Generalised Attractors in Five Dimensional Gauged Supergravity

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arXiv: 1206.3887

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Plan of the talk

Introduction:

- Attractor mechanism plays a crucial role in understanding the origin of black hole entropy in supergravity theories *Ferrara et.al '95*.
- Moduli fields in black hole background are attracted to specific charge dependent values on the horizon.
- Attractor values are determined by solving sets of algebraic equations.
- Macroscopic entropy is determined in terms of charges - independent of asymptotic values of moduli.
- Agrees with microscopic results in string theory.

Introduction:

- Attractor mechanism is a consequence of near horizon geometry rather than supersymmetry *Ferrara et.al '97*.
- Also extends to non-supersymmetric cases *Goldstein et.al '05*.
- Recently, Attractor mechanism generalised for $\mathcal{N} = 2, d = 4$ gauged supergravity *Klemm et.al 09, Kachru et.al 11*.
- Generalised attractors: solutions of equations of motion that reduce to algebraic equations when all the fields, curvature tensors are constants in tangent space.

Introduction:

- Lifshitz, Schrödinger geometries are examples of generalised attractors.
- Such geometries are near horizon geometries of extremal black branes *Goldstein et.al '09*.
- Bianchi attractors: Classification of near horizon geometries of homogeneous extremal black branes in $d = 5$ *Iizuka et.al '11*.
- Lifshitz, Schrödinger geometries belong to Bianchi Type I in the classification.

Motivation: Why Bianchi type metrics?

- Bianchi type metrics are candidates for gravity duals in the context of AdS/CMT *Liu et.al '11*.
- Specifically, exhibit vanishing entropy at zero temperature - obey third 'law' of thermodynamics.
- Approach of *Iizuka et.al '11*,focussed on exhaustive classification.
- Any possible application in AdS/CFT requires supergravity embedding to start with.

Motivation: Why gauged sugra?

- Bianchi attractors arise in 5d Einstein-Maxwell systems with massive gauge fields.
- Explicit mass terms break SUSY and are not allowed in Sugra.
- Typical scalar kinetic term of Gauged supergravities,

$$g_{\tilde{x}\tilde{y}}\mathcal{D}_\mu\phi^{\tilde{x}}\mathcal{D}^\mu\phi^{\tilde{y}}; \quad \mathcal{D}_\mu\phi^{\tilde{x}} \equiv \partial_\mu\phi^{\tilde{x}} + gA^I_\mu K^{\tilde{x}}_I(\phi).$$

- At attractor points scalars are constant, terms like

$$g^2 g_{\tilde{x}\tilde{y}} K^{\tilde{x}}_I K^{\tilde{y}}_J A^I_\mu A^{J\mu}$$

  act as effective mass term for the gauge field.

- Gauged sugras (not all) : from string theory via flux compactifications. (hope for string embedding!)

Our Work

- We extend the work of *Kachru et.al 11* to $\mathcal{N} = 2, d = 5$ gauged supergravity.
- We show that near horizon geometries of homogeneous extremal black branes *Iizuka et.al '11* are attractor solutions of gauged supergravity.
- Examples: Using a simple gauged sugra model of *Zagermann et.al 00*, we realise a $z = 3$ Lifshitz solution, a Bianchi Type II and a Bianchi Type VI solution as attractors.

Gauged Sugra and Generalised Attractors

Gauged Sugra: Symmetries

- The most general $\mathcal{N} = 2, d = 5$ gauged sugra has gravity coupled to vector, tensor and hypermultiplets *Dall'Agata 00*.
- The scalars in the theory parametrise a manifold that factorises into a direct product of a very special and quaternionic manifold,

$$\mathcal{M}_{scalar} = \mathcal{S}(n_v + n_T) \otimes \mathcal{Q}(n_H).$$

- The R symmetry group is $SU(2)_R$.

