Scattering in supersymmetric Chern-Simons matter theories at large N

Karthik Inbasekar

TEL AVIV UNIVERSITY אוניברסיטת תל אביב

Neve Shalom Seminar
Jan 17, 2017

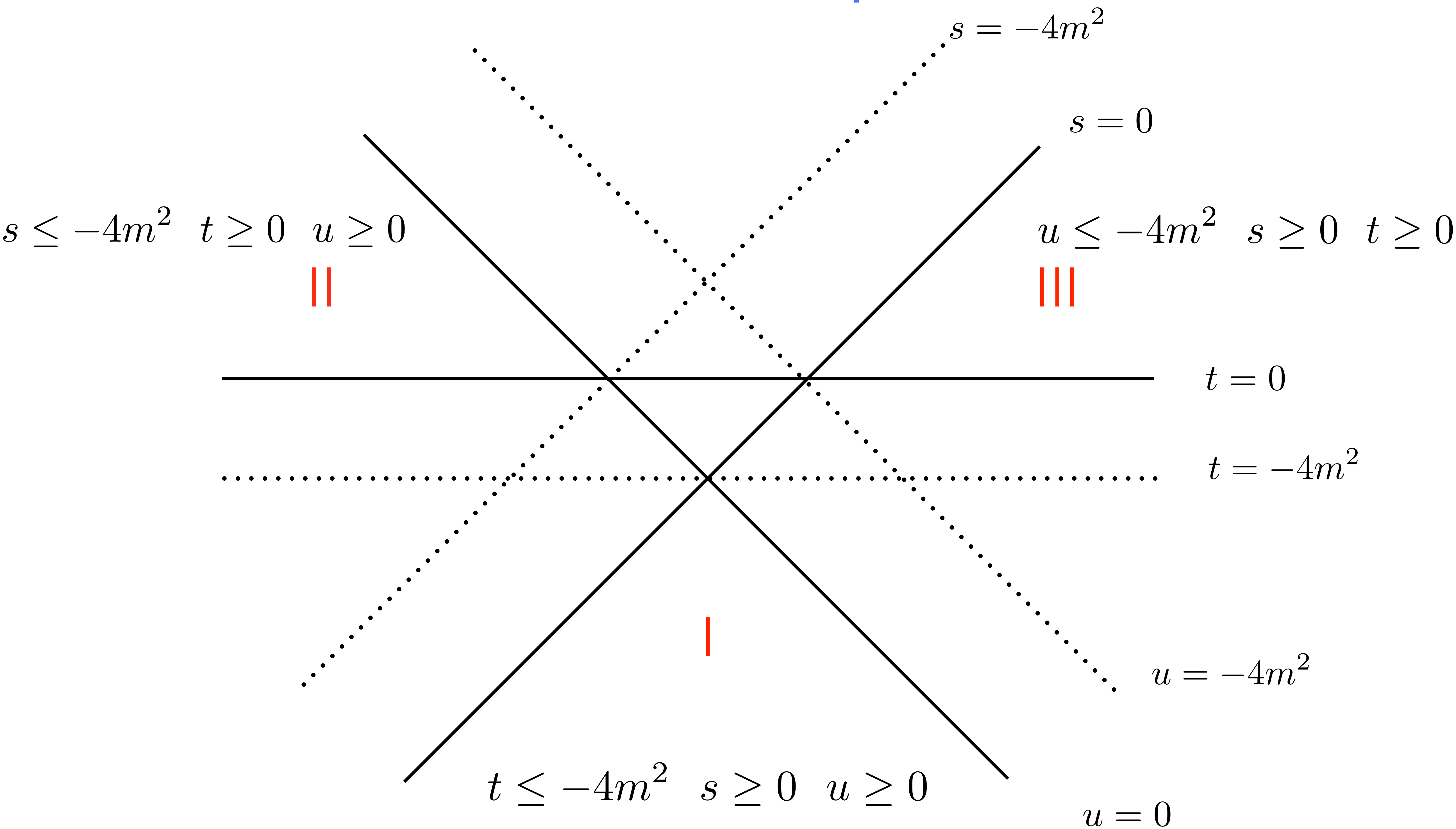
Crossing symmetry

- According to traditional wisdom in a relativistic QFT the scattering amplitude has crossing symmetry.
- Crossing symmetry is a statement of analytic continuation of amplitudes. Eg:

$$S(e^+e^- \to \mu^+\mu^-) \leftrightarrow S(e^-\mu^- \to e^-\mu^-)$$

- The S matrix for any process involving a particle of momentum p, in the initial state is equal to the S matrix for a process with an anti-particle of momentum k=-p in the final state.
- Crossing symmetry follows from the general principles (Gribov)
  - S matrix is a function of kinematical invariants.
  - Analytic function of its arguments.
  - All singularities are determined by physical masses of intermediate state particles.

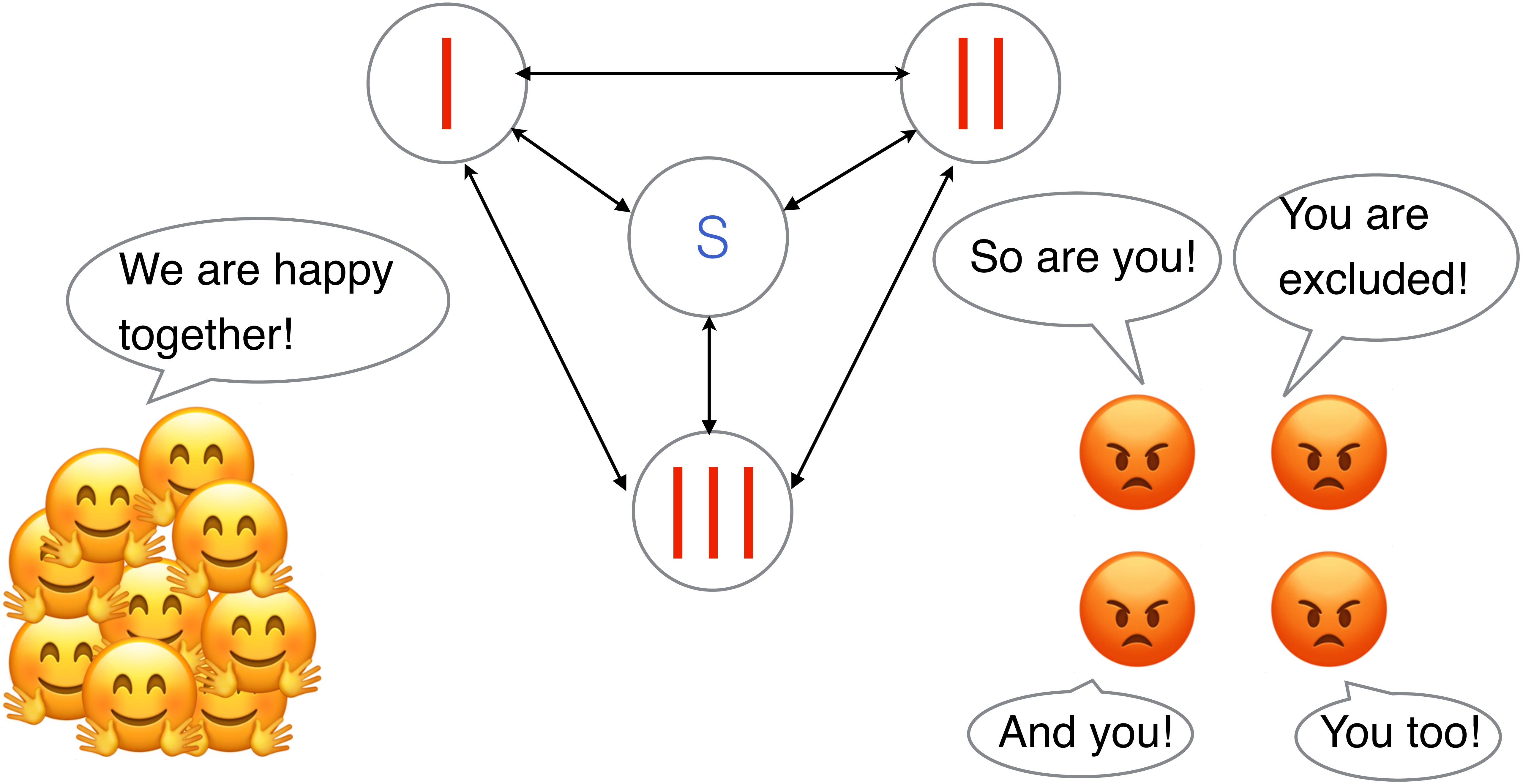
Mandelstam plane



$$s + t + u = -4m^2$$

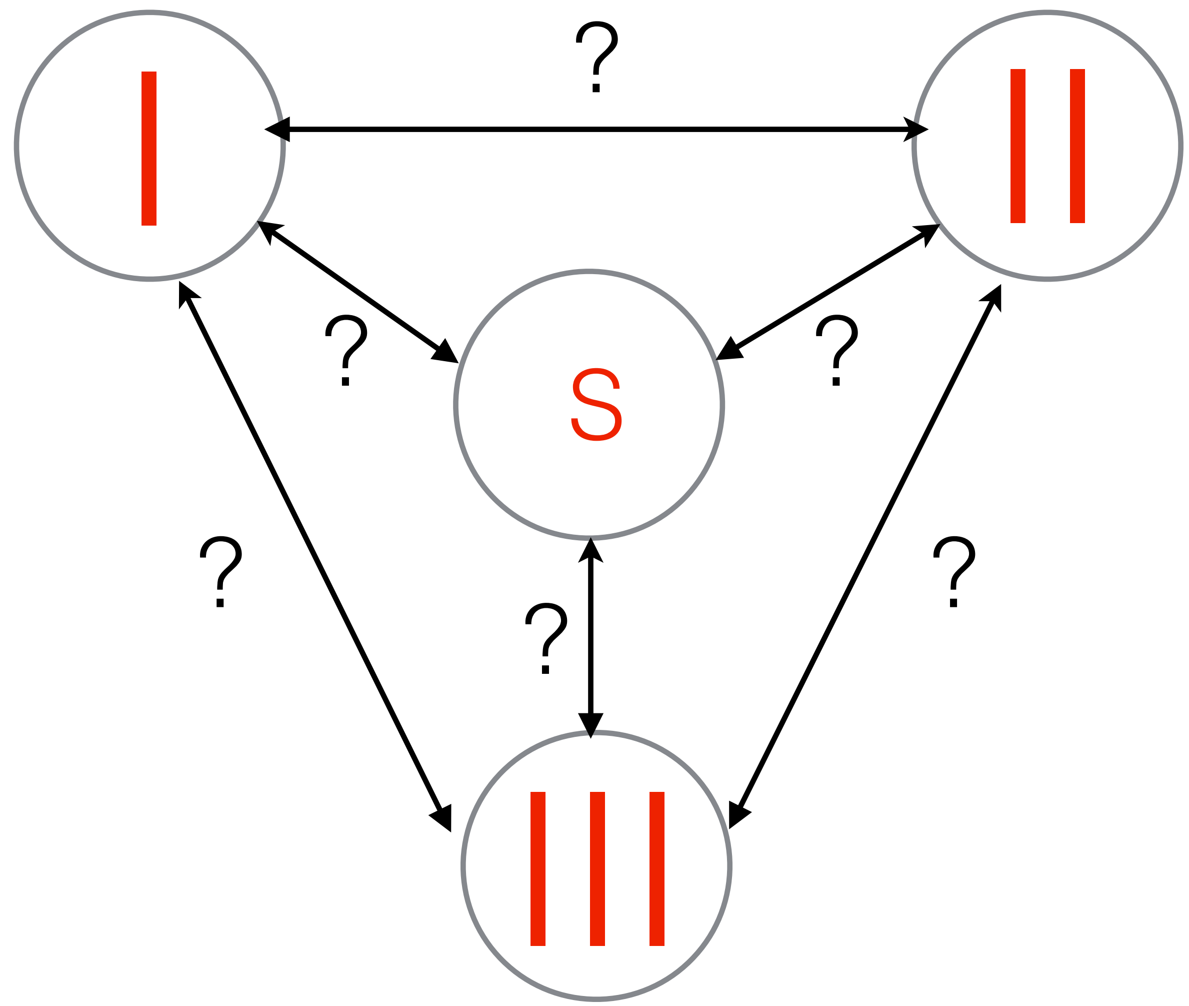
There exists a single function that can be analytically continued to different channels of scattering.

