Duality, Unitarity and recursions in the scattering of N=2 CS matter theory

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Motivation: Bosonization duality

- 2+1 d bosonization duality in U(N) Chern-Simons matter theories

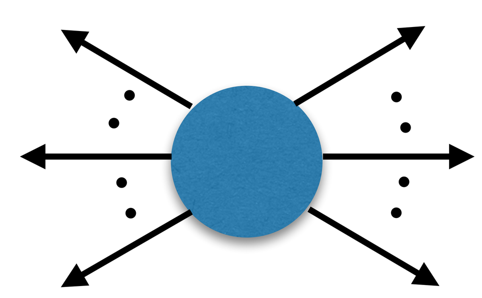
$$U(N)\ \text{CS+fundamental boson at Wilson Fisher limit}$$
$$\Leftarrow \text{dual} \Rightarrow$$
$$U(N)\ \text{CS+fundamental fermion}$$

Aharony, Giombi, Giveon, Gur-Ari, Gurucharan, Kutasov, Jain, Maldacena, Minwalla, Prakash, Yacoby, Yin, Yokoyama, Wadia, Zhiboedov

- Plenty of tests: Spectrum of single trace primaries, Three point functions, Thermal partition functions, 2 → 2 S matrices match under duality.
- All tests performed in the large N, large κ limit, 't Hooft coupling: $\lambda = \frac{N}{\kappa}$
- Strong-weak nature of the duality is difficult to test.
- Computations of 2 → 2 S matrices already yielded plenty of surprises: Anyonic effects that require modification of usual crossing rules to preserve unitarity!

Aim

- To compute to n → n planar scattering amplitudes in Chern-Simons matter theories.



- To provide evidence for the bosonization duality.

Background

2 → 2 Scattering, Duality, and unitarity in CS matter theories

- Channels of scattering: $R_1 \otimes R_2 = \sum_m R_m$

$$\text{Fundamental} \otimes \text{Fundamental} \to \text{Symm}(U_d) \oplus \text{Asymm}(U_e)$$
$$\text{Fundamental} \otimes \text{Antifundamental} \to \text{Adjoint}(T) \oplus \text{Singlet}(S)$$
$$T_{Asym} \sim T_{Sym} \sim T_{Adj} \sim O\left(\frac{1}{N}\right)\ ,\ T_{sing} \sim O(1)$$

- Anyonic phase in the m'th channel of scattering

$$\nu_m = \frac{4\pi}{\kappa} T_1^a T_2^a = \frac{2\pi}{\kappa}\left(C_2(R_1) + C_2(R_2) - C_2(R_m)\right)$$
$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right)\ , \nu_{Sing} \sim O(\lambda)$$

- The singlet channel is effectively anyonic in the large N ,large κ limit.
- Observation: Naive crossing rules from any of the non-anyonic channels to the singlet channel leads to a non unitary S matrix.
- Conjecture: Singlet channel S matrices have the form

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$

Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama

- Delta function and modified crossing rules appear to be universal.
- Conjectured S matrix obeys duality, unitarity and reduces to Aharanov-Bohm scattering in non-relativistic limit, in all the examples considered so far.

N=2 supersymmetric Chern-Simons matter theory

- Renormalizable $\mathcal{N}=2$ Chern-Simons coupled to fundamental matter in U(N)

$$\mathcal{S}^L_{\mathcal{N}=2} = \int d^3x\left[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\text{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right) + \bar\psi(i\not{D} + m_0)\psi - \mathcal{D}^\mu\bar\phi\mathcal{D}_\mu\phi - m_0^2\bar\phi\phi\right.$$
$$\left. - \frac{4\pi m_0}{\kappa}(\bar\phi\phi)^2 - \frac{4\pi}{\kappa^2}(\bar\phi\phi)^3 + \frac{4\pi}{\kappa}(\bar\phi\phi)(\bar\psi\psi) + \frac{2\pi}{\kappa}(\bar\psi\phi)(\bar\phi\psi)\right]$$

- Has Integer parameters N, κ and exhibits a strong-weak self-duality under the map.

$$\lambda' = \lambda - \text{Sgn}(\lambda)\ ,\ N' = |\kappa| - N + 1\ ,\ \kappa' = -\kappa\ , m' = -m$$

Aharony, Gur-Ari, Yacoby ;Jain, Minwalla, Yokoyama

- In the symmetric, anti-symmetric and adjoint channels of scattering, the 2 → 2 S matrices are tree level exact to all orders in λ in the planar limit.