Gauged Sugra: Gauging the Symmetries

- Gauging: Suitable subgroup K of the isometry group G of the full scalar manifold $\mathcal{M}_{scalar}$, and the $SU(2)_R$ symmetry group.
- Ordinary derivatives on scalar and fermions are replaced with K-covariant derivatives.

$$\partial_\mu \phi^{\tilde{x}} \to \mathcal{D}_\mu \phi^{\tilde{x}} \equiv \partial_\mu \phi^{\tilde{x}} + g A^I_\mu K^{\tilde{x}}_I(\phi)$$
$$\partial_\mu q^X \to \mathcal{D}_\mu q^X \equiv \partial_\mu q^X + g A^I_\mu K^X_I(q)$$
$$\nabla_\mu B^M_{\nu\rho} \to \mathcal{D}_\mu B^M_{\nu\rho} \equiv \nabla_\mu B^M_{\nu\rho} + g A^I_\mu \Lambda^M_{IN} B^N_{\nu\rho},$$

- Gauging the $SU(2)_R$ Symmetry:

$$\nabla_\mu \psi_{\nu i} \to \nabla_\mu \psi_{\nu i} + g_R A^I_\mu {P_{Ii}}^j(q) \psi_{\nu j}.$$

Gauged Sugra: Lagrangian

The bosonic part of the five dimensional $\mathcal{N} = 2$ gauged supergravity *Dall'Agata 00*:

$$\begin{aligned}
\hat{e}^{-1}\mathcal{L}^{\mathcal{N}=2}_{Bosonic} = & -\frac{1}{2}R - \frac{1}{4}a_{\tilde{I}\tilde{J}}\mathcal{H}^{\tilde{I}}_{\mu\nu}\mathcal{H}^{\tilde{J}\mu\nu} - \frac{1}{2}g_{XY}\mathcal{D}_\mu q^X \mathcal{D}^\mu q^Y \\
& - \frac{1}{2}g_{\tilde{x}\tilde{y}}\mathcal{D}_\mu\phi^{\tilde{x}}\mathcal{D}^\mu\phi^{\tilde{y}} + \frac{\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\tau}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\tau \\
& + \frac{\hat{e}^{-1}}{4g}\epsilon^{\mu\nu\rho\sigma\tau}\Omega_{MN}B^M_{\mu\nu}\mathcal{D}_\rho B^N_{\sigma\tau} - \mathcal{V}(\phi, q).
\end{aligned}$$

$$\begin{aligned}
&\mathcal{H}^{\tilde{I}}_{\mu\nu} = (F^I_{\mu\nu}, B^M_{\mu\nu}), && \mu = 0, \ldots, 4 \\
&M = 1, \ldots, n_T, && I = 0, 1, \ldots, n_V \\
&\tilde{x} = 0, 1, \ldots n_v + n_T, && X = 1, 2, \ldots 4n_H.
\end{aligned}$$

Gauged Sugra: Potential and fermionic shifts

$$\mathcal{V}(\phi, q) = 2g^2 W^{\tilde{a}} W^{\tilde{a}} - g_R^2 [2P_{ij}P^{ij} - P_{ij}^{\tilde{a}} P^{\tilde{a}ij}] + 2g^2 \mathcal{N}_{iA}\mathcal{N}^{iA}$$

$$P_{ij} \equiv h^I P_{Iij}, \qquad P_{ij}^{\tilde{a}} \equiv h^{\tilde{a}I} P_{Iij}$$

$$W^{\tilde{a}} \equiv \frac{\sqrt{6}}{4} h^I K_I^{\tilde{x}} f_{\tilde{x}}^{\tilde{a}}, \qquad \mathcal{N}^{iA} \equiv \frac{\sqrt{6}}{4} h^I K_I^X f_X^{Ai}.$$

Bosonic part of supersymmetry transformations:

$$\delta_\epsilon \psi_{\mu i} = \sqrt{6}\nabla_\mu \epsilon_i + \frac{i}{4} h_{\tilde{I}} (\gamma_{\mu\nu\rho}\epsilon_i - 4g_{\mu\nu}\gamma_\rho \epsilon_i)\mathcal{H}^{\nu\rho\tilde{I}} + i g_R P_{ij} \gamma_\mu \epsilon^j$$

$$\delta_\epsilon \lambda_i^{\tilde{a}} = -\frac{i}{2} f_{\tilde{x}}^{\tilde{a}} \gamma^\mu \epsilon_i \mathcal{D}_\mu \phi^{\tilde{x}} + \frac{1}{4} h_{\tilde{I}}^{\tilde{a}} \gamma^{\mu\nu} \epsilon_i \mathcal{H}_{\mu\nu}^{\tilde{I}} + g_R P_{ij}^{\tilde{a}} \epsilon^j + g W^{\tilde{a}} \epsilon_i$$

$$\delta_\epsilon \zeta^A = -\frac{i}{2} f_{iX}^A \gamma^\mu \epsilon^i \mathcal{D}_\mu q^X + g\mathcal{N}_i^A \epsilon^i.$$

The potential can be written as squares of fermionic shifts.

