All tree level amplitudes in Chern-Simons theories with fundamental matter

Karthik Inbasekar

TEL AVIV UNIVERSITY אוניברסיטת תל אביב

אוניברסיטת בן-גוריון בנגב
Ben-Gurion University of the Negev

June 1, 2019

University of Connecticut

Summer school on strong interactions beyond simple factorization:
Collectivity at high energy from initial to final state.

Based on

All tree level scattering amplitudes in Chern-Simons theories with fundamental matter

K.I, Jain, Nayak, Umesh Phys.Rev.Lett. 121 (2018) no.16, 161601

Recursion relations: Why?

Feynman diagrams required grow with the number of particles involved.

eg in QCD gluon amplitudes

	Feynman Diagrams
$g+g \to g+g$	4
$g+g \to g+g+g$	25
$g+g \to g+g+g+g$	220

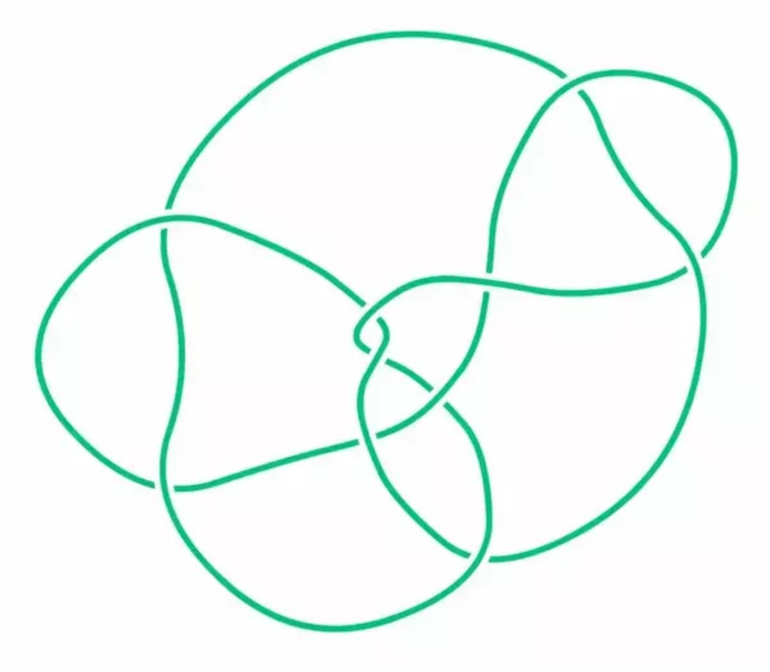
Recursion relations are efficient methods to compute amplitudes in cases where conventional Feynman diagram approaches become cumbersome.

One such method is **BCFW (Britto-Cachazo-Feng-Witten)** recursion.

Mostly applicable for scattering amplitudes at tree level.

In this talk we will apply recursion methods to **arbitrary n point tree level amplitudes in Chern-Simons matter theories in 2+1 dimensions**.

Chern-Simons matter theories



"Anyone who knows anyons cannot describe them.
Anyone who can describe anyons does not know them."

Chern-Simons theories

Physics in 2+1 dimensions has interesting features and intriguing surprises.

There exists a new type of gauge theory completely different from the usual Maxwell theory called **Chern-Simons theory**.

The Pontryagin density in 3+1 dimensions can be written as a total derivative

Raju's Talk

$$\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma}) = 4\partial_\sigma(\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(A_\mu\partial_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho)$$

The boundary term has the same form as the Chern-Simons Lagrangian in 2+1 dimensions.

Pure Chern-Simons theory is **topological**. Source free classical equations of motion

$$L = \frac{k}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$$

$$\frac{k}{2}\epsilon^{\mu\nu\rho}F_{\nu\rho} = 0$$

Solutions in the free theory are **pure gauge!**

Chern-Simons matter theories: Aharanov-Bohm phase

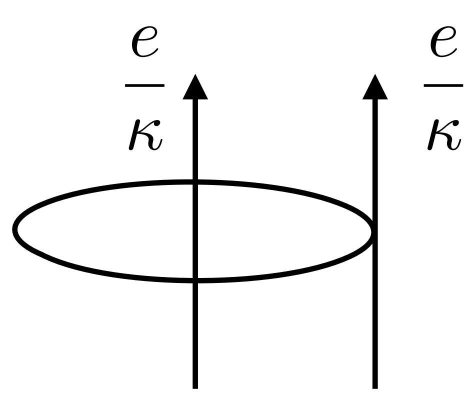
When **coupled to charged matter**

$$\frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - A_\mu J^\mu_{matter}$$

