Exact Scattering amplitudes in $\mathcal{N} = 3$ supersymmetric Chern-Simons matter theories at large N

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Based on:

Exact scattering amplitudes

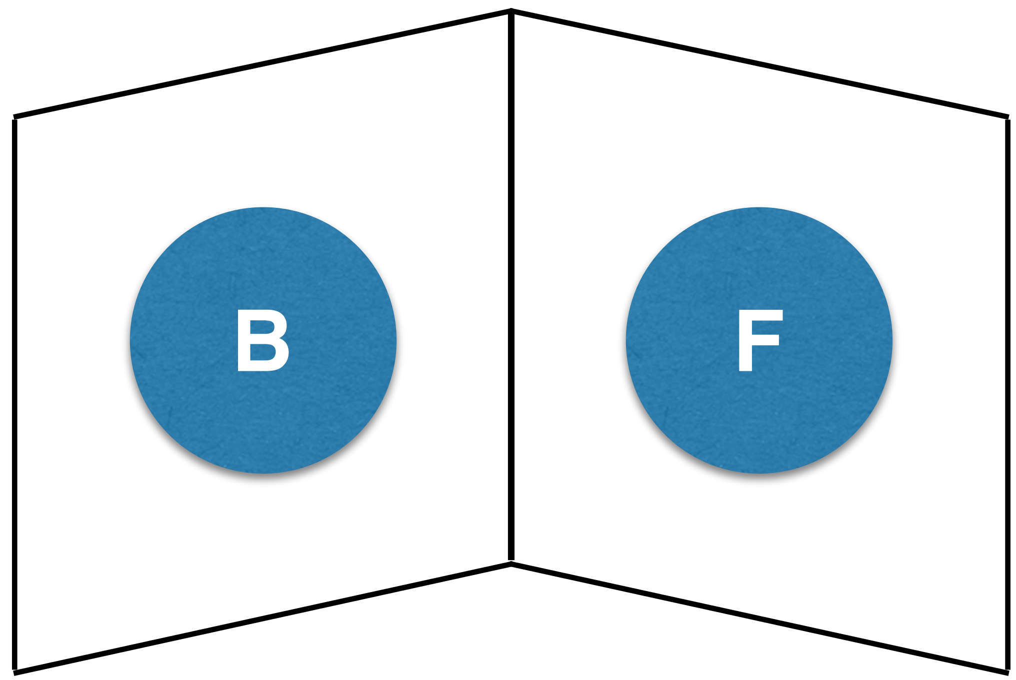
K.I, Janagal, Shukla; arXiv: 2001.02363 (Submitted to JHEP)

K.I, Janagal, Shukla; Phys.Rev. D100 (2019) no.8, 085008

Dual superconformal symmetry, Yangian and BCFW recursions.

