Amplitudes and hidden symmetries in N=2 Chern-Simons Matter theory

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Nov 9, 2017

Based on

- K.I, S.Jain, P.Nayak, V.Umesh, **arXiv :1710.04227**
- K.I, S.Jain, S.Majumdar, P.Nayak, T.Neogi, T.Sharma, R.Sinha, V.Umesh, **arXiv : 1711.mmnnp**
- K.I, S.Jain, T.Sharma, V.Umesh, **arXiv : 1711.mmnnp**

- K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh, S.Yokoyama, **arXiv: 1505.06571**, JHEP 1510 (2015) 176.
- S.Jain, M.Mandlik, S.Minwalla, T.Takimi, S.Wadia, S.Yokoyama, **arXiv:1404.6373,** JHEP 1504 (2015) 129.

Part I

Introduction

N=2 Chern-Simons matter theory

- General renormalizable $\mathcal{N}=2$ theory with one fundamental multiplet

$$\mathcal{S}^L_{\mathcal{N}=2} = \int d^3x \Big[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right)$$
$$+ \bar{\psi} i \not{D}\psi - \mathcal{D}^\mu\bar{\phi}\mathcal{D}_\mu\phi + \frac{4\pi^2}{\kappa^2}(\bar{\phi}\phi)^3 + \frac{4\pi}{\kappa}(\bar{\phi}\phi)(\bar{\psi}\psi) + \frac{2\pi}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi)\Big]$$

- The theory exhibits a **strong-weak self duality** under the duality map

$$\kappa' = -\kappa\ ,\ \ N' = |\kappa| - N + 1\ ,\ \ \lambda' = \lambda - \mathrm{Sgn}(\lambda)$$

- **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** $2 \to 2$ scattering amplitudes to all orders in the 't Hooft coupling.

- In the **(non-anyonic)** symmetric, anti-symmetric and adjoint channels of scattering the amplitude is tree-level exact to all orders in λ.

- In the **(anyonic)** singlet channel the coupling dependence is extremely simple.

2→2 scattering amplitude to all orders in λ

- Tree level super amplitude

$$T_{\rm tree} = \frac{4\pi}{\kappa}\frac{\langle 12\rangle}{\langle 23\rangle}\delta(\sum_{i=1}^{4} p_i)\delta^2\left(\mathcal{Q}\right)$$

$$\delta^2(Q) = \sum_{i<j=1}^{n} \langle ij\rangle \eta_i\eta_j$$

- **All loop** super amplitude **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama**

$$T^{\rm non-anyonic}_{\rm all-loop} = T_{\rm tree}$$

$$T^{\rm anyonic}_{\rm all-loop} = N\frac{\sin(\pi\lambda)}{\pi\lambda}T_{\rm tree}$$

$$S^{\rm non-anyonic} = I + i\, T^{\rm non-anyonic}_{\rm all-loop}$$

$$S^{\rm anyonic} = \cos\left(\pi\lambda\right) I + i\, T^{\rm anyonic}_{\rm all-loop}$$

- Passes all consistency checks: **Unitarity and Duality**

Motivation

- Why is the $2 \to 2$ particle scattering in the non-anyonic channels **tree level exact**? and why does it have a very **simple coupling dependence in anyonic channel**?
- Maybe some **powerful symmetry** that protects the amplitude from renormalization.
- Is it possible to compute **all loop $m \to n$ scattering amplitudes** in the N=2 theory at least in the planar limit?
- Does the **non-renormalization** results of the $2 \to 2$ scattering continue to persist for arbitrary higher point amplitudes?
- What are the **generalization of the crossing rules** for the anyonic channels in an arbitrary $m \to n$ scattering.
- These computations would **test the duality** in regions un-probed by large N perturbation theory yet.

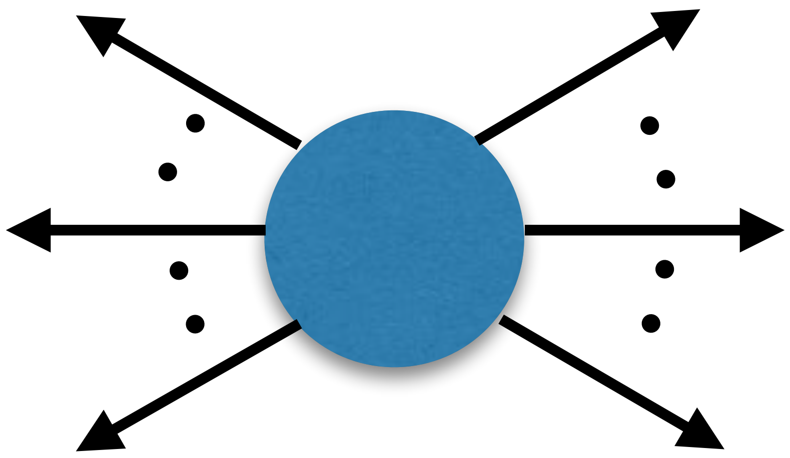
What we do

- As a first step towards the all loop m→n scattering, is it possible to write down **arbitrary m → n tree level amplitudes** ?
- We are able to achieve this via **BCFW recursions** **K.I, Jain, Nayak, Umesh**
- As a first step towards thinking about higher point loop amplitudes we identify a **hidden symmetry** in the 2 → 2 amplitude computed earlier that might explain the non-renormalization.
- This symmetry is known as **dual superconformal symmetry**.

  K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh
- The superconformal symmetry and dual superconformal symmetry together generate an infinite dimensional symmetry known as the **Yangian**.
- This suggests that the theory we are dealing with may be **integrable!**

Part II

All tree level amplitudes



- K.I, S.Jain, P.Nayak, V.Umesh, **arXiv :1710.04227**

BCFW recursions in 2+1 dimensions

- Recursion relations enable to construct **n point tree level scattering amplitudes from lower point tree level amplitudes.**

Britto, Cachazo, Feng, Witten

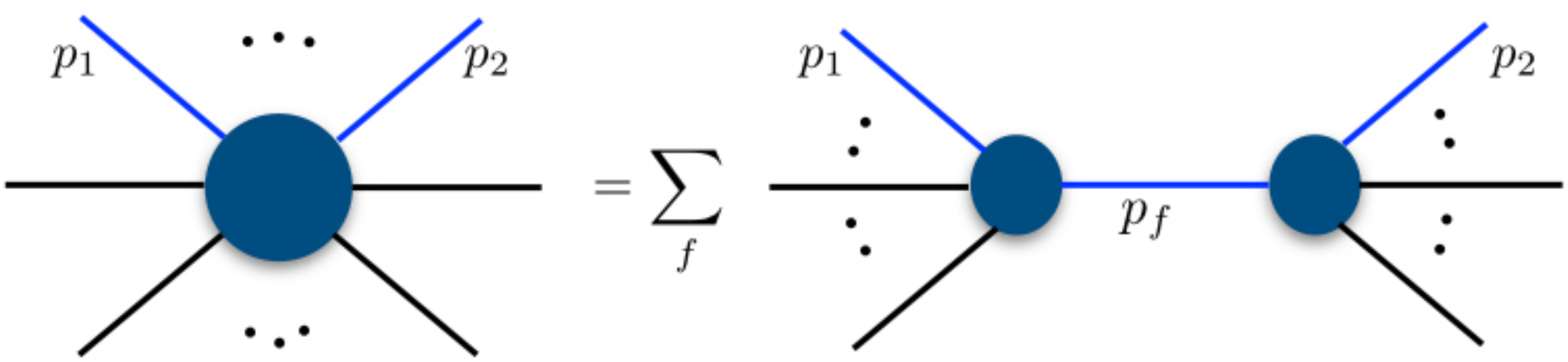
- Central idea:
  - Tree level amplitudes are **continuously deformable** analytic functions of momenta.
  - Only type of singularities that can appear at tree level are **simple poles**
  - One can **reconstruct amplitudes** for generic scattering kinematics knowing its behavior in **singular kinematics**.
  - In these singular regions **amplitudes factorize** into causally disconnected amplitudes with fewer legs, connected by an i**ntermediate onshell state**.