the **CS gauge field attaches magnetic fluxes to the particles**

$$\rho = \kappa B \ , \ J^i = \kappa\epsilon^{ij}E_j$$

Adiabatic excursion of such particles leads to the **Aharanov-Bohm effect**



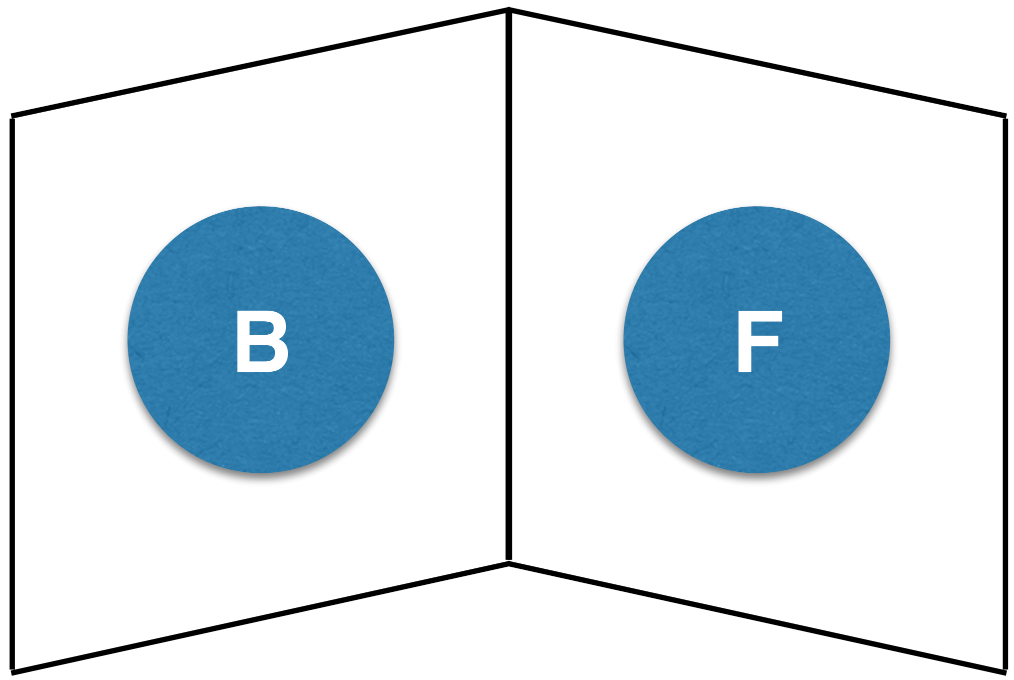
$$\text{AB phase} = e^{ie\int A.dx} = e^{i\frac{e^2}{\kappa}}$$

The phase: interpreted as **point particle explanation of anyonic statistics.**

This effect **modifies traditional crossing symmetry** QFT

Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama, JHEP 1510 (2015) 176

Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama, JHEP 1504 (2015) 129



Bosonization duality

Aharony,Bardeen,Benini,Chang,Frishman,Giombi,Giveon,Gur-Ari,Gurucharan,K.I,
Karch,Kutasov,Jain,Maldacena,Mandlik,Minwalla,Moshe,Sharma,Prakash,Takimi,Trivedi,
Seiberg,Sonnenschein,Tong,Yacoby,Yin,Yokoyama,Wadia,Witten,Zhiboedov

Chern-Simons coupled to fermions

$U(N_F)$ Chern-Simons coupled to fundamental fermions (regular fermion)

$$S = \int d^3x \left(i\epsilon^{\mu\nu\rho} \frac{\kappa_F}{4\pi} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) + \bar{\psi}^i \gamma^\mu D_\mu \psi_i + m_f \bar{\psi}^i \psi_i \right)$$

Complex fermion $\psi_i \longrightarrow$ Transforms as a vector under $U(N_F)$

(Fundamental representation)

$\bar{\psi}^i \longrightarrow$ **(Anti-Fundamental representation)**

Gauge field $A^a (T^a)_i^{\ j} \longrightarrow$ **(Adjoint representation)**

For eg $\kappa = 1, N_F = 1$ describes a topological insulator. $\kappa = 0, N_F = 1$ describes a Dirac metal. Also relevant for models of Quantum Hall Effect.

Son, Senthil, Melitsky, Vishwanath

Rob's talk

Chern-Simons coupled to bosons

$U(N_B)$ Chern-Simons coupled to **fundamental bosons**

$$S = \int d^3x \Big(i\epsilon^{\mu\nu\rho}\frac{\kappa_B}{4\pi}\mathrm{Tr}(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho) + D_\mu\bar{\phi}D^\mu\phi$$
$$+\sigma\bar{\phi}\phi + N_B\frac{m_B^2}{b_4}\sigma - N_B\frac{\sigma^2}{2b_4}\Big)$$

Complex scalar $\phi_i \longrightarrow$ Transforms as a vector under $U(N_B)$ **(Fundamental representation)**

$\bar{\phi}^i \longrightarrow$ **(Anti-Fundamental representation)**

Gauge field $A^a(T^a)_i^{\ j} \longrightarrow$ **(Adjoint representation)**

$\sigma \longrightarrow$ **(Auxiliary field)**

Wilson-Fisher limit (critical boson), strongly coupled

$$b_4 \to \infty\ ,\ m_B \to \infty\ ,\ 4\pi\frac{m_B^2}{b_4} = \text{fixed}$$

Large N bosonization duality

t' Hooft large N limit $\lambda_B = \lim_{\substack{N_B \to \infty \\ \kappa_B \to \infty}} \frac{N_B}{\kappa_B}$ $\lambda_F = \lim_{\substack{N_F \to \infty \\ \kappa_F \to \infty}} \frac{N_F}{\kappa_F}$