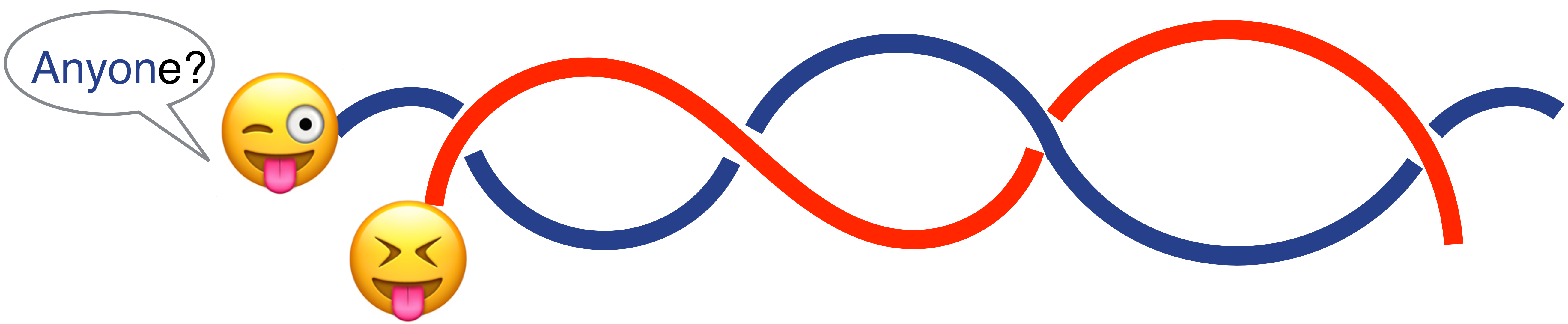
Crossing Symmetry



Depending on whether the scattering quanta is bosonic or fermionic the amplitude analytically continues up to an overall phase.

Crossing Symmetry





Does crossing symmetry hold when the scattering particles have anyonic statistics?

How do we go about answering this question?

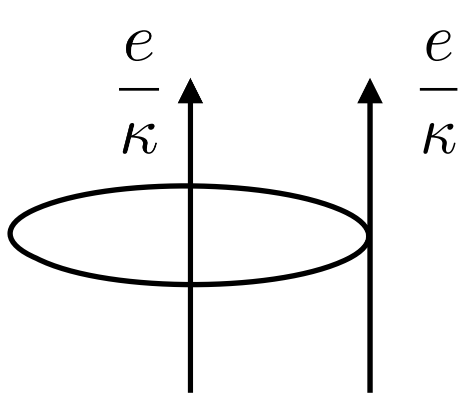
- Pure Chern-Simons theory in 2+1 dimensions is topological.
- When coupled to charged matter

$$\frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - A_\mu J^\mu_{matter}$$

- the CS gauge field attaches magnetic fluxes to the particles

$$\rho = \kappa B \ , \ J^i = \kappa\epsilon^{ij}E_j$$

- Adiabatic excursion of such particles leads to the Aharanov-Bohm effect



$$\text{AB phase} = e^{ie\int A.dx} = e^{i\frac{e^2}{\kappa}}$$

- We will study scattering in U(N) Chern-Simons matter theories in the large N, large k limit. In certain channels of scattering we will see that the particles are effectively anyonic in the large N limit. We will see how application of naive crossing rules conflict with unitarity and how modified crossing rules restore it.

Based on

- K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh, S.Yokoyama, arXiv: 1505.06571, JHEP 1510 (2015) 176.
- K.I, S.Jain, L.Janagal, S. Minwalla, A.Shukla, work in progress.
- S.Jain, M.Mandlik, S.Minwalla, T.Takimi, S.Wadia, S.Yokoyama, arXiv:1404.6373, JHEP 1504 (2015) 129

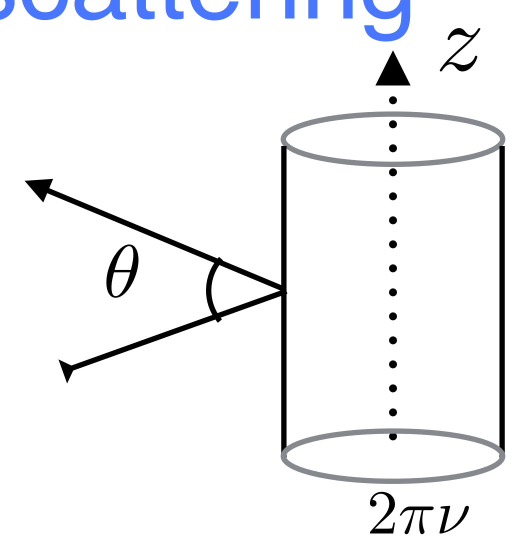
Outline

- Introduction
  - Non-relativistic Aharonov-Bohm Scattering and unitarity
  - Scattering in U(N) Chern-Simons matter theories.
  - Conjectured crossing rules in the Anyonic channels.
  - Tests of the conjecture.
- Scattering in susy Chern-Simons matter theories
  - $\mathcal{N} = 1, 2$ theories
  - Onshell supersymmetry
  - Methods: Dyson-Schwinger series, Large N.
  - S matrices.
  - $\mathcal{N} = 3$ theory
  - Preliminary results
- Unitarity
- Summary and future outlook

Part I

Aharanov-Bohm scattering and unitarity

Non-Relativistic Aharanov-Bohm scattering



- Scattering of a unit charged particle of a flux tube

- Schrödinger problem

$$\left(-\frac{1}{2m}(\nabla + 2\pi i G\nu)^2 - \frac{\kappa^2}{2m}\right)\psi = 0$$

$$G_{ij} = \frac{\epsilon_{ij}}{2\pi}\partial_j \ln r$$

- Boundary conditions:
  - wavefunction regularity at the origin
  - at large r, reduces to the incoming wave

- Scattering amplitude

$$h(\theta) = 2\pi(\cos\pi\nu - 1)\delta(\theta) + \sin\pi\nu(\mathrm{Pv}\cot\frac{\theta}{2} - i\mathrm{Sgn}(\nu))$$

- has a peculiar delta function piece modulated by the anyonic phase.

Aharanov-Bohm scattering and unitarity

$$h(\theta) = 2\pi(\cos\pi\nu - 1)\delta(\theta) + \sin\pi\nu(\mathrm{Pv}\cot\frac{\theta}{2} - i\mathrm{Sgn}(\nu))$$

- The delta function piece was originally missed by Aharanov-Bohm
- Without the delta function term, the scattering amplitude fails unitarity Ruijsennars; Bak, Jackiw, Pi
- Take an S matrix of general form $S(s,\theta) = I + iT(s,\theta)$
- the Unitarity condition takes the form

$$-i(T(s,\theta) - T^*(s,-\theta)) = \frac{1}{8\pi\sqrt{s}}\int d\alpha T(s,\alpha)T^*(s,-(\alpha-\theta))$$

- let us write

$$T(\sqrt{s},\theta) = H(\sqrt{s})\ x(\theta) + W_1(\sqrt{s}) - iW_2(\sqrt{s})\delta(\theta)$$

Aharanov-Bohm scattering and unitarity

- The unitarity conditions take the form

$$H - H^* = \frac{1}{8\pi\sqrt{s}}(W_2 H^* - H W_2^*)$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H H^*)$$

$$W_1 - W_1^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_1^* - W_2^* W_1) - \frac{i}{4\sqrt{s}}(H H^* - W_1 W_1^*)$$

- For Aharanov-Bohm scattering

$$H = 4\sqrt{s}\sin(\pi\nu)\ ,\ W_1 = -4\sqrt{s}\sin(\pi\nu)\mathrm{Sgn}(\nu)\ ,\ W_2 = 8\pi\sqrt{s}(\cos(\pi\nu) - 1)$$

- The first and third equations are trivially obeyed.

- The second equation is obeyed due to the trignometric identity.

$$2(1 - \cos(\pi\nu)) = (1 - \cos(\pi\nu))^2 + \sin^2(\pi\nu)$$

- without the $\cos\pi\nu$ term unitarity condition is not satisfied.

Scattering in U(N) Chern-Simons matter theories

- Consider $2 \to 2$ scattering of particles in representations R_1 and R_2 of U(N)

$$R_1 \times R_2 = \sum_m R_m$$

- The S matrix takes the schematic form

$$S = \sum_m P_m S_m$$

$$P_m : \text{ projector in } m^{th} \text{ rep, } S_m \text{ is scattering in } m^{th} \text{ channel.}$$

- The Aharanov-Bohm phase of the particle R_1 as it circles around particle R_2 is $2\pi\nu_m$ where

$$\nu_m = \frac{4\pi}{\kappa} T_1^a T_2^a = \frac{2\pi}{\kappa}\left(C_2(R_1) + C_2(R_2) - C_2(R_m)\right)$$

- Scattering amplitude in the m'th exchange channel: Aharanov-Bohm scattering of a unit charge particle off a flux tube of flux $2\pi\nu_m$

Scattering in U(N) Chern-Simons matter theories

- Channels of scattering

$$\text{Fundamental} \otimes \text{Fundamental} \to \text{Symm}(U_d) \oplus \text{Asymm}(U_e)$$

$$\text{Fundamental} \otimes \text{Antifundamental} \to \text{Adjoint}(T) \oplus \text{Singlet}(S)$$

- The quadratic Casimirs

$$C_2(F) = C_2(A) = \frac{N^2-1}{2N} \ , \ C_2(Sym) = \frac{N^2+N-2}{N}$$

$$C_2(ASym) = \frac{N^2-N-2}{N} \ , \ C_2(Adj) = N \ , \ C_2(Sing) = 0$$

- Anyonic phase

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa} \ , \ \nu_{Asym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}$$

$$\nu_{Adj} = \frac{1}{N\kappa} \ , \ \nu_{Sin} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

- In the large N, large κ limit, define 't Hooft coupling $\lambda = \frac{N}{\kappa}$

Scattering in U(N) Chern-Simons matter theories

- Anyonic phases in the large N, large κ limit

$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right) \;, \nu_{Sing} \sim O(\lambda)$$

- The T matrices themselves have the large N behavior

$$T_{Asym} \sim T_{Sym} \sim T_{Adj} \sim O\left(\frac{1}{N}\right) \;, \; T_{sing} \sim O(1)$$

- The singlet channel is effectively anyonic in the large N ,large κ limit.

- Unitarity $i(T^\dagger - T) = TT^\dagger$ is a non-trivial check only for the singlet channel. In other channels it follows from hermiticity.