K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh; JHEP 1906 (2019) 016

K.I, Jain, Nayak, Umesh; Phys.Rev.Lett. 121 (2018) no.16, 161601

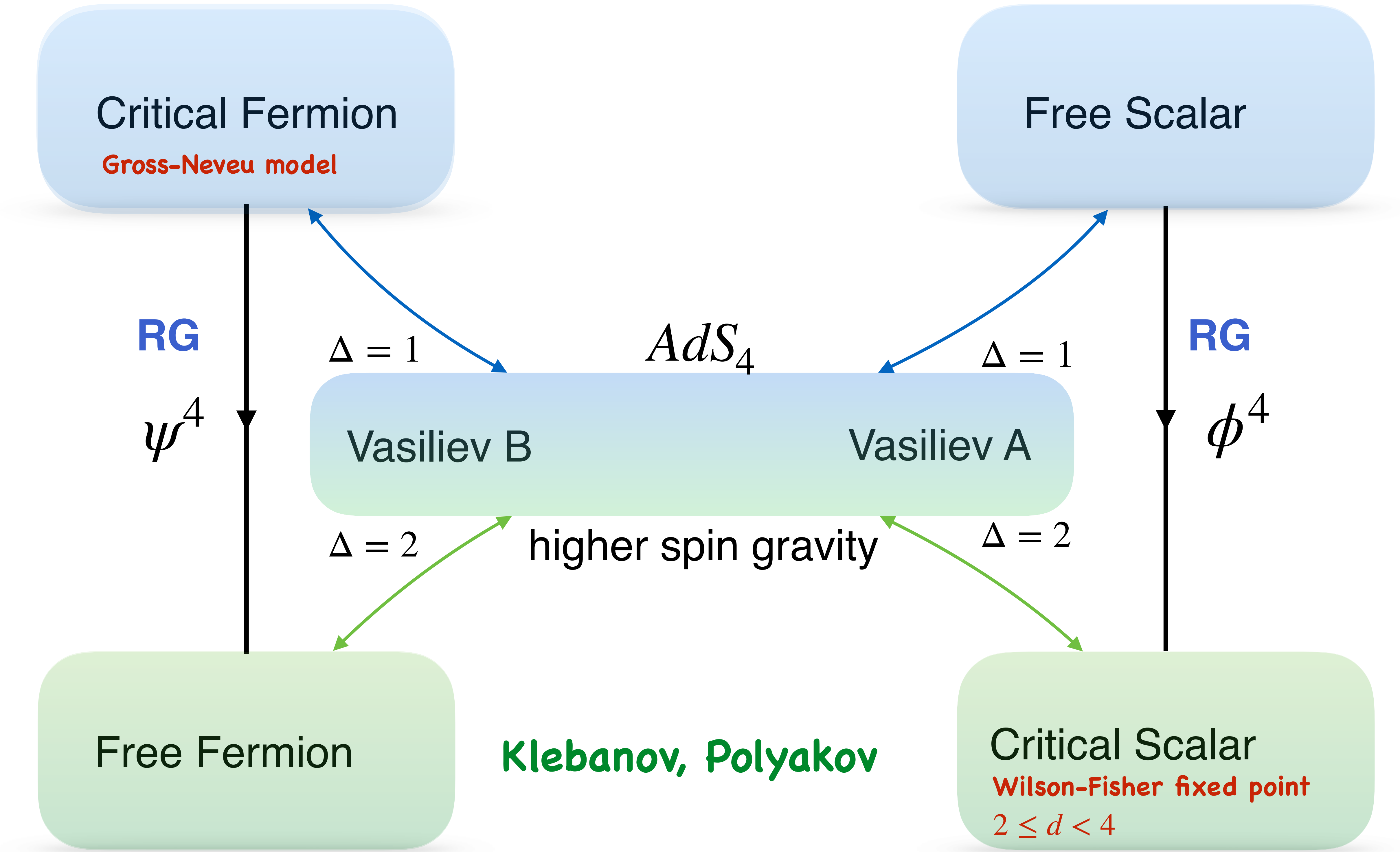


Motivation

1. Bosonization duality

Aharony,Benini,Chang,Frishman,Giombi,Giveon,Gur-Ari,Gurucharan,Hsin,K.I, Jain
Karch,Kutasov,Maldacena,Mandlik,Minwalla,Moshe,Nayak,Prakash,
Radicevic,Sharma,Seiberg,Takimi,Trivedi,Tong,Yacoby,Yin,Yokoyama,Wadia,Witten,
Zhiboedov

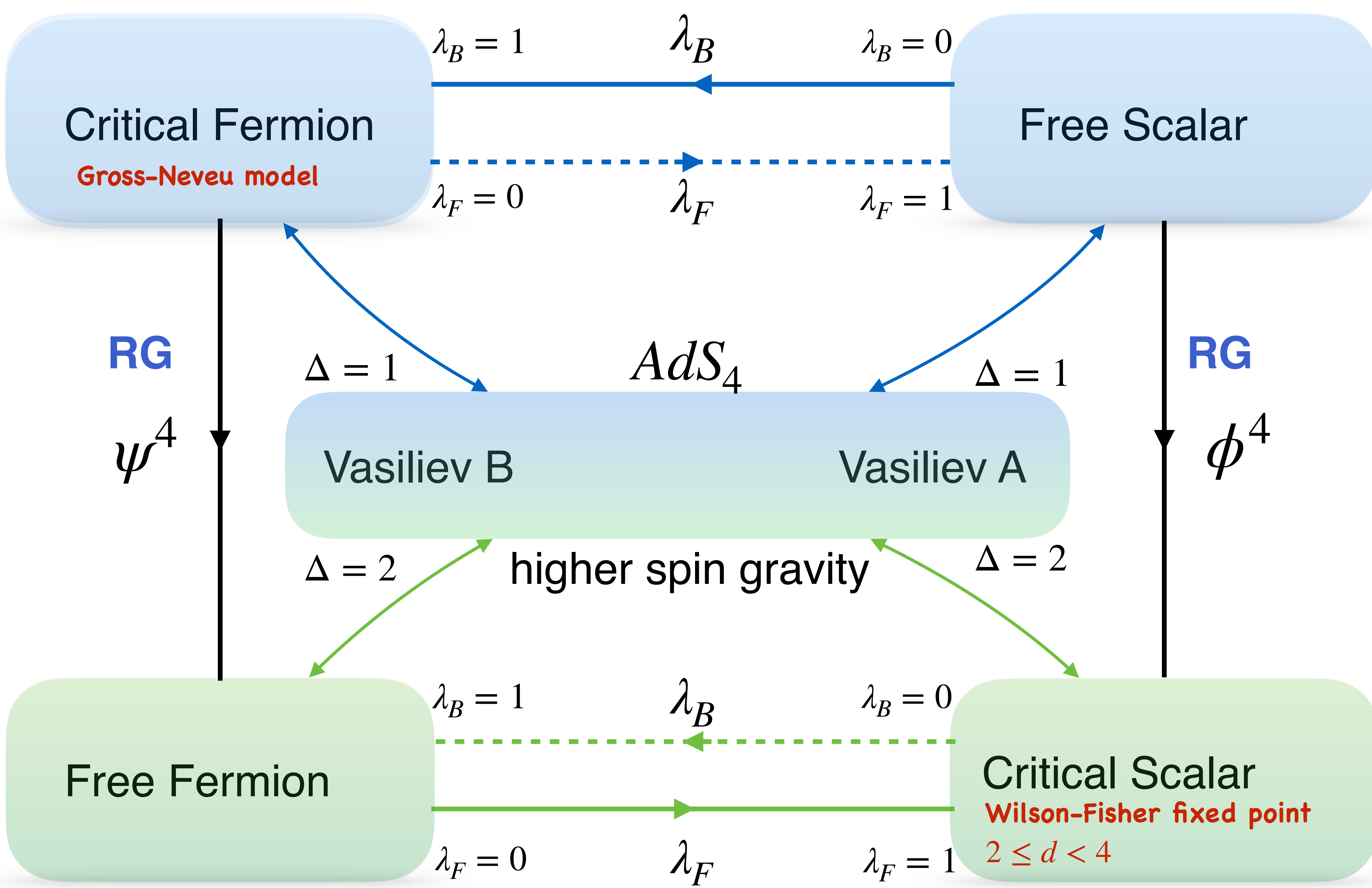
duality in Vector models: 2+1 dimensions