Generalised Attractors: Definition

Ansatz:

- In tangent space, all the bosonic fields in the theory take constant values at the attractor points.

$$\phi^{\tilde{z}} = \mathrm{const}\ ; q^Z = \mathrm{const}\ ; A^I_a = \mathrm{const}\ ;$$
$$B^M_{ab} = \mathrm{const}\ ; {c_{bc}}^a = \mathrm{const}.$$

- The attractor geometries are characterised by constant anholonomy coefficients.

$$[e_a, e_b] = {c_{ab}}^c e_c\ ; \quad {c_{ab}}^c = e^\mu_a e^\nu_b (\partial_\mu e^c_\nu - \partial_\nu e^c_\mu)$$

Generalised Attractors: Features

- Gauge field, Tensor field and Einstein equations reduce to algebraic equations at the attractor point.
- Scalar field equations reduce to a minimisation condition on an attractor potential.
- The attractor potential is also independently constructed from squares of fermionic shifts.
- Constant anholonomy $\Rightarrow$ regular geometries.

Gauge field equation

- Since ${c_{ab}}^c = const$,

$$F_{ab} = e_a^\mu e_b^\nu (\partial_\mu e_\nu^c - \partial_\nu e_\mu^c) A_c = {c_{ab}}^c A_c$$

- The Gauge field equation of motion,

$$\partial_\mu(\hat{e} a_{I\tilde{J}} \mathcal{H}^{\tilde{J}\mu\nu}) = -\frac{1}{2\sqrt{6}} C_{I\tilde{J}\tilde{K}} \epsilon^{\nu\mu\rho\sigma\tau} \mathcal{H}^{\tilde{J}}_{\mu\rho} \mathcal{H}^{\tilde{K}}_{\sigma\tau} + g\hat{e}\big[g_{XY} K_I^X \mathcal{D}^\nu q^Y + g_{\tilde{x}\tilde{y}} K_I^{\tilde{x}} \mathcal{D}^\nu \phi^{\tilde{y}}\big]$$

in tangent space, is an algebraic equation at the attractor points

$$\hat{e}\ a_{I\tilde{J}}[{\omega_{a,}}^a{}_c \mathcal{H}^{cb\tilde{J}} + {\omega_{a,}}^b{}_c \mathcal{H}^{ac\tilde{J}}] = -\frac{1}{2\sqrt{6}} C_{I\tilde{J}\tilde{K}} \epsilon^{bacde} \mathcal{H}^{\tilde{J}}_{ac} \mathcal{H}^{\tilde{K}}_{de} + g^2 \hat{e}\big[g_{XY} K_I^X K_J^Y + g_{\tilde{x}\tilde{y}} K_I^{\tilde{x}} K_J^{\tilde{y}}\big] A^{Jb}.$$

Tensor field equation

- The tensor field equation is,

$$\frac{1}{g}\epsilon^{\mu\nu\rho\sigma\tau}\Omega_{MP}\mathcal{D}_{\rho}B^{M}_{\mu\nu} + \hat{e}a_{\tilde{I}P}\mathcal{H}^{\tilde{I}\sigma\tau} = 0.$$

- In tangent space,

$$\frac{1}{g}\epsilon^{abcde}\left[c_{ac}{}^{f}B^{M}_{fb} + gA^{I}_{c}\Lambda^{M}_{IN}B^{N}_{ab}\right]\Omega_{MP} + \hat{e}a_{\tilde{I}P}\mathcal{H}^{\tilde{I}de} = 0.$$

is an algebraic equation at the attractor points,

Einstein equation

- The Einstein equation at the attractor point:

$$R_{ab} - \frac{1}{2} R \eta_{ab} = T^{attr}_{ab}$$

- In the absence of torsion, The left handside is algebraic:

$$R_{abc}{}^{d} = \partial_a \omega_{bc}{}^{d} - \partial_b \omega_{ac}{}^{d} - \omega_{ac}{}^{e} \omega_{be}{}^{d} + \omega_{bc}{}^{e} \omega_{ae}{}^{d} - c_{ab}{}^{e} \omega_{ec}{}^{d}$$

$$\omega_{a,bc} = \frac{1}{2}[c_{ab,c} - c_{ac,b} - c_{bc,a}]$$

- The stress energy tensor at the attractor point:

$$T^{attr}_{ab} = \mathcal{V}_{attr}(\phi, q)\eta_{ab} - \Big[a_{\tilde{I}\tilde{J}} \mathcal{H}^{\tilde{I}}_{ac} \mathcal{H}_{b}{}^{c\tilde{J}} + g^2 [g_{XY} K^X_I K^Y_J + g_{\tilde{x}\tilde{y}} K^{\tilde{x}}_I K^{\tilde{y}}_J] A^I_a A^J_b \Big].$$

- The Einstein equations are algebraic at the attractor points.

