Exact amplitudes in Supersymmetric Chern-Simons matter theories

Karthik Inbasekar

TEL AVIV UNIVERSITY אוניברסיטת תל אביב

אוניברסיטת בן-גוריון בנגב
Ben-Gurion University of the Negev

June 6, 2019

Brown University, Department of Physics

Based on

K.I, S.Jain, L.Janagal, S. Minwalla, A.Shukla, **work in progress**.

References:

Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama, JHEP 1510 (2015) 176

Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama, JHEP 1504 (2015) 129

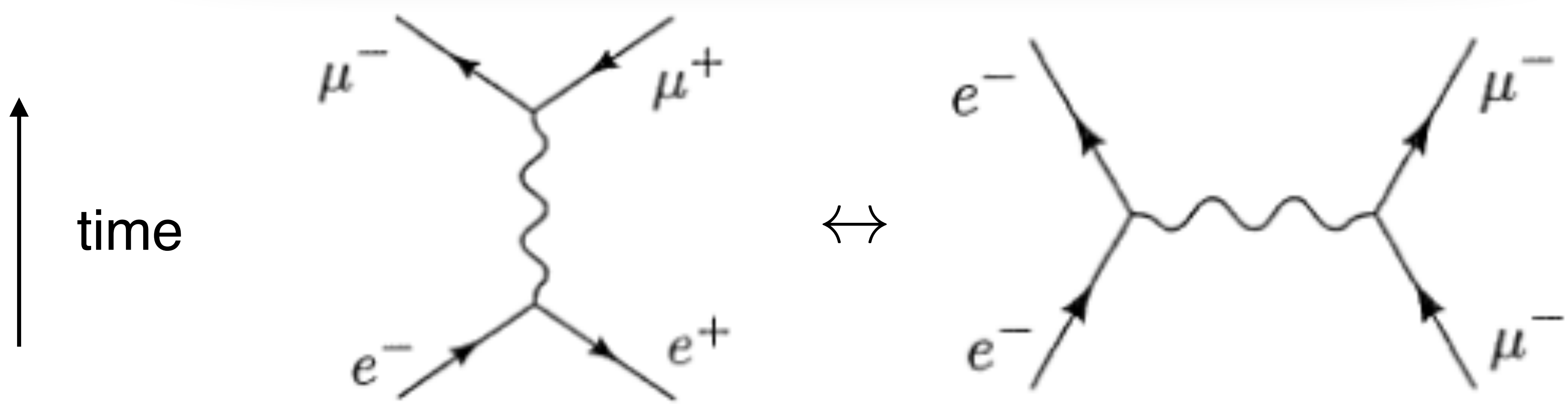
Crossing Symmetry

Crossing Symmetry

According to traditional wisdom in a relativistic QFT the **scattering amplitude exhibits crossing symmetry**.

Crossing symmetry is essentially a **statement of analytic continuation of amplitudes**. Eg:

$$S(e^+e^- \to \mu^+\mu^-) \leftrightarrow S(e^-\mu^- \to e^-\mu^-)$$



The S matrix for any process involving a **particle of momentum p, in the initial state** is identical to the S matrix for a process with **an anti-particle of momentum k=-p in the final state.**

Crossing Symmetry

In a general QFT Crossing symmetry can be shown from the following general principles.

Gribov

The S matrix is a **function of its kinematical invariants.**

$$S(s, t, u, g_i)$$

Causality: The **S matrix is an analytic function of its arguments**. Terms like $\theta(p_0)$ do not appear.

All **singularities are determined by physical masses** of participating intermediate state particles