BCFW recursions in 2+1 dimensions

- Promote the amplitude into a one complex parameter family of amplitudes

$$A(z) = A(p_1, \ldots p_i(z), p_{i+1}, \ldots p_l(z), \ldots, p_{2n})$$

- The necessary and sufficient conditions are:
  - The momentum shift should preserve on-shell conditions and momentum conservation.
  - The amplitude should be asymptotically well behaved.



- A higher point amplitude factorizes into lower point amplitudes!

Preserving onshell conditions

- In 3d the momentum shift is non-linear in z

$$\begin{pmatrix} \hat{\lambda}_1 \\ \hat{\lambda}_2 \end{pmatrix} = R \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \qquad R = \begin{pmatrix} \frac{z+z^{-1}}{2} & -\frac{z-z^{-1}}{2\ i} \\ \frac{z-z^{-1}}{2\ i} & \frac{z+z^{-1}}{2} \end{pmatrix}$$

$$p_i \to \frac{p_{ij}}{2} + qz^2 + \tilde{q}z^{-2} \qquad q^{\alpha\beta} = \frac{1}{4}(\lambda_2 + i\lambda_1)^\alpha (\lambda_2 + i\lambda_1)^\beta$$

$$p_j \to \frac{p_{ij}}{2} - qz^2 - \tilde{q}z^{-2} \qquad \tilde{q}^{\alpha\beta} = \frac{1}{4}(\lambda_2 - i\lambda_1)^\alpha (\lambda_2 - i\lambda_1)^\beta \ .$$

- and **preserves the onshell condition**

$$p_i^2 = 0 \ , p_j^2 = 0$$

$$q.\tilde{q} = -\frac{1}{4} p_i.p_j \ , \ \ q + \tilde{q} = \frac{1}{2}(p_i - p_j) \ , q.p_{ij} = 0 \ , \ \ \tilde{q}.p_{ij} = 0$$

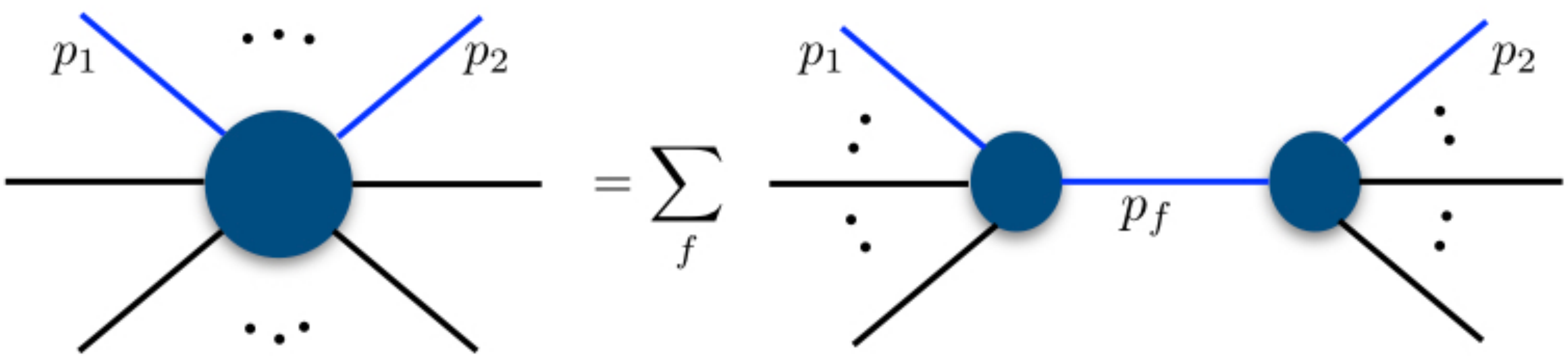
Gang, Huang, Koh, Lee, Lipstein

Asymptotic behavior

- Onshell susy methods, encode the **component amplitudes into a superamplitude**.
- **Susy ward identities** relate various component amplitudes and reduce the number of independent amplitudes.
- Susy also ensures that **if the independent component amplitudes are well behaved then the entire superamplitude is well behaved**.
- Using two independent methods we showed that the superamplitude is well behaved
    - **Background field expansion**
    - **Explicit Feynman diagram computation** of component amplitudes.
- The recursion formula then follows from **Cauchy residue theorem**.

The recursion formula for arbitrary 2n point superamplitude

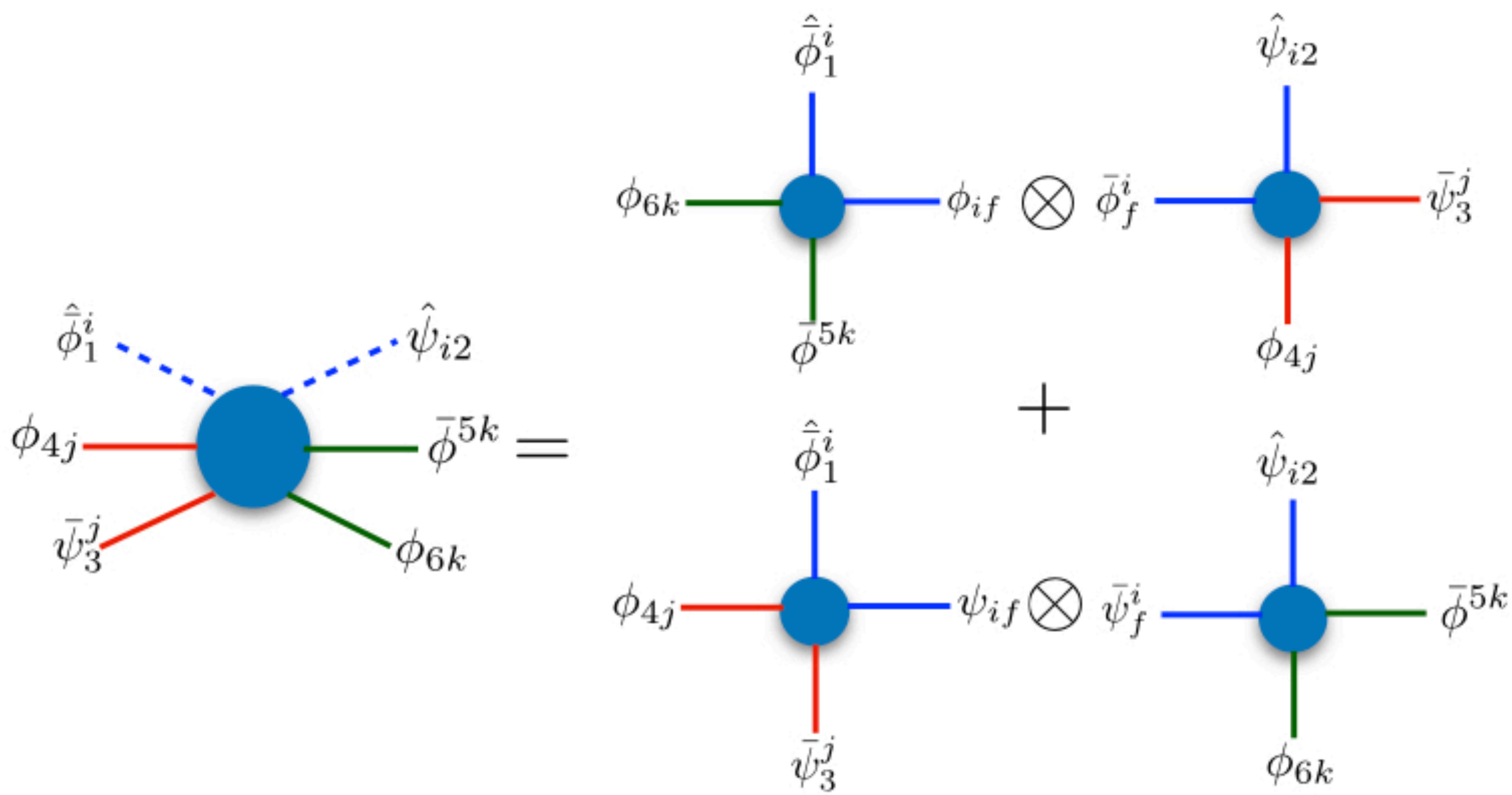
$$A_{2n}(z=1) = \sum_f \int \frac{d\theta}{p_f^2} \Bigg(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} A_L(z_{a;f}, \theta) A_R(z_{a;f}, i\theta) + (z_{a;f} \leftrightarrow z_{b;f}) \Bigg)$$