What happens when you turn on Chern-Simons interactions?

Bosonization duality in 2+1 dimensions

$$N \to \infty \,,\, \kappa \to \infty \,,\, \lambda = \frac{N}{\kappa}$$



How does one test a strong-weak duality?

At **strong coupling** (at critical points), no Lagrangian description.

How do we compute anything at all?

At weak coupling description (away from critical points), Lagrangian description. We can compute physical observables. But how to get the results at strong coupling?

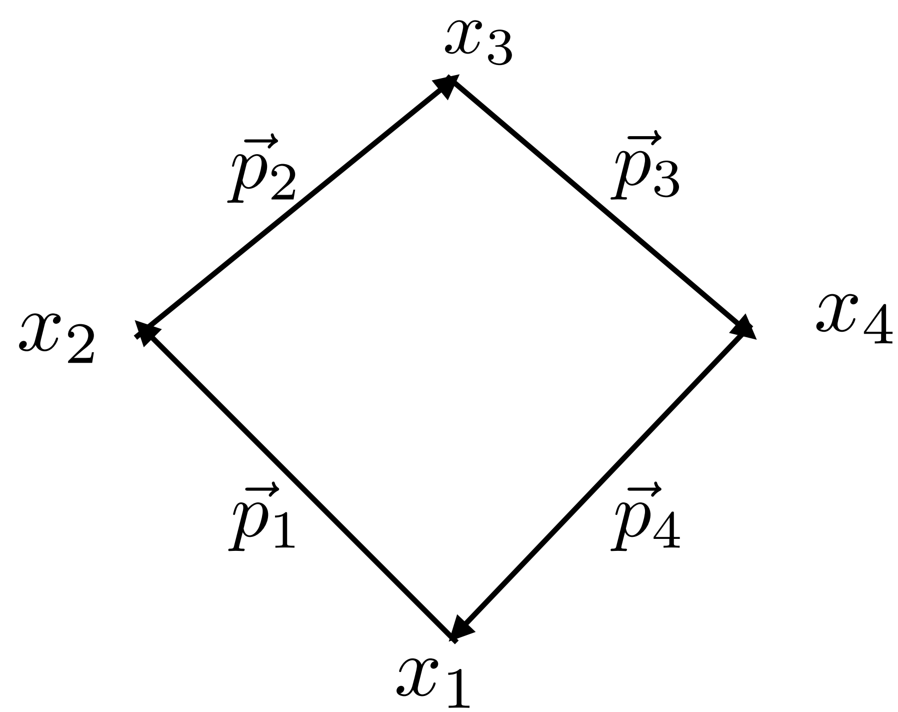
In weak coupling regime, compute observables to all orders in 't Hooft coupling λ in the large N limit

- **Spectrum of single trace primary operators (matching operator dimensions).**
- **Thermal partition functions.**
- **Correlation functions of gauge invariant currents.**
- **$2 \to 2$ S matrices**

… …

Motivation

2. Symmetries in amplitudes



Dual superconformal symmetry in amplitudes

Amplitudes in $\mathcal{N}=2$ supersymmetric Chern-Simons matter theories have remarkably simple properties