- Observation: Naive crossing symmetry rules from any of the non-anyonic channels to the singlet channel leads to a non unitary S matrix.

S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

- Conjecture: Singlet channel S matrices have the form

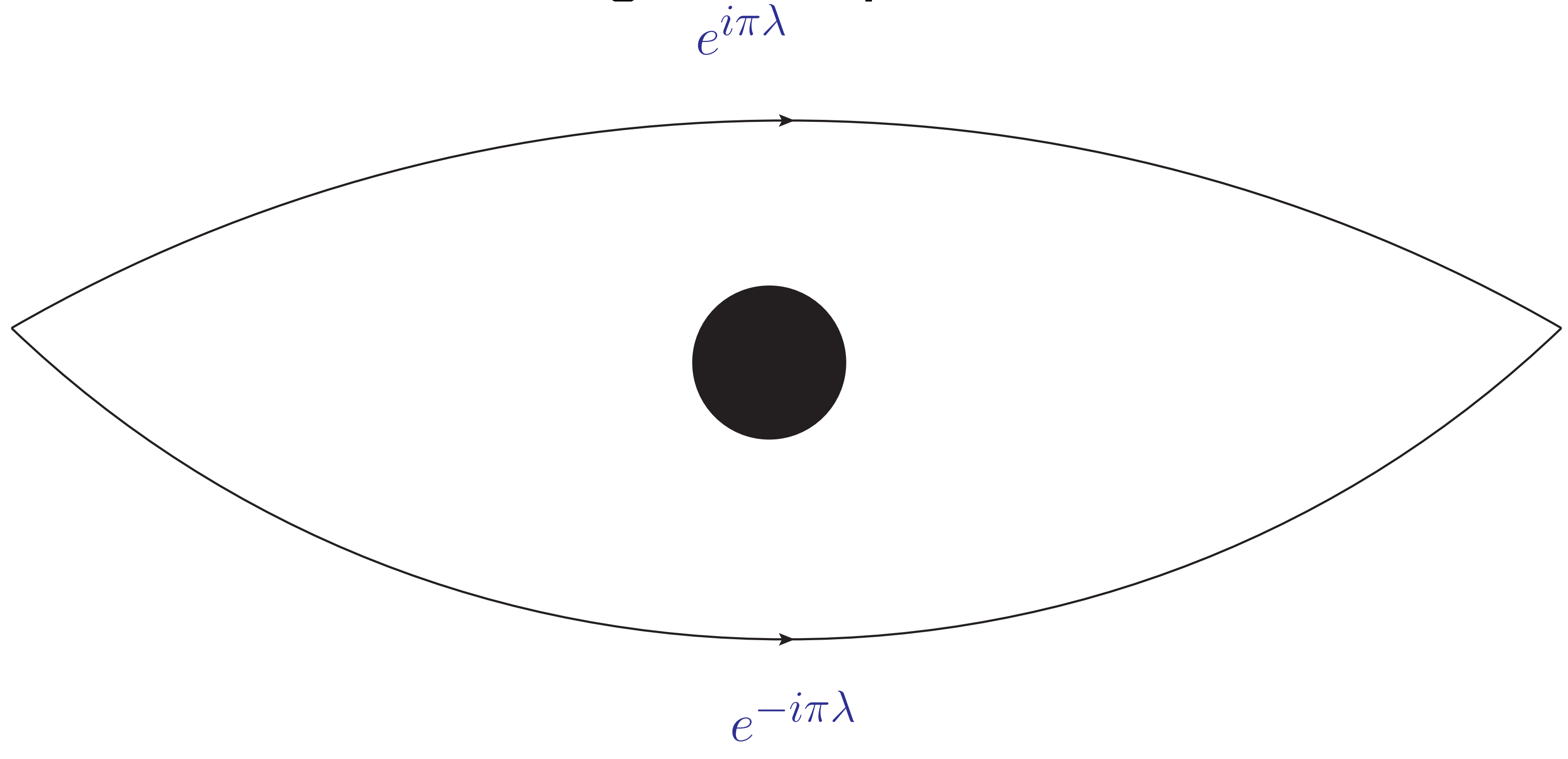
$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\mathrm{naive}}(s,\theta)$$

- $\mathcal{T}^{S;\mathrm{naive}}$ is the matrix obtained from naive analytic continuation of particle-particle scattering.

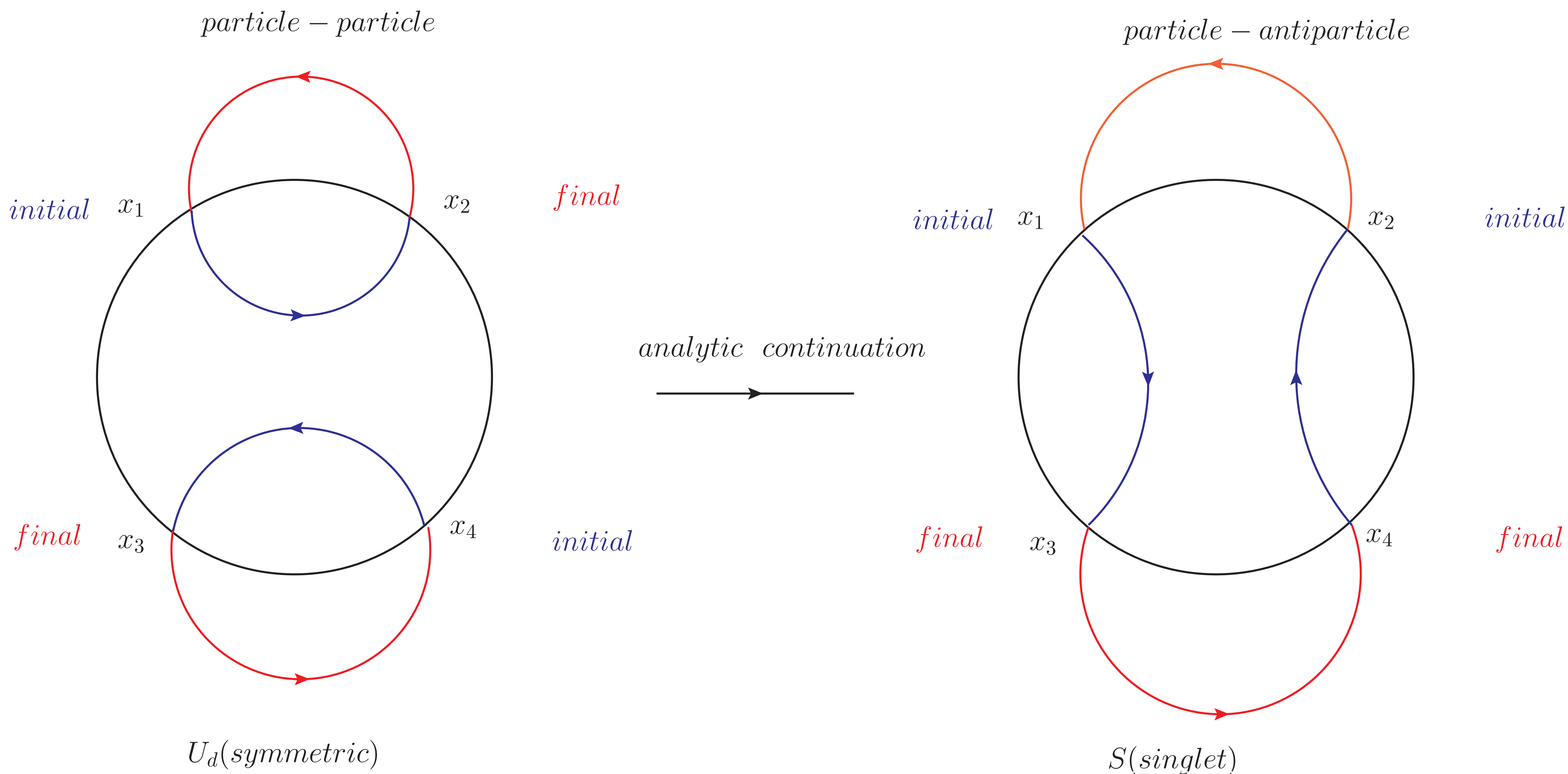
Nature of the conjecture: Delta function

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\mathrm{naive}}(s,\theta)$$

- The conjectured S matrix has a non-analytic delta function piece.
- The delta function modulated by anyonic phase is already known to be necessary to unitarize Aharonov-Bohm scattering.
- $\cos\pi\lambda$ in the identity term is due to the interference of the Aharanov-Bohm phases of the incoming wave packets.



Modified crossing rules: Heuristic explanation



- Attach Wilson lines to make correlators gauge invariant

$$T_{U_d} W_{U_d} \to T_S W_S$$

$$\frac{W_{U_d}}{W_S} = \frac{\oint \text{with 2 circular Wilson lines}}{\oint \text{with 1 circular Wilson line}} = \frac{\sin(\pi\lambda)}{\pi\lambda}$$ Witten

Tests of the conjecture

- Delta function and modified crossing rules appear to be universal.
- Tests:
  - Unitarity of the S matrix
  - 3d Bosonization duality Aharony,Bardeen,Benini,Chang,Frishman,Giombi,Giveon, Gur-Ari,Gurucharan,Kutasov,Jain,Maldacena,Mandlik,Minwalla,Moshe,Sharma,Prakash, Takimi,Trivedi,Seiberg,Sonnenschein,Yacoby,Yin,Yokoyama,Wadia,Witten,Zhiboedov
  - Non-relativistic limit should be consistent with Aharanov-Bohm result.

- All the tests have been explicitly verified for
  - U(N) Chern-Simons coupled to fundamental bosons.
  - U(N) Chern-Simons coupled to fundamental fermions.

    S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama
  - $\mathcal{N} = 1, 2$ supersymmetric Chern-Simons matter theories.

    K.I, S.Jain, S.Mazumdar, S.Minwalla, V. Umesh, S.Yokoyama
- Preliminary computations in $\mathcal{N} = 3$ theories already show consistency with unitarity and 3d bosonization duality. K.I, S.Jain, L.Janagal, S.Minwalla, A.Shukla

- Further checks for $\mathcal{N} = 3$ and computations for $\mathcal{N} = 4, 5, 6$ are in progress.

Rest of the talk..

- Test the conjecture in most general renormalizable $\mathcal{N}=1,2,3$ supersymmetric Chern-Simons matter theories.

- Work in superspace - manifest supersymmetry.

- Work in large N, large κ limit - only planar diagrams.

- Compute exact propagator and off-shell four point correlator to all orders in 't Hooft coupling.

- Take onshell limit and read off the S matrices in symmetric, antisymmetric, and adjoint channels.

- Apply the conjecture to the singlet channel and test it for unitarity, non-relativistic limit…

Main results

- Results for $\mathcal{N} = 1, 2, 3$ consistent with 3d Bosonization duality

- Unitarity requires delta function term at forward scattering and modified crossing rules for the singlet channel in all these theories exactly as conjectured by
  S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama
- Substantial evidence for universality of the conjecture.

- For $\mathcal{N} = 2, 3$ theories
  - The S matrix computed to all orders in the 't Hooft coupling in the non-anyonic channels are tree level exact - non renormalization of propagator and vertex.