- $z_{a;f}, z_{b;f}$ are zeroes of $p_f^2(z) = 0$
- The formula can be recursively applied to write down any **higher point superamplitude as products of four point superamplitude**.

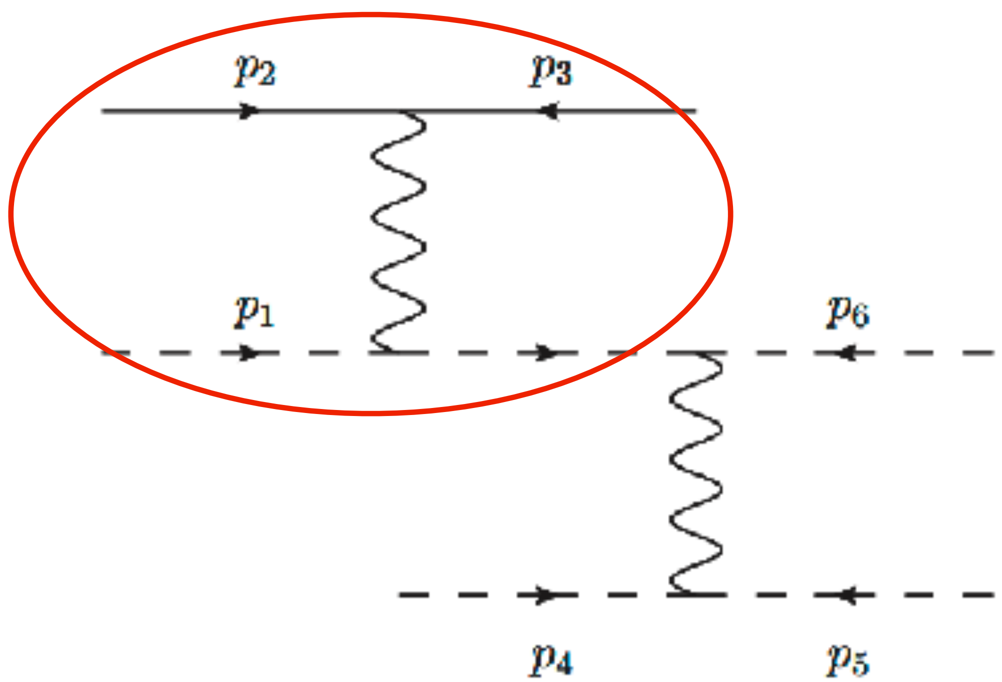
Eg: Six point amplitude as product of four point amplitudes

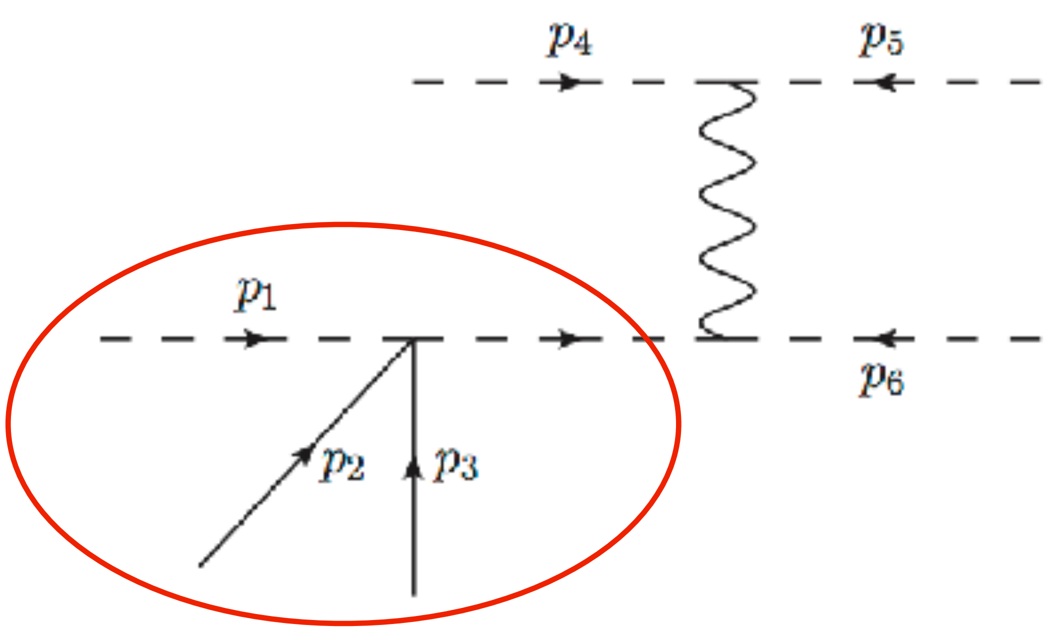
$$
\begin{aligned}
&\langle \bar\phi_1 \psi_2 \bar\psi_3 \phi_4 \bar\phi_5 \phi_6 \rangle = \\
&\left(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} \langle \hat{\bar\phi}_1 \hat\phi_f \bar\phi_5 \phi_6 \rangle_{z_{a;f}} \langle \hat{\bar\phi}_{(-f)} \hat\psi_2 \bar\psi_3 \phi_4 \rangle_{z_{a;f}} + (z_{a;f} \leftrightarrow z_{b;f}) \right) \frac{i}{p_f^2} \Big|_{p_f = p_{234}} \\
&+ \left(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} \langle \hat{\bar\phi}_1 \hat\psi_f \bar\psi_3 \phi_4 \rangle_{z_{a;f}} \langle \hat{\bar\psi}_{(-f)} \hat\psi_2 \bar\phi_5 \phi_6 \rangle_{z_{a;f}} + (z_{a;f} \leftrightarrow z_{b;f}) \right) \frac{i}{p_f^2} \Big|_{p_f = p_{256}}
\end{aligned}
$$



Eg: Six point amplitude: Asymptotic behavior

- The **Asymptotic behavior involves very precise cancellations of divergences** in the Feynman diagram approach.
- For eg, the process $\langle\bar{\psi}_1\phi_2\bar{\phi}_3\psi_4\bar{\phi}_5\phi_6\rangle$ gets contribution from 15 diagrams.
- 5 of them are well behaved, the remaining 10 are **individually divergent**, However the **divergences cancel pair wise**.
- Typical cancellations involve look like





$$\sim -\frac{8\pi^2 iz}{\kappa^2}\frac{\langle q3\rangle\langle 45\rangle\langle 56\rangle\langle 46\rangle}{p_{45}^2 p_{123}^2} + \mathcal{O}(\frac{1}{z})\ ,\ \sim \frac{8\pi^2 iz}{\kappa^2}\frac{\langle q3\rangle\langle 45\rangle\langle 56\rangle\langle 46\rangle}{p_{45}^2 p_{123}^2} + \mathcal{O}(\frac{1}{z})$$

Recursion relations for non-supersymmetric theories!