The all loop result

$$T^{all\ loop}_{symm} = T^{all\ loop}_{Asymm} = T^{all\ loop}_{Adj} = T_{tree}$$

$$T^{all\ loop}_{singlet} = N\frac{\sin\pi\lambda}{\pi\lambda}T_{tree}$$

$$T_{\rm tree} = \frac{4\pi}{\kappa}\frac{\langle 12\rangle}{\langle 23\rangle}\delta(\sum_{i=1}^{4}p_i)\delta^2\left(\mathcal{Q}\right)$$

$$\delta^2(Q) = \sum_{i<j=1}^{n}\langle ij\rangle\eta_i\eta_j$$

$$A_i = a_i + \eta_i\alpha_i$$

$$A_i^\dagger = a_i^\dagger\eta_i + \alpha_i^\dagger$$

$$S = \langle 0|A_4A_3A_2^\dagger A_1^\dagger|0\rangle$$

exhibits dual superconformal symmetry!

K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh

Yangian symmetry K.I, Jain, Nayak, Sharma (in progress)

It is natural to expect such symmetries in higher susy theories.

Perturbative amplitudes in N=6 ABJM theory

At **tree level n point** ABJM amplitudes are **Yangian invariant**.

Gang, Huang, Koh, Lee, Lipstein

At **One loop, four point amplitudes vanish in $\mathcal{O}(\epsilon)$,** in dimensional regularisation $3 - 2\epsilon$.

Bianchi, Leoni, Mauri, Penati, Santambrogio

At **two loops**

$$\mathcal{A}_4^{\text{2-loop}} = \left(\frac{N}{k}\right)^2 \mathcal{A}_4^{\text{tree}} \left[-\frac{\left(-\mu^{-2} y_{13}^2\right)^{-\epsilon} + \left(-\mu^{-2} y_{24}^2\right)^{-\epsilon}}{(2\epsilon)^2} + \frac{1}{2} \ln^2\left(\frac{y_{13}^2}{y_{24}^2}\right) + 4\zeta_2 - 3\ln^2 2 + \mathcal{O}(\epsilon) \right]$$

Elvang & Huang

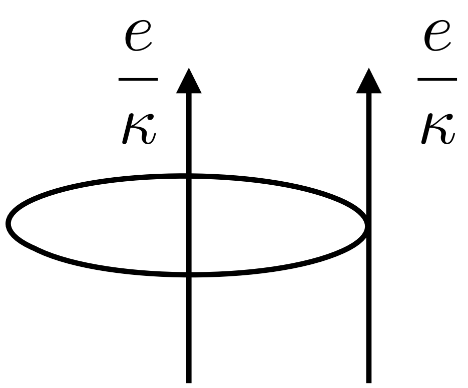
This is **inconsistent with perturbative unitarity**, the cuts in the two loop amplitudes cannot be saturated by a vanishing one loop result.

Curiously, the three loop amplitude is non-vanishing. Bianchi, Leoni

Perhaps a careful study of amplitudes **including anyonic effects** in CS matter theories could clarify these results.

Motivation

3. Anyonic effects in CS matter theories



$$\text{AB phase} = e^{ie\int A.dx} = e^{i\frac{e^2}{\kappa}}$$

Anyonic effects in CS matter theories

Consider particles in representations R_1, R_2 of the gauge group

$$R_1 \otimes R_2 = \sum_m R_m$$

The Aharanov-Bohm phase of the particle R_1 as it circles around particle R_2 is $2\pi\nu_m$ in the mth channel of decomposition

$$\nu_m = \frac{4\pi}{\kappa}\mathrm{Tr}\left(T_{R_1}T_{R_2}\right) = \frac{2\pi}{\kappa}\left(C_2(R_1) + C_2(R_2) - C_m(R_m)\right)$$

Eg in SU(N)

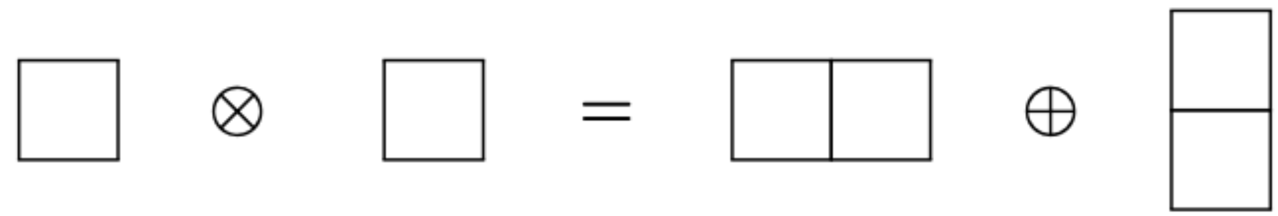
$$\text{Particles }(\mathbf{N}) \quad \equiv \quad \square\,, \quad \text{Anti-Particles }(\overline{\mathbf{N}}) \quad \equiv \left.\begin{array}{c}\square\\ \square\\ \vdots\\ \square\end{array}\right\} N-1$$