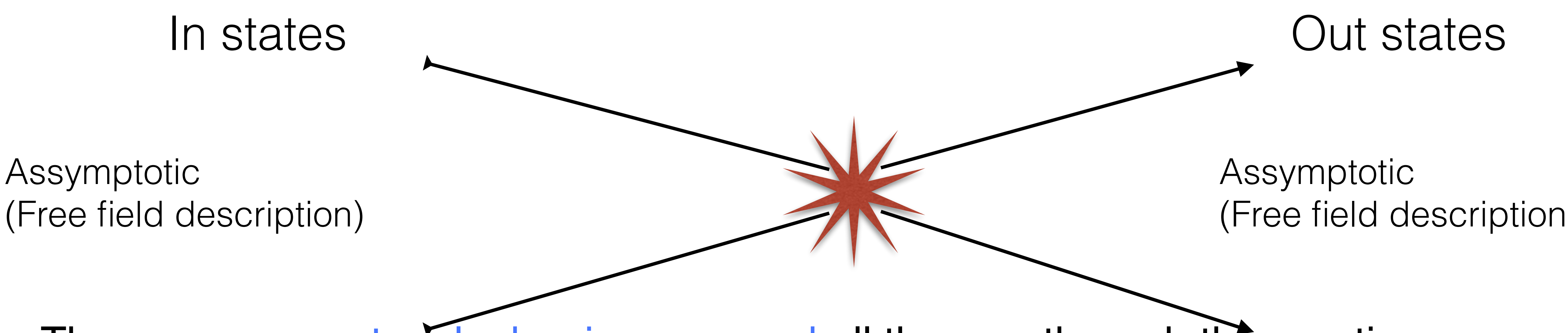
  - Following the conjecture: The singlet channel S matrix is not tree level exact, but continues to be very simple.

  - Illustrates the conflict between naive crossing symmetry and unitarity in a simple setting.

Part IIA

Scattering in supersymmetric Chern-Simons matter theories

Supersymmetric scattering



- The supersymmetry algebra is preserved all the way through the reaction - use to characterize states and all processes.

- Superspace: Manifest supersymmetry, efficient packaging.

- The S matrix in onshell superspace is constrained to obey supersymmetric ward identities. Relates component processes in terms of few independent ones.

- For $\mathcal{N} \geq 3$, R symmetry group is non-abelian, no manifest superspace formalism exists - possible to reformulate the theories in $\mathcal{N} = 1$ superspace.

- Advantage: Similar computation. Disadvantage: loss of manifest susy, forced to check for covariance by going to components.

$\mathcal{N}=1,2$ susy Chern-Simons matter theories

- General renormalizable $\mathcal{N}=1,2$ theory with one fundamental multiplet

$$\mathcal{S}_{\mathcal{N}=1} = -\int d^3x d^2\theta \Big[\frac{\kappa}{2\pi} Tr\Big(-\frac{1}{4} D_\alpha \Gamma^\beta D_\beta \Gamma^\alpha - \frac{1}{6} D^\alpha \Gamma^\beta \{\Gamma_\alpha, \Gamma_\beta\} - \frac{1}{24} \{\Gamma^\alpha, \Gamma^\beta\}\{\Gamma_\alpha, \Gamma_\beta\} \Big)$$
$$- \frac{1}{2} (D^\alpha \bar{\Phi} + i \bar{\Phi} \Gamma^\alpha)(D_\alpha \Phi - i \Gamma_\alpha \Phi) + m_0 \bar{\Phi}\Phi + \frac{\pi w}{\kappa} (\bar{\Phi}\Phi)^2 \Big]$$

- Φ is a complex scalar superfield, Γ^α is a real superfield.

$$\Phi = \phi + \theta\psi - \theta^2 F \ , \bar{\Phi} = \bar{\phi} + \theta\bar{\psi} - \theta^2 \bar{F} \ ,$$
$$\Gamma^\alpha = \chi^\alpha - \theta^\alpha B + i\theta^\beta A_\beta{}^\alpha - \theta^2 (2\lambda^\alpha - i\partial^{\alpha\beta}\chi_\beta) \ .$$

- Integer parameters N, κ, matter self coupling w, bare mass m_0. At $w=1$ the supersymmetry is enhanced to $\mathcal{N}=2$.

- The theory exhibits a self duality under the duality map

$$\lambda' = \lambda - \mathrm{Sgn}(\lambda) \ , \ w' = \frac{3-w}{1+w} \qquad m_0' = \frac{-2m_0}{1+w}$$
$$N' = |\kappa| - N + 1 \ , \ \kappa' = -\kappa$$

S.Jain, S.Minwalla, S.Yokoyama

Scattering a superfield

$$\begin{pmatrix} \Phi(\theta_1, p_1) \\ \bar{\Phi}(\theta_2, p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\Phi}(\theta_3, p_3) \\ \Phi(\theta_4, p_4) \end{pmatrix}$$

$$\mathcal{S}_B : \begin{pmatrix} \phi(p_1) \\ \bar{\phi}(p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\phi}(p_3) \\ \phi(p_4) \end{pmatrix} \;,\; \mathcal{S}_F : \begin{pmatrix} \psi(p_1) \\ \bar{\psi}(p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\psi}(p_3) \\ \psi(p_4) \end{pmatrix}$$

$$H_1 : \begin{pmatrix} \phi(p_1) \\ \bar{\phi}(p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\psi}(p_3) \\ \psi(p_4) \end{pmatrix} \;,\; H_2 : \begin{pmatrix} \psi(p_1) \\ \bar{\psi}(p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\phi}(p_3) \\ \phi(p_4) \end{pmatrix}$$

$$H_3 : \begin{pmatrix} \phi(p_1) \\ \bar{\psi}(p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\phi}(p_3) \\ \psi(p_4) \end{pmatrix} \;,\; H_4 : \begin{pmatrix} \psi(p_1) \\ \bar{\phi}(p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\psi}(p_3) \\ \phi(p_4) \end{pmatrix}$$

$$H_5 : \begin{pmatrix} \phi(p_1) \\ \bar{\psi}(p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\psi}(p_3) \\ \phi(p_4) \end{pmatrix} \;,\; H_6 : \begin{pmatrix} \psi(p_1) \\ \bar{\phi}(p_2) \end{pmatrix} \to \begin{pmatrix} \bar{\phi}(p_3) \\ \psi(p_4) \end{pmatrix}$$

- Supersymmetric ward identities relate the some of the processes in terms of others. Not all processes are independent.

Onshell susy for $\mathcal{N}=1,2$ theories

- Asymptotic in/out states satisfy free field equation

$$\left(D^2+m\right)\Phi=0$$

- Solution

$$\Phi(x,\theta)=\int\frac{d^2p}{\sqrt{2p^0}(2\pi)^2}\bigg[\Big(a(\mathbf{p})(1+m\theta^2)+\theta^\alpha u_\alpha(\mathbf{p})\alpha(\mathbf{p})\Big)e^{ip.x}$$
$$+\Big(a^{c\dagger}(\mathbf{p})(1+m\theta^2)+\theta^\alpha v_\alpha(\mathbf{p})\alpha^{c\dagger}(\mathbf{p})\Big)e^{-ip.x}\bigg]$$

- Action of off-shell susy operator on onshell superfields

$$[Q_\alpha^{off},\Phi]=Q_\alpha^{off}\Phi=i\left(\frac{\partial}{\partial\theta^\alpha}-i\theta^\beta\partial_{\beta\alpha}\right)\Phi$$

- Onshell representation of the supercharge Q

$$-iQ_\alpha^{on}=u_\alpha(\mathbf{p}_i)\left(\alpha\partial_a+\alpha^c\partial_{a^c}\right)+u_\alpha^*(\mathbf{p}_i)\left(a\partial_\alpha+a^c\partial_{\alpha^c}\right)$$
$$-v_\alpha^*(\mathbf{p}_i)\left(a^\dagger\partial_{\alpha^\dagger}+(a^c)^\dagger\partial_{(\alpha^c)^\dagger}\right)+v_\alpha(\mathbf{p}_i)\left(\alpha^\dagger\partial_{a^\dagger}+(\alpha^c)^\dagger\partial_{(a^c)^\dagger}\right)$$

Onshell susy for $\mathcal{N} = 1, 2$ theories

- Introduce creation and annihilation operator superfields

$$A_i(\mathbf{p}) = a_i(\mathbf{p}) + \alpha_i(\mathbf{p})\theta_i \ ,$$
$$A_i^\dagger(\mathbf{p}) = a_i^\dagger(\mathbf{p}) + \theta_i \alpha_i^\dagger(\mathbf{p}) \ .$$

- Action of onshell susy operator on these

$$[Q_\alpha^{on}, A_i(\mathbf{p}_i, \theta_i)] = Q_\alpha^1 A_i(\mathbf{p}_i, \theta_i)$$
$$[Q_\alpha^{on}, A_i^\dagger(\mathbf{p}_i, \theta_i)] = Q_\alpha^2 A_i^\dagger(\mathbf{p}_i, \theta_i)$$

$$Q_\beta^1 = i\left(-u_\beta(\mathbf{p})\frac{\overrightarrow{\partial}}{\partial\theta} - v_\beta(\mathbf{p})\theta\right)$$
$$Q_\beta^2 = i\left(v_\beta(\mathbf{p})\frac{\overrightarrow{\partial}}{\partial\theta} - u_\beta(\mathbf{p})\theta\right) \ .$$

Onshell susy for $\mathcal{N}=1,2$ theories

- $2 \to 2$ S matrix $p_1 + p_2 \to p_3 + p_4$

$$S(\mathbf{p}_1,\theta_1,\mathbf{p}_2,\theta_2,\mathbf{p}_3,\theta_3,\mathbf{p}_4,\theta_4)\sqrt{(2p_1^0)(2p_2^0)(2p_3^0)(2p_4^0)} =$$
$$\langle 0|A_4(\mathbf{p}_4,\theta_4)A_3(\mathbf{p}_3,\theta_3)UA_2^\dagger(\mathbf{p}_2,\theta_2)A_1^\dagger(\mathbf{p}_1,\theta_1)|0\rangle$$

- Supersymmetric ward identity for the superspace S matrix

$$\Big(\overrightarrow{Q}^1_\alpha(\mathbf{p}_1,\theta_1)+\overrightarrow{Q}^1_\alpha(\mathbf{p}_2,\theta_2)$$
$$+\overrightarrow{Q}^2_\alpha(\mathbf{p}_3,\theta_3)+\overrightarrow{Q}^2_\alpha(\mathbf{p}_4,\theta_4)\Big)S(\mathbf{p}_1,\theta_1,\mathbf{p}_2,\theta_2,\mathbf{p}_3,\theta_3,\mathbf{p}_4,\theta_4)=0$$

Onshell susy for $\mathcal{N}=1,2$ theories

- The solution satisfying the supersymmetric ward identity for the $\mathcal{N}=1$ theory is given in terms of two independent functions $\mathcal{S}_B$ and $\mathcal{S}_F$.

$$
\begin{aligned}
S(\mathbf{p}_1,\theta_1,\mathbf{p}_2,\theta_2,\mathbf{p}_3,\theta_3,\mathbf{p}_4,\theta_4) = \mathcal{S}_B + \mathcal{S}_F\ \theta_1\theta_2\theta_3\theta_4 + \\
\left(\frac{1}{2}C_{12}\mathcal{S}_B - \frac{1}{2}C^*_{34}\mathcal{S}_F\right)\ \theta_1\theta_2 + \left(\frac{1}{2}C_{13}\mathcal{S}_B - \frac{1}{2}C^*_{24}\mathcal{S}_F\right)\ \theta_1\theta_3 \\
+ \left(\frac{1}{2}C_{14}\mathcal{S}_B + \frac{1}{2}C^*_{23}\mathcal{S}_F\right)\ \theta_1\theta_4 + \left(\frac{1}{2}C_{23}\mathcal{S}_B + \frac{1}{2}C^*_{14}\mathcal{S}_F\right)\ \theta_2\theta_3 \\
+ \left(\frac{1}{2}C_{24}\mathcal{S}_B - \frac{1}{2}C^*_{13}\mathcal{S}_F\right)\ \theta_2\theta_4 + \left(\frac{1}{2}C_{34}\mathcal{S}_B - \frac{1}{2}C^*_{12}\mathcal{S}_F\right)\ \theta_3\theta_4
\end{aligned}
$$

- no $\boldsymbol{\theta}$ term is four boson scattering, the four $\boldsymbol{\theta}$ term is four fermion scattering. C_{ij} are functions of $u_\alpha(p), v_\alpha(p)$.