- BCFW does not apply to the **non-susy CS coupled to fermions/bosons** since the amplitudes **do not have good asymptotic behavior**.

- It is possible to extract the recursion relations for non-susy fermionic/bosonic CS matter theories from the N=2 results!! Eg:

    - At tree level, the Feynman diagrams for an **all fermion amplitude are same** for susy/non-susy theory.

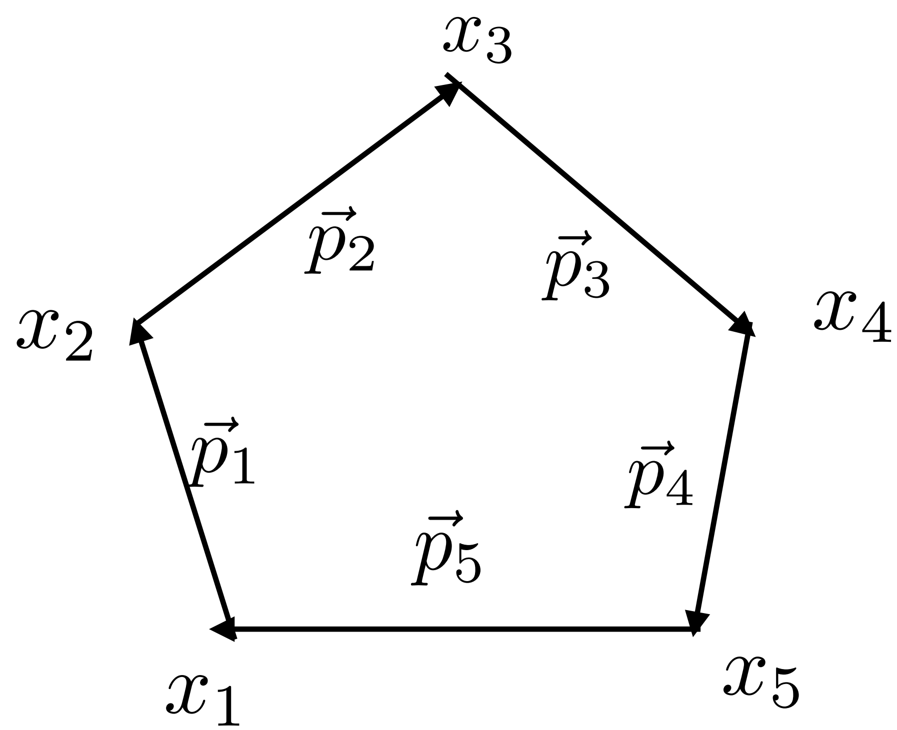
    - **Susy ward identity**: The four point super amplitude is completely specified by one function, choose it to be the four fermion amplitude.

    - Use this information recursively in the BCFW formula!

- An arbitrary higher point tree level amplitude in the fermionic CS matter theory can be entirely written in terms of **4 fermion amplitude**.

Part III

Dual superconformal invariance



- K.I, S.Jain, S.Majumdar, P.Nayak, T.Neogi, T.Sharma, R.Sinha, V.Umesh, **arXiv : 1711.mmnnp**

Dual variables

- The dual variables realize momentum conservation linearly in the x variables

$$x_{i,i+1}^{\alpha\beta} = x_i^{\alpha\beta} - x_{i+1}^{\alpha\beta} = p_i^{\alpha\beta} = \lambda_i^\alpha \lambda_i^\beta$$
$$\theta_{i,i+1}^{\alpha} = \theta_i^{\alpha} - \theta_{i+1}^{\alpha} = q_i^{\alpha} = \lambda_i^\alpha \eta_i$$

- momentum and supermomentum conservation imply

$$P^{\alpha\beta} = \sum_i p_i^{\alpha\beta} = x_{n+1}^{\alpha\beta} - x_1^{\alpha\beta} = 0,$$
$$\mathcal{Q}^{\alpha} = \sum_i q_i^{\alpha} = \theta_{n+1}^{\alpha} - \theta_1^{\alpha} = 0.$$

- The **four point super amplitude in dual space**

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\,\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- Goal is to show that this is **invariant under the superconformal symmetry in the dual variables**.

Superconformal algebra in dual space

- The N=2 superconformal algebra is generated by

$$\{P_{\alpha\beta},\ M_{\alpha\beta},\ D,\ K_{\alpha\beta},\ R,\ Q_\alpha,\ \bar{Q}_\alpha,\ S_\alpha,\ \bar{S}_\alpha\}$$

$$P_{\alpha\beta} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i^{\alpha\beta}}, \quad D = -\sum_{i=1}^{n}\left(x_i^{\alpha\beta}\frac{\partial}{\partial x_i^{\alpha\beta}} + \frac{1}{2}\theta_i^\alpha \frac{\partial}{\partial \theta_i^\alpha}\right),$$

$$Q_\alpha = \sum_{i=1}^{n} \frac{\partial}{\partial \theta_i^\alpha}, \quad \bar{Q}_\alpha = \sum_{i=1}^{n} \theta_i^\beta \frac{\partial}{\partial x_i^{\beta\alpha}},$$

$$M_{\alpha\beta} = \sum_{i=1}^{n}\left(x_{i\alpha}{}^{\gamma}\frac{\partial}{\partial x_i^{\gamma\beta}} + \frac{1}{2}\theta_{i\alpha}\frac{\partial}{\partial \theta_i^\beta}\right), \quad R = \sum_{i=1}^{n} \theta_i^\alpha \frac{\partial}{\partial \theta_i^\alpha}$$

- The remaining generators can be expressed using the inversion operator

$$I\left[x_i^{\alpha\beta}\right] = \frac{x_i^{\alpha\beta}}{x_i^2}, \quad I\left[\theta_i^\alpha\right] = \frac{x_i^{\alpha\beta}\theta_{i\beta}}{x_i^2}$$

$$K_{\alpha\beta} = I P_{\alpha\beta} I, \quad S_\alpha = I Q_\alpha I, \quad \bar{S}_\alpha = I \bar{Q}_\alpha I.$$

Dual superconformal invariance of the superamplitude

- A general four point amplitude in N=2 theory written in dual space

$$\mathcal{A}_4 = F(x_{i,j}^2)\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- Note that the **delta functions transform under the inversion** as

$$I\left[\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)\right] = x_1^4\ \delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- Whereas in the **N=4 and ABJM case, the corresponding delta function is invariant under the inversion**!, and the corresponding F function is invariant under the symmetry generators with homogeneous weights.

$$\tilde{K}^{\alpha\beta}\mathcal{A}^{(4)}_{ABJM} = \left(K^{\alpha\beta} + \frac{1}{2}\sum_{j=1}^{4}\Delta_j x_j^{\alpha\beta}\right)\mathcal{A}^{(4)}_{ABJM} = 0\ ,\ \ \Delta_i = \{1,1,1,1\}$$

- So **it was expected that the superamplitude in the N=2 theory would not have any dual superconformal invariance** at all.

