

# $2 \rightarrow 2$ scattering in supersymmetric matter Chern-Simons theories at large $N$

Karthik Inbasekar



ETH, Zurich

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## Based on

- K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh,  
S.Yokoyama: Arxiv [1505.06571](#), *JHEP* **1510** (2015) 176

## Related earlier work

- S.Jain, M.Mandlik, S.Minwalla, T.Takimi, S.Wadia,  
S.Yokoyama: Arxiv [1404.6373](#), *JHEP* **1504** (2015) 129

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# CS matter theories: Preliminaries

2 → 2 scattering  
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- Non-Abelian  $U(N)$  gauge theories in 2+1 dimensions are rich.

- Yang-Mills + Chern-Simons action

$$\frac{i\kappa}{4\pi} \int \text{Tr} \left( A dA + \frac{2}{3} A^3 \right) - \frac{1}{4g_{YM}^2} \int d^3x \text{Tr} F_{\mu\nu}^2$$

- Describes massive gluons with mass  $\propto \kappa g_{YM}^2$ .
- Low energies : pure Chern-Simons theory, topological.
- Chern-Simons gauge theory coupled to matter gives rise to interesting dynamics.

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# CS matter theories: Preliminaries

- Equations of motion for **abelian theory with scalar matter** of unit charge

$$\kappa \varepsilon^{\mu\nu\rho} F_{\nu\rho} = 2\pi J^\mu$$

- Chern-Simons interaction ties  $\frac{1}{\kappa}$  units of **flux** to the charged scalar.
- Aharonov-Bohm** effect: **Exchange** of two unit charge particles result in a **phase**  $\frac{\pi}{\kappa}$ .
- Chern-Simons** gauge field interacting **with matter** turns them into **anyons** with anyonic phase  $\pi\nu = \frac{\pi}{\kappa}$ .
- non-abelian** case: for eg **exchange** of  $U(N)$  matter quanta  $R_1$  and  $R_2$  gives a **phase operator**  $(R_1 \times R_2 = \sum_m R_m)$

$$\nu_m = \frac{T_{R_1} \cdot T_{R_2}}{\kappa} = \frac{C_2(R_1) + C_2(R_2) - C_2(R_m)}{2\kappa}$$

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# CS matter theories: Preliminaries

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- Lots of activity in  $U(N)$  Chern-Simons theories with fundamental matter.
- Motivations: AdS/CFT, [Vasiliev duality](#), limit of ABJ theory.
- [Solvability](#): The theory is [solvable in large  \$N\$](#)  limit.
- [2+1 d bosonization](#): Bose-fermi duality even in the absence of supersymmetry.

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# Level rank duality in CS matter theory

- $U(N_B)$  Chern-Simons theory coupled to fundamental boson [Giombi, Minwalla, Prakash, Trivedi, Wadia]

$$S = \int d^3x \left( i\epsilon^{\mu\nu\rho} \frac{\kappa_B}{4\pi} \text{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) \right. \\ \left. D_\mu \bar{\phi} D^\mu \phi + \sigma \bar{\phi} \phi + N_B \frac{m_B^2}{b_4} \sigma - N_B \frac{\sigma^2}{2b_4} \right)$$

- Wilson-Fisher limit

$$b_4 \rightarrow \infty, \quad m_B \rightarrow \infty, \quad 4\pi \frac{m_B^2}{b_4} = \text{fixed}$$

- $U(N_F)$  Chern-Simons theory coupled to fundamental fermion

$$S = \int d^3x \left( i\epsilon^{\mu\nu\rho} \frac{\kappa_F}{4\pi} \text{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) \right. \\ \left. + \bar{\psi} \gamma^\mu D_\mu \psi + m_F \bar{\psi} \psi \right)$$

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# Level rank duality in CS matter theory

- Statement of duality

$U(N_B)$  CS+fundamental boson at Wilson Fisher limit

$\Leftarrow$  dual  $\Rightarrow$

$U(N_F)$  CS+fundamental fermion

- under the duality map

$$\kappa_F = -\kappa_B$$

$$N_F = |\kappa_B| - N_B$$

$$\lambda_B = \lambda_F - \text{sgn}(\lambda_F)$$

$$m_F = -m_B^{\text{cri}} \lambda_B$$

- with condition

$$\lambda_F m_F > 0$$

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# Evidence for duality

- Spectrum of single trace operators and three point functions on both sides match.  
[Giombi, Minwalla, Prakash, Trivedi, Wadia] ,  
[Aharony, Gur-Ari, Yacoby], [Maldacena, Zhiboedov]
- Thermal partition functions on both sides match.  
[Jain, Trivedi, Wadia, Yokoyama] ,  
[Aharony, Giombi, Gur-Ari, Maldacena, Yacoby], [Takimi]
- Duality follows from a deformation of Giveon-Kutasov duality in supersymmetric theory.  
[Jain, Minwalla, Yokoyama], [Gur-Ari, Yacoby]
- Most recent:  $2 \rightarrow 2$  S matrices in C.S.+bosonic and C.S.+fermionic theories map to one another.

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# Conjectured Duality for susy matter CS

- Jain, Minwalla, Yokoyama conjectured that  $\mathcal{N} = 1, 2$  supersymmetric matter coupled Chern-Simons theories are self dual

$$Theory(\lambda', w', m') \iff Theory(\lambda, w, m)$$

- under the map

$$\lambda' = \lambda - \text{Sgn}(\lambda), \quad w' = \frac{3 - w}{1 + w} \quad m'_0 = \frac{-2m_0}{1 + w}$$

$$N' = |\kappa| - N + 1, \quad \kappa' = -\kappa$$

- with a pole mass

$$m = \frac{2m_0}{2 + (-1 + w)\lambda \text{Sgn}(m)}$$

- $m' = -m$  under duality and  $\lambda m(m_0, w) \geq 0$

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# Scattering in CS matter theories

- The statement of **duality** is actually a statement of **bosonization of fermions**.
- Bosonic and fermionic S matrices related by **duality** is equivalent to a **bosonization map**.
- Such a mapping is **possible in 2+1 dimensions**: Dirac equation **uniquely** determines the **polarization spinors** as a function of the **momentum**.
- In **large N** limit, only **planar diagrams** contribute. Possible to get **exact results** as a function of 't Hooft coupling  $\lambda$ .
- It has been shown that the **S matrices** for  $2 \rightarrow 2$  processes in the **CS+bosonic** theory map to the **CS+fermionic theory** under **duality**.

[Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama]

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# Scattering in CS matter theories: Peculiarities

- Scattering results **consistent with duality**.
- In **singlet channel** (particle-Antiparticle)  $S$  matrices obtained from **naive crossing symmetry** rules are **inconsistent with unitarity** and have **incorrect non-relativistic limit**.
- **Consistency with unitarity** requires
  - **Delta function** term at forward scattering.
  - **Modified crossing symmetry** rules.
- **Conjecture**: Singlet channel  $S$  matrices have the form

$$S = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s, \theta)$$

- $\mathcal{T}^{S;\text{naive}}$  is the matrix obtained from **naive analytic continuation** of **particle-particle** scattering.

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# Nature of the conjecture: Delta function and modified crossing rules

$$S = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s, \theta)$$

- The conjectured S matrix has a **non-analytic  $\delta(\theta)$**  piece.
- **delta function** is already known to be **necessary to unitarize** non-relativistic **Aharonov-Bohm scattering** [Ruijsenaars; Bak,Jackiw,Pi].
- **$\cos(\pi\lambda)$**  is due to the **interference of the Aharonov-Bohm phases** of the wave packets.
- modified crossing factor  $\frac{\sin(\pi\lambda)}{\pi\lambda}$  is the **expectation value of circular wilson loop** on  $S^3$  in **pure Chern-Simons theory** [Witten].