Particles: Fundamental, Anti-Particles: Anti-Fundamental

Anyonic effects in CS matter theories

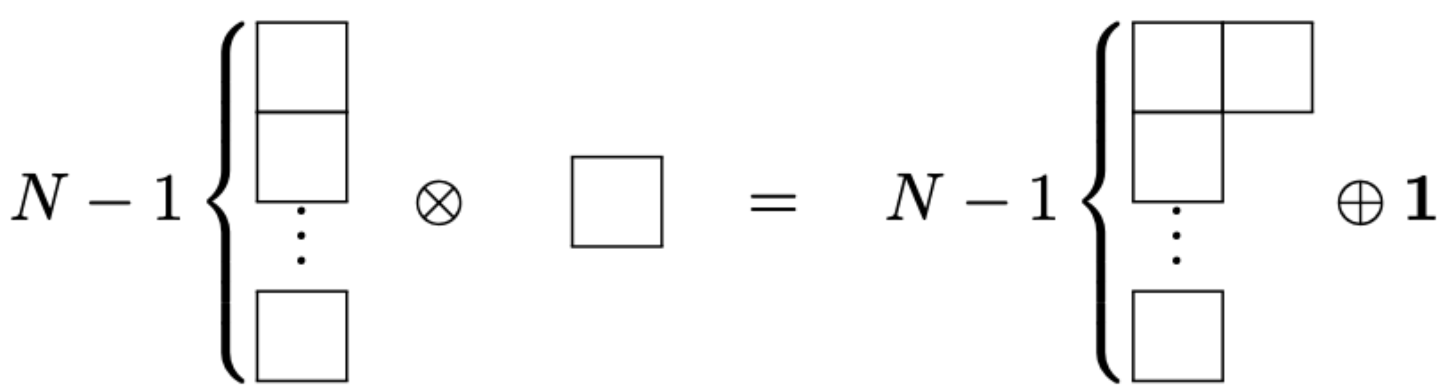
Eg in SU(N)

For exchange of particles



Fundamental $\otimes$ Fundamental $=$ Symmetric $\oplus$ Anti-Symmetric

For exchange of particle and an anti-particle



Anti-Fundamental $\otimes$ Fundamental $=$ Adjoint $\oplus$ Singlet

Anyonic effects in CS matter theories

$$C_2(F) = C_2(A) = \frac{N^2-1}{2N}\,, \qquad C_2(Sym) = \frac{N^2+N-2}{N}\,, \qquad C_2(ASym) = \frac{N^2-N-2}{N}\,,$$

$$C_2(Adj) = N\,, \qquad C_2(Sing) = 0$$

AB phases

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa}\,, \; \nu_{ASym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}\,, \; \nu_{Adj} = \frac{1}{N\kappa}\,, \; \nu_{Sing} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

Let us simplify life a bit...

In the limit $N \to \infty\,, \kappa \to \infty$ the 't Hooft coupling $\lambda = \dfrac{N}{\kappa}$

$$\nu_{Sym} \sim \nu_{ASym} \sim \nu_{Adj} \sim \mathcal{O}\left(\frac{1}{N}\right)\,, \; \nu_{Sing} \sim -\lambda$$

Anyonic effects in CS matter theories

Observation: **Naive crossing symmetry** rules from any of the **non-anyonic channels to the singlet channel** leads to a **non unitary** S matrix.

S.Jain, M. Mandlik, S.Minwalla , T.Takimi S.Wadia, S.Yokoyama

Conjectured S matrix in anyonic channel for any CS matter theory

$$S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\nu)\ \delta(\theta) + i\frac{\sin(\pi\nu)}{\pi\nu}T^{S;naive}(s,\theta)$$

eg: for the singlet channel in $SU(N)$ CS matter theory

$$\nu_{Sing} = -\lambda \qquad S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\ \delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}T^{S;naive}(s,\theta)$$

Tests of the conjecture

The conjecture appears to be **universal and generic** to any Chern-Simons matter theory.

Tests

Unitarity of the S matrix

3d Bosonization duality

Verifications

U(N) Chern-Simons coupled to fundamental bosons.
U(N) Chern-Simons coupled to fundamental fermions.

S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

$\mathcal{N} = 1{,}2$ supersymmetric Chern-Simons matter theories.

K.I, S.Jain, S.Mazumdar, S.Minwalla, V. Umesh, S.Yokoyama

$\mathcal{N} = 3$ **supersymmetric Chern-Simons matter theory.**

K.I, L.Janagal, A. Shukla

Exact $2 \to 2$ amplitude in $\mathcal{N} = 3$

Supersymmetric Chern-Simons matter theory

N=3 theory in N=1 superspace

The N=3 Superconformal theory is usually formulated in N=2 superspace consists of a **pair of chiral multiplets transforming in conjugate representations of the gauge group.** **D. Gaiotto, X. Yin**

$$(Q_i, \tilde{Q}^i)$$

We re-formulate the theory in N=1 superspace following **Gaiotto, Witten**

The reason is **purely technical** (no intuitive or easy way to implement supersymmetric light cone gauge in N=2 superspace)

The **price we pay is loss of manifest SU(2) R symmetry in superspace**, However this **can be easily recovered** by going to components.