- However, in the N=2 case, **dual superconformal invariance, still works but the weights become non-homogeneous.**

Dual superconformal invariance of the superamplitude

- The four point amplitude in the N=2 theory is

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- In this form the **translation, Lorentz invariance and supersymmetry invariance of the amplitude is manifest.**

- The amplitude is just a function of the square differences in the x variable.

- Under Dilatations it transforms as a eigenfunction of weight 4.

- Under R symmetry it transforms as eigenfunction of weight 2.

- To show the dual superconformal invariance it is sufficient to show the invariance under $K_{\alpha\beta}, S_\alpha, \bar{S}_\alpha$

Dual superconformal invariance of the superamplitude

$$
\begin{aligned}
K_{\alpha\beta}\left[\mathcal{A}_4\right] &= I P_{\alpha\beta} I \left[\mathcal{A}_4\right] \\
&= I \sum_{i=1}^{4} \partial_{i\alpha\beta} \left[x_1^4 \sqrt{\frac{x_2^2 x_4^2}{x_1^2 x_3^2}}\, \mathcal{A}_4 \right] \\
&= I \left[-\frac{1}{2} x_1^4 \sqrt{\frac{x_2^2 x_4^2}{x_1^2 x_3^2}} \left(3\frac{x_1^{\alpha\beta}}{x_1^2} + \frac{x_2^{\alpha\beta}}{x_2^2} + \frac{x_4^{\alpha\beta}}{x_4^2} - \frac{x_3^{\alpha\beta}}{x_3^2} \right) \mathcal{A}_4 \right] \\
&= -\frac{1}{2} \left(3x_1^{\alpha\beta} + x_2^{\alpha\beta} + x_4^{\alpha\beta} - x_3^{\alpha\beta} \right) \mathcal{A}_4 \\
&= -\frac{1}{2} \left(\sum_{j=1}^{4} \Delta_j x_j^{\alpha\beta} \right) \mathcal{A}_4 \qquad \text{w/} \quad \{\Delta_j\} = \{3, 1, -1, 1\}
\end{aligned}
$$

- So the invariance under $K_{\alpha\beta}, S_\alpha, \bar{S}_\alpha$

$$
\tilde{K}^{\alpha\beta} \mathcal{A}^{(4)} = \left(K^{\alpha\beta} + \frac{1}{2} \sum_{j=1}^{4} \Delta_j x_j^{\alpha\beta} \right) \mathcal{A}^{(4)} = 0 \qquad \tilde{\bar{S}}_\alpha \left[\mathcal{A}_4\right] = \left(\bar{S}_\alpha + \frac{1}{2} \left(\sum_{j=1}^{4} \Delta_j \theta_{j\alpha} \right) \right) \left[\mathcal{A}_4\right] = 0
$$

$$
S_\alpha \left[\mathcal{A}_4\right] = I Q_\alpha I \left[\mathcal{A}_4\right] = I Q_\alpha \left[x_1^4 \sqrt{\frac{x_2^2 x_4^2}{x_1^2 x_3^2}}\, \mathcal{A}_4 \right] = 0.
$$

Dual superconformal invariance at all loops

- We showed that the function A4 is dual superconformal invariant!

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- The **tree level superamplitude is dual superconformal invariant**.

$$T_{tree} = \frac{4\pi}{\kappa}\mathcal{A}_4$$

- The **all loop results** computed in **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** are **also dual superconformal invariant.**

$$T_{sym}^{all\ loop} = T_{Asym}^{all\ loop} = T_{Adj}^{all\ loop} = T_{tree}$$

$$T_{sing}^{all\ loop} = N\frac{\sin\pi\lambda}{\pi\lambda}T_{tree}$$

- Now that we know this symmetry exists, **can we reverse the argument and do an S matrix bootstrap to fix the general structure of the amplitude**?

Constraining amplitudes from dual superconformal symmetry

- The **four point amplitude in momentum space** can be interpreted as a **four point correlator in dual space**, then dual conformal invariance fixes

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$
$$= \frac{1}{x_{12}^{\Delta_1+\Delta_2} x_{34}^{\Delta_3+\Delta_4}} \left(\frac{x_{24}}{x_{14}}\right)^{\Delta_1-\Delta_2} \left(\frac{x_{14}}{x_{13}}\right)^{\Delta_3-\Delta_4} f(u,v,\kappa,\lambda)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}.$$

- However, the operators are null separated since

$$x_{12}^2 = p_1^2 = 0, \quad x_{14}^2 = p_4^2 = 0,$$
$$x_{34}^2 = p_3^2 = 0, \quad x_{23}^2 = p_2^2 = 0$$

- The correlator is understood in a limiting sense $u \to 0\ ,\ v \to 0\ ,\ \dfrac{u}{v} = \text{fixed}$
- **If dual superconformal symmetry is exact it fixes the momentum dependence completely***

$$f(u,v,\kappa,\lambda) = g(\kappa,\lambda)$$

Constraining amplitudes from dual superconformal symmetry

- In general the S matrix could get complicated functions with poles and branch cuts.

- If dual conformal invariance is an exact symmetry at loop level then no such behavior appears.

- Non trivial momentum dependence could still appear from $x_{i,j}^{\Delta}$ when Δ gets correction from loops.

- This can give rise to log dependence for instance, However these do not appear if there are no IR divergences.

- **If we assume that there are no IR divergences, and that the dual conformal invariance is an exact symmetry, then the momentum dependence is fixed**.

4 point amplitude as a free field correlator in dual space

- Recall that $\{\Delta_1, \Delta_2, \Delta_3, \Delta_4\} = \frac{1}{2}\{4-1, 1, -1, 1\}$
- The factor of 4 is due to momentum+supermomentum conservation and can be removed.

$$\Delta_1 = \frac{1}{2}(4-1) = 2 - \tilde{\Delta}_1,$$

- With this identification the operator dimensions are

$$\tilde{\Delta}_1 = \Delta_3 = -\frac{1}{2}$$

$$\Delta_2 = \Delta_4 = \frac{1}{2}$$

- The four point correlator in dual space gets fixed to (cancellations in limiting sense)

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = g(\kappa, \lambda)\sqrt{\frac{x_{13}^2}{x_{24}^2}}$$

- Same as the amplitude without the delta functions!

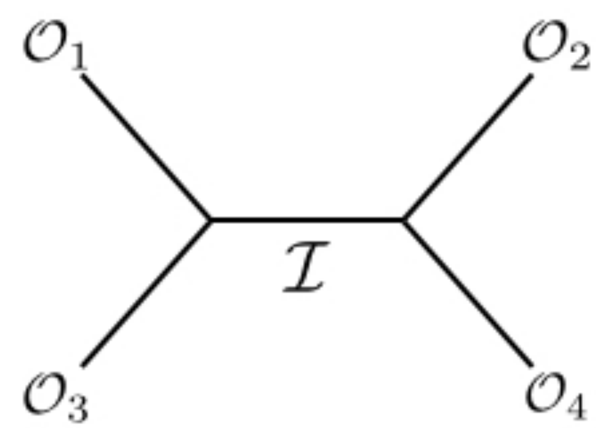
$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

4 point amplitude as a free field correlator in dual space

- In a general CFT, in the **double light cone limit, only Identity operators are expected to contribute**!