- Thus $\mathcal{N}=1$ susy determines 6 of 8 processes in terms of 2 independent functions.

Onshell susy for $\mathcal{N}=2$ theories

- The $\mathcal{N}=2$ S matrix is already $\mathcal{N}=1$ supersymmetric, but obeys additional constraint from $\mathcal{N}=2$ susy.
- This can also be formulated in $\mathcal{N}=1$ onshell superspace

$$\left(\sum_{i=1}^{4} Q^i_\alpha(\mathbf{p}_i,\theta_i)+\bar{Q}^i_\alpha(\mathbf{p}_i,\theta_i)\right) S(\mathbf{p}_i,\theta_i)=0$$

$$\left(\sum_{i=1}^{4} Q^i_\alpha(\mathbf{p}_i,\theta_i)-\bar{Q}^i_\alpha(\mathbf{p}_i,\theta_i)\right) S(\mathbf{p}_i,\theta_i)=0$$

- Additional constraint relates $\mathcal{S}_B$ and $\mathcal{S}_F$

$$\begin{aligned}&\mathcal{S}_B\left(C_{13}u_\alpha(\mathbf{p}_3)+C_{14}u_\alpha(\mathbf{p}_4)+C_{12}v_\alpha(\mathbf{p}_2)+v^*_\alpha(\mathbf{p}_1)\right)\\&=\mathcal{S}_F(C^*_{24}u_\alpha(\mathbf{p}_3)-C^*_{23}u_\alpha(\mathbf{p}_4)+C^*_{34}v_\alpha(\mathbf{p}_2))\end{aligned}$$

- The $\mathcal{N}=2$ S matrix is completely specified by one function. Eg:

$$p_1=p+q, p_2=-k-q, p_3=p, p_4=-k$$

$$\mathcal{S}_B=\mathcal{S}_F\frac{-2m(k-p)_-+iq_3(k+p)_-}{2m(k-p)_-+iq_3(k+p)_-}\ .$$

Part IIB

Dyson-Schwinger series, Offshell correlators and S matrices at large N

Supersymmetric light cone gauge

- There exists a supersymmetric generalization of the light cone gauge

$$\Gamma_- = 0 \;\Rightarrow\; A_- = A_1 + iA_2 = 0$$

- All gauge self interactions (in superspace) vanish

$$S = -\int d^3x d^2\theta \Big[-\frac{\kappa}{8\pi} Tr(\Gamma^- i\partial_{--}\Gamma^-) - \frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi \\ - \frac{i}{2}\Gamma^-(\bar{\Phi} D_- \Phi - D_- \bar{\Phi}\Phi) + m_0 \bar{\Phi}\Phi + \frac{\pi w}{\kappa}(\bar{\Phi}\Phi)^2 \Big]$$

- Susy light cone gauge maintains manifest susy.

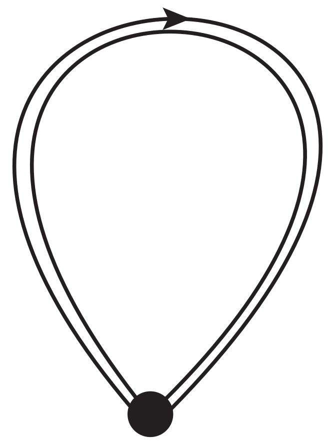
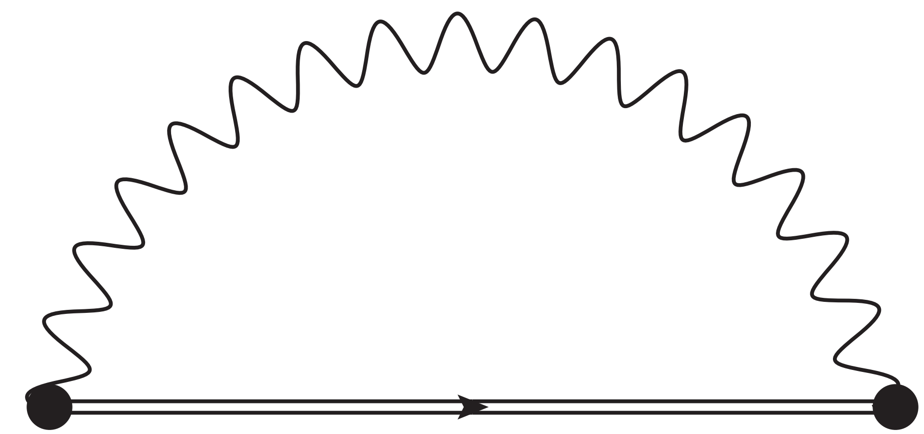
- Bare propagators

$$\langle \bar{\Phi}(\theta_1, p)\Phi(\theta_2, -p')\rangle = \frac{D^2_{\theta_1,p} - m_0}{p^2 + m_0^2}\delta^2(\theta_1 - \theta_2)(2\pi)^3\delta^3(p - p')$$

$$\langle \Gamma^-(\theta_1, p)\Gamma^-(\theta_2, -p')\rangle = -\frac{8\pi}{\kappa}\frac{\delta^2(\theta_1 - \theta_2)}{p_{--}}(2\pi)^3\delta^3(p - p')$$

Exact propagator to all orders in λ

- Integral equation for self energy

$\Sigma(p, \theta_1, \theta_2) \;=$  $+$ 

- Both $P(r, \theta_1, \theta_2)$ and $\Sigma(r, \theta_1, \theta_2)$ satisfy off-shell susy ward identities and are determined upto unknown functions of momenta.

$$\begin{aligned}
\Sigma(p, \theta_1, \theta_2) = 2\pi\lambda w \int \frac{d^3 r}{(2\pi)^3} \delta^2(\theta_1 - \theta_2) P(r, \theta_1, \theta_2) \\
- 2\pi\lambda \int \frac{d^3 r}{(2\pi)^3} D_-^{\theta_2, -p} D_-^{\theta_1, p} \left(\frac{\delta^2(\theta_1 - \theta_2)}{(p-r)_{--}} P(r, \theta_1, \theta_2) \right) \\
+ 2\pi\lambda \int \frac{d^3 r}{(2\pi)^3} \frac{\delta^2(\theta_1 - \theta_2)}{(p-r)_{--}} D_-^{\theta_1, r} D_-^{\theta_2, -r} P(r, \theta_1, \theta_2)
\end{aligned}$$

- The integral equation determines these unknown functions.

Exact propagator to all orders in λ

- Solution to the exact propagator is extremely simple

$$P(p,\theta_1,\theta_2) = \frac{D^2 - m}{p^2 + m^2}\delta^2(\theta_1 - \theta_2)$$

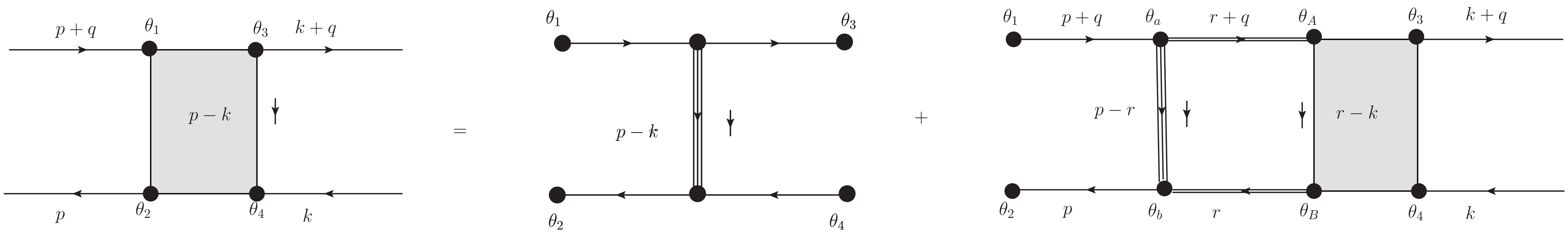
- Same structure as bare propagator with m_0 replaced by m

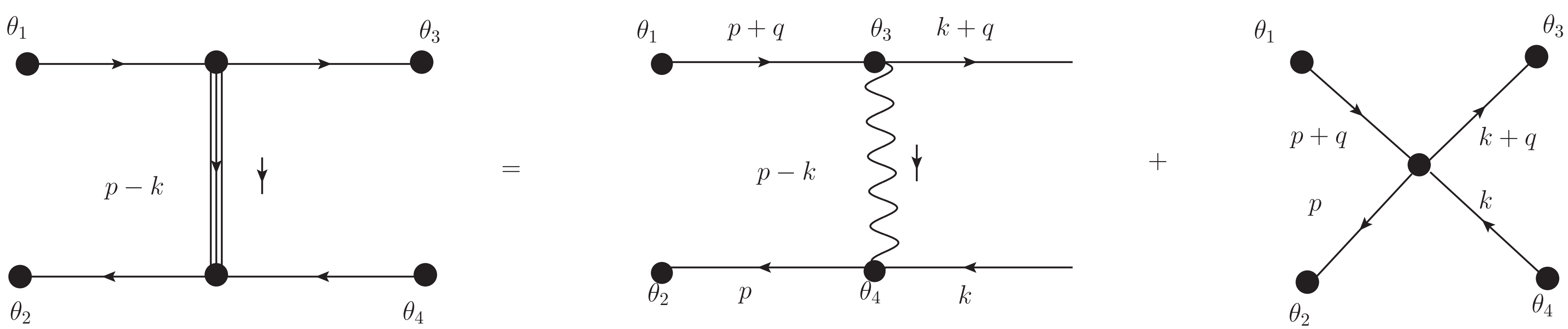
$$m = \frac{2m_0}{2 + (-1+w)\lambda\ \mathrm{Sgn(m)}}$$

- m is the pole mass. It is invariant under the bosonization duality.

- For the $\mathcal{N} = 2$ theory, $w = 1$, there is no mass renormalization.

Exact four point function to all orders in λ





- The integral equation can be schematically written as

$$V(\theta_i, p_i) = V_0(\theta_i, p_i) + \int \frac{d^3 r}{(2\pi)^3} d^2\theta_j' V_0(\theta_i, \theta_j', p_i, r)$$
$$P(\theta_j', p_i + r) P(\theta_j', r) V(\theta_j', \theta_i, p_i)$$

- Solved the integral equations exactly in the large N limit, for arbitrary values of the ’t Hooft coupling λ and determined the off-shell four point function in the kinematic regime $q_\pm = 0$

S matrix in the non-anyonic channels for the $\mathcal{N}=1$ theory

- The onshell limit directly gives the S matrix for the T (adjoint), U_d symmetric and U_e (anti-symmetric) channels of scattering.