Scalar equation

- The scalar $\phi^{\tilde{x}}$ field equations,

$$\hat{e}^{-1}\partial_\mu\left[\hat{e}\ g_{\tilde{z}\tilde{y}}\mathcal{D}^\mu\phi^{\tilde{y}}\right] - \frac{1}{2}\frac{\partial g_{\tilde{x}\tilde{y}}}{\partial\phi^{\tilde{z}}}\mathcal{D}_\mu\phi^{\tilde{x}}\mathcal{D}^\mu\phi^{\tilde{y}}$$

$$-gA^I_\mu g_{\tilde{x}\tilde{y}}\frac{\partial K^{\tilde{x}}_I}{\partial\phi^{\tilde{z}}}\mathcal{D}^\mu\phi^{\tilde{y}} - \frac{1}{4}\frac{\partial a_{\tilde{I}\tilde{J}}}{\partial\phi^{\tilde{z}}}\mathcal{H}^{\tilde{I}}_{\mu\nu}\mathcal{H}^{\tilde{J}\mu\nu} - \frac{\partial\mathcal{V}(\phi,q)}{\partial\phi^{\tilde{z}}} = 0.$$

- For the quaternion q^Z, the equation of motion is

$$\hat{e}^{-1}\partial_\mu\left[\hat{e}\ g_{ZY}\mathcal{D}^\mu q^Y\right] - \frac{1}{2}\frac{\partial g_{XY}}{\partial q^Z}\mathcal{D}_\mu q^X\mathcal{D}^\mu q^Y$$

$$-gA^I_\mu g_{XY}\frac{\partial K^X_I}{\partial q^Z}\mathcal{D}^\mu q^Y - \frac{\partial\mathcal{V}(\phi,q)}{\partial q^Z} = 0.$$

Scalar equations and Attractor potential

Using attractor ansatz,

- Equation of motion for $\phi^{\tilde{x}}$ reduces to,

$$\frac{\partial}{\partial \phi^{\tilde{z}}}\left[\mathcal{V}(\phi,q)+\frac{1}{2}g^2\, g_{\tilde{x}\tilde{y}}K_I^{\tilde{x}}K_J^{\tilde{y}}A^{Ia}A_a^J+\frac{1}{4}a_{\tilde{I}\tilde{J}}\mathcal{H}^{\tilde{I}}_{ab}\mathcal{H}^{\tilde{J}ab}\right]=0.$$

- Equation of motion for q^Z reduces to,

$$\frac{\partial}{\partial q^Z}\left[\mathcal{V}(\phi,q)+\frac{1}{2}g^2 g_{XY}K_I^X K_J^Y A^{aI}A_a^J\right]=0.$$

Attractor Potential

- Scalar field equations reduce to a minimisation condition on an attractor potential.

$$\mathcal{V}_{attr}(\phi, q) = \Big[\mathcal{V}(\phi, q) + \frac{1}{4} a_{\tilde{I}\tilde{J}} \mathcal{H}^{\tilde{I}}_{ab} \mathcal{H}^{\tilde{J}ab} + \frac{1}{2} g^2 [\, g_{\tilde{x}\tilde{y}} K^{\tilde{x}}_I K^{\tilde{y}}_J + g_{XY} K^X_I K^Y_J] A^{Ia} A^J_a \Big]$$

- The attractor potential gives rise to the attractor values of the scalars upon extremization.

