + r^{2v}]dz^2\Big]$$

$$e_2 = \partial_x \qquad e_3 = \partial_y \qquad e_4 = x\partial_y + \partial_z$$

$$[e_2, e_4] = e_3 \qquad [e_2, e_3] = 0 \qquad [e_3, e_4] = 0$$

$$u = \sqrt{2}; \qquad v = w = \frac{1}{2\sqrt{2}};$$

$$L = \sqrt{\frac{2}{3}\frac{(\phi_c^1)^4}{g}}; \qquad A^t = \frac{1}{Lr^u}\sqrt{\frac{5}{8}}\frac{1}{(\phi_c^1)^2};$$

$$\phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}; \quad V_0V_1 > 0; \quad \frac{23}{2(\phi_c^1)^4} \le 1.$$

Bianchi Type VI

Bianchi Type VI specified by gauging parameters g, V_0, V_1 and h

$$ds^2 = L^2\left[-r^{2u}dt^2 + \frac{dr^2}{r^2} + dx^2 + e^{-2x}r^{2v}dy^2 + e^{-2hx}r^{2w}dz^2\right]$$

$$e_2 = \partial_x \qquad e_3 = e^x\partial_y \qquad e_4 = e^{hx}\partial_z$$

$$[e_2, e_3] = e_3 \qquad [e_2, e_4] = he_4 \qquad [e_3, e_4] = 0$$

$$u = \frac{1}{\sqrt{2}}(1-h); \quad v = -\frac{1}{\sqrt{2}}h; \quad w = \frac{1}{\sqrt{2}}; \quad L = \frac{(\phi_c^1)^4}{\sqrt{6}g}(1-h);$$

$$A^t = \frac{1}{Lr^u}\sqrt{\frac{-2h}{(-1+h)^2}}\frac{1}{(\phi_c^1)^2}; \quad h < 0; \quad h \neq 0, 1;$$

$$\phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}; \quad V_0 V_1 > 0; \quad \frac{8(3-h+3h^2)}{(\phi_c^1)^4(-1+h)^2} \leq 1$$

Summary

- We studied the generalised attractors in $\mathcal{N} = 2, d = 5$ gauged supergravity.
- Generalised attractors are defined by constant anholonomy, constant gauge fields, constant tensor fields and constant scalars at the attractor points.
- Gauge field, Tensor field and Einstein equations reduce to algebraic equations at the attractor point.
- Scalar field equations reduce to a minimisation condition on an attractor potential.
- The attractor potential is also independently constructed from squares of fermionic shifts.

Summary

- The attractor geometries are characterised by constant anholonomy coefficients.

- We showed that near horizon geometries of homogeneous extremal black branes are generalised attractor solutions of gauged supergravity.

- Examples: Using a simple gauged sugra model, we realise a $z = 3$ Lifshitz solution, a Bianchi Type II and a Bianchi Type VI solution as attractors.

- Other Bianchi type metrics can be realised as attractor solutions of more generic gauged supergravity models.

Comments and Caveats

- Toplogical terms: Chern-Simons, Tensor fields do not contribute.
- Bianchi type V and type III metrics which are limiting cases of type VI do not seem to be valid attractors of the gauged supergravity model considered here.
- Caution: Attractor equations, attractor geometries in black hole case exist at local minima of potential. Here they exist at critical points.
- So far, we have considered only abelian gauging, Non abelian gauging will impose further restrictions on the parameters V_I.

Future Outlook

- Completion: Find suitable models to embed rest of the Bianchi metrics.
- Stability: Several Bianchi attractors exist for same range of parameters even in the gauged supergravity embedding. Is there a stability criteria ?
- String embedding: String theory/M-theory compactification to obtain a suitable gauged supergravity.
- Applications: Field theory duals of all Bianchi attractors.
- Entropy function: Can generalised attractors be understood from entropy function ?

Useful References:

- Gauged sugra reviews: hep-th/9605032, hep-th/0102114
- 5d gauged sugra models: hep-th/9912027, hep-th/0002228
- Generalised attractors: 1104.2884, 1206.3887.
- Bianchi metrics: Homogeneous Relativistic Cosmologies - Shepley, 1201.4861.

Thank You!