- The final answer can be covariantized and is gauge invariant.

$$
\begin{aligned}
\mathcal{T}_B =& \frac{4i\pi}{\kappa} \epsilon_{\mu\nu\rho} \frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} + J_B(q,\lambda) \ , \\
\mathcal{T}_F =& \frac{4i\pi}{\kappa} \epsilon_{\mu\nu\rho} \frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} + J_F(q,\lambda) \ ,
\end{aligned}
$$

$$
\begin{aligned}
J_B(q,\lambda) =& \frac{4\pi q}{\kappa} \frac{N_1 N_2 + M_1}{D_1 D_2} \ , \\
J_F(q,\lambda) =& \frac{4\pi q}{\kappa} \frac{N_1 N_2 + M_2}{D_1 D_2} \ ,
\end{aligned}
$$

S matrix in the non-anyonic channels for the $\mathcal{N}=1$ theory

$$
\begin{aligned}
N_1 &= \left(\left(\frac{2|m|+iq}{2|m|-iq}\right)^{-\lambda} (w-1)(2m+iq) + (w-1)(2m-iq) \right) \ , \\
N_2 &= \left(\left(\frac{2|m|+iq}{2|m|-iq}\right)^{-\lambda} (q(w+3)+2im(w-1)) + (q(w+3)-2im(w-1)) \right) \ , \\
M_1 &= -\, 8mq((w+3)(w-1)-4w) \left(\frac{2|m|+iq}{2|m|-iq}\right)^{-\lambda} \ , \\
M_2 &= -\, 8mq(1+w)^2 \left(\frac{2|m|+iq}{2|m|-iq}\right)^{-\lambda} \ , \\
D_1 &= \left(i \left(\frac{2|m|+iq}{2|m|-iq}\right)^{-\lambda} (w-1)(2m+iq) - 2im(w-1) + q(w+3) \right) \ , \\
D_2 &= \left(\left(\frac{2|m|+iq}{2|m|-iq}\right)^{-\lambda} (-q(w+3)-2im(w-1)) + (w-1)(q+2im) \right) \ .
\end{aligned}
$$

S matrix in the non-anyonic channels for the $\mathcal{N}=2$ theory

- Remarkable simplification in the $\mathcal{N}=2$ theory (w=1)

$$\mathcal{T}_B^{\mathcal{N}=2} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} - \frac{8\pi m}{\kappa} ,$$
$$\mathcal{T}_F^{\mathcal{N}=2} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} + \frac{8\pi m}{\kappa}$$

- The S matrix is tree level exact to all orders in λ.

- No loop corrections in the planar limit: non-renormalization.

Duality invariance

- Under the duality transformation

$$w' = \frac{3-w}{w+1}, \lambda' = \lambda - \mathrm{sgn}(\lambda), m' = -m, \kappa' = -\kappa$$

$$J_B(q, \kappa', \lambda', w', m') = -J_F(q, \kappa, \lambda, w, m) \ ,$$
$$J_F(q, \kappa', \lambda', w', m') = -J_B(q, \kappa, \lambda, w, m) \ .$$

- Duality maps the bosonic and fermionic S matrices into one another upto an overall phase.

- Concrete evidence for duality.

- Supersymmetric ward identity guarantees duality invariance of all other processes in the theory.

S matrix in the singlet channel

- We cannot extract the singlet channel S matrix directly because of the kinematic restriction $q_\pm = 0$.
- Naive crossing symmetry rules give rise to a non-unitary S matrix.
- Applying the conjecture of S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

$$\mathcal{S}_B^S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\left(4\pi i\lambda\sqrt{s}\cot(\theta/2) + J_B(\sqrt{s},\lambda)\right) ,$$

$$\mathcal{S}_F^S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\left(4\pi i\lambda\sqrt{s}\cot(\theta/2) + J_F(\sqrt{s},\lambda)\right) .$$

$$J_B(\sqrt{s},\lambda) = -\,4\pi i\lambda\sqrt{s}\frac{N_1N_2 + M_1}{D_1D_2} ,$$

$$J_F(\sqrt{s},\lambda) = -\,4\pi i\lambda\sqrt{s}\frac{N_1N_2 + M_2}{D_1D_2}$$

Simple check: Non relativistic limit

- Non relativistic limit of the singlet channel S matrix $\sqrt{s} \to 2m$ while keeping other parameters fixed.

$$
\mathcal{T}_B^S(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\delta(\theta) + 4\sqrt{s}\sin(\pi\lambda)\left(i\cot(\theta/2)-1\right) \ ,
$$
$$
\mathcal{T}_F^S(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\delta(\theta) + 4\sqrt{s}\sin(\pi\lambda)\left(i\cot(\theta/2)+1\right) \ .
$$

- This is precisely the Aharanov-Bohm result.

- Surprisingly this result coincides with the w=1, S matrix that corresponds to the $\mathcal{N}=2$ theory.

- Presumably susy enhancement in the non-relativistic limit.

Unitarity

- Recall that the superspace S matrix was completely specified by two functions

$$S(\mathbf{p}_1,\theta_1,\mathbf{p}_2,\theta_2,\mathbf{p}_3,\theta_3,\mathbf{p}_4,\theta_4) = \mathcal{S}_B + \mathcal{S}_F\ \theta_1\theta_2\theta_3\theta_4 +$$
$$\left(\frac{1}{2}C_{12}\mathcal{S}_B - \frac{1}{2}C^*_{34}\mathcal{S}_F\right)\ \theta_1\theta_2 + \left(\frac{1}{2}C_{13}\mathcal{S}_B - \frac{1}{2}C^*_{24}\mathcal{S}_F\right)\ \theta_1\theta_3$$
$$+\left(\frac{1}{2}C_{14}\mathcal{S}_B + \frac{1}{2}C^*_{23}\mathcal{S}_F\right)\ \theta_1\theta_4 + \left(\frac{1}{2}C_{23}\mathcal{S}_B + \frac{1}{2}C^*_{14}\mathcal{S}_F\right)\ \theta_2\theta_3$$
$$+\left(\frac{1}{2}C_{24}\mathcal{S}_B - \frac{1}{2}C^*_{13}\mathcal{S}_F\right)\ \theta_2\theta_4 + \left(\frac{1}{2}C_{34}\mathcal{S}_B - \frac{1}{2}C^*_{12}\mathcal{S}_F\right)\ \theta_3\theta_4$$

- Formally one defines a hermitian conjugate and a multiplication rule.

- It follows then, that it is sufficient to check the unitarity equations for the no $\boldsymbol{\theta}$ terms ($\bar{\phi}\phi \to \bar{\phi}\phi$) and four $\boldsymbol{\theta}$ terms ($\bar{\psi}\psi \to \bar{\psi}\psi$).

- Supersymmetric ward identities guarantee that the rest of the processes will obey the unitarity conditions.

- The above automatically accounts for the contribution of processes like $\bar{\phi}\phi \to \bar{\psi}\psi$ to the 4 boson scattering unitarity equation.

Unitarity in symmetric, antisymmetric and adjoint channels

- The T matrices for the symmetric, anti-symmetric and adjoint channels are all $O\left(\frac{1}{N}\right)$ - unitarity equation is linear.

$$\mathcal{T}_B(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \mathcal{T}_B^*(\mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_1, \mathbf{p}_2) \ ,$$
$$\mathcal{T}_F(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \mathcal{T}_F^*(\mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_1, \mathbf{p}_2)$$

- Linearity - no branch cuts in the physical domain of scattering.

- It can be explicitly checked that the unitarity conditions are obeyed using

$$J_B(q,\lambda) = J_B^*(-q,\lambda) \ , \ J_F(q,\lambda) = J_F^*(-q,\lambda)$$

- T matrix in the Singlet channel is O(1), unitarity equation is non-linear.

- The unitarity equation for any component process of interest can be derived starting from the superspace S matrix that was determined by ward identities.

Unitarity in singlet channel

- After doing lot of superspace processing, the unitarity equations can be written using $T(\theta) = i\cot(\theta/2)$.

$$\mathcal{T}_B^S = H_B T(\theta) + W_B - iW_2\delta(\theta)\ ,\ \mathcal{T}_F^S = H_F T(\theta) + W_F - iW_2\delta(\theta)\ ,$$

- bosonic unitarity equation

$$H_B - H_B^* = \frac{1}{8\pi\sqrt{s}}(W_2 H_B^* - H_B W_2^*)\ ,$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H_B H_B^*)\ ,$$

$$W_B - W_B^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_B^* - W_2^* W_B) - \frac{i}{4\sqrt{s}}(H_B H_B^* - W_B W_B^*) - \frac{iY}{4\sqrt{s}}(W_B - W_F)(W_B^* - W_F^*)$$

- Fermionic unitarity equation

$$H_F - H_F^* = \frac{1}{8\pi\sqrt{s}}(W_2 H_F^* - H_F W_2^*)\ ,$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H_F H_F^*)\ ,$$

$$W_F - W_F^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_F^* - W_2^* W_F) - \frac{i}{4\sqrt{s}}(H_F H_F^* - W_F W_F^*) - \frac{iY}{4\sqrt{s}}(W_B - W_F)(W_B^* - W_F^*)$$

Unitarity in singlet channel

- Unitarity equations are verified using

$$H_B = H_F = 4\sqrt{s}\sin(\pi\lambda),\ W_2 = 8\pi\sqrt{s}(\cos(\pi\lambda)-1),\ T(\theta) = i\cot(\theta/2)$$

$$W_B = J_B(\sqrt{s},\lambda)\frac{\sin(\pi\lambda)}{\pi\lambda}\ ,$$
$$W_F = J_F(\sqrt{s},\lambda)\frac{\sin(\pi\lambda)}{\pi\lambda}\ .$$

- Important fact is that the S matrix in the singlet channel must have the delta function modulated by the anyonic phase and modified crossing rules exactly as conjectured. Else unitarity fails.

- It is easier to understand what happens with the $\mathcal{N}=2$ theory.

- The symmetric, antisymmetric and adjoint channel S matrices for the $\mathcal{N}=2$ theory are tree level exact.

- Naive crossing would then imply that the singlet channel is also tree level exact. But if this is true $i(T^\dagger - T) = TT^\dagger$ would never be obeyed (LHS is zero)

Unitarity in singlet channel

- This is because, tree level S matrices do not have the singularities necessary to satisfy Cutkosky rules.

- Modified crossing rules and anyonic phase modulated identity piece resolve this puzzle

$$\mathcal{T}_B^{S;\mathcal{N}=2}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2) - 8m) \ ,$$
$$\mathcal{T}_F^{S;\mathcal{N}=2}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2) + 8m).$$

- The S matrix continues to be simple but is not tree level exact in the singlet channel.

- The non analytic piece makes $\mathcal{T}_B, \mathcal{T}_F$ not Hermitian - the term proportional to identity is imaginary.

- Thus unitarity equation is satisfied in the singlet channel for the N=2 theory as well.