$U(N_B)$ Chern-Simons coupled to fundamental bosons at Wilson-Fisher limit

dual

$U(N_F)$ Chern-Simons coupled to fundamental Fermions

duality map
$$\begin{cases} \kappa_F = -\kappa_B \\ N_F = |\kappa_B| - N_B \\ \lambda_B = \lambda_F - \mathrm{Sign}(\lambda_F) \\ m_F = -m_B^{Crit} \lambda_B \end{cases}$$

There is a conjectured/untested finite N and k duality map

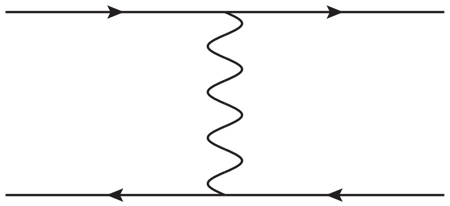
Physical observables computed on one side, match with observables on the other side under the duality map.

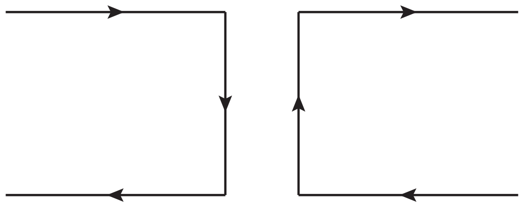
Strong-weak duality: Observables need to be computed to all loops!

S matrix at large N,k

Yuri's Talk

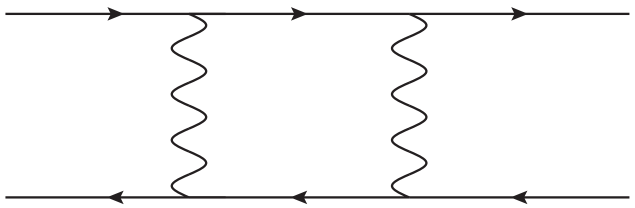
Tree

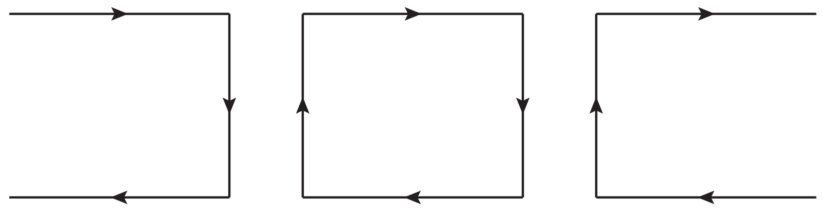




$$\frac{1}{\kappa} = \frac{\lambda}{N} \qquad \mathcal{O}\left(\frac{1}{N}\right)$$

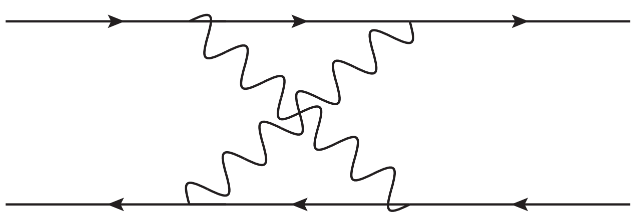
one loop (planar)

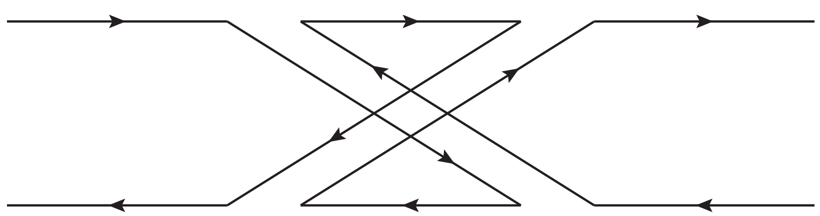




$$\frac{N}{\kappa^2} = \frac{\lambda^2}{N} \qquad \mathcal{O}\left(\frac{1}{N}\right)$$

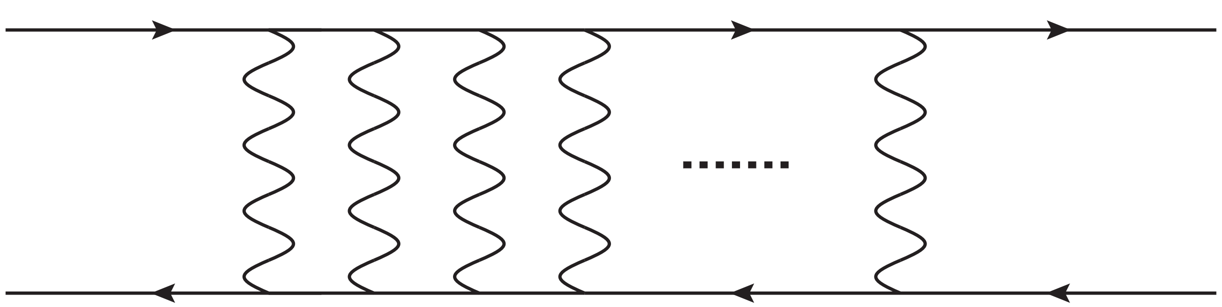
one loop (non planar)





$$\frac{1}{\kappa^2} = \frac{\lambda^2}{N^2} \qquad \mathcal{O}\left(\frac{1}{N^2}\right)$$

All loops (planar)



$$\frac{N^l}{k^{l+1}} = \frac{\lambda^{l+1}}{N} \qquad \mathcal{O}\left(\frac{1}{N}\right)$$

Our work

It is possible to compute four point amplitudes exactly to all loops in the planar limit.

Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama, JHEP 1510 (2015) 176

Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama, JHEP 1504 (2015) 129

However, it is a formidable task to repeat the procedure for higher point amplitudes!

Such a computation would help test the bosonization duality at higher point amplitudes.

Understanding higher point amplitudes is also key to check duality beyond the planar limit.

First: **construct higher point tree amplitudes** before going to loops.

We compute arbitrary n-point tree level amplitudes in Chern-Simons theories coupled to fermions/bosons using BCFW recursions.

Our work

However: The non-supersymmetric Chern-Simons theories coupled to bosons/fermions **do not satisfy the necessary conditions** required for BCFW factorisation.

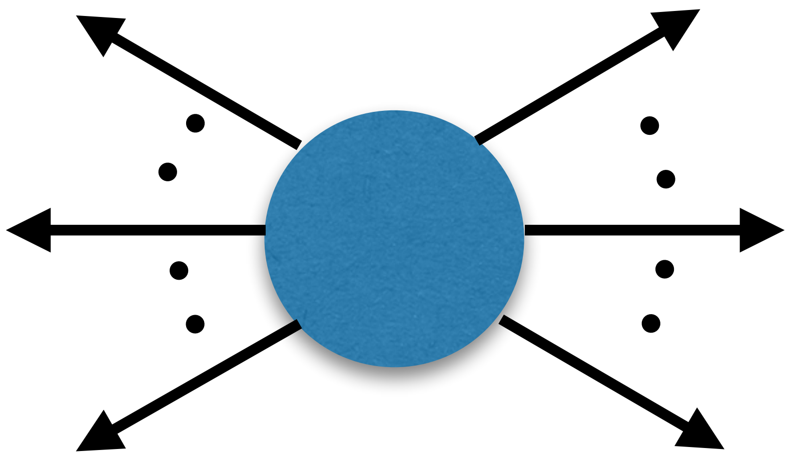
What we will do is to **derive the recursion relations for N=2 supersymmetric Chern-Simons matter theories that obey BCFW requirements**.

Then use the fact, that **at tree level purely bosonic/fermionic amplitudes in the supersymmetric theory are identical to their non-supersymmetric counterparts**.

The task then reduces to extracting relevant component amplitudes from a superamplitude constructed using BCFW.

Since we are at tree level, these are valid for any N and k.

All tree level amplitudes



Inbasekar, Jain, Nayak, Umesh Phys.Rev.Lett. 121 (2018) no.16, 161601

“Spinor helicity” variables in 2+1 dimension

Spinor helicity variables are efficient methods for writing onshell quantities, without the need to find explicit representations!

Consider the momentum in 2+1 dimension $p_\mu \sigma^\mu_{ab} = p_{ab} = \begin{pmatrix} -p_0 + p_3 & p_1 \\ p_1 & -p_0 - p_3 \end{pmatrix}$

real and symmetric

The “onshell” condition for a massless particle $det(p_{ab}) = -(-p_0^2 + p_1^2 + p_3^2) = 0$

Two real degrees of freedom, can be encoded in a purely real or purely imaginary two component commuting variable $\lambda^i_a \equiv |i\rangle$

In four dimensions there are Weyl spinors, and the formalism requires two of them $\lambda^i_a \equiv |i\rangle, \tilde{\lambda}^i_{\dot{a}} \equiv |i]$

Any onshell momentum can be represented by

$$p^i_{ab} = |i\rangle|i\rangle \equiv \lambda^i_a \lambda^i_b$$

$$\det(p^i_{ab}) = \epsilon^{ab}\lambda^i_a \lambda^i_b = \langle ii\rangle = 0$$

Mandelstam variables

$$s_{ij} = (p_i + p_j)^2 = 2p_i \, . \, p_j = \langle ij\rangle^2$$

Supersymmetric Chern-Simons matter theory

$d=3\,,\mathcal{N}=2$ susy Chern-Simons theory coupled to matter in fundamental representation of U(N)

$$\mathcal{S}^{L}_{\mathcal{N}=2}=\int d^3x\Big[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho-\frac{2i}{3}A_\mu A_\nu A_\rho\right)$$
$$+\bar{\psi}i\not{\mathcal{D}}\psi-\mathcal{D}^\mu\bar{\phi}\mathcal{D}_\mu\phi+\frac{4\pi^2}{\kappa^2}(\bar{\phi}\phi)^3+\frac{4\pi}{\kappa}(\bar{\phi}\phi)(\bar{\psi}\psi)+\frac{2\pi}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi)\Big]$$

Susy encodes various component amplitudes into a superamplitude.

boson

fermion

$$A_i(\mathbf{p})=a_i(\mathbf{p})+\eta_i\alpha_i(\mathbf{p})$$
$$A_i^\dagger(\mathbf{p})=a_i^\dagger(\mathbf{p})\eta_i+\alpha_i^\dagger(\mathbf{p})$$

Formally the superamplitude is

$$S_{2n}(\mathbf{p}_1,\ldots,\mathbf{p}_{2n},\eta_1,\ldots,\eta_{2n})=\langle 0|A_{2n}\ldots A_{n+1}A_n^\dagger\ldots A_1^\dagger|0\rangle$$

Symmetries constrain the general kinematic structure of the superamplitude.