Gauge field equation

- Since ${c_{ab}}^c = const$,

$$F_{ab} = e_a^\mu e_b^\nu (\partial_\mu e_\nu^c - \partial_\nu e_\mu^c) A_c = {c_{ab}}^c A_c$$

- The Gauge field equation of motion,

$$\begin{aligned}\partial_\mu(\hat{e} a_{I\tilde{J}} \mathcal{H}^{\tilde{J}\mu\nu}) = &- \frac{1}{2\sqrt{6}} C_{I\tilde{J}\tilde{K}} \epsilon^{\nu\mu\rho\sigma\tau} \mathcal{H}^{\tilde{J}}_{\mu\rho} \mathcal{H}^{\tilde{K}}_{\sigma\tau} \\ &+ g\hat{e}\left[g_{XY} K_I^X \mathcal{D}^\nu q^Y + g_{\tilde{x}\tilde{y}} K_I^{\tilde{x}} \mathcal{D}^\nu \phi^{\tilde{y}}\right]\end{aligned}$$

in tangent space, is an algebraic equation at the attractor points

$$\begin{aligned}\hat{e}\ a_{I\tilde{J}}[\omega_{a,}{}^a{}_c \mathcal{H}^{cb\tilde{J}} + \omega_{a,}{}^b{}_c \mathcal{H}^{ac\tilde{J}}] = &- \frac{1}{2\sqrt{6}} C_{I\tilde{J}\tilde{K}} \epsilon^{bacde} \mathcal{H}^{\tilde{J}}_{ac} \mathcal{H}^{\tilde{K}}_{de} \\ &+ g^2 \hat{e}\left[g_{XY} K_I^X K_J^Y \right. \\ &\left. + g_{\tilde{x}\tilde{y}} K_I^{\tilde{x}} K_J^{\tilde{y}}\right] A^{Jb}.\end{aligned}$$

Tensor field equation

- The tensor field equation is,

$$\frac{1}{g}\epsilon^{\mu\nu\rho\sigma\tau}\Omega_{MP}\mathcal{D}_\rho B^M_{\mu\nu} + \hat{e}a_{\tilde{I}P}\mathcal{H}^{\tilde{I}\sigma\tau} = 0.$$

- In tangent space,

$$\frac{1}{g}\epsilon^{abcde}\left[c_{ac}{}^f B^M_{fb} + gA^I_c\Lambda^M_{IN}B^N_{ab}\right]\Omega_{MP} + \hat{e}a_{\tilde{I}P}\mathcal{H}^{\tilde{I}de} = 0.$$

is an algebraic equation at the attractor points,

Einstein equation

- The Einstein equation at the attractor point:

$$R_{ab} - \frac{1}{2} R \eta_{ab} = T_{ab}^{attr}$$

- In the absence of torsion, The left handside is algebraic:

$$R_{abc}{}^{d} = \partial_a \omega_{bc}{}^{d} - \partial_b \omega_{ac}{}^{d} - \omega_{ac}{}^{e} \omega_{be}{}^{d} + \omega_{bc}{}^{e} \omega_{ae}{}^{d} - c_{ab}{}^{e} \omega_{ec}{}^{d}$$

$$\omega_{a,bc} = \frac{1}{2} [c_{ab,c} - c_{ac,b} - c_{bc,a}]$$

- The stress energy tensor at the attractor point:

$$T_{ab}^{attr} = \mathcal{V}_{attr}(\phi, q) \eta_{ab} - \Big[a_{\tilde{I}\tilde{J}} \mathcal{H}_{ac}^{\tilde{I}} \mathcal{H}_{b}{}^{c\tilde{J}} + g^2 [g_{XY} K_I^X K_J^Y + g_{\tilde{x}\tilde{y}} K_I^{\tilde{x}} K_J^{\tilde{y}}] A_a^I A_b^J \Big].$$

- The Einstein equations are algebraic at the attractor points.

Killing spinor integrability conditions

- KSI expressible in terms of fermionic shifts. Defining $M_{abc} = \gamma_{abc} - 4\eta_{ab}\gamma_c$,

$$
\begin{aligned}
-\frac{1}{4}R_{ae}{}^{cd}\gamma_{cd}\epsilon_i = &-\frac{1}{\sqrt{6}}(\Sigma_{i|j})^{fc}[\omega_{a,}{}^{b}{}_{f}M_{e[bc]} - \omega_{e,}{}^{b}{}_{f}M_{a[bc]}]\epsilon^j \\
&+ \frac{1}{6}\Big\{[(\Sigma_{i|j})^{bc}M_{abc} + \gamma_a S_{ij}][(\Sigma_{k|l})^{gh}M_{egh} + \gamma_e S_{kl}] \\
&- [(\Sigma_{i|j})^{bc}M_{ebc} + \gamma_e S_{ij}][(\Sigma_{k|l})^{gh}M_{agh} + \gamma_a S_{kl}]\Big\}\epsilon^{jk}\epsilon^l
\end{aligned}
$$

- All shifts vanish $\Rightarrow$ Maximal supersymmetry (AdS_5 vacuum, unique). [hep-th/0304064]

$$
-\frac{1}{4}R_{ae}{}^{cd}\gamma_{cd}\epsilon_i = \frac{1}{6}S_{ij}S_{kl}\gamma_{ae}\epsilon^{jk}\epsilon^l
$$

Killing spinor integrability conditions

- Some shifts vanish $\Rightarrow$ partially broken supersymmetry (Lifshitz, Bianchi types)
- cases with only vector multiplets: Either $1/2$ BPS or $1/4$ BPS solutions. [hep-th/0304064]
- Lifshitz solutions: known to be $1/4$ BPS [1102.5344].
- We expect Bianchi attractors to be $1/4$ BPS.