SO(2) R charges **SU(2) doublets**

$$\begin{pmatrix} Q_i \\ \bar{\tilde{Q}}_i \end{pmatrix} \rightarrow \begin{pmatrix} \Phi_i^+ \\ \Phi_i^- \end{pmatrix} \quad \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \qquad \phi_i^A = \begin{pmatrix} \phi_i^+ \\ \phi_i^- \end{pmatrix}, \bar{\phi}_A^i = \begin{pmatrix} (\bar{\phi}^+)^i \\ (\bar{\phi}^-)^i \end{pmatrix}$$

Chang, Minwalla, Yin, Sharma

Mass deformed N=3 theory in N=1 superspace

In N=1 Superspace, this appears similar to the **twisted/untwisted mass deformations** (with opposite signs)

$$
\begin{aligned}
\mathcal{S}^{E}_{\mathcal{N}=3} = -\int d^3x d^2\theta \Big[& \frac{\kappa}{4\pi} Tr\Big(-\frac{1}{4} D^\alpha \Gamma^\beta D_\beta \Gamma_\alpha + \frac{i}{6} D^\alpha \Gamma^\beta \{\Gamma_\alpha, \Gamma_\beta\} + \frac{1}{24} \{\Gamma^\alpha, \Gamma^\beta\}\{\Gamma_\alpha, \Gamma_\beta\}\Big) \\
& - \frac{1}{2}(D^\alpha \bar{\Phi}^+ + i\bar{\Phi}^+ \Gamma^\alpha)(D_\alpha \Phi^+ - i\Gamma_\alpha \Phi^+) - \frac{1}{2}(D^\alpha \bar{\Phi}^- + i\bar{\Phi}^- \Gamma^\alpha)(D_\alpha \Phi^- - i\Gamma_\alpha \Phi^-) \\
& - \frac{\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^+\Phi^+\right) - \frac{\pi}{\kappa}\left(\bar{\Phi}^-\Phi^-\right)\left(\bar{\Phi}^-\Phi^-\right) + \frac{4\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^-\Phi^-\right) \\
& + \frac{2\pi}{\kappa}\left(\bar{\Phi}^+\Phi^-\right)\left(\bar{\Phi}^-\Phi^+\right) - \left(m_0 \bar{\Phi}^+\Phi^+ - m_0 \bar{\Phi}^-\Phi^-\right)\Big]
\end{aligned}
\tag{2.1}
$$

N3eucS

$\Phi^+ \leftrightarrow \Phi^-$ $\qquad$ $m_0 \leftrightarrow -m_0$

This is essentially N=3 supersymmetry in disguise and is a symmetry of all n point correlators in the theory.

Supersymmetric light cone gauge

$$\Gamma_- = 0 \;\Rightarrow\; A_- = A_1 + iA_2 = 0$$

In this gauge, the action simplifies to

$$\mathcal{S}^E_{\mathcal{N}=3} = -\int d^3x d^2\theta \Big[\frac{\kappa}{16\pi} Tr(\Gamma^- i\partial_{--}\Gamma^-) - \sum_{a=\pm} \frac{1}{2} D^\alpha \bar\Phi^a D_\alpha \Phi^a - \frac{i}{2}\Gamma^-(\bar\Phi^a D_- \Phi^a - D_-\bar\Phi^a \Phi^a)$$

$$- \frac{\pi}{\kappa}\left(\bar\Phi^+\Phi^+\right)\left(\bar\Phi^+\Phi^+\right) - \frac{\pi}{\kappa}\left(\bar\Phi^-\Phi^-\right)\left(\bar\Phi^-\Phi^-\right) + \frac{4\pi}{\kappa}\left(\bar\Phi^+\Phi^+\right)\left(\bar\Phi^-\Phi^-\right) + \frac{2\pi}{\kappa}\left(\bar\Phi^+\Phi^-\right)\left(\bar\Phi^-\Phi^+\right)$$

$$- \left(m_0\bar\Phi^+\Phi^+ - m_0\bar\Phi^-\Phi^-\right)\Big]$$

The exact propagators are

$$P_{\bar\Phi^\pm\Phi^\pm}(\theta_1,\theta_2,p) \equiv \langle\bar\Phi^\pm(\theta_1,p)\Phi^\pm(\theta_2,-p')\rangle = \frac{D^2_{\theta_1,p} \pm m}{p^2+m^2}\delta^2(\theta_1-\theta_2)(2\pi)^3\delta^3(p-p') \quad \text{exact}$$