Attractor Potential from fermion shifts

- Generalised Fermion shifts:

$$\Sigma^{\tilde{a}}{}_{i|j} = g_R P^{\tilde{a}}{}_{ij} - g W^{\tilde{a}} \epsilon_{ij} \quad ; \quad (\Sigma^A{}_{|j}) = g \mathcal{N}_j{}^A$$

$$(\Sigma^{\tilde{a}}{}_{i|j})^a = \frac{i}{2} g f^{\tilde{a}}_{\tilde{x}} K^{\tilde{x}}_I A^{Ia} \epsilon_{ij} \quad ; \quad (\Sigma^A{}_{|j})^a = -\frac{i}{2} f^A_{jX} K^X_I A^{aI}$$

$$(\Sigma^{\tilde{a}}{}_{i|j})^{ab} = -\frac{1}{4} h^{\tilde{a}}_{\tilde{I}} \mathcal{H}^{\tilde{I}ab} \epsilon_{ij} \quad ; \quad (\Sigma_{i|j})^{bc} = -\frac{i}{4} h_{\tilde{I}} \mathcal{H}^{bc\tilde{I}} \epsilon_{ij}$$

$$S_{ij} = i g_R P_{ij}$$

- Susy transformations at attractor points:

$$\delta\psi_{ai} = \sqrt{6} D_a \epsilon_i + (\Sigma_{i|j})^{bc} (\gamma_{abc} - 4\eta_{ab}\gamma_c) \epsilon^j + \gamma_a S_{ij} \epsilon^j$$

$$\delta\lambda^{\tilde{a}}_i = \Sigma^{\tilde{a}}{}_{i|j} \epsilon^j + (\Sigma^{\tilde{a}}{}_{i|j})^a \gamma_a \epsilon^j + (\Sigma^{\tilde{a}}{}_{i|j})^{ab} \gamma_{ab} \epsilon^j$$

$$\delta\zeta^A = (\Sigma^A{}_{|j}) \epsilon^j + (\Sigma^A{}_{|j})^a \gamma_a \epsilon^j$$

Attractor Potential from fermion shifts

- The attractor potential can be constructed independently from squares of fermionic shifts

$$-\mathcal{V}_{attr}\frac{\epsilon^{l}{}_{k}}{4} = \bar{S}^{i}{}_{k}S_{i}{}^{l} - \epsilon^{lj}\Big\{\big[(\overline{\Sigma^{A}{}_{|k}})(\Sigma_{A|j}) + \frac{1}{2}(\overline{\Sigma^{\tilde{a}i}{}_{|k}})(\Sigma^{\tilde{a}}{}_{i|j})\big] + \big[(\overline{\Sigma^{A}{}_{|k}})_{a}(\Sigma_{A|j})^{a} + \frac{1}{2}(\overline{\Sigma^{\tilde{a}i}{}_{|k}})_{a}(\Sigma^{\tilde{a}}{}_{i|j})^{a}\big] + \big[(\overline{\Sigma^{i}{}_{|k}})_{ab}(\Sigma_{i|j})^{ab} + (\overline{\Sigma^{\tilde{a}i}{}_{|k}})_{ab}(\Sigma^{\tilde{a}}{}_{i|j})^{ab}\big]\Big\},$$

which can be shown to reproduce,

$$\mathcal{V}_{attr}(\phi, q) = \Big[\mathcal{V}(\phi, q) + \frac{1}{4}a_{\tilde{I}\tilde{J}}\mathcal{H}^{\tilde{I}}_{ab}\mathcal{H}^{\tilde{J}ab} + \frac{1}{2}g^{2}\big[\, g_{\tilde{x}\tilde{y}}K^{\tilde{x}}_{I}K^{\tilde{y}}_{J} + g_{XY}K^{X}_{I}K^{Y}_{J}\big]A^{Ia}A^{J}_{a}\Big]$$

Killing spinor integrability conditions

- KSI expressible in terms of fermionic shifts. Defining $M_{abc} = \gamma_{abc} - 4\eta_{ab}\gamma_c$,

$$
\begin{aligned}
-\frac{1}{4}R_{ae}{}^{cd}\gamma_{cd}\epsilon_i = &-\frac{1}{\sqrt{6}}(\Sigma_{i|j})^{fc}[\omega_{a,}{}^b{}_f M_{e[bc]} - \omega_{e,}{}^b{}_f M_{a[bc]}]\epsilon^j \\
&+ \frac{1}{6}\Big\{[(\Sigma_{i|j})^{bc}M_{abc} + \gamma_a S_{ij}][(\Sigma_{k|l})^{gh}M_{egh} + \gamma_e S_{kl}] \\
&- [(\Sigma_{i|j})^{bc}M_{ebc} + \gamma_e S_{ij}][(\Sigma_{k|l})^{gh}M_{agh} + \gamma_a S_{kl}]\Big\}\epsilon^{jk}\epsilon^l
\end{aligned}
$$