Supersymmetric Chern-Simons matter theory

eg Kinematics of four point amplitude is fixed in terms of one function of momentum and couplings.

$$S_4 = f(\mathbf{p}_i, \lambda_i)\left(\sum_{i<j=1}^{4} \langle ij\rangle \eta_i \eta_j\right)\delta\left(\sum_{i=1}^{4} p_i\right)$$

Six point amplitude is fixed in terms of two functions

$$S_6 = \left(f_1(\mathbf{p}_i, \lambda_i)\sum_{a=1}^{3} \epsilon^{abc}\langle ab\rangle \eta_c + f_2(\mathbf{p}_i, \lambda_i)\sum_{m=4}^{6} \epsilon^{def}\langle de\rangle \eta_f\right)\left(\sum_{i<j=1}^{6} \langle ij\rangle \eta_i \eta_j\right)\delta\left(\sum_{i=1}^{6} p_i\right)$$

The unknown function is computed using Feynman diagrams.

$$T_4 = \frac{4\pi i}{\kappa}\frac{\langle 12\rangle}{\langle 23\rangle}\left(\sum_{i<j=1}^{4} \langle ij\rangle \eta_i \eta_j\right)\delta\left(\sum_{i=1}^{4} p_i\right)$$

However the six and higher point amplitudes require many more diagrams, We will derive recursive relations.

BCFW recursions

Recursion relations enable to construct n point tree level scattering amplitudes from lower point tree level amplitudes.

Britto, Cachazo, Feng, Witten

Central idea:

Dixon

Tree amplitudes: continuously deformable **analytic functions of momenta**.

Only type of singularities that appear at tree level are **simple poles**.

One can reconstruct amplitudes for generic scattering kinematics knowing its behaviour in singular kinematics. **Complex analysis and analytic functions**

In these singular regions amplitudes factorize into causally disconnected amplitudes with fewer legs, connected by an intermediate onshell state.

We will focus on situation where all the external particles are massless.

Onshell methods are not so well developed for massive cases, but it is possible to write recursions in the massive cases as well.

BCFW recursions in 2+1 dimensions

Promote the amplitude into a one complex parameter family of amplitudes

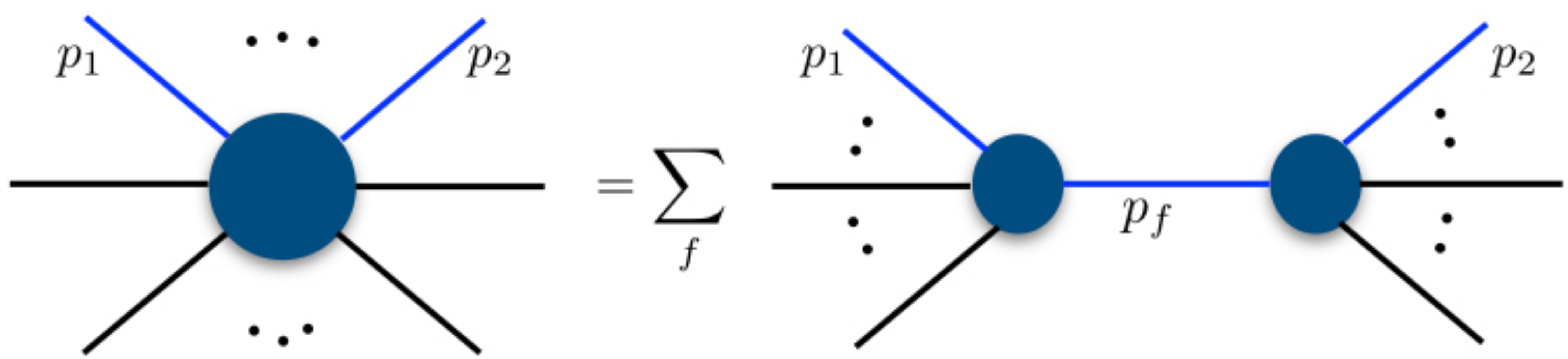
$$A(z) = A(p_1, \ldots p_i(z), p_{i+1}, \ldots p_l(z), \ldots, p_{2n})$$

The necessary and sufficient conditions are:

Britto, Cachazo, Feng, Witten

All deformed momenta preserve on-shell conditions, and momentum is conserved.

The amplitude should be asymptotically well behaved under the deformation.



A higher point amplitude factorizes into lower point amplitudes!