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# Unitarity and anyonic behavior in Singlet Channel

- **Unitarity**  $i(T^\dagger - T) = TT^\dagger$ : **non-trivial** only for **singlet channel** in the large N limit.

$$T_{Asym} \sim T_{Sym} \sim T_{Adj} \sim O\left(\frac{1}{N}\right), \quad T_{sing} \sim O(1)$$

- The anyonic phase operator  $\nu_m = \frac{C_2(R_1) + C_2(R_2) - C_2(R_m)}{2\kappa}$

$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right), \quad \nu_{Sing} \sim O(\lambda)$$

- **symmetric/antisymmetric** channels and **adjoint** channel are effectively **non-anyonic** in large N.
- Particle-Antiparticle scattering in the **singlet channel** is effectively **anyonic** - **usual crossing rules fail unitarity**.
- **Remedy**: **delta function** and **modified crossing rules**.

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# Universality and tests

- delta function and modified crossing rules - appear to be universal
- Tests:
  - Unitarity of the S matrix
  - Bose-Fermi duality
  - Non-relativistic limit gives Aharonov-Bohm
- All the tests have been explicitly verified for
  - $U(N)$  Chern-Simons coupled to fundamental bosons
  - $U(N)$  Chern-Simons coupled to fundamental fermions
  - $\mathcal{N} = 1, 2$  Supersymmetric  $U(N)$  Chern-Simons matter theories
- Further investigations ongoing for  $\mathcal{N} = 3, 4, 5, 6$  susy CS matter theories [K.I, S.Jain, S.Minwalla, S.Yokoyama]

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# Our work

- Test the **conjecture** in the most general renormalizable supersymmetric  $\mathcal{N} = 1$  Chern-Simons matter theory.
- Superspace - **manifest supersymmetry**
- Work in **large  $N$**  - only **planar diagrams** .
- Compute **off-shell four point correlator**, take **on-shell limit** and extract the  **$S$  matrix**.
- Provide evidence for **duality** and subject the **conjecture** to stringent **unitarity test**.

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# Main results

- Results in perfect agreement with duality.
- Unitarity requires the delta function at forward scattering and crossing symmetry rules modified by exactly the same way as conjectured in [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama]
- Substantial evidence for universality of the conjecture.

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# Bonus results

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- Results of  $\mathcal{N} = 2$  theory obtained at **special value** of quartic scalar coupling.
- **Non-renormalization of pole mass and vertex for  $\mathcal{N} = 2$  theory** - good things happen with more susy .
- $\mathcal{N} = 1$   $S$  matrix has **interesting pole structure**, with **vanishing pole mass** on a self-dual codimension one surface in the space of couplings.

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# Supersymmetric scattering

- $2 \rightarrow 2$  scattering amplitude: transition between free incoming and free outgoing onshell particles.
- Initial and final states of  $\Phi_i$  are effectively subject to free equations of motion

$$(D^2 + m) \Phi = 0$$

- Solution

$$\Phi(x, \theta) = \int \frac{d^2 p}{\sqrt{2p^0} (2\pi)^2} \left[ \left( a(\mathbf{p})(1 + m\theta^2) + \theta^\alpha u_\alpha(\mathbf{p}) \alpha(\mathbf{p}) \right) e^{ip \cdot x} + \left( a^{c\dagger}(\mathbf{p})(1 + m\theta^2) + \theta^\alpha v_\alpha(\mathbf{p}) \alpha^{c\dagger}(\mathbf{p}) \right) e^{-ip \cdot x} \right]$$

- action of off-shell supersymmetry operator on onshell superfields  $[Q_\alpha^{off}, \Phi] = Q_\alpha^{off} \Phi = i \left( \frac{\partial}{\partial \theta^\alpha} - i\theta^\beta \partial_{\beta\alpha} \right) \Phi$

$$\begin{aligned} -iQ_\alpha^{on} &= u_\alpha(\mathbf{p}_i) (\alpha \partial_a + \alpha^c \partial_{a^c}) + u_\alpha^*(\mathbf{p}_i) (a \partial_\alpha + a^c \partial_{\alpha^c}) \\ &\quad - v_\alpha^*(\mathbf{p}_i) \left( a^\dagger \partial_{\alpha^\dagger} + (a^c)^\dagger \partial_{(\alpha^c)^\dagger} \right) v_\alpha(\mathbf{p}_i) \left( \alpha^\dagger \partial_{a^\dagger} + (\alpha^c)^\dagger \partial_{(a^c)^\dagger} \right) \end{aligned}$$

# Onshell superspace

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- Introduce creation and annihilation operator superfields

$$A_i(\mathbf{p}) = a_i(\mathbf{p}) + \alpha_i(\mathbf{p})\theta_i ,$$

$$A_i^\dagger(\mathbf{p}) = a_i^\dagger(\mathbf{p}) + \theta_i \alpha_i^\dagger(\mathbf{p}) .$$

- Action of supersymmetry operator

$$[Q_\alpha^{on}, A_i(\mathbf{p}_i, \theta_i)] = Q_\alpha^1 A_i(\mathbf{p}_i, \theta_i)$$

$$[Q_\alpha^{on}, A_i^\dagger(\mathbf{p}_i, \theta_i)] = Q_\alpha^2 A_i^\dagger(\mathbf{p}_i, \theta_i)$$

$$Q_\beta^1 = i \left( -u_\beta(\mathbf{p}) \frac{\overrightarrow{\partial}}{\partial \theta} - v_\beta(\mathbf{p}) \theta \right)$$

$$Q_\beta^2 = i \left( v_\beta(\mathbf{p}) \frac{\overrightarrow{\partial}}{\partial \theta} - u_\beta(\mathbf{p}) \theta \right) .$$

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# Supersymmetry and dual supersymmetry

- Action of **bose-fermi duality**

$$a^D = \alpha, \quad \alpha^D = a \quad m^D = -m$$

- **dual supersymmetry** operator has the form

$$(Q^D)_\beta^1 = i \left( -u_\beta(\mathbf{p}, -m) \frac{\overrightarrow{\partial}}{\partial \theta} - v_\beta(\mathbf{p}, -m) \theta \right),$$

$$(Q^D)_\beta^2 = i \left( v_\beta(\mathbf{p}, -m) \frac{\overrightarrow{\partial}}{\partial \theta} - u_\beta(\mathbf{p}, -m) \theta \right)$$

- using  $u(m, p) = -v(-m, p)$ ,  $v(m, p) = -u(-m, p)$   
and  $\theta \leftrightarrow \frac{\partial}{\partial \theta}$

$$(Q^D)^1 \propto Q^1, \quad (Q^D)^2 \propto Q^2$$

- Quantities invariant under usual **supersymmetry** also invariant under **dual supersymmetry**.