- All shifts vanish $\Rightarrow$ Maximal supersymmetry (AdS_5 vacuum, unique). *Gauntlett '03*

$$
-\frac{1}{4}R_{ae}{}^{cd}\gamma_{cd}\epsilon_i = \frac{1}{6}S_{ij}S_{kl}\gamma_{ae}\epsilon^{jk}\epsilon^l
$$

Killing spinor integrability conditions

- Some shifts vanish $\Rightarrow$ partially broken supersymmetry (Lifshitz, Bianchi types)
- cases with only vector multiplets: Either $1/2$ BPS or $1/4$ BPS solutions. *Gauntlett '03*
- Lifshitz solutions: known to be $1/4$ BPS *Cassani '11*.
- We expect Bianchi attractors *Iizuka '11* to be $1/4$ BPS.

Examples

Homogeneity implies constant anholonomy

- Homogeneity implies Constant anholonomy and vice-versa *Ellis et.al 1969*.
- Consider homogeneous 5d spacetimes with spacelike hypersurfaces of dimension three.
- basis of Killing vectors that generate a simply transitive group of dimension three.

$$[\xi_\mu, \xi_\nu] = \tilde{C}_{\mu\nu}{}^{\lambda} \xi_\lambda.$$

- There exist unique invariant vector fields e_a that commute with the Killing vectors

$$[\xi_\mu, e_a] = 0.$$

Homogeneity implies constant anholonomy

- Jacobi identity between (e_a, ξ_μ, ξ_ν) imply $\tilde{C}_{\mu\nu}{}^{\lambda}$ are constants in spacetime.
- Jacobi identity between (e_b, e_a, ξ_μ) imply $c_{ab}{}^{c}$ are constants on the surface of transitivity.

$$[e_a, e_b] = c_{ab}{}^{c} e_c$$

- Bianchi connection: Algebra of invariant vectors isomorphic to real Lie Algebras of dimension 3. *Shepley*
- Bianchi classification: 9 types of Lie algebras
- Corresponding Lie groups generate homogeneous symmetries along the 3 space like directions of 5d black branes *Iizuka '11*.

Bianchi Attractors in a simple gauged sugra model

- For illustration, take a gauged supergravity model with one vector and two tensor multiplets *Gunaydin et.al 00*.
- Within this model, we realise a $z = 3$ Lifshitz solution, a Bianchi Type II and a Bianchi Type VI solution as attractors.
- The other Bianchi type metrics can be realised as attractor solutions of more generic gauged supergravity models.

Model dependent data

- Moduli space

$$G = SO(1,1) \times \frac{SO(2,1)}{SO(2)}.$$

- Metric on moduli space $g_{\tilde{x}\tilde{y}}$, $a_{\tilde{I}\tilde{J}}$.

- Gauging: $SO(2)$ subgroup of G using a single vector A^0 (graviphoton).

- R-Symmetry: $A_\mu[U(1)_R] = A^0_\mu V_0 + A^1_\mu V_1$

- derived data: Killing vector

$$K_0^{\tilde{x}} = \left\{ -\frac{\phi^1}{||\phi||^2}, \frac{\phi^2}{||\phi||^2}, \frac{\phi^3}{||\phi||^2} \right\}.$$

Model dependent data

- Potential:

$$\mathcal{V}(\phi) = \frac{g^2}{8}\left[\frac{[(\phi^2)^2 + (\phi^3)^2]}{||\phi||^6}\right] - 2g_R^2\left[2\sqrt{2}\frac{\phi^1}{||\phi||^2}V_0V_1 + ||\phi||^2V_1^2\right].$$

- Conditions for $\mathcal{N} = 2$ supersymmetry and AdS vaccum:

$$\phi^2 = 0, \quad \phi^3 = 0, \quad \phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}, \quad V_0V_1 > 0, \quad 32\frac{g_R^2}{g^2}V_0^2 \leq 1.$$

- potential evaluated at these values gives the AdS cosmological constant $\mathcal{V}_{AdS} = -6g_R^2(\phi_c^1)^2V_1^2$.

- Bianchi attractors exist for values of scalars for which the theory would also have an *AdS* Vacuum.