Recursion formula for an arbitrary 2n point amplitude

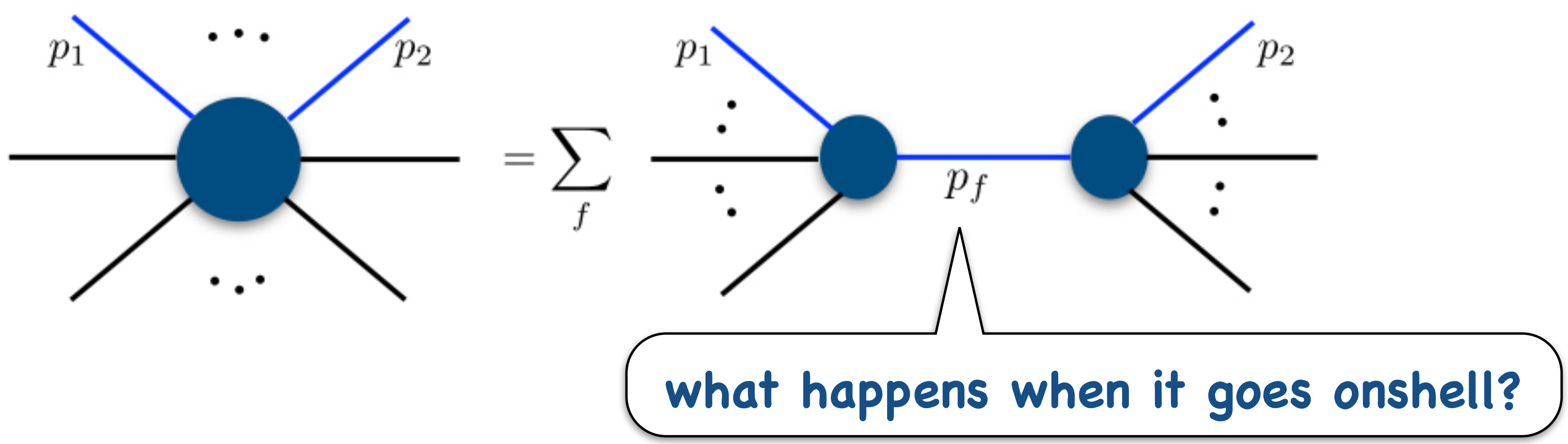
Write a contour integral representation for the amplitude

$$\frac{1}{2\pi i}\oint_{C_{z=1}} \frac{dz}{z-1} A(z)$$

Deform the contour to z→∞, If A(z) has no poles at infinity, the integral vanishes

$$A(z=1) = -\sum_{\text{poles}:z^i} \text{Res}_{z=z^i} \frac{A(z)}{z-1}$$

Cauchy residue theorem



Most general form of A(z)

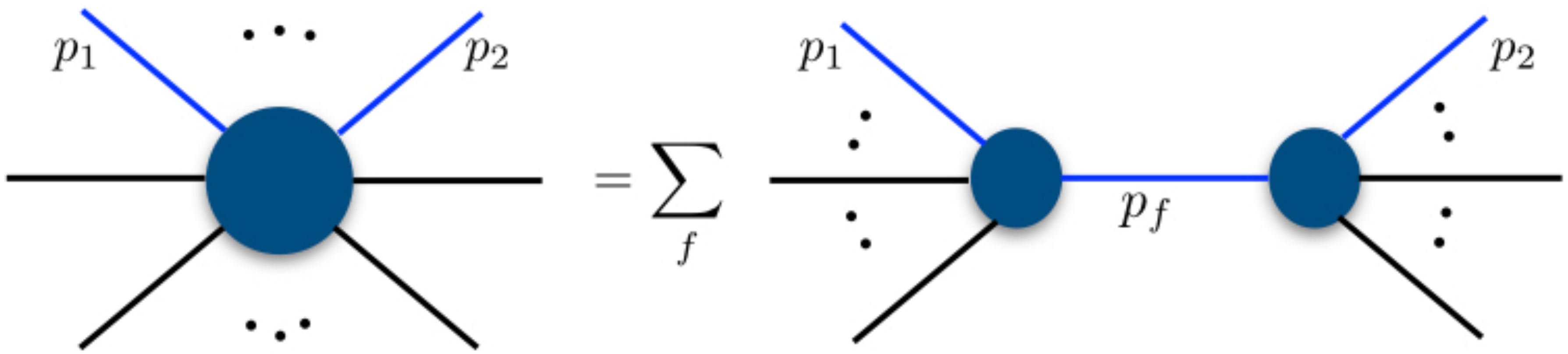
$$A(z) = \frac{A_L(z) A_R(z)}{p_f(z)^2}$$

remember

all the deformed momenta satisfy the onshell conditions!

Only singularities that can appear are simple poles!