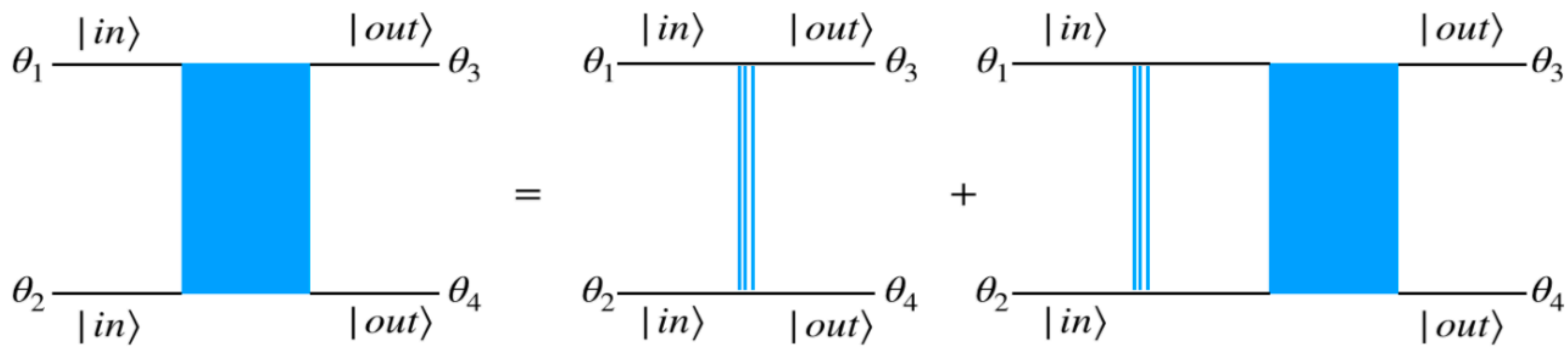
SUSY

$$P_\Gamma(\theta_1,\theta_2,p) \equiv \langle\Gamma^-(\theta_1,p)\Gamma^-(\theta_2,-p')\rangle = -\frac{8\pi}{\kappa}\frac{\delta^2(\theta_1-\theta_2)}{p_{--}}(2\pi)^3\delta^3(p-p')$$