Unitarity in singlet channel for the N=2 theory

- It is also very easy to demonstrate in an explicit calculation

$$H_B - H_B^* = \frac{1}{8\pi\sqrt{s}}(W_2 H_B^* - H_B W_2^*) \ ,$$

$$W_2 + W_2^* = -\frac{1}{8\pi\sqrt{s}}(W_2 W_2^* + 4\pi^2 H_B H_B^*) \ ,$$

$$W_B - W_B^* = \frac{1}{8\pi\sqrt{s}}(W_2 W_B^* - W_2^* W_B) - \frac{i}{4\sqrt{s}}(H_B H_B^* - W_B W_B^*) - \frac{iY}{4\sqrt{s}}(W_B - W_F)(W_B^* - W_F^*)$$

- For the N=2 theory

$$H_B = H_F = 4\sqrt{s}\sin(\pi\lambda) \ , \ W_B = -8m\sin(\pi\lambda) \ ,$$

$$W_F = 8m\sin(\pi\lambda) \ , \ W_2 = 8\pi\sqrt{s}(\cos(\pi\lambda) - 1) \ , \ Y(s) = \frac{-s + 4m^2}{16m^2}$$

- The first equation is trivially satisfied, the third one is satisfied because

$$(H_B H_B^* - W_B W_B^*) = -16\sin^2(\pi\lambda)(-s + 4m^2)$$

$$Y(W_B - W_F)(W_B^* - W_F^*) = 16\sin^2(\pi\lambda)(-s + 4m^2)$$

- The second equation is satisfied due to the same trignometric identity as in the Aharanov-Bohm case.

Part IIC

Scattering in N=3 theory (work in progress)

N=3 theory in N=1 superspace

- The $\mathcal{N}=3$ superconformal theory in the language of $\mathcal{N}=2$ superspace consists of a pair of chiral multiplets $(Q_i, \tilde{Q}^i)$ transforming in conjugate representations of the gauge group. D. Gaiotto, X. Yin

- We wish to formulate the theory in $\mathcal{N}=1$ superspace
  - It is not clear what is the analogue of supersymmetric light cone gauge in $\mathcal{N}=2$ superspace.
  - The machinery for scattering already is well formulated in $\mathcal{N}=1$ superspace.
  - Care: In this formalism the $U(1)$ sub group of the full $SU(2)$ R symmetry acts as a global symmetry.
  - Introduce pair of $\mathcal{N}=1$ superfields, carrying the $U(1)$ charges, to form $SU(2)$ doublets

$$\begin{pmatrix} Q_i \\ \bar{\tilde{Q}}_i \end{pmatrix} \rightarrow \begin{pmatrix} \Phi_i^+ \\ \Phi_i^- \end{pmatrix} \qquad \phi_i^A = \begin{pmatrix} \phi_i^+ \\ \phi_i^- \end{pmatrix}, \bar{\phi}_A^i = \begin{pmatrix} (\bar{\phi}^+)^i \\ (\bar{\phi}^-)^i \end{pmatrix}$$

Mass deformed N=3 theory in N=1 superspace

- The theory has one unique mass deformation : The mass deformation is a triplet under the $SU(2)$ R symmetry. C. Cardova, T. Dumitrescu, K. Intriligator

- A triplet mass deformation would appear in the component lagrangian as

$$-\bar{\phi}^A M_A{}^D M_D{}^E \phi_E + \bar{\psi}^{\beta A} M_A{}^D \psi_{\beta D}$$

$$M_A{}^B = \sum_{i=1}^{3} m_i (\sigma^i)_A{}^B$$

- We can always use the $SU(2)$ R symmetry to rotate to the σ^3 frame.

- The superspace action is

$$\begin{aligned}
\mathcal{S}^E_{\mathcal{N}=3} = -\int d^3x d^2\theta \Big[& \frac{\kappa}{4\pi} Tr\Big(-\frac{1}{4} D^\alpha \Gamma^\beta D_\beta \Gamma_\alpha + \frac{i}{6} D^\alpha \Gamma^\beta \{\Gamma_\alpha, \Gamma_\beta\} + \frac{1}{24}\{\Gamma^\alpha, \Gamma^\beta\}\{\Gamma_\alpha, \Gamma_\beta\}\Big) \\
& - \frac{1}{2}(D^\alpha \bar{\Phi}^+ + i\bar{\Phi}^+ \Gamma^\alpha)(D_\alpha \Phi^+ - i\Gamma_\alpha \Phi^+) - \frac{1}{2}(D^\alpha \bar{\Phi}^- + i\bar{\Phi}^- \Gamma^\alpha)(D_\alpha \Phi^- - i\Gamma_\alpha \Phi^-) \\
& - \frac{\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^+\Phi^+\right) - \frac{\pi}{\kappa}\left(\bar{\Phi}^-\Phi^-\right)\left(\bar{\Phi}^-\Phi^-\right) + \frac{4\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^-\Phi^-\right) + \frac{2\pi}{\kappa}\left(\bar{\Phi}^+\Phi^-\right)\left(\bar{\Phi}^-\Phi^+\right) \\
& - \left(m_0 \bar{\Phi}^+\Phi^+ - m_0 \bar{\Phi}^-\Phi^-\right)\Big]
\end{aligned}$$

Mass deformed N=3 theory

- The component action in the WZ gauge

$$
\begin{aligned}
S^L_{\mathcal{N}=3} = \int d^3x \Big[&\mathrm{Tr}\left(-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right)\right) \\
&+ i\bar\psi^A \not{\mathcal{D}}\psi_A + m_0\bar\psi^A(\sigma^3)_A{}^B\psi_B - \mathcal{D}^\mu\bar\phi_A\mathcal{D}_\mu\phi^A - m_0^2\bar\phi_A\phi^A \\
&- \frac{4\pi^2}{\kappa^2}(\bar\phi_A\phi^B)(\bar\phi_B\phi^C)(\bar\phi_C\phi^A) + \frac{4\pi}{\kappa}(\bar\phi_A\phi^B)(\bar\psi^A\psi_B) + \frac{2\pi}{\kappa}(\bar\psi^A\phi_B)(\bar\phi^B\psi_A) \\
&- \frac{4\pi}{\kappa}(\bar\psi^A\phi_A)(\bar\phi^B\psi_B) + \frac{2\pi}{\kappa}(\bar\psi^A\phi_A)(\bar\psi^B\phi_B) + \frac{2\pi}{\kappa}(\bar\phi^A\psi_A)(\bar\phi^B\psi_B) \\
&+ \frac{4\pi m_0}{\kappa}(\bar\phi^A\phi_A)(\bar\phi^C(\sigma_3)_C{}^D\phi_D)\Big]
\end{aligned}
$$

- has the N=3 supersymmetry invariance

$$
\begin{aligned}
Q_{BC\alpha}\phi_A =& \psi_{\alpha(B}\,\epsilon_{C)A}, \\
Q_{BC\alpha}\bar\phi^A =& -\bar\psi_{\alpha(B}\,\delta_{C)}{}^A, \\
Q_{BC\alpha}\psi_{\beta A} =& -i\mathcal{D}_{\alpha\beta}\phi_{(B}\,\epsilon_{C)A} + m_0 C_{\alpha\beta}\phi_{(B}(\sigma^3)_{C)A} \\
&+ \frac{2\pi}{\kappa}C_{\alpha\beta}(\bar\phi_A\phi_{(B})\phi_{C)} + \frac{2\pi}{\kappa}C_{\alpha\beta}(\bar\phi_{(B}\phi_{C)})\phi_A, \\
Q_{BC\alpha}\bar\psi^{\beta A} =& i\mathcal{D}_\alpha{}^\beta\bar\phi_{(B}\,\delta_{C)}{}^A + m_0\delta_\alpha{}^\beta\bar\phi_{(B}(\sigma_3)_{C)}{}^A \\
&+ \frac{2\pi}{\kappa}\delta_\alpha{}^\beta(\bar\phi_{(B}\phi^A)\bar\phi_{C)} - \frac{2\pi}{\kappa}\delta_\alpha{}^\beta(\bar\phi_{(B}\phi_{C)})\bar\phi^A, \\
Q_{BC\alpha}A^a_\mu =& -\frac{4\pi}{\kappa}(\gamma_\mu)_\alpha{}^\beta\bar\phi^i_{(B}(T^a)_i{}^j\psi_{C)\beta j} - \frac{4\pi}{\kappa}(\gamma_\mu)_\alpha{}^\beta\bar\psi^i_{\beta(B}(T^a)_i{}^j\phi_{C)j}
\end{aligned}
$$

Scattering in N=3 theory

- We can characterize the in/out states in terms of their $U(1)$ charges.

$$|\theta_1\theta_2\rangle \equiv |++\rangle\ ,\ |--\rangle\ ,\ |+-\rangle\ ,\ |-+\rangle$$

$$\langle\theta_3\theta_4| \equiv \langle++|\ ,\ \langle--|\ ,\ \langle-+|\ ,\ \langle+-|$$

- Neutral sector: the in/out states separately have $U(1)$ charge zero

$$A_1 = \langle +-|S_1|-+\rangle \equiv \Phi^+(\theta_3)\bar{\Phi}^+(\theta_4) \leftarrow \bar{\Phi}^+(\theta_1)\Phi^+(\theta_2)$$

$$A_2 = \langle -+|S_2|+-\rangle \equiv \Phi^-(\theta_3)\bar{\Phi}^-(\theta_4) \leftarrow \bar{\Phi}^-(\theta_1)\Phi^-(\theta_2)$$

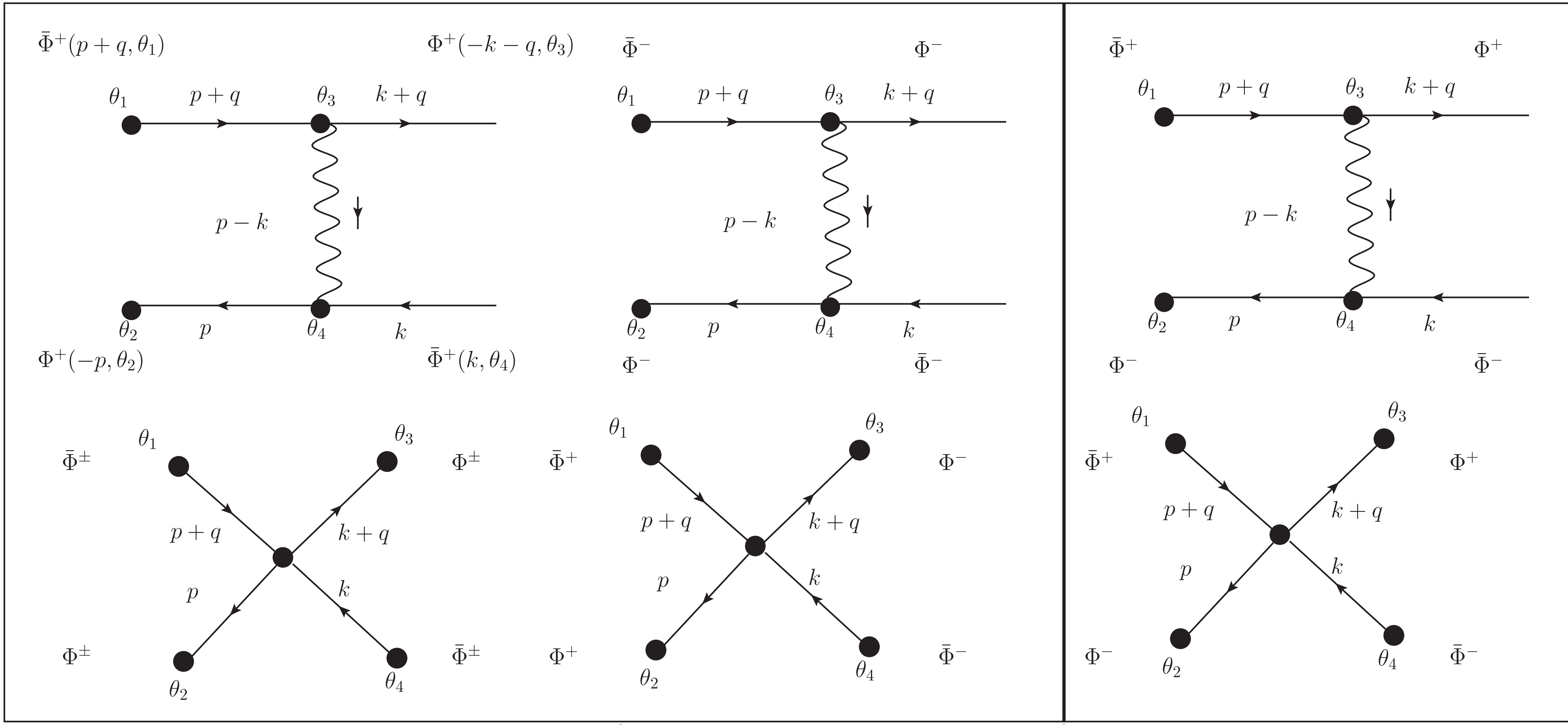
$$A_3 = \langle +-|S_3|+-\rangle \equiv \Phi^+(\theta_3)\bar{\Phi}^+(\theta_4) \leftarrow \bar{\Phi}^-(\theta_1)\Phi^-(\theta_2)$$

- Charged sector: the in/out states have $U(1)$ charge $\pm/\mp$

$$A_5 = \langle --|S_5|++\rangle \equiv \Phi^-(\theta_3)\bar{\Phi}^+(\theta_4) \leftarrow \bar{\Phi}^-(\theta_1)\Phi^+(\theta_2)$$

- On-shell supersymmetry analysis for the N=3 theory, predicts two independent functions, that characterize the full S matrix of the theory. Charged and neutral sectors do not mix - one function for each.