Recursion formula for an arbitrary 2n point amplitude



$$A(z=1) = -\sum_f \sum_{\text{poles}: z_f^i} \mathrm{Res}_{z=z_f^i} \frac{1}{z-1} \frac{A_L(p_1 \ldots p_i(z), \ldots p_n) A_R(p_{n+1} \ldots p_j(z), \ldots p_{2n})}{\hat{p}_f^2(z)}$$

For the N=2 supersymmetric Chern-Simons theory the formula takes the explicit form

$$A_{2n}(z=1) = \sum_f \int \frac{d\theta}{p_f^2} \Bigg(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} A_L(z_{a;f}, \theta) A_R(z_{a;f}, i\theta) \\ + (z_{a;f} \leftrightarrow z_{b;f}) \Bigg)$$

Necessary and sufficient conditions for BCFW

Onshell susy methods, encode the component amplitudes into a superamplitude.

Susy ward identities relate various component amplitudes and reduce the number of independent amplitudes.

Susy also ensures that if the independent component amplitudes are well behaved then the entire superamplitude is well behaved.

Using two independent methods we showed that the superamplitude in the N=2 theory is **well behaved asymptotically**

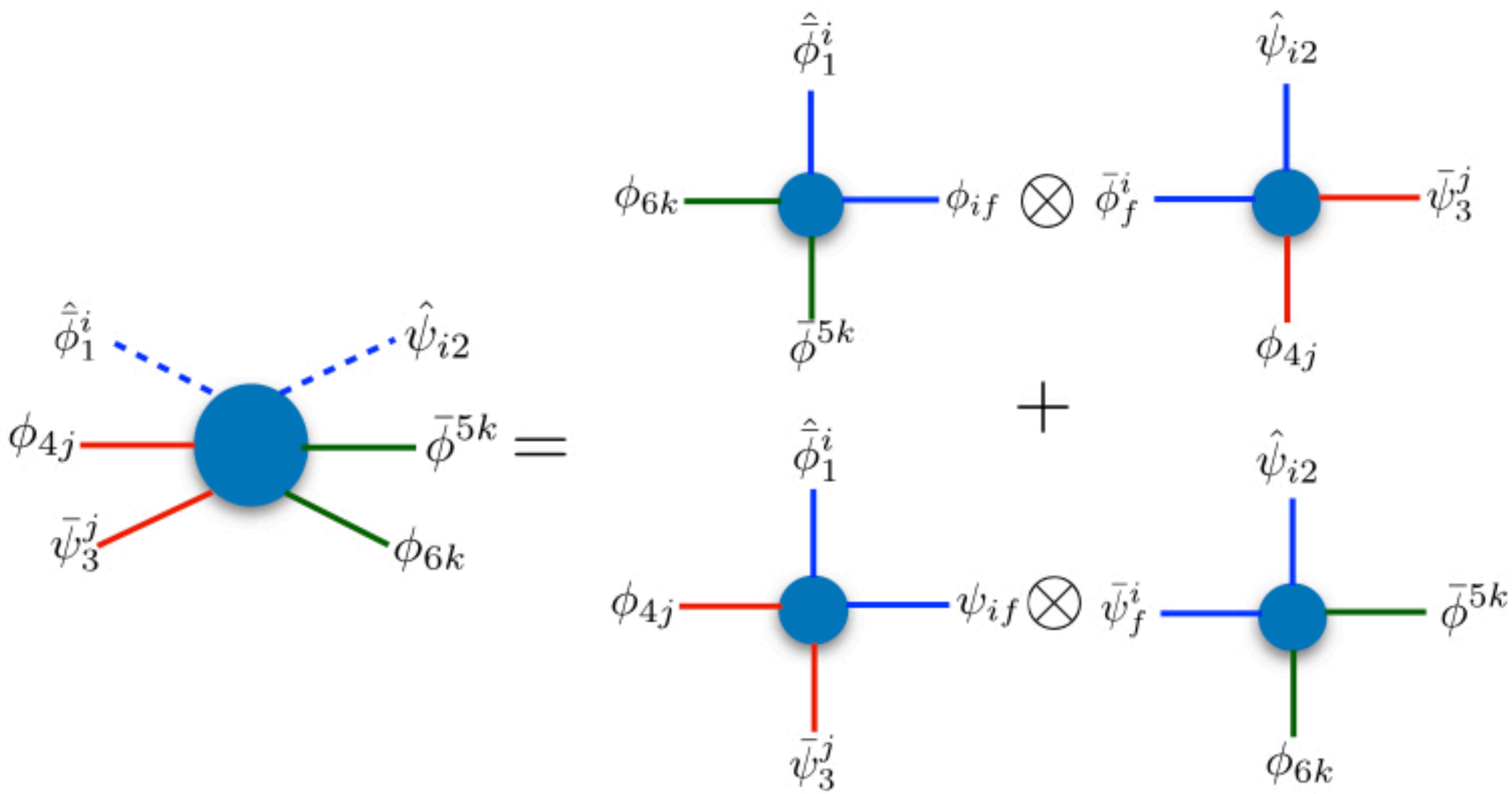
Background field expansion. Arkani-Hamed, Kaplan

z **Deformation of divergent Feynman diagrams** of component amplitudes. (tedious, but helps do numerical checks for any given higher point amplitude).