Methodology

- Take metric ansatz: Bianchi types,
- gauge field ansatz: time like gauge field

$$A^{0t} = e^{t}_{\bar{0}} A^{0\bar{0}} = \frac{\hat{r}^{-u}}{L} A^{0\bar{0}}$$

- Assume all tensor fields vanish!
- Run through the machinery and solve the algebraic field equations!

Bianchi Type I - Lifshitz

Bianchi Type I specified by gauging parameters g, V_0, V_1.

$$ds^2 = L^2\left[-\hat{r}^{2u}d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + \hat{r}^2(d\hat{x}^2 + d\hat{y}^2 + d\hat{z}^2)\right]$$

$$[e_2, e_4] = 0 \qquad [e_2, e_3] = 0 \qquad [e_2, e_4] = 0$$

$$u = 3; \quad A^{0t} = \frac{1}{L\hat{r}^u}\sqrt{\frac{2}{3}}\frac{1}{(\phi_c^1)^2}; \quad L = \sqrt{3}\frac{(\phi_c^1)^4}{g};$$

$$\phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}; \quad V_0 V_1 > 0; \qquad \frac{32}{3(\phi_c^1)^4} \leq 1.$$

Bianchi Type II

Bianchi Type II specified by gauging parameters g, V_0, V_1.

$$ds^2 = L^2\Big[-\hat{r}^{2u}d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + \hat{r}^{2w}d\hat{x}^2 + \hat{r}^{2(v+w)}d\hat{y}^2 - 2\hat{x}\hat{r}^{2(v+w)}d\hat{y}d\hat{x} + [\hat{r}^{2(v+w)}\hat{x}^2 + \hat{r}^{2v}]d\hat{z}^2\Big]$$

$$[e_2, e_3] = 0 \qquad [e_2, e_4] = 0 \qquad [e_3, e_4] = e_2$$

$$u = \sqrt{2}; \qquad v = w = \frac{1}{2\sqrt{2}};$$

$$L = \sqrt{\frac{2}{3}\frac{(\phi_c^1)^4}{g}}; \qquad A^{0t} = \frac{1}{L\hat{r}^u}\sqrt{\frac{5}{8}}\frac{1}{(\phi_c^1)^2};$$

$$\phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}; \quad V_0V_1 > 0; \quad \frac{23}{2(\phi_c^1)^4} \leq 1.$$

Bianchi Type VI

Bianchi Type VI specified by gauging parameters g, V_0, V_1 and h

$$ds^2 = L^2\left[-\hat{r}^{2u}d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + d\hat{x}^2 + e^{-2\hat{x}}\hat{r}^{2v}d\hat{y}^2 + e^{-2h\hat{x}}\hat{r}^{2w}d\hat{z}^2\right]$$

$$[e_2, e_4] = e_2 \qquad [e_3, e_4] = he_3$$

$$u = \frac{1}{\sqrt{2}}(1-h); \quad v = -\frac{1}{\sqrt{2}}h; \quad w = \frac{1}{\sqrt{2}}; \quad L = \frac{(\phi_c^1)^4}{\sqrt{6}g}(1-h);$$

$$A^{0t} = \frac{1}{L\hat{r}^u}\sqrt{\frac{-2h}{(-1+h)^2}\frac{1}{(\phi_c^1)^2}}; \quad h < 0; \quad h \neq 0, 1;$$

$$\phi_c^1 = \left(\sqrt{2}\frac{V_0}{V_1}\right)^{\frac{1}{3}}; \quad V_0 V_1 > 0; \quad \frac{8(3 - h + 3h^2)}{(\phi_c^1)^4(-1+h)^2} \leq 1$$

Summary

- We studied the generalised attractors in $\mathcal{N} = 2, d = 5$ gauged supergravity.
- Generalised attractors are defined by constant anholonomy, constant gauge fields, constant tensor fields and constant scalars at the attractor points.
- Gauge field, Tensor field and Einstein equations reduce to algebraic equations at the attractor point.
- Scalar field equations reduce to a minimisation condition on an attractor potential.
- The attractor potential is also independently constructed from squares of fermionic shifts.

Summary

- The attractor geometries are characterised by constant anholonomy coefficients.
- Homogeneity implies Constant anholonomy and vice-versa *Ellis et.al 1969*.
- We showed that near horizon geometries of homogeneous extremal black branes *Iizuka et.al '11* are attractor solutions of gauged supergravity.
- Examples: Using a simple gauged sugra model of *Gunaydin et.al 00*, we realise a $z = 3$ Lifshitz solution, a Bianchi Type II and a Bianchi Type VI solution as attractors.
- Other Bianchi type metrics can be realised as attractor solutions of more generic gauged supergravity models.