- **Onshell supersymmetry commutes with duality**

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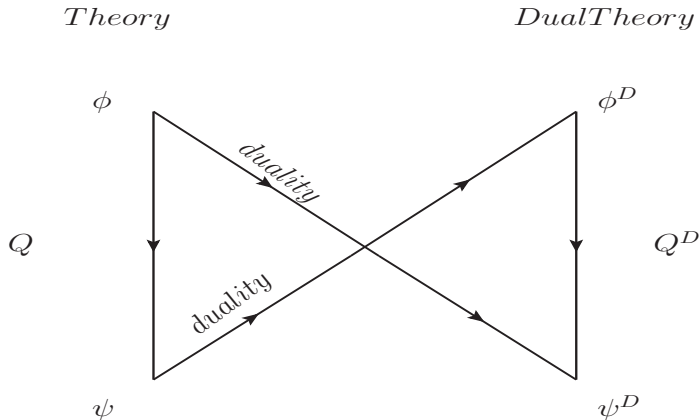
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# S matrix in onshell superspace

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- 2  $\rightarrow$  2 S matrix:  $p_1 + p_2 \rightarrow p_3 + p_4$

$$S(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) \sqrt{(2p_1^0)(2p_2^0)(2p_3^0)(2p_4^0)} = \\ \langle 0 | A_4(\mathbf{p}_4, \theta_4) A_3(\mathbf{p}_3, \theta_3) U A_2^\dagger(\mathbf{p}_2, \theta_2) A_1^\dagger(\mathbf{p}_1, \theta_1) | 0 \rangle$$

- Supersymmetric ward identity for the S matrix

$$\left( \vec{Q}_\alpha^1(\mathbf{p}_1, \theta_1) + \vec{Q}_\alpha^1(\mathbf{p}_2, \theta_2) \right. \\ \left. + \vec{Q}_\alpha^2(\mathbf{p}_3, \theta_3) + \vec{Q}_\alpha^2(\mathbf{p}_4, \theta_4) \right) S(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) = 0$$

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## S matrix in onshell superspace

- S matrix **solution** (in-states:  $p_1, p_2$ , out-states  $p_3, p_4$ ) is determined **in terms of two functions**  $\mathcal{S}_B$  and  $\mathcal{S}_F$  of momenta, couplings and mass.

$$\begin{aligned} S(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) = & \mathcal{S}_B + \mathcal{S}_F \theta_1 \theta_2 \theta_3 \theta_4 + \\ & \left( \frac{1}{2} C_{12} \mathcal{S}_B - \frac{1}{2} C_{34}^* \mathcal{S}_F \right) \theta_1 \theta_2 + \left( \frac{1}{2} C_{13} \mathcal{S}_B - \frac{1}{2} C_{24}^* \mathcal{S}_F \right) \theta_1 \theta_3 \\ & + \left( \frac{1}{2} C_{14} \mathcal{S}_B + \frac{1}{2} C_{23}^* \mathcal{S}_F \right) \theta_1 \theta_4 + \left( \frac{1}{2} C_{23} \mathcal{S}_B + \frac{1}{2} C_{14}^* \mathcal{S}_F \right) \theta_2 \theta_3 \\ & + \left( \frac{1}{2} C_{24} \mathcal{S}_B - \frac{1}{2} C_{13}^* \mathcal{S}_F \right) \theta_2 \theta_4 + \left( \frac{1}{2} C_{34} \mathcal{S}_B - \frac{1}{2} C_{12}^* \mathcal{S}_F \right) \theta_3 \theta_4 \end{aligned}$$

- **No  $\theta$  term:** **four boson** scattering, **four  $\theta$  term :** **four fermion** scattering.
- All **other processes** (two boson to two fermion etc) **determined completely** in terms of the two independent functions  $\mathcal{S}_B$  and  $\mathcal{S}_F$ .

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$$\begin{aligned}\frac{1}{2}C_{12} &= -\frac{1}{4m}v^*(\mathbf{p}_1)v^*(\mathbf{p}_2) & \frac{1}{2}C_{23} &= -\frac{1}{4m}v^*(\mathbf{p}_2)u^*(\mathbf{p}_3) \\ \frac{1}{2}C_{13} &= -\frac{1}{4m}v^*(\mathbf{p}_1)u^*(\mathbf{p}_3) & \frac{1}{2}C_{24} &= -\frac{1}{4m}v^*(\mathbf{p}_2)u^*(\mathbf{p}_4) \\ \frac{1}{2}C_{14} &= -\frac{1}{4m}v^*(\mathbf{p}_1)u^*(\mathbf{p}_4) & \frac{1}{2}C_{34} &= -\frac{1}{4m}u^*(\mathbf{p}_3)u^*(\mathbf{p}_4)\end{aligned}$$

$$\begin{aligned}\frac{1}{2}C_{12}^* &= \frac{1}{4m}v(\mathbf{p}_1)v(\mathbf{p}_2) & \frac{1}{2}C_{23}^* &= \frac{1}{4m}v(\mathbf{p}_2)u(\mathbf{p}_3) \\ \frac{1}{2}C_{13}^* &= \frac{1}{4m}v(\mathbf{p}_1)u(\mathbf{p}_3) & \frac{1}{2}C_{24}^* &= \frac{1}{4m}v(\mathbf{p}_2)u(\mathbf{p}_4) \\ \frac{1}{2}C_{14}^* &= \frac{1}{4m}v(\mathbf{p}_1)u(\mathbf{p}_4) & \frac{1}{2}C_{34}^* &= \frac{1}{4m}u(\mathbf{p}_3)u(\mathbf{p}_4)\end{aligned}$$

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# On onshell supersymmetry for $\mathcal{N} = 2$ S matrix

- The  $\mathcal{N} = 2$  S matrix is already  $\mathcal{N} = 1$  supersymmetric.
- obeys additional constraint from  $\mathcal{N} = 2$  supersymmetry.
- Action of off-shell  $Q_\alpha, \bar{Q}_\alpha$  on on-shell chiral/anti-chiral superfields determines action of on-shell  $\mathcal{N} = 2$  supersymmetry.
- conditions for on-shell  $\mathcal{N} = 2$  susy of S matrix

$$\left( \sum_{i=1}^4 Q_\alpha^i(\mathbf{p}_i, \theta_i) + \bar{Q}_\alpha^i(\mathbf{p}_i, \theta_i) \right) S(\mathbf{p}_i, \theta_i) = 0$$
$$\left( \sum_{i=1}^4 Q_\alpha^i(\mathbf{p}_i, \theta_i) - \bar{Q}_\alpha^i(\mathbf{p}_i, \theta_i) \right) S(\mathbf{p}_i, \theta_i) = 0$$

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- Additional constraint relates the functions  $\mathcal{S}_B$  and  $\mathcal{S}_F$

$$\begin{aligned} \mathcal{S}_B (C_{13} u_\alpha(\mathbf{p}_3) + C_{14} u_\alpha(\mathbf{p}_4) + C_{12} v_\alpha(\mathbf{p}_2) + v_\alpha^*(\mathbf{p}_1)) \\ = \mathcal{S}_F (C_{24}^* u_\alpha(\mathbf{p}_3) - C_{23}^* u_\alpha(\mathbf{p}_4) + C_{34}^* v_\alpha(\mathbf{p}_2)) \end{aligned}$$

- $\mathcal{N} = 2$  S matrix is completely specified by one function.

- eg:  $p_1 = p + q, p_2 = -k - q, p_3 = p, p_4 = -k$

$$\mathcal{S}_B = \mathcal{S}_F \frac{-2m(k-p)_- + iq_3(k+p)_-}{2m(k-p)_- + iq_3(k+p)_-}.$$

- Already  $\mathcal{N} = 2$  supersymmetry is quite constraining.
- Expect all the component S matrices in higher susy cases to be determined by one function.