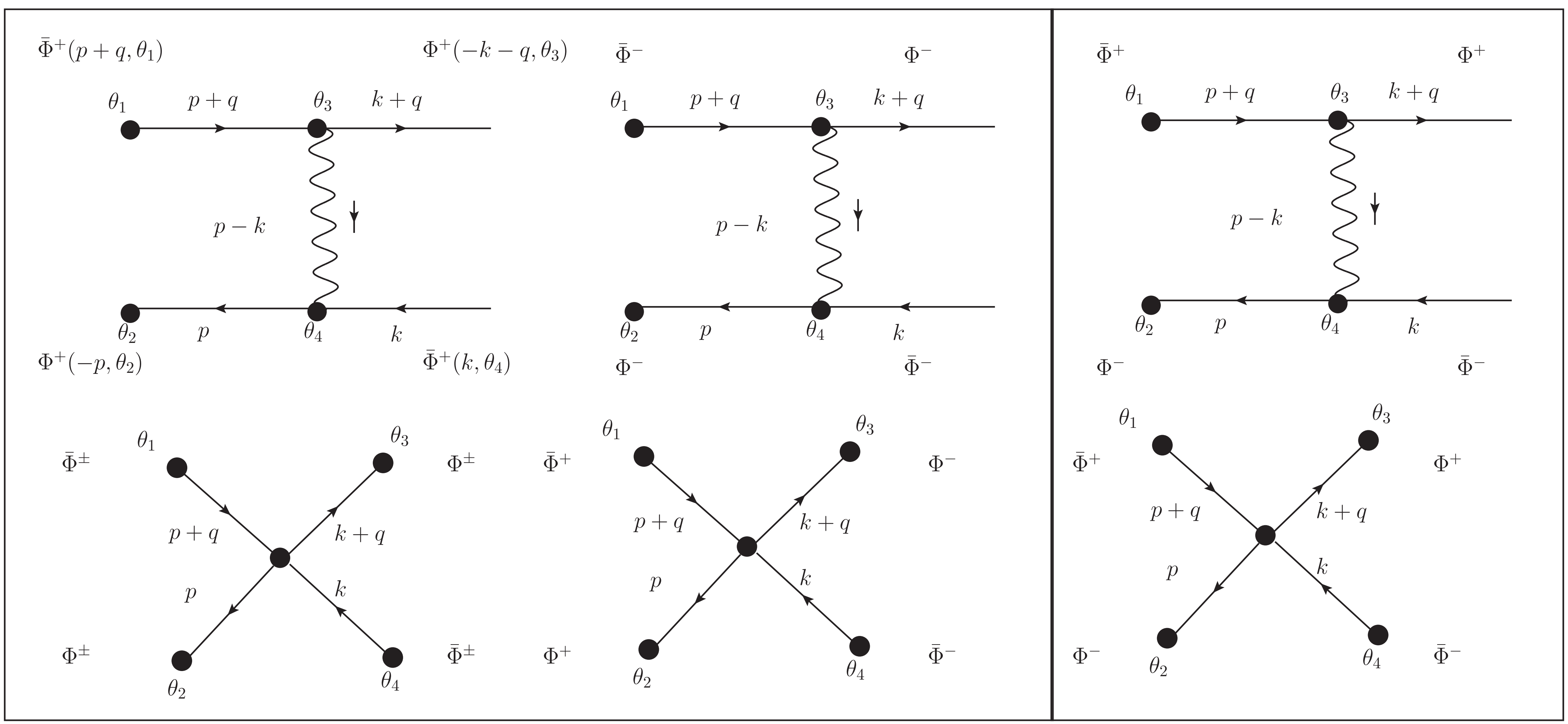
Large N

The **mass term** is the central charge in the susy algebra and hence **the two point function of matter superfields is protected.**

Dyson-Schwinger equations for Four point correlator



Tree amplitudes



Dyson-schwinger equations - summary

We have solved the Dyson-Schwinger equations for the exact offshell four point correlator to **all orders in the 't Hooft coupling λ in the kinematic regime $q_{\pm} = 0$**

S matrix is read off by **taking onshell limit**.

This **allows extraction of S matrix in the symmetric, anti-symmetric and adjoint channels** of scattering (q_{μ} is momentum transfer)

However, it is **impossible to extract the singlet channel** directly, since q is center of mass energy and cannot be spacelike. (Also the channel is anyonic!)

We **use the conjectured crossing rules to obtain the singlet channel** S matrices.

S matrices

In the symmetric, anti-symmetric and adjoint channels of scattering

$$\mathcal{T}_B^{\mathcal{N}=3} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2}$$
$$\mathcal{T}_F^{\mathcal{N}=3} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2}$$

The S matrices computed to all orders in λ are tree level exact!

$$T_{Sym} \sim T_{ASym} \sim T_{Adj} \sim \mathcal{O}\left(\frac{1}{N}\right)$$

Unitarity is guaranteed by Hermiticity in these channels.

Conjectured S matrix: simple but not tree level exact!!

$$T_B^{\mathcal{N}=3}(s,\theta) = T_F^{\mathcal{N}=3}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\ \delta(\theta) + 4i\sqrt{s}\sin(\pi\lambda)\ \cot\left(\frac{\theta}{2}\right)$$

Tree level exactness in the singlet that would violate unitarity! $i(T-T^\dagger) = TT^\dagger$ $\qquad T_{Sing} \sim \mathcal{O}(1)$

Summary

Summary

We computed $2 \to 2$ amplitudes in $SU(N)$ $\mathcal{N} = 3$ supersymmetric Chern-Simons matter theory at large N **exactly to all orders in the 't Hooft coupling λ.**

Our results are **consistent with supersymmetry, unitarity and Bosonization duality**.

The amplitude in the symmetric, anti-symmetric and adjoint channels of scattering are tree level exact to all orders in the 't Hooft coupling λ.

Conjectured S matrix in singlet channel is simple and is minimally modified to be consistent with unitarity and duality.

Summary

In the massless limit the bosonic and fermionic S matrices of the $\mathcal{N}=3$ theory are identical to that of the $\mathcal{N}=2$ theory.

We expect that the following results for the $\mathcal{N}=2$ theory generalise to the $\mathcal{N}=3$ theory as well.

dual superconformal symmetry of the four point amplitude

K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh

Yangian symmetry K.I, Jain, Nayak, Sharma (in progress)

For $\mathcal{N}\geq 4$ the gauge group has to be bi-fundamental, diagrams are more complicated and it is hard to set up a closed Dyson-Schwinger equation.

However, We do expect some simplicity in the all loop amplitudes.

Open questions

At finite N and k, all the channels are anyonic!

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa} \;, \quad \nu_{Asym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}$$
$$\nu_{Adj} = \frac{1}{N\kappa} \;, \quad \nu_{Sin} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

It would be valuable to have a first principle derivation of the crossing rules.

Anyonic phases in ABJM theory

$$U(N)_k \times U(N)_{-k}$$

Remarkable simplicity even at finite N and k

$$\nu_{\mathrm{Adj}_N,\mathrm{Adj}_N} = 0$$
$$\nu_{\mathrm{Adj}_N,\mathrm{Sing}_N} = \frac{N}{\kappa}$$
$$\nu_{\mathrm{Sing}_N,\mathrm{Adj}_N} = -\frac{N}{\kappa}$$
$$\nu_{\mathrm{Sing}_N,\mathrm{Sing}_N} = 0$$

Thank you!!

At large N reduces to those of vector like models.

It appears that the crossing rules should be simpler in the ABJM theory even for finite N and k.