- In the channel where $(\mathcal{O}_1, \mathcal{O}_3)$ and $(\mathcal{O}_2, \mathcal{O}_4)$ are brought together

$$\tilde{\Delta}_1 = \Delta_3 = -\frac{1}{2}$$
$$\Delta_2 = \Delta_4 = \frac{1}{2}$$



- The four point amplitude can be accounted for by an **identity exchange**.

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \langle \mathcal{O}_1(x_1)\mathcal{O}_3(x_3)\rangle\langle \mathcal{O}_2(x_2)\mathcal{O}_4(x_4)\rangle$$
$$= c_1 c_2 \sqrt{\frac{x_{13}^2}{x_{24}^2}}$$

- This suggests $g(\kappa, \lambda) = c_1 c_2.$

- It would be interesting to understand the CFT interpretation of these operators, and also to see what happens in the cross channel.

Part III
Summary

Summary

- We started with a goal of computing **arbitrary m→n tree level scattering amplitudes** in U(N) $\mathcal{N}=2$ Chern-Simons matter theories with fundamental matter.

- We achieved this via **BCFW recursion relations**, this enabled us to express arbitrary n point amplitudes as products of four point amplitudes!

- We saw an explicit example where the six point amplitude is expressed as a product of two four point amplitudes via two factorization channels.

- The non-susy amplitudes do not satisfy the BCFW requirements.

- However we were able to use the fact that the four point superamplitude in the $\mathcal{N}=2$ theory is specified by one function and that the tree level amplitudes are identical to the non-susy case to write recursions for the non-susy theory as well.

- We saw an explicit example for the six fermion amplitude in the fermion coupled Chern-Simons theory.

Summary

- We showed that the **all loop 2→2 scattering amplitude** computed in **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** is **dual superconformal invariant**.
- Thus **dual superconformal symmetry is all loop exact**, at least for the 4 point amplitude.
- The presence of dual conformal symmetry then allows us to interpret the **amplitude in momentum space as a correlator in dual space**.
- We argued that **if dual conformal symmetry was an exact symmetry it fixed the momentum dependence of the amplitude completely**.
- We interpreted the four point amplitude in dual space as a free field correlator where the identity operator exchange accounted for it.
- However, **general principles such as unitarity, duality and dual conformal symmetry are insufficient to fix the overall coupling dependence**.

Immediate things to do: Yangian

- The presence of the **superconformal and dual superconformal symmetries indicate a Yangian symmetry** in the amplitude.
- A Yangian superalgebra is a **set of level-zero and level-one generators** that obey a graded commutation relation

$$[J_A^{(0)}, J_B^{(0)}\} = f_{AB}^{C} J_C^{(0)}, \quad [J_A^{(1)}, J_B^{(0)}\} = f_{AB}^{C} J_C^{(1)}$$

- The **level zero generators** are identified with the **space time superconformal generators,** and the structure constant is that of the $Osp(2|4)$ superconformal algebra.
- Aside from Jacobi identity, the level 1 and level 0 generators also satisfy an identity known as the Serre relation.
- At level 1 a general ansatz **Checherin, Kazakov,Loebert, Muller, Zhong**

$$J_A^{(1)} = \frac{1}{2} f_A^{BC} \sum_{j<k} J_{jC}^{(0)} J_{kB}^{(0)} + \sum_k v^l J_{lA}^{(0)}$$

- The goal is now to **identify the dual superconformal generators with the level 1 generators**, successive commutations produce higher level generators leading to an infinite dimensional algebra.

K.I, Jain, Sharma, Umesh… in progress

Things to do: Yangian for higher point amplitudes

- As an extension of the above story, **dual conformal invariance for higher point amplitudes and Yangian invariance**.
- A general n point correlator has n(n-3)/2 independent cross ratios, for eg six point function out of 9 crossratios, three survive onshell

$$u = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2}, \quad v = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{14}^2}, \quad , w = \frac{x_{35}^2 x_{26}^2}{x_{36}^2 x_{25}^2}.$$

- The six point amplitude can be determined upto a function of 3 variables. in principle this can allow for much more complicated dependence on the momentum.
- In general **higher point amplitudes** need not be proportional to the tree level answer as it was the case in the four point amplitude, and they **can also get non-trivial renormalization**.
- If the Yangian story goes through, it would be interesting to see if the **symmetry can be seen at the level of the Lagrangian**!