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## Theory

- General renormalizable  $\mathcal{N} = 1$  theory coupled to single fundamental matter multiplet  $\Phi$

$$\begin{aligned} \mathcal{S}_{\mathcal{N}=1} = - \int d^3x d^2\theta \left[ \frac{\kappa}{2\pi} \text{Tr} \left( -\frac{1}{4} D_\alpha \Gamma^\beta D_\beta \Gamma^\alpha \right. \right. \\ \left. \left. - \frac{1}{6} D^\alpha \Gamma^\beta \{ \Gamma_\alpha, \Gamma_\beta \} - \frac{1}{24} \{ \Gamma^\alpha, \Gamma^\beta \} \{ \Gamma_\alpha, \Gamma_\beta \} \right) \right. \\ \left. - \frac{1}{2} (D^\alpha \bar{\Phi} + i \bar{\Phi} \Gamma^\alpha) (D_\alpha \Phi - i \Gamma_\alpha \Phi) \right. \\ \left. + m_0 \bar{\Phi} \Phi + \frac{\pi w}{\kappa} (\bar{\Phi} \Phi)^2 \right] \end{aligned}$$

- $\Phi$  : complex superfield,  $\Gamma_\alpha$ : real superfield

$$\Phi = \phi + \theta\psi - \theta^2 F, \quad \bar{\Phi} = \bar{\phi} + \theta\bar{\psi} - \theta^2 \bar{F},$$

$$\Gamma^\alpha = \chi^\alpha - \theta^\alpha B + i\theta^\beta A_\beta{}^\alpha - \theta^2 (2\lambda^\alpha - i\partial^{\alpha\beta} \chi_\beta).$$

- Integer parameters  $N, \kappa$ , matter coupling constant  $w$ , 't Hooft coupling  $\lambda = \frac{N}{\kappa}$ .

# Supersymmetric light cone gauge

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- Supersymmetric generalisation of light cone gauge

$$\Gamma_- = 0 \Rightarrow A_- = A_1 + iA_2 = 0$$

- Gauge self interactions **vanish**

$$S = - \int d^3x d^2\theta \left[ - \frac{\kappa}{8\pi} \text{Tr}(\Gamma^- i \partial_- \Gamma^-) - \frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi \right. \\ \left. - \frac{i}{2} \Gamma^- (\bar{\Phi} D_- \Phi - D_- \bar{\Phi} \Phi) \right. \\ \left. + m_0 \bar{\Phi} \Phi + \frac{\pi W}{\kappa} (\bar{\Phi} \Phi)^2 \right]$$

- Susy light cone gauge maintains **manifest supersymmetry**.

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# Strategy for computing $S$ matrix

- Use off-shell supersymmetry to constrain the structure of two point and four-point functions in superspace.
- Use these structures to set up a Dyson-Schwinger series for exact propagator and exact off-shell four point function.
- work only with diagrams that contribute to leading order in the large  $N$  limit.
- read off  $S$  matrices from off-shell four point function by taking on-shell limits.

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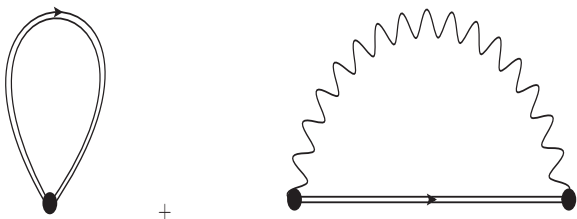
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# Exact propagator in large N limit

- Integral equation for self-energy

$$\Sigma(p, \theta_1, \theta_2) =$$


The image shows two Feynman diagrams representing terms in the self-energy integral equation. The first diagram is a tadpole loop, consisting of a double line with an arrow pointing clockwise, forming a teardrop shape. The second diagram is a bubble loop, consisting of a wavy line (representing a scalar or ghost) on top and a double line with an arrow pointing clockwise on the bottom, forming a semi-circular shape. Both diagrams have two external vertices represented by black dots.

$$\begin{aligned} \Sigma(p, \theta_1, \theta_2) = & 2\pi\lambda w \int \frac{d^3r}{(2\pi)^3} \delta^2(\theta_1 - \theta_2) P(r, \theta_1, \theta_2) \\ & - 2\pi\lambda \int \frac{d^3r}{(2\pi)^3} D_-^{\theta_2, -p} D_-^{\theta_1, p} \left( \frac{\delta^2(\theta_1 - \theta_2)}{(p-r)_{--}} P(r, \theta_1, \theta_2) \right) \\ & + 2\pi\lambda \int \frac{d^3r}{(2\pi)^3} \frac{\delta^2(\theta_1 - \theta_2)}{(p-r)_{--}} D_-^{\theta_1, r} D_-^{\theta_2, -r} P(r, \theta_1, \theta_2) \end{aligned}$$

# Exact propagator in large N limit

- Solution to **exact propagator** is extremely simple

$$P(p, \theta_1, \theta_2) = \frac{D^2 - m}{p^2 + m^2} \delta^2(\theta_1 - \theta_2)$$

- Same structure as the **bare propagator** with  $m_0$  replaced by  $m$ .
- $m$  is the **pole mass**

$$m = \frac{2m_0}{2 + (-1 + w)\lambda \text{Sgn}(m)}$$

is **duality invariant**, agrees with the pole mass computed by **Jain, Minwalla, Yokoyama**

- **Bonus:** In the  $\mathcal{N} = 2$  limit ( $w = 1$ ), **no mass renormalization for  $\mathcal{N} = 2$  theory** !

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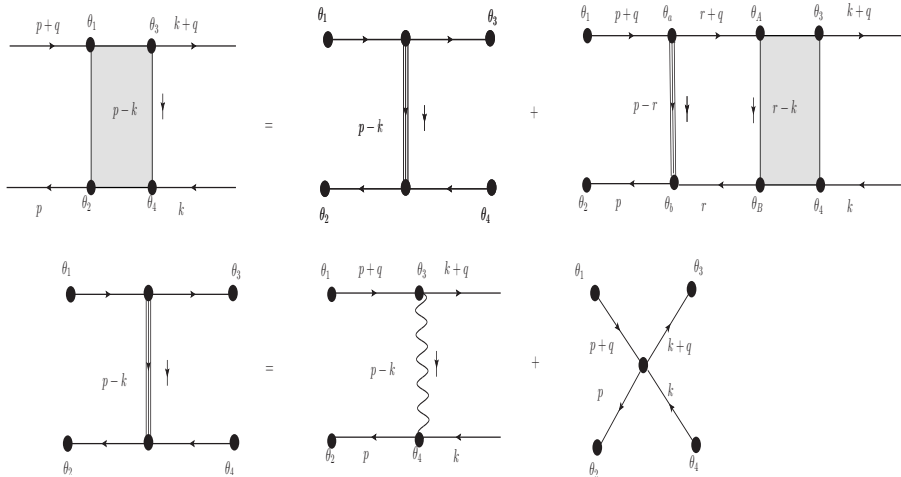
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# An integral equation for the four point function



**Figure:** The diagrams in the first line pictorially represents the Schwinger-Dyson equation for offshell four point function. The second line represents the tree level contributions from the gauge superfield interaction and the quartic interactions.

# An integral equation for the four point function

- Schematically the integral equations are of the form

$$V(\theta_i, p_i) = V_0(\theta_i, p_i) + \int \frac{d^3 r}{(2\pi)^3} d^2 \theta'_j V_0(\theta_i, \theta'_j, p_i, r) \\ P(\theta'_j, p_i + r) P(\theta'_j, r) V(\theta'_j, \theta_i, p_i)$$

- solved** integral equations **exactly** in large N limit for **arbitrary** values of the **t'Hooft coupling**  $\lambda$  and determined the **offshell four point function** in the kinematic regime  $q_{\pm} = 0$ .
- Onshell limit** directly gives the **S matrix** for **T (adjoint)**,  **$U_d$  (symm)** and  **$U_e$  (Asymm)** channels of scattering ( $q_{\mu}$  is momentum transfer).
- Impossible to extract **S (singlet)** channel S matrix since  $q_{\mu}$  is center of mass energy (cannot be spacelike).