Tree level amplitudes



$$\bar{\Phi}^\pm\Phi^\pm \to \bar{\Phi}^\pm\Phi^\pm$$

$$T_B^{tree} = \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} - \frac{8m\pi}{\kappa}$$

$$T_F^{tree} = \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} + \frac{8m\pi}{\kappa}$$

$$\bar{\Phi}^+\Phi^+ \to \bar{\Phi}^-\Phi^-$$

$$T_B^{tree} = T_F^{tree} = 0$$

$$\bar{\Phi}^+\Phi^- \to \bar{\Phi}^-\Phi^+$$

$$T_B^{tree} = T_F^{tree} = \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-}$$

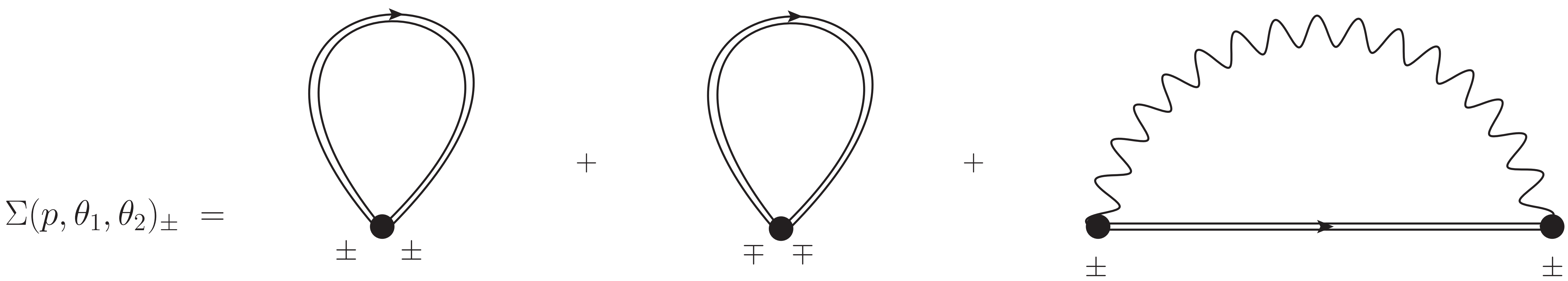
Exact propagator to all orders in λ

- Bare propagators

$$\langle \bar{\Phi}^{\pm}(\theta_1, p)\Phi^{\pm}(\theta_2, -p')\rangle = \frac{D^2_{\theta_1,p} \pm m_0}{p^2 + m_0^2}\delta^2(\theta_1 - \theta_2)(2\pi)^3\delta^3(p - p')$$

$$\langle \Gamma^{-}(\theta_1, p)\Gamma^{-}(\theta_2, -p')\rangle = -\frac{8\pi}{\kappa}\frac{\delta^2(\theta_1 - \theta_2)}{p_{--}}(2\pi)^3\delta^3(p - p')$$

- Integral equation for the self energy

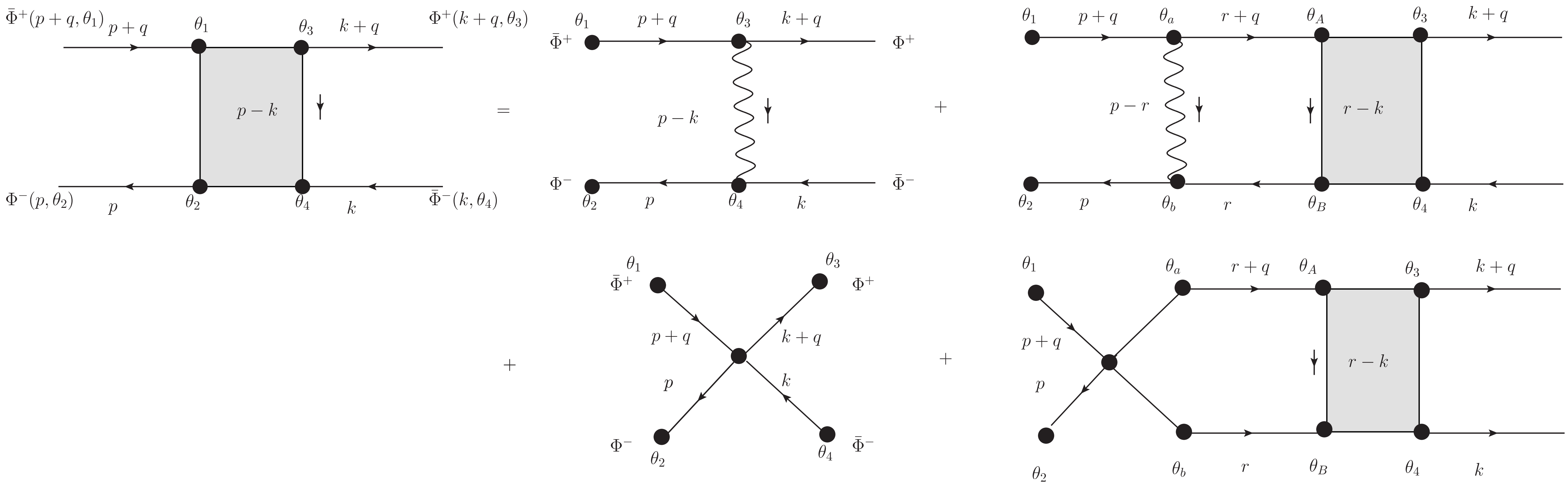


- All the contributions cancel among each other, and there is no mass renormalization as it happened for the N=2 theory.

- Thus the exact propagators to all orders in ’t Hooft coupling is same as the bare propagators.

Exact four point function to all orders in λ

- The integral equation in the charged sector is similar to the N=1,2 theory



- The integral equation in the neutral sector is a 2 x 2 matrix equation in superspace and is more involved (work in progress) but appears solvable.

- Nevertheless, in the charged sector we are able to solve the integral equations in the kinematic regime $q_\pm = 0$.

Charged Sector : S matrices

- The S matrix in the symmetric, antisymmetric and adjoint channels is very simple

$$\mathcal{T}_B^{\mathcal{N}=3} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2}$$
$$\mathcal{T}_F^{\mathcal{N}=3} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2}$$

- In fact this is same as the N=2 S matrix at m=0, thus in this sector the N=3 S matrices in the non-anyonic channels are not renormalized to all orders in ’t Hooft coupling λ.

- Applying the conjecture, the singlet channel S matrix is given by

$$\mathcal{T}_B^{S;\mathcal{N}=3}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2)) \ ,$$
$$\mathcal{T}_F^{S;\mathcal{N}=3}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2))$$

- All the checks such as duality, unitarity and non-relativistic limit go through in a straightforward manner in the charged sector.

Neutral Sector : S matrices

- Since the full N=3 S matrix must have SU(2) covariance, it is natural to expect that the S matrices in the neutral sector are also tree level exact in the symmetric, antisymmetric and adjoint channels.

- And the modified crossing rules must be applied for the singlet channel as before.

- All the consistency checks, such as duality, unitarity and non-relativistic limit should hold sector by sector.

- However, it would be very satisfying (to the eye and to the mind) if the presentation is formulated in a N=3 covariant way.

…work in progress

K.I, S.Jain, L.Janagal, S. Minwalla, A.Shukla

Part III

Summary

Summary

- In QFT crossing symmetry is the statement that there exists a single function from which all the channels of scattering can be obtained by analytic continuation.

- Naive crossing symmetry works when the scattering particles are of bosonic or fermionic statistics.

- When the scattering quanta are anyonic, the usual crossing rules lead to non-unitary S matrices.

- General structure of S matrices in the anyonic channels is of the form

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\mathrm{naive}}(s,\theta)$$

  - S matrix has an identity term modulated by the anyonic phase.
  - The usual crossing rules are modified.
  - modifications appear to be universal for all CS matter theories.

Summary

- We presented evidence for universality by studying $2 \to 2$ scattering in supersymmetric Chern-Simons theories at large N.

- The symmetric, antisymmetric and adjoint channels are non-anyonic in the large N and the usual crossing rules hold.

- The singlet channel is anyonic in the large N limit and the usual crossing rules lead to non-unitary S matrices.

- The conjectured modifications lead to S matrices that are unitary, invariant under bosonization duality and have the correct non-relativistic limit.

- We presented our tests in $\mathcal{N} = 1, 2, 3$ supersymmetric Chern-Simons matter theories, and find substantial evidence for the conjecture of

  S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

Summary

- For the $\mathcal{N} = 2, 3$ theories, the S matrices in the non-anyonic channel are not renormalized to all orders in 't Hooft coupling λ.

- In the anyonic channel the S matrix is not tree level exact, but continues to be simple. We saw how the modifications suggested resolve the issue of unitarity in this connection.

- It would be very interesting and tempting to conjecture that:
  - In CS matter theories with $\mathcal{N} \geq 2$ susy, the S matrices in the non-anyonic channels are not renormalized to all orders in λ.

  - In the anyonic channel, the identity term gets modulated by anyonic phase and modified crossing rules apply as expected.

  - However for $\mathcal{N} \geq 4$ the theories have bi-fundamental matter, and care has to be taken while extrapolating any statement.

Summary

- It would be worthwhile to rigorously prove the delta function term and modified crossing rules.

- The modified crossing factor $\frac{\sin \pi\lambda}{\pi\lambda}$ appears all over the place in CS matter theories. It is in fact the expectation value of a circular Wilson loop on S^3. Understanding how this factor appears in the S matrix through an honest computation in the singlet channel may be key.

- It is important to observe that at finite N and κ all the scattering channels are anyonic!

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa} \ , \ \nu_{Asym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}$$
$$\nu_{Adj} = \frac{1}{N\kappa} \ , \ \nu_{Sin} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

- Understanding or even conjecturing the crossing rules in this case, may have practical applications!


תודה