$$\mathcal{T}_B^{\mathcal{N}=2} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} - \frac{8\pi m}{\kappa}\ ,$$
$$\mathcal{T}_F^{\mathcal{N}=2} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} + \frac{8\pi m}{\kappa}$$

- Duality is straightforward! Unitarity guaranteed by hermiticity.
- The singlet channel S matrix continues to be simple, but not tree level exact!

$$\mathcal{T}_B^{S;\mathcal{N}=2}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda) - 1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2) - 8m)\ ,$$
$$\mathcal{T}_F^{S;\mathcal{N}=2}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda) - 1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2) + 8m).$$

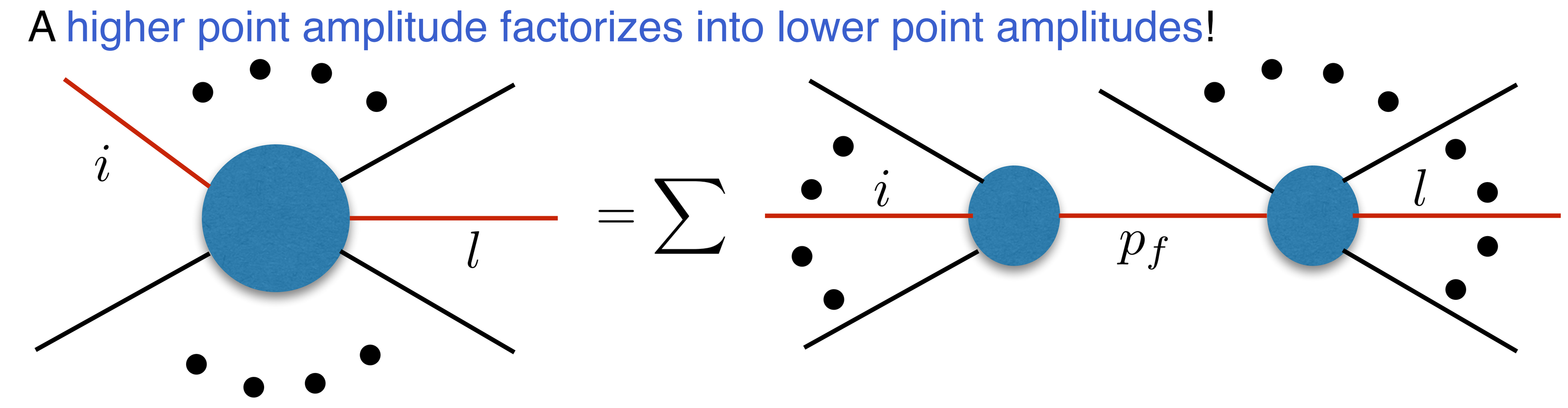
K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama

- Naive crossing would have implied that the singlet channel is also tree level exact!! But if this is true $i(T^\dagger - T) = TT^\dagger$ would never be obeyed!! $(T_{Sing} \sim O(1))$

Method

BCFW recursion relations in 2+1 dimensions

- Recursion relations enable to construct n point tree level scattering amplitudes from lower point tree level amplitudes. Britto, Cachazo, Feng, Witten
- Basic idea: consider the analytic properties of the amplitude as a complex function by shifting the external momenta. $A(z) = A(p_1, \ldots p_i(z), p_{i+1}, \ldots p_l(z), \ldots, p_{2n})$
- The necessary and sufficient conditions are:
  - The momentum shift preserves on-shell conditions and momentum conservation. In 2+1 dimensions this shift is non-linear in z.
  - The amplitude is asymptotically well behaved $A(z) \to 0$ as $z \to \infty$.
- A higher point amplitude factorizes into lower point amplitudes!



Recursion formula

Non-renormalization of the 2n point amplitude

- For particle only scattering: one does not need to worry about anyonic effects and crossing in the planar limit.
- Since the 2 → 2 amplitude is not renormalized in this sector, it is reasonable to expect that the n →n amplitude is also not renormalized.
- In the large N, large κ limit the off-shell 2n point function is made of blocks of off-shell four point functions. The on-shell limit gives the n →n point scattering amplitude.
- We explicitly computed the all loop 3→3 amplitude this way (for particle only scattering) and it is tree level exact as expected.