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- S matrix: onshell limit of off-shell four point correlator

$$\mathcal{T}_B = \frac{4i\pi}{\kappa} \epsilon_{\mu\nu\rho} \frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} + J_B(q, \lambda) ,$$

$$\mathcal{T}_F = \frac{4i\pi}{\kappa} \epsilon_{\mu\nu\rho} \frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} + J_F(q, \lambda) ,$$

$$J_B(q, \lambda) = \frac{4\pi q}{\kappa} \frac{N_1 N_2 + M_1}{D_1 D_2} ,$$

$$J_F(q, \lambda) = \frac{4\pi q}{\kappa} \frac{N_1 N_2 + M_2}{D_1 D_2} ,$$

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## S matrix in $T$ , $U_d$ , $U_e$ channels of scattering

$$N_1 = \left( \left( \frac{2|m| + iq}{2|m| - iq} \right)^{-\lambda} (w-1)(2m+iq) + (w-1)(2m-iq) \right) ,$$

$$N_2 = \left( \left( \frac{2|m| + iq}{2|m| - iq} \right)^{-\lambda} (q(w+3) + 2im(w-1)) + (q(w+3) - 2im(w-1)) \right) ,$$

$$M_1 = -8mq((w+3)(w-1) - 4w) \left( \frac{2|m| + iq}{2|m| - iq} \right)^{-\lambda} ,$$

$$M_2 = -8mq(1+w)^2 \left( \frac{2|m| + iq}{2|m| - iq} \right)^{-\lambda} ,$$

$$D_1 = \left( i \left( \frac{2|m| + iq}{2|m| - iq} \right)^{-\lambda} (w-1)(2m+iq) - 2im(w-1) + q(w+3) \right) ,$$

$$D_2 = \left( \left( \frac{2|m| + iq}{2|m| - iq} \right)^{-\lambda} (-q(w+3) - 2im(w-1)) + (w-1)(q+2im) \right) .$$

# Bonus: S matrix in $T$ , $U_d$ , $U_e$ channels for $\mathcal{N} = 2$ theory

- Remarkable **simplification** in the  $\mathcal{N} = 2$  limit ( $w=1$ )

$$\mathcal{T}_B^{\mathcal{N}=2} = \frac{4i\pi}{\kappa} \epsilon_{\mu\nu\rho} \frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} - \frac{8\pi m}{\kappa} ,$$

$$\mathcal{T}_F^{\mathcal{N}=2} = \frac{4i\pi}{\kappa} \epsilon_{\mu\nu\rho} \frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} + \frac{8\pi m}{\kappa}$$

- All orders S matrix is just tree level - **no loop corrections** - non renormalization.

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# Duality invariance of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ S matrices

- Under the duality transformation

$$w' = \frac{3-w}{w+1}, \lambda' = \lambda - \text{sgn}(\lambda), m' = -m, \kappa' = -\kappa$$

$$J_B(q, \kappa', \lambda', w', m') = -J_F(q, \kappa, \lambda, w, m),$$

$$J_F(q, \kappa', \lambda', w', m') = -J_B(q, \kappa, \lambda, w, m).$$

- Duality maps the purely bosonic and purely fermionic S matrices into one another upto overall phase.
- Concrete evidence for duality.
- Supersymmetric ward identity guarantees the duality invariance of all other processes.

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# S matrix in (singlet) S channel

- We cannot extract the S channel S matrix directly because of kinematic restriction  $q_{\pm} = 0$ .
- Usual rules of crossing symmetry in quantum field theory predict particle - anti particle scattering from particle particle scattering or vice-versa
- Naive analytic continuation gives a non-unitary S matrix in the S channel as observed in earlier work.
- Any analytic continuation cannot produce the non-analytic delta function piece required for unitarization.
- Remedy: Modify crossing symmetry rules as conjectured in [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama]

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## Conjectured $S$ matrix in $S$ channel $\mathcal{N} = 1$ theory

- Conjectured  $S$  matrix for the  $\mathcal{N} = 1$  theory in center of mass frame

$$\mathcal{S}_B^S(s, \theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda} \left( 4\pi i\lambda\sqrt{s}\cot(\theta/2) + J_B(\sqrt{s}, \lambda) \right) ,$$

$$\mathcal{S}_F^S(s, \theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda} \left( 4\pi i\lambda\sqrt{s}\cot(\theta/2) + J_F(\sqrt{s}, \lambda) \right) .$$

$$J_B(\sqrt{s}, \lambda) = -4\pi i\lambda\sqrt{s}\frac{N_1N_2 + M_1}{D_1D_2} ,$$

$$J_F(\sqrt{s}, \lambda) = -4\pi i\lambda\sqrt{s}\frac{N_1N_2 + M_2}{D_1D_2}$$

## Conjectured $S$ matrix in $S$ channel $\mathcal{N} = 1$ theory

$$N_1 = \left( (w-1)(2m + \sqrt{s}) + (w-1)(2m - \sqrt{s})e^{i\pi\lambda} \left( \frac{\sqrt{s} + 2|m|}{\sqrt{s} - 2|m|} \right)^\lambda \right),$$

$$N_2 = \left( (-i\sqrt{s}(w+3) + 2im(w-1)) + (-i\sqrt{s}(w+3) - 2im(w-1))e^{i\pi\lambda} \left( \frac{\sqrt{s} + 2|m|}{\sqrt{s} - 2|m|} \right)^\lambda \right),$$

$$M_1 = 8mi\sqrt{s}((w+3)(w-1) - 4w)e^{i\pi\lambda} \left( \frac{\sqrt{s} + 2|m|}{\sqrt{s} - 2|m|} \right)^\lambda,$$

$$M_2 = 8mi\sqrt{s}(1+w)^2 e^{i\pi\lambda} \left( \frac{\sqrt{s} + 2|m|}{\sqrt{s} - 2|m|} \right)^\lambda,$$

$$D_1 = \left( i(w-1)(2m + \sqrt{s}) - (2im(w-1) + i\sqrt{s}(w+3))e^{i\pi\lambda} \left( \frac{\sqrt{s} + 2|m|}{\sqrt{s} - 2|m|} \right)^\lambda \right),$$

$$D_2 = \left( (\sqrt{s}(w+3) - 2im(w-1)) + (w-1)(-i\sqrt{s} + 2im)e^{i\pi\lambda} \left( \frac{\sqrt{s} + 2|m|}{\sqrt{s} - 2|m|} \right)^\lambda \right)$$

## Straightforward non-relativistic limit of the $\mathcal{N} = 1$ S matrix

- Non-rel limit:  $\sqrt{s} \rightarrow 2m$  with all other parameters held fixed.

$$\mathcal{T}_B^S(s, \theta) = -8\pi i \sqrt{s} (\cos(\pi\lambda) - 1) \delta(\theta) + 4\sqrt{s} \sin(\pi\lambda) (i \cot(\theta/2) - 1) ,$$

$$\mathcal{T}_F^S(s, \theta) = -8\pi i \sqrt{s} (\cos(\pi\lambda) - 1) \delta(\theta) + 4\sqrt{s} \sin(\pi\lambda) (i \cot(\theta/2) + 1) .$$

- conjectured S channel S matrix has simple non-relativistic limit leading to known Aharonov-Bohm result.
- Surprisingly this result is also same as the  $\mathcal{N} = 2$  S channel S matrix.
- Presumably supersymmetry enhancement in non-relativistic limit.

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# Unitarity equation

- Define on-shell superfield  $S^\dagger$  as

$$S^\dagger(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) = S^*(\mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4, \mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2)$$

- Supersymmetric ward identity for  $S^\dagger$  implies  $S^\dagger$  is supersymmetric if and only if  $S$  is supersymmetric.
- The supersymmetric unitarity equation is

$$(S \star S^\dagger - I) = 0$$

- Recall that the superfield expansion for  $S$  is completely specified by  $\mathcal{S}_B$  and  $\mathcal{S}_F$ .
- Sufficient to check the LHS for no  $\theta$  and four  $\theta$  terms.
- Supersymmetric ward identity guarantees the rest of the terms will obey the unitarity equations.