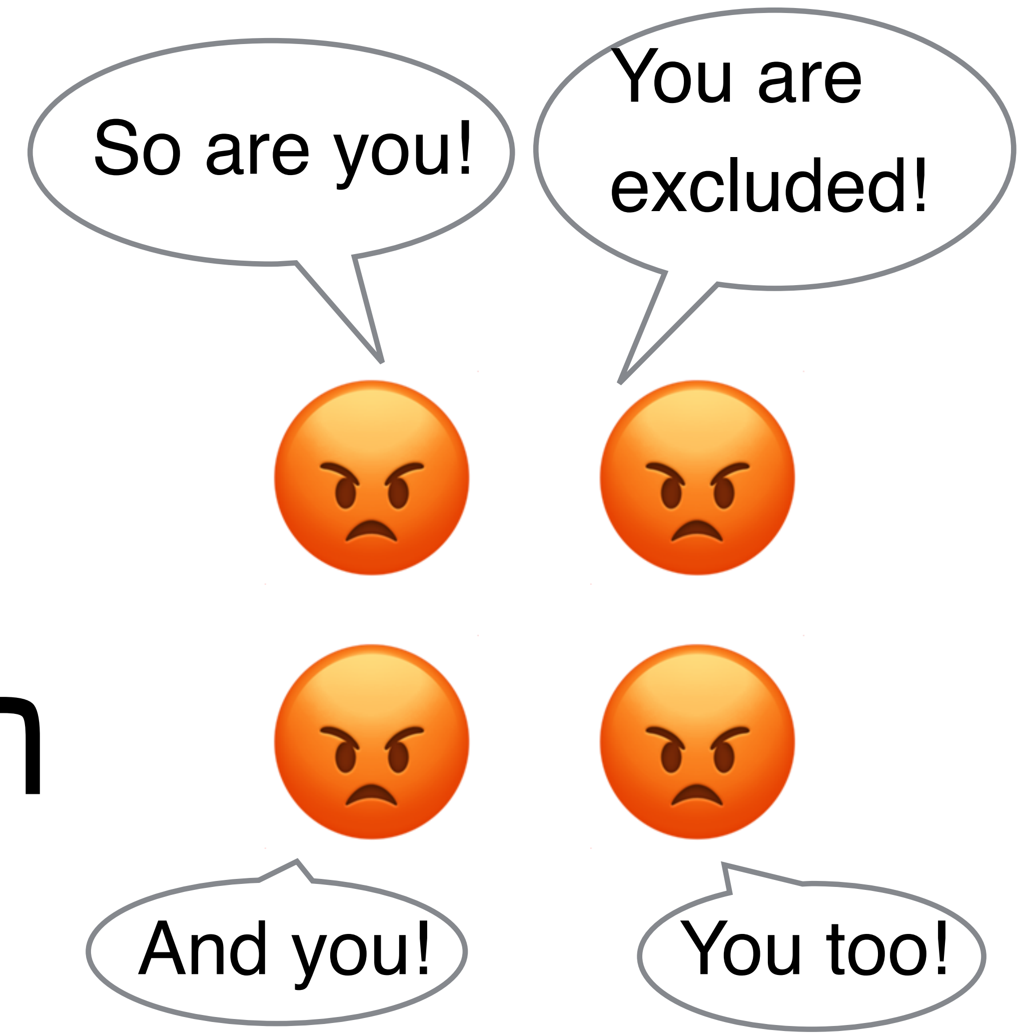
Beisert,Garrus, Russo

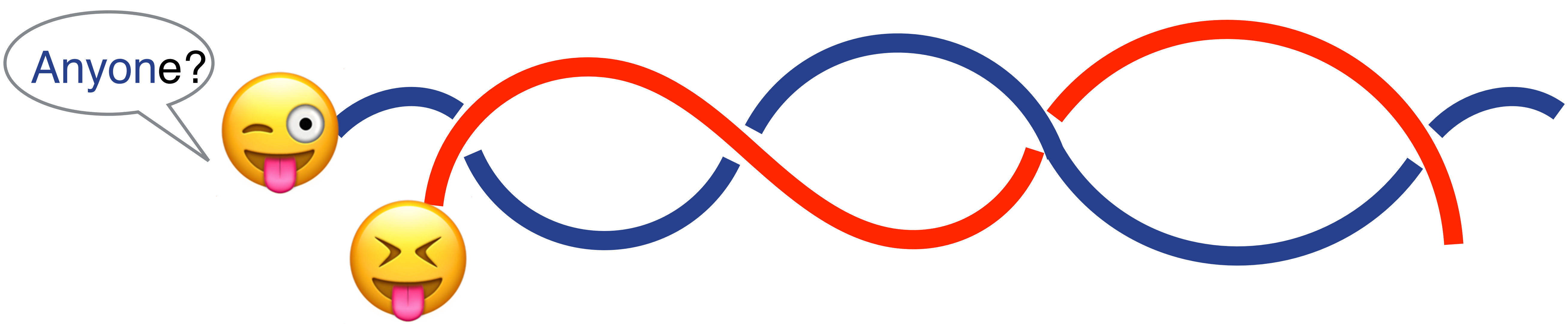
Even more things to do…

- **Amplitude-Wilson loop duality, Orthogonal grassmanian** representation of the amplitudes.
- More **stringent tests of duality**, computing arbitrary higher point correlators, alternative to Feynman diagrams?
- What are the c**rossing rules for anyonic channels in arbitrary higher point amplitudes**?
- What is the anyonic phase structure for an **n-particle Aharonov-Bohm scattering**?
- Does dual conformal invariance exist for non-supersymmetric amplitudes, if so can it constrain the form of the amplitude?


We are happy together!

תודה רבה!


So are you!
You are excluded!
And you!
You too!


Anyone?

Aside: Spinor helicity basis

- Spinor helicity representation

$$p_i^{\alpha\beta} = p_i^{\mu}\sigma_{\mu}^{\alpha\beta} = \lambda_i^{\alpha}\lambda_i^{\beta}$$

$$(p_i + p_j)^2 = 2p_i.p_j = -\langle\lambda_i^{\alpha}\lambda_{j,\alpha}\rangle^2 = \langle ij\rangle^2$$

- Onshell Supersymmetry generators

$$\mathcal{Q} = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} \lambda_i \eta_i,$$

$$\bar{\mathcal{Q}} = \sum_{i=1}^{n} \bar{q}_i = \sum_{i=1}^{n} \lambda_i \partial_{\eta_i}$$

Scattering in U(N) Chern-Simons matter theories

- Consider $2 \to 2$ scattering of particles in representations R_1 and R_2 of U(N)

$$R_1 \times R_2 = \sum_m R_m$$

- The S matrix takes the schematic form

$$S = \sum_m P_m S_m$$

$$P_m : \text{ projector in } m^{th} \text{ rep, } S_m \text{ is scattering in } m^{th} \text{ channel.}$$

- The Aharanov-Bohm phase of the particle R_1 as it circles around particle R_2 is $2\pi\nu_m$ where

$$\nu_m = \frac{4\pi}{\kappa} T_1^a T_2^a = \frac{2\pi}{\kappa} \left(C_2(R_1) + C_2(R_2) - C_2(R_m) \right)$$

- Scattering amplitude in the m'th exchange channel: Aharanov-Bohm scattering of a unit charge particle off a flux tube of flux $2\pi\nu_m$

Scattering in U(N) Chern-Simons matter theories

- Channels of scattering

$$\text{Fundamental} \otimes \text{Fundamental} \to \text{Symm}(U_d) \oplus \text{Asymm}(U_e)$$

$$\text{Fundamental} \otimes \text{Antifundamental} \to \text{Adjoint}(T) \oplus \text{Singlet}(S)$$

- The quadratic Casimirs

$$C_2(F) = C_2(A) = \frac{N^2-1}{2N}\ ,\ C_2(Sym) = \frac{N^2+N-2}{N}$$

$$C_2(ASym) = \frac{N^2-N-2}{N}\ ,\ C_2(Adj) = N\ ,\ C_2(Sing) = 0$$

- Anyonic phase

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa}\ ,\ \nu_{Asym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}$$

$$\nu_{Adj} = \frac{1}{N\kappa}\ ,\ \nu_{Sin} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

- In the large N, large κ limit, define 't Hooft coupling $\lambda = \frac{N}{\kappa}$

Scattering in U(N) Chern-Simons matter theories

- Anyonic phases in the large N, large κ limit

$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right) \; , \nu_{Sing} \sim O(\lambda)$$

- The T matrices themselves have the large N behavior

$$T_{Asym} \sim T_{Sym} \sim T_{Adj} \sim O\left(\frac{1}{N}\right) \; , \; T_{sing} \sim O(1)$$

- The singlet channel is effectively anyonic in the large N ,large κ limit.

- Unitarity $i(T^\dagger - T) = TT^\dagger$ is a non-trivial check only for the singlet channel. In other channels it follows from hermiticity.

- Observation: Naive crossing symmetry rules from any of the non-anyonic channels to the singlet channel leads to a non unitary S matrix.