BCFW conditions for the N=2 theory

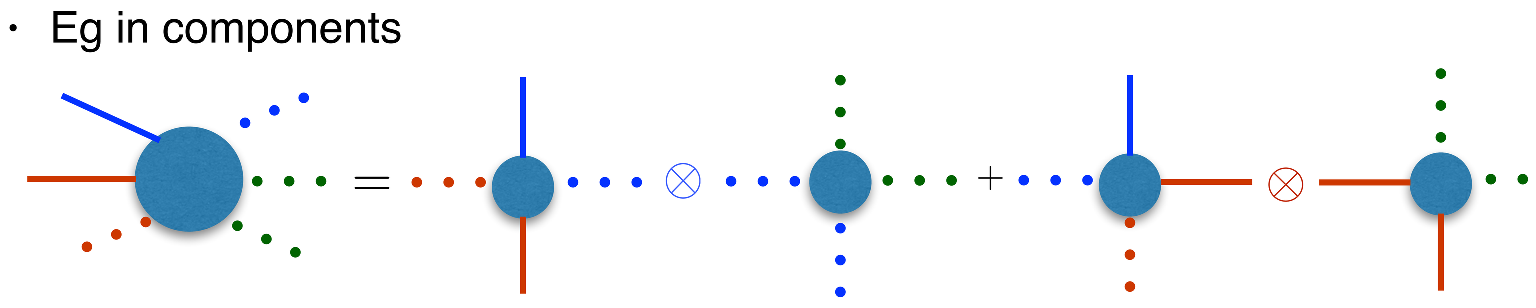
- Onshell susy methods, encode the component amplitudes into a superamplitude.
- Susy ward identities relate various component amplitudes and reduce the number of independent amplitudes. Eg 2 → 2 is characterized by one independent function. 3 → 3 is characterized by 2 functions….. n → n is characterized by n-1 functions.
- Susy also ensures that if the independent component amplitudes are well behaved then the entire superamplitude is well behaved. In practice it is sufficient to check asymptotic behavior of one component amplitude.
- Using two independent methods we showed that the superamplitude is well behaved
  - Background field expansion.
  - Explicit Feynman diagram computation of component amplitudes.

Recursion for six point amplitude and factorization channels

- The recursion formula for the six point amplitude for eg takes the form

$$A_6(\Lambda_1\ldots,\Lambda_6) = \sum_f \sum_{z_f = z_{f_i}} \int d\eta \frac{A_4^L(\Lambda_1,\ldots,\Lambda_f,z_f)A_4^R(\Lambda_f,\ldots,\Lambda_6,z_f)}{p_f^2}$$

- Eg in components



Recursion formula for the 2n point superamplitude

- Schematically the recursion relation for a general 2n point amplitude is a factorization in terms of 4 point amplitudes

$$A_{2n}(\Lambda_1\ldots,\Lambda_{2n}) = \sum_{f_1}\ldots\sum_{f_{n-2}}\sum_{z_{f_1}=z^i_{f_1}}\ldots\sum_{z_{f_{n-2}}=z^i_{f_{n-2}}}\int d\eta_1\ldots d\eta_{n-2}$$
$$\frac{A_4(\Lambda_1,\ldots,\Lambda_{f_1},z_{f_1},\eta_1)A_4(\Lambda_{f_1},\ldots,\Lambda_{f_2},z_{f_1},z_{f_2},\eta_2)\ldots A_4(\Lambda_{f_{n-2}},\ldots,\Lambda_{2n},z_{f_1}\ldots,z_{f_{n-2}})}{p_{f_1}^2\ldots p_{f_{n-2}}^2}$$

Results, Conclusions and Future directions

- Using BCFW recursions, we constructed n → n amplitudes in terms of 2 → 2 amplitudes in N=2 Chern-Simons matter theories.
- Since the 2 → 2 amplitudes are tree level exact, the n → n amplitudes are also tree level exact to all orders in the 't Hooft coupling (In the particle only sector).
- We have shown that duality invariance of the four point amplitude implies duality invariance of the 2n point amplitude - maximum possible test so far in planar limit!
- Unitarity: follows from hermiticity exactly as in 2 → 2 scattering.
- Our results extend to tree level diagrams of non-supersymmetric Chern-Simons theories coupled to fermions.
- For the anyonic channels, the factorization result holds true for tree level amplitudes.
- Work in Progress: Duality, unitarity and recursions for the all loop anyonic channels?