## Unitarity equations for $T$ , $U_d$ and $U_e$ channels

- Writing  $\mathcal{S}_B = I + i\mathcal{T}_B$ ,  $\mathcal{S}_F = I + i\mathcal{T}_f$
- The  $S$  matrices in the  $T$ ,  $U_d$  and  $U_e$  channels are all  $O(1/N)$  - unitarity equation is linear

$$\mathcal{T}_B(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \mathcal{T}_B^*(\mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_1, \mathbf{p}_2) ,$$

$$\mathcal{T}_F(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \mathcal{T}_F^*(\mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_1, \mathbf{p}_2)$$

- Linearity: No branch cuts in the physical domain of scattering in these channels.
- Explicitly checked that unitarity conditions are obeyed using

$$J_B(q, \lambda) = J_B^*(-q, \lambda) , \quad J_F(q, \lambda) = J_F^*(-q, \lambda)$$

- The  $S$  matrix in the  $S$  channel is  $O(1)$  - the unitarity conditions are non-linear

# Unitarity equations in the S channel

- Consider the general structure ( $T(\theta) = i \cot(\theta/2)$ .)

$$\mathcal{T}_B^S = H_B T(\theta) + W_B - iW_2 \delta(\theta) , \quad \mathcal{T}_F^S = H_F T(\theta) + W_F - iW_2 \delta(\theta) ,$$

- first unitarity equation

$$H_B - H_B^* = \frac{1}{8\pi\sqrt{s}}(W_2 H_B^* - H_B W_2^*) ,$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H_B H_B^*) ,$$

$$W_B - W_B^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_B^* - W_2^* W_B) - \frac{i}{4\sqrt{s}}(H_B H_B^* - W_B W_B^*) - \frac{iY}{4\sqrt{s}}(W_B - W_F)(W_B^* - W_F^*)$$

- Second unitarity equation

$$H_F - H_F^* = \frac{1}{8\pi\sqrt{s}}(W_2 H_F^* - H_F W_2^*) ,$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H_F H_F^*) ,$$

$$W_F - W_F^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_F^* - W_2^* W_F) - \frac{i}{4\sqrt{s}}(H_F H_F^* - W_F W_F^*) - \frac{iY}{4\sqrt{s}}(W_B - W_F)(W_B^* - W_F^*)$$



## Unitarity equation in the S channel

- Unitarity equations are verified using

$$H_B = H_F = 4\sqrt{s} \sin(\pi\lambda), \quad W_2 = 8\pi\sqrt{s}(\cos(\pi\lambda)-1), \quad T(\theta) = i \cot(\theta/2)$$

$$W_B = J_B(\sqrt{s}, \lambda) \frac{\sin(\pi\lambda)}{\pi\lambda},$$

$$W_F = J_F(\sqrt{s}, \lambda) \frac{\sin(\pi\lambda)}{\pi\lambda}.$$

- **Algebraic-miracle**: Non-linear unitarity equations obeyed by very complicated functions.
- unitarity is an extremely sensitive test <sup>1</sup>.
- Important to note that the **crossing symmetry rules** have to be modified exactly as conjectured in [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama], else the unitarity test fails.

---

<sup>1</sup>Tagline: one sign is doom

## Unitarity in the S channel - $\mathcal{N} = 2$ case

- The  $\mathcal{N} = 2$   $T$  matrix is tree level exact in  $T, U$  channels.
- Naive crossing symmetry would imply the same for  $S$  channel, unitarity equation  $i(T^\dagger - T) = TT^\dagger$  would never be obeyed (LHS would be zero).
- modified crossing rules resolve this puzzle:

$$\mathcal{T}_B^{S;\mathcal{N}=2}(s, \theta) = -8\pi i \sqrt{s} (\cos(\pi\lambda) - 1) \delta(\theta) + \sin(\pi\lambda) (4i\sqrt{s} \cot(\theta/2) - 8m) ,$$

$$\mathcal{T}_F^{S;\mathcal{N}=2}(s, \theta) = -8\pi i \sqrt{s} (\cos(\pi\lambda) - 1) \delta(\theta) + \sin(\pi\lambda) (4i\sqrt{s} \cot(\theta/2) + 8m) .$$

- Non-analytic piece makes  $\mathcal{T}_B, \mathcal{T}_F$  not Hermitian, both LHS and RHS are non-zero and non-linear unitarity equation is obeyed.

# Pole structure of the S matrix

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Delta function and modified crossing

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## Onshell supersymmetry and the S matrix

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S matrix in onshell superspace

## Exact computation of all orders S matrix

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Exact propagator in large N limit

Exact four point correlator in large N limit

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# Pole structure of the singlet channel S matrix

- Both bosonic and fermionic S matrices have a pole at threshold ( $s = 4m^2$ ) for  $w \leq -1$ . For  $w \leq -1 + \epsilon$  the pole is close to threshold.

- As  $w$  is decreased further and as it hits a critical value  $w = w_c$  the pole becomes massless!

$$w = w_c(\lambda) = 1 - \frac{2}{|\lambda|}$$

- As  $w$  is further decreased and as  $w \rightarrow -\infty$  the pole approaches threshold once again.
- To summarize, a one parameter tuning of the superpotential interaction parameter  $w$  - sufficient to produce massless bound states in a massive theory.
- $w$  can be scaled to  $w_c$  - possible decoupled QFT description of light states.

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# Summary

- Computations and conjectures for the all orders  $2 \rightarrow 2$   $S$  matrix in the general renormalizable  $\mathcal{N} = 1$  Chern-Simons matter theory with a single fundamental matter multiplet.
- Used [supersymmetric ward identity](#) to derive conditions and constraints on off-shell correlators, on-shell  $S$  matrices and derive unitarity conditions.
- Computed [exact offshell four point correlators in the large  \$N\$  limit](#) in kinematic regime  $q_{\pm} = 0$ .
- Obtained  $S$  matrices by taking [onshell limit](#) of offshell four point correlator.
- [Conjectured  \$S\$  matrix in the singlet channel.](#)

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# Summary

- Results are consistent with duality.
- Results are consistent with unitarity if and only if we assume that the usual results of crossing symmetry are modified in precisely the manner proposed in [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama].
- Non-relativistic limit of the S matrix reproduces the known Aharonov-Bohm result.
- The S channel S matrix has an interesting analytic structure. In a certain range of superpotential parameters the S matrix has a bound state pole.
- A one parameter tuning of superpotential parameters can be used to set the pole mass to zero.

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# Open questions

- $\mathcal{N} = 2$   $S$  matrices are tree level exact in non-anyonic channels and depend on  $\lambda$  very simple way in the anyonic channel - can it reproduced from general principles and  $\mathcal{N} = 2$  supersymmetry?
- Generalisation to higher supersymmetry - mass deformed  $\mathcal{N} = 3, 4, 5$ , and mass deformed  $\mathcal{N} = 6$  ABJ theory - in progress [K.I, S.Jain, S.Minwalla, S. Yokoyama]
- Four point correlator: useful in computation of 2,3,4 point functions of gauge invariant currents - explicit computation in  $\mathcal{N} = 2$  theory?, possible  $\mathcal{N} = 2$  generalisation of Maldacena-Zhiboedov solutions - in progress [K.I, S.Jain, P.Nayak]
- decoupled gapless sector: effective field theory for the massless bound states of the  $S$  matrix.

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# Open questions

- Rigorous proof of delta function and modified crossing rules, generalisation to finite  $N$  and  $\kappa$ .
- From perturbative pov modified crossing rules could be related to IR divergences.
- IR divergences can be summed up and exponentiated [Grammer,Yennie; Bern,Dickson,Smirnov]
- Modified crossing factor  $\frac{\sin(\pi\lambda)}{\pi\lambda}$  is identical to the expectation value of circular wilson loop in pure Chern-Simons theory on  $S^3$ .
- To explore: Path integral derivation of Witten's result, crossing and fusion rules in RCFT's.