S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

- Conjecture: Singlet channel S matrices have the form

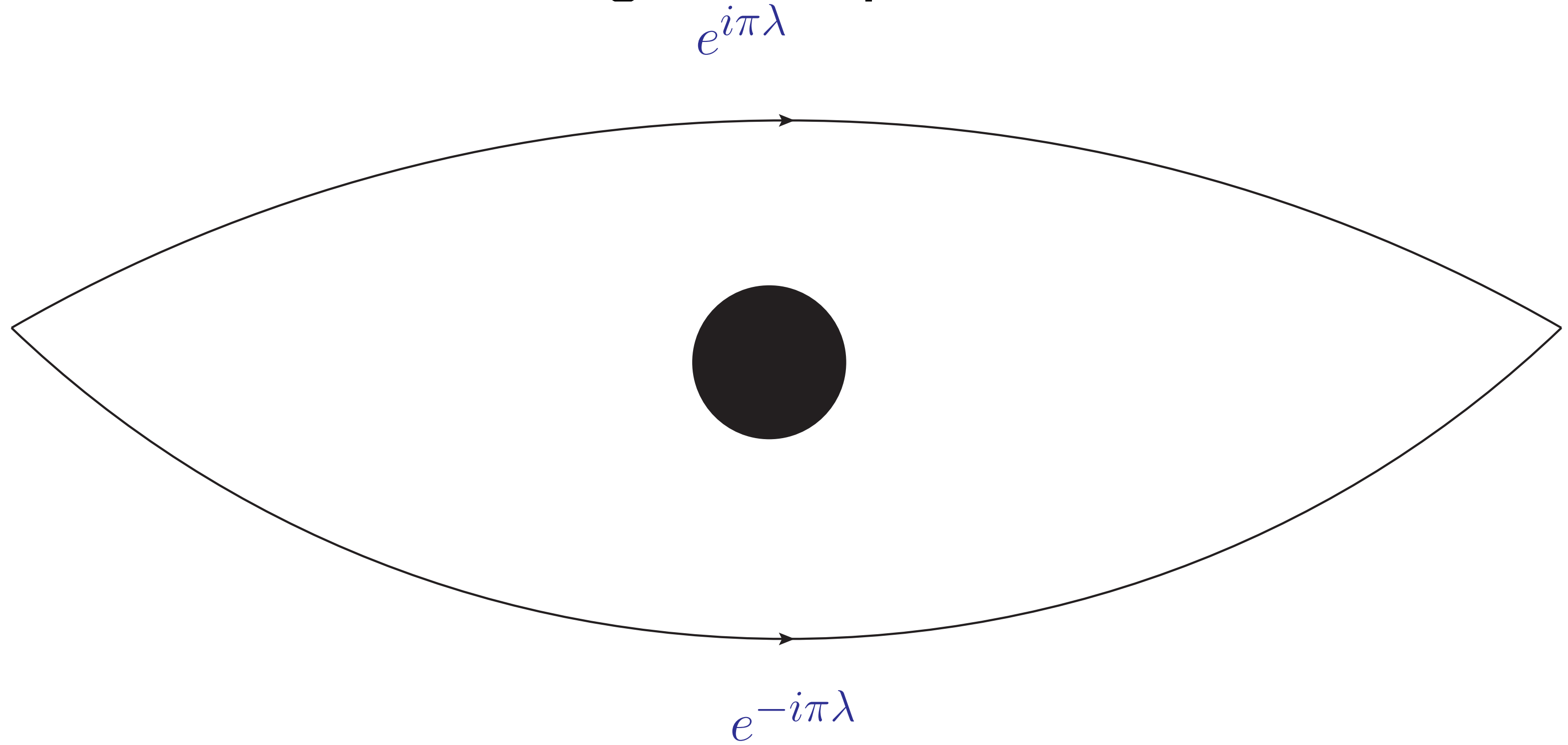
$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\mathrm{naive}}(s,\theta)$$

- $\mathcal{T}^{S;\mathrm{naive}}$ is the matrix obtained from naive analytic continuation of particle-particle scattering.

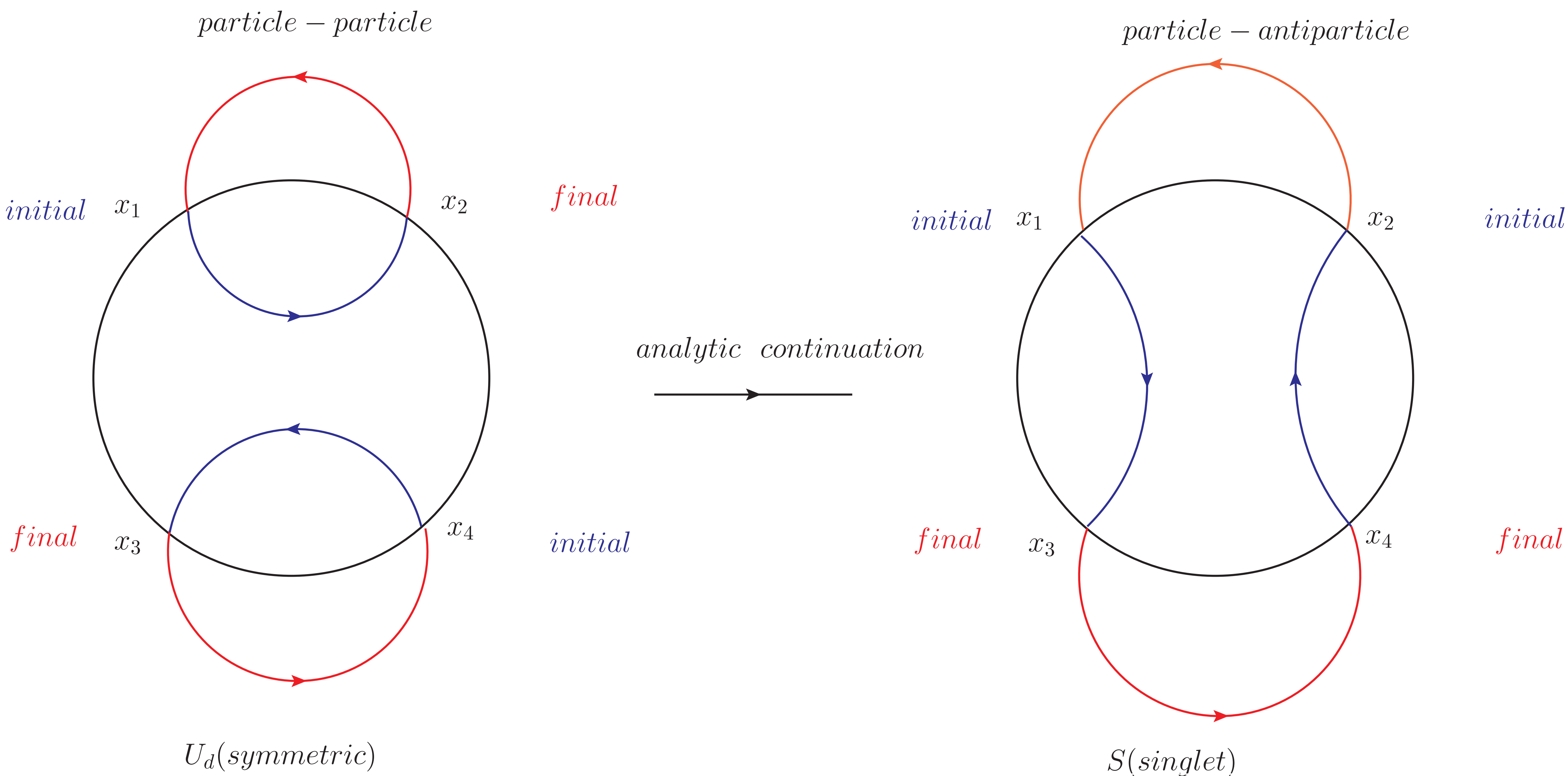
Nature of the conjecture: Delta function

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$

- The conjectured S matrix has a non-analytic delta function piece.
- The delta function modulated by anyonic phase is already known to be necessary to unitarize Aharonov-Bohm scattering.
- $\cos\pi\lambda$ in the identity term is due to the interference of the Aharanov-Bohm phases of the incoming wave packets.



Modified crossing rules: Heuristic explanation



- Attach Wilson lines to make correlators gauge invariant

$$T_{U_d} W_{U_d} \to T_S W_S$$

$$\frac{W_{U_d}}{W_S} = \frac{\oint \text{with 2 circular Wilson lines}}{\oint \text{with 1 circular Wilson line}} = \frac{\sin(\pi\lambda)}{\pi\lambda}$$

Witten