$AdS_2 \times \mathbb{R}^3$ from $U(1)_R$ gauged supergravity

$$ds^2 = L^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + dx^2 + dy^2 + dz^2\right]$$

$$e_2 = \partial_x \qquad e_3 = \partial_y \qquad e_4 = \partial_z$$

$$[e_2, e_3] = 0 \qquad [e_2, e_4] = 0 \qquad [e_3, e_4] = 0$$

$$A_0^t = \frac{1}{Lr}Q_0; \quad A_1^t = \frac{1}{Lr}Q_1; \quad \frac{Q_0}{Q_1} = \frac{1}{\sqrt{2}(\phi_c^1)^3} = \frac{1}{2}\frac{V_1}{V_0};$$

$$L^2 = -\frac{1}{2\Lambda}; \quad \Lambda = -6g_R^2V_1^2(\phi_c^1)^2; \quad \phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}$$

$$V_0V_1 > 0$$

Stability analysis: Preliminary results

- Stability: Several Bianchi attractors exist for same range of parameters even in the gauged supergravity embedding. Is there a stability criteria ?
- Fluctuation analysis: radial scalar field perturbation about critical value.

$$\phi_c^{\tilde{z}} + \epsilon\ \delta\phi^{\tilde{z}}(r)$$

- Assume that the gauge fields have components only along time t or only along spatial (x, y, z) directions.
- Set all tensor fields to zero.

Stability analysis: Preliminary results

- Scalar field equation upto $O(\epsilon)$:

$$\nabla_\mu \nabla^\mu \delta\phi^{\tilde{x}} - M_{\tilde{y}}{}^{\tilde{x}} \delta\phi^{\tilde{y}} = 0$$

$$M_{\tilde{y}}{}^{\tilde{x}} \equiv g^{\tilde{z}\tilde{x}} \frac{\partial^2 \mathcal{V}_{att}}{\partial\phi^{\tilde{z}} \partial\phi^{\tilde{y}}} \bigg|_{\phi^{\tilde{y}} = \phi_c^{\tilde{y}}}$$

  ∇_μ is with respect to the full extremal black brane metric $g_{\mu\nu}$.

- Expanding the metric about the horizon

$$g_{\mu\nu} \sim g^0_{\mu\nu}(r - r_h) + \epsilon\, g^1_{\mu\nu}(r - r_h) + O(\epsilon^2) + \dots$$

  $g^0_{\mu\nu}$ is the near horizon metric.

- In scalar equation, one can ignore the higher order terms in the metric fluctuation as long as there is no backreaction at $O(\epsilon)$.

Stability analysis: Preliminary results

- The trace of the stress energy tensor at $O(\epsilon)$ after using attractor equations,

$$T_\mu{}^\mu(\phi_c+\delta\phi) = T(\phi_c)_{att} + g^2 A^I_\mu A^{J\mu} \left.\frac{\partial K_{IJ}}{\partial \phi^{\tilde{z}}}\right|_{\phi=\phi_c} \epsilon\, \delta\phi^{\tilde{z}} + O(\epsilon^2)$$

$$K_{IJ}(\phi) = g_{\tilde{x}\tilde{y}} K^{\tilde{x}}_I K^{\tilde{y}}_J$$

suggests that in general there will be finite backreaction even at first order from the "effective mass term" for the gauge fields.

- For starters, let us assume that the highlighted term is very small compared to the cosmological constant and analyse the behavior of $\delta\phi$ in scalar equation.

- Note that for pure $U(1)_R$ gauged sugra there is no backreaction at $O(\epsilon)$.

Stability analysis: Preliminary results

- For Bianchi type metrics, the scalar equation will take the general form

$$r^2\partial_r^2(\delta\phi^{\tilde{x}}) + m\ r\partial_r(\delta\phi^{\tilde{x}}) - \lambda_{\tilde{x}}\delta\phi^{\tilde{x}} = 0 \quad ; m > 0$$

$$M_{\tilde{y}}{}^{\tilde{x}}\delta\phi^{\tilde{y}} = \lambda_{\tilde{x}}\delta\phi^{\tilde{x}}$$

- There exists a converging solution for $\delta\phi$ as one approaches the horizon ($r \to 0$), if $\lambda_{\tilde{x}} > 0$.

- This is same as the condition

$$\frac{\partial^2 \mathcal{V}_{att}}{\partial\phi^{\tilde{z}}\partial\phi^{\tilde{y}}} > 0$$

which was obtained in [hep-th/0507096] in the context of non-supersymmetric attractors.

- Given a model, this condition can be applied to check for stability of solutions. However this does not tell if one solution is preferred over the other.