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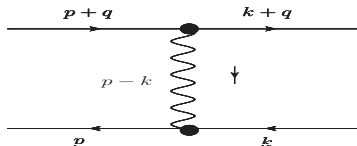
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**Thank You!**

# Need for a conjecture

- $P_i(p_1) + A^j(p_2) \rightarrow P_m(p_3) + A^n(p_4)$



- Work in light-cone gauge in the frame  $q_{\pm} = 0$ .
- Adjoint** channel (from top) -  $q_{\pm} = 0$  is a frame choice, full answer can be obtained by covariantizing.

$$p_1 = p + q, \quad p_2 = -k - q, \quad p_3 = p, \quad p_4 = -k$$

- Singlet** channel (from left), **exchange momentum** cannot be spacelike!

$$p_1 = p + q, \quad p_2 = -p, \quad p_3 = k + q, \quad p_4 = -k$$

- Cannot compute singlet channel directly.
- Using **usual crossing symmetry** gives a **non-unitary S matrix** for **singlet** channel.

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# Bare Propagators

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- The **bare scalar superfield propagator**:

$$\langle \bar{\Phi}(\theta_1, p) \Phi(\theta_2, -p') \rangle = \frac{D_{\theta_1, p}^2 - m_0^2}{p^2 + m_0^2} \delta^2(\theta_1 - \theta_2) (2\pi)^3 \delta^3(p - p')$$

- The **gauge superfield propagator**:

$$\langle \Gamma^-(\theta_1, p) \Gamma^-(\theta_2, -p') \rangle = -\frac{8\pi}{\kappa} \frac{\delta^2(\theta_1 - \theta_2)}{p_{--}} (2\pi)^3 \delta^3(p - p')$$

where  $p_{--} = -(p_1 + ip_2) = -p_-$ .

- Gauge field **component propagators** have same form as **non-susy light cone gauge**

$$\langle A_+(p) A_3(-p') \rangle = \frac{4\pi i}{\kappa} \frac{1}{p_-} (2\pi)^3 \delta^3(p - p')$$

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# Susy constraints on two-point correlator

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- Supersymmetric ward identity for two point correlator

$$(Q_{\theta_1,p} + Q_{\theta_2,-p})\langle\bar{\Phi}(\theta_1,p)\Phi(\theta_2,-p)\rangle = 0$$

- Exact propagator solves the ward identity

$$\langle\bar{\Phi}(p,\theta_1)\Phi(-p',\theta_2)\rangle = (2\pi)^3\delta^3(p-p')P(\theta_1,\theta_2,p)$$

$$P(\theta_1,\theta_2,p) = (C_1(p^\mu)D_{\theta_1,p}^2 + C_2(p^\mu))\delta^2(\theta_1 - \theta_2)$$

- eg for bare propagator

$$C_1 = \frac{1}{p^2 + m_0^2}, \quad C_2 = \frac{m_0}{p^2 + m_0^2}$$

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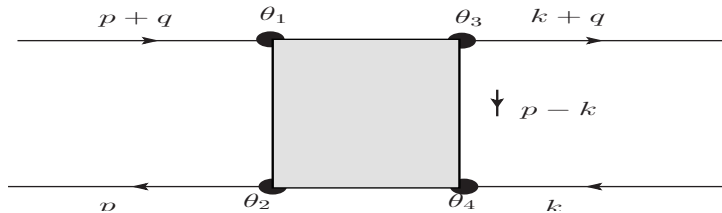
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## Susy constraints on four-point function



- Supersymmetric ward identity for four point function

$$(Q_{\theta_1, p+q} + Q_{\theta_2, -p} + Q_{\theta_3, -k-q} + Q_{\theta_4, k})V(\theta_1, \theta_2, \theta_3, \theta_4, p, k, q) = 0$$

$$\begin{aligned} \langle \bar{\Phi}((p+q + \frac{l}{4}), \theta_1) \Phi(-p + \frac{l}{4}, \theta_2) \Phi(-(k+q) + \frac{l}{4}, \theta_3) \bar{\Phi}(k + \frac{l}{4}, \theta_4) \rangle \\ = (2\pi)^3 \delta(l) V(\theta_1, \theta_2, \theta_3, \theta_4, p, k, q) \end{aligned}$$

# Susy constraints on four-point function

- Solution of the ward identity

$$V = \exp \left( \frac{1}{4} X \cdot (p \cdot X_{12} + q \cdot X_{13} + k \cdot X_{43}) \right) F(X_{12}, X_{13}, X_{43}, p, q, k)$$

$$X = \sum_{i=1}^4 \theta_i \ , \ X_{ij} = \theta_i - \theta_j \ ,$$

- $F$  is a **shift invariant function**  $\theta_i \rightarrow \theta_i + \gamma$ .
- $V$  may be taken to be invariant under the  **$\mathbb{Z}_2$  symmetry**

$$\begin{aligned} p &\rightarrow k + q, \ k \rightarrow p + q, \ q \rightarrow -q \ , \\ \theta_1 &\rightarrow \theta_4, \ \theta_2 \rightarrow \theta_3, \ \theta_3 \rightarrow \theta_2, \ \theta_4 \rightarrow \theta_1 \end{aligned}$$

# An integral equation for the four point function

- Most general form of  $F$  can be parameterized in terms of 32 bosonic functions of  $p, k$  and  $q$ .
- leads to 32 coupled integral equations - tedious.
- In the kinematic regime  $q_{\pm} = 0$  the ansatz

$$V = \exp\left(\frac{1}{4}X.(p.X_{12} + q.X_{13} + k.X_{43})\right) F(X_{12}, X_{13}, X_{43}, p, q, k)$$
$$F = X_{12}^+ X_{43}^+ \left( A(p, k, q) X_{12}^- X_{43}^- X_{13}^+ X_{13}^- + B(p, k, q) X_{12}^- X_{43}^- \right. \\ \left. + C(p, k, q) X_{12}^- X_{13}^+ + D(p, k, q) X_{13}^+ X_{43}^- \right)$$

is closed under the multiplication rule induced by the RHS of the integral equation.



## Product of $S$ matrices

- General multiplication rule for two  $S$  matrices

$$S_1 \star S_2 \equiv \int d\Gamma S_1(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{k}_3, \phi_1, \mathbf{k}_4, \phi_2) \\ \exp(\phi_1\phi_3 + \phi_2\phi_4) 2k_1^0 (2\pi)^2 \delta^{(2)}(\mathbf{k}_3 - \mathbf{k}_1) 2k_2^0 (2\pi)^2 \delta^{(2)}(\mathbf{k}_4 - \mathbf{k}_2) \\ S_2(\mathbf{k}_1, \phi_3, \mathbf{k}_2, \phi_4, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4)$$

$$d\Gamma = \frac{d^2 k_3}{2k_3^0 (2\pi)^2} \frac{d^2 k_4}{2k_4^0 (2\pi)^2} \frac{d^2 k_1}{2k_1^0 (2\pi)^2} \frac{d^2 k_2}{2k_2^0 (2\pi)^2} d\phi_1 d\phi_3 d\phi_2 d\phi_4$$

- supersymmetry invariant Identity operator

$$I(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) = \exp(\theta_1\theta_3 + \theta_2\theta_4) I(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)$$

$$I(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = 2p_3^0 (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{p}_3) 2p_4^0 (2\pi)^2 \delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_4)$$

- Multiplication rule with Identity operator

$$S \star I = I \star S = S$$

- More generally product of two supersymmetric  $S$  matrices is supersymmetric.