The recursion formula then follows from Cauchy residue theorem.

eg: six point amplitude as product of four point amplitudes

$$\langle\bar{\phi}_1\psi_2\bar{\psi}_3\phi_4\bar{\phi}_5\phi_6\rangle = \left(\frac{32\pi^2 i}{\kappa^2}\right)\Big[\frac{\langle 2|p_4|3\rangle p_{12}\cdot p_{56} - \langle 2|p_1|3\rangle p_{34}\cdot p_{56}}{p_{256}^2 p_{124}^2}$$
$$+\Big(\langle 3|p_{12}|5\rangle\big(\langle 2|p_1|5\rangle p_{34}\cdot p_{56} - \langle 2|p_6|5\rangle p_{34}\cdot p_{12}\big)$$
$$-\langle 34\rangle\langle 12\rangle\big(\langle 1|p_6|4\rangle p_{12}\cdot p_{56} - \langle 1|p_2|5\rangle\langle 4|p_6|5\rangle\big)\Big)\frac{1}{p_{234}^2 p_{123}^2 p_{126}^2}\Big]$$



Necessary and sufficient conditions for BCFW

BCFW does not directly apply to the **non-susy CS coupled to fermions/ bosons** since the theories do not satisfy the necessary/sufficient conditions

It is possible to extract the recursion relations for non-susy fermionic/ bosonic CS matter theories from the N=2 results!!

At tree level, the Feynman diagrams for an all fermion amplitude are same for susy/non-susy theory.

Susy ward identity: The four point super amplitude is completely specified by one function, choose it to be the four fermion amplitude.

Use this information recursively in the BCFW formula at any order.

Eg: Six fermion amplitude in susy/non-susy theory

The six fermion amplitude in the N=2 theory is

$$\langle\bar\psi_1\psi_2\bar\psi_3\psi_4\bar\psi_5\psi_6\rangle = \left(\frac{16\pi^2 i}{\kappa^2}\right)\left(\frac{1}{p_{124}^2p_{125}^2p_{256}^2}-\frac{1}{p_{123}^2p_{126}^2p_{234}^2}\right)$$
$$\Big[-\langle 1|p_{34}|2\rangle\langle 3|p_{56}|4\rangle\langle 5|p_{12}|6\rangle+\langle 12\rangle\langle 34\rangle\langle 56\rangle$$
$$\times\big(\langle 12\rangle\langle 1|p_{34}|2\rangle+\langle 34\rangle\langle 3|p_{56}|4\rangle+\langle 5|p_{12}|6\rangle\langle 56\rangle\big)\Big]$$

Even though this result is obtained in the susy theory, due to identical diagrammatics at TREE LEVEL, it is also the result in the non-supersymmetric Chern-Simons coupled to fermions.

A similar relation can be written down for the bosonic theory as as well.

Summary

Summary

We started with a goal of computing arbitrary m→n tree level scattering amplitudes in U(N) Chern-Simons matter theories with fundamental matter.

We achieved this via BCFW recursion relations, this enabled us to express arbitrary n point amplitudes as products of four point amplitudes!

We saw an explicit example where the six point amplitude is expressed as a product of two four point amplitudes via two factorization channels.

We can also extract the recursion relations for non-supersymmetric theories even though BCFW does not directly apply.

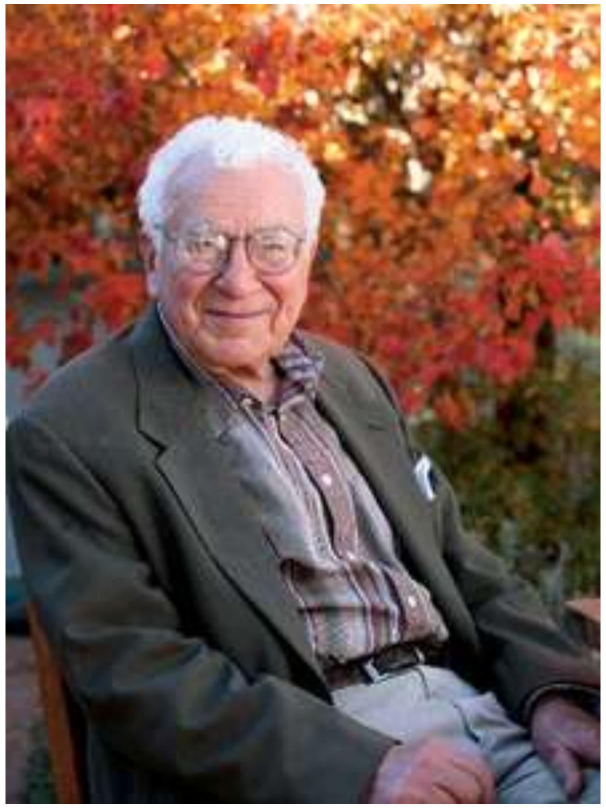
The central idea of BCFW is that when hard particles scatter in a sea of soft particles, factorisation occurs.

There are different kinds of deformations and all of them rely on one crucial property: **analyticity of the amplitude**.

It would be interesting to see if such methods can be useful in QCD.

Thank you!

תודה רבה!



நன்றி!

Three quarks for Muster Mark!
Sure he hasn't got much of a bark
And sure any he has it's all beside the mark!