Comments and Caveats

- Toplogical terms: Chern-Simons, Tensor fields do not contribute.
- Bianchi type V and type III metrics do not seem to be valid attractors of the gauged supergravity considered here.
- Caution: Attractor equations, attractor geometries in black hole case exist at local minima of potential. Here they exist at critical points.
- Nevertheless, same terminology is used! Stability - has to be tested!.

Stability analysis: Preliminary results

- Fluctuation analysis: scalar field perturbation about critical value.

$$\phi_c^{\tilde{z}} + \delta\phi^{\tilde{z}}(r)$$

- For simplicity take the metric of the form:

$$ds^2 = L^2\left[-a(r)^2 dt^2 \frac{dr^2}{b(r)^2} + b(r)^2(dx^2 + dy^2 + dz^2)\right]$$

- metric captures near horizon geometries (eg Bianchi Type I, Type VII) and asymptotics AdS_5.

- Assume time like gauge fields, set all tensor fields to zero.

Stability analysis: Preliminary results

- Backreaction can be ignored upto first order perturbation.

- Scalar field equation upto first order is:

$$\frac{g_{\tilde{z}\tilde{y}}(\phi_c)}{L^2}\left[\frac{1}{ab^2}\partial_r(ab^4)\partial_r(\delta\phi^{\tilde{y}})+b^2\partial_r^2(\delta\phi^{\tilde{y}})\right]-M_{\tilde{z}\tilde{y}}\delta\phi^{\tilde{y}}=0$$

$$M_{\tilde{z}\tilde{y}}\equiv\left.\frac{\partial^2\mathcal{V}_{att}}{\partial\phi^{\tilde{z}}\partial\phi^{\tilde{y}}}\right|_{\phi^{\tilde{y}}=\phi_c^{\tilde{y}}}$$

- The above equation has well behaved solutions as long as $M_{\tilde{z}\tilde{y}}$ has positive eigenvalues!

- This is reminiscent of the condition obtained in *Goldstein '05* for Non-Supersymetric black hole attractors.

Stability analysis: Preliminary results

- For gauged sugra model *Gunaydin et.al 00*.
- Perturbation at asymptotic AdS

$$r^2\partial_r^2(\delta\phi^1) + 5r\partial_r(\delta\phi^1) + 4\delta\phi^1 = 0$$

- has well behaved solutions as $r \to \infty$

$$\delta\phi^1 \simeq \frac{C_1}{r^2} + C_2\frac{\log r}{r^2}$$

- Perturbation at the horizon (for eg Lifshitz) gives the equation:

$$r^2\partial_r^2(\delta\phi^1) + 7r\partial_r(\delta\phi^1) + 40\delta\phi^1 = 0$$

- has oscillatory solutions as $r \to 0$!

$$\delta\phi^1 \simeq C_1\frac{\sin(\sqrt{31}\log r)}{r^3} + C_2\frac{\cos(\sqrt{31}\log r)}{r^3}$$

Stability analysis: preliminary results

- It appears to us that the sign of $M_{\tilde{z}\tilde{y}}$ will always be negative for time like gauge fields.
- This would imply that purely electrically charged solutions are unstable.
- This could also be a consequence of the fact that the "critical" values of the scalars, correspond to a saddle point of the potential in this model.
- However, check for further dependencies (model dependent artifacts, higher order effects etc), yet to be done.

Future Outlook

- Necessary to find a good string theory/M-theory compactification of a suitable gauged supergravity for string embedding (ongoing).
- For Black holes the 4d and 5d attractors are related by a lift *Gaiotto et.al 05*, to explore a similar understanding in this case.
- CMT duals of Bianchi attractors - SLQL ?
- Can generalised attractors be understood from Entropy function formalism ? *Sen 07*.

Useful References:

- General gauged sugra reviews: *hep-th/9605032, Andrianopoli et.al*; *hep-th/0102114, Fre*
- Simple 5d gauged sugra models: *hep-th/9912027,hep-th/0004117, Gunaydin et.al*
- Generalised attractors: *1104.2884, Kachru et.al* ; *1206.3887, Inbasekar et.al*.
- Bianchi metrics: *Homogeneous Relativistic Cosmologies, Shepley*; *1201.4861, Iizuka et.al* .

Thank You!