# Unitarity equation in center of mass frame

- Writing  $\mathcal{S}_B = I + i\mathcal{T}_B$ ,  $\mathcal{S}_F = I + i\mathcal{T}_f$

- No theta term:

$$\frac{1}{8\pi\sqrt{s}} \int d\theta \left( -Y(s)(\mathcal{T}_B(s, \theta) + 4Y(s)\mathcal{T}_f(s, \theta))(\mathcal{T}_B^*(s, -(\alpha - \theta)) + 4Y(s)\mathcal{T}_f^*(s, -(\alpha - \theta))) \right. \\ \left. + \mathcal{T}_B(s, \theta)\mathcal{T}_B^*(s, -(\alpha - \theta)) \right) = i(\mathcal{T}_B^*(s, -\alpha) - \mathcal{T}_B(s, \alpha))$$

- Four theta term:

$$\frac{1}{8\pi\sqrt{s}} \int d\theta \left( Y(s)(\mathcal{T}_B(s, \theta) + 4Y(s)\mathcal{T}_f(s, \theta))(\mathcal{T}_B^*(s, -(\alpha - \theta)) + 4Y(s)\mathcal{T}_f^*(s, -(\alpha - \theta))) \right. \\ \left. - 16Y(s)^2\mathcal{T}_f(s, \theta)\mathcal{T}_f^*(s, -(\alpha - \theta)) \right) = i4Y(s)(-\mathcal{T}_f(s, \alpha) + \mathcal{T}_f^*(s, -\alpha))$$

$$Y(s) = \frac{-s + 4m^2}{16m^2}$$

# Unitarity equations in the S channel

- The  $S$  matrix in the S channel is  $O(1)$  - the unitarity conditions are non-linear

$$\frac{1}{8\pi\sqrt{s}} \int d\theta \left( -Y(s)(\mathcal{T}_B^S(s, \theta) - \mathcal{T}_F^S(s, \theta))(\mathcal{T}_B^{S*}(s, -(\alpha - \theta)) - \mathcal{T}_F^{S*}(s, -(\alpha - \theta))) \right. \\ \left. + \mathcal{T}_B^S(s, \theta)\mathcal{T}_B^{S*}(s, -(\alpha - \theta)) \right) = i(\mathcal{T}_B^{S*}(s, -\alpha) - \mathcal{T}_B^S(s, \alpha))$$

$$\frac{1}{8\pi\sqrt{s}} \int d\theta \left( Y(s)(\mathcal{T}_B^S(s, \theta) - \mathcal{T}_F^S(s, \theta))(\mathcal{T}_B^{S*}(s, -(\alpha - \theta)) - \mathcal{T}_F^{S*}(s, -(\alpha - \theta))) \right. \\ \left. - \mathcal{T}_F^S(s, \theta)\mathcal{T}_F^{S*}(s, -(\alpha - \theta)) \right) = i(\mathcal{T}_F^S(s, \alpha) - \mathcal{T}_F^{S*}(s, -\alpha))$$

- Under duality  $\mathcal{T}_B \rightarrow \mathcal{T}_F$  and vice versa; both the equations map to one another.
- Unitarity conditions are compatible with duality.

# Unitarity equations in the S channel

- In the **center of mass frame**, the supersymmetric unitarity equations are

$$\frac{1}{8\pi\sqrt{s}} \int d\theta \left( -Y(s)(\mathcal{T}_B^S(s, \theta) - \mathcal{T}_F^S(s, \theta))(\mathcal{T}_B^{S*}(s, -(\alpha - \theta)) - \mathcal{T}_F^{S*}(s, -(\alpha - \theta))) \right. \\ \left. + \mathcal{T}_B^S(s, \theta)\mathcal{T}_B^{S*}(s, -(\alpha - \theta)) \right) = i(\mathcal{T}_B^{S*}(s, -\alpha) - \mathcal{T}_B^S(s, \alpha))$$

$$\frac{1}{8\pi\sqrt{s}} \int d\theta \left( Y(s)(\mathcal{T}_B^S(s, \theta) - \mathcal{T}_F^S(s, \theta))(\mathcal{T}_B^{S*}(s, -(\alpha - \theta)) - \mathcal{T}_F^{S*}(s, -(\alpha - \theta))) \right. \\ \left. - \mathcal{T}_F^S(s, \theta)\mathcal{T}_F^{S*}(s, -(\alpha - \theta)) \right) = i(\mathcal{T}_F^S(s, \alpha) - \mathcal{T}_F^{S*}(s, -\alpha))$$

- Under **duality**  $\mathcal{T}_B \rightarrow \mathcal{T}_F$  and vice versa; both the equations map to one another.
- **Unitarity conditions are compatible with duality.**

# Poles of the $S$ matrix

- Both bosonic and fermionic  $S$  matrix have a **pole at threshold for  $w \leq -1$** . Near  $w = -1 - \delta w, y = 1 - \delta y$  the  $S$  matrix has the pole structure ( $y = \sqrt{s}/2m$ )

$$\mathcal{T}_B \sim \frac{\left(\frac{\delta y}{2}\right)^{|\lambda|}}{\delta w - 2\left(\frac{\delta y}{2}\right)^{|\lambda|}}, \quad \mathcal{T}_F \sim \frac{\left(\frac{\delta y}{2}\right)^{1+|\lambda|}}{\delta w - 2\left(\frac{\delta y}{2}\right)^{|\lambda|}}$$

- As  $w$  is decreased further and as it hits a **critical value  $w = w_c$**

$$w = w_c(\lambda) = 1 - \frac{2}{|\lambda|}$$

- the **pole becomes massless!**. Near  $w = w_c - \delta w$  and  $y = \delta y$  the poles **approach zero mass quadratically**

$$\mathcal{T}_B \sim \mathcal{T}_F - \frac{64|m|\sin(\pi\lambda)(-1 + |\lambda|)}{|\lambda|(\delta w^2\lambda^2 - 4\delta y^2(1 - |\lambda|)^2)}$$

# Poles of the $S$ matrix

- As  $w$  is further decreased and as  $w \rightarrow -\infty$  the pole approaches threshold once again. Near  $w = -\frac{1}{\delta w}$ ,  $y = 1 - \delta y$  the  $S$  matrix has the pole structure

$$\mathcal{T}_B \sim \frac{\left(\frac{\delta y}{2}\right)^{2-|\lambda|}}{\delta w - \frac{1}{2} \left(\frac{\delta y}{2}\right)^{1-|\lambda|}}, \quad \mathcal{T}_F \sim \frac{\left(\frac{\delta y}{2}\right)^{1-|\lambda|}}{\delta w - \frac{1}{2} \left(\frac{\delta y}{2}\right)^{1-|\lambda|}}$$

- To summarize, a one parameter tuning of the superpotential interaction parameter  $w$  - sufficient to produce massless bound states in our massive theory.
- $w$  can be scaled to  $w_c$  - possible decoupled QFT description of light states.
- Is this a  $\mathcal{N} = 1$  Wilson-Fischer theory made of single real superfield?

# Analytic structure of S channel $S$ matrix

- The  $S$  matrix in the singlet channel has an interesting analytic structure.
- As a function of  $s$  (at fixed  $t$ ), there is the expected two particle branch cut starting at  $s = 4m^2$ .
- For smaller but positive values of  $s$  there exist poles in the  $S$  matrix for a range of coupling parameters.
- These poles represent bound states that exist at large but finite  $N$ .
- At some critical value of the scalar coupling  $w = w_c(\lambda)$  the pole becomes massless!
- To summarize, a one parameter tuning of the superpotential interaction parameter  $w$  - sufficient to produce massless bound states in a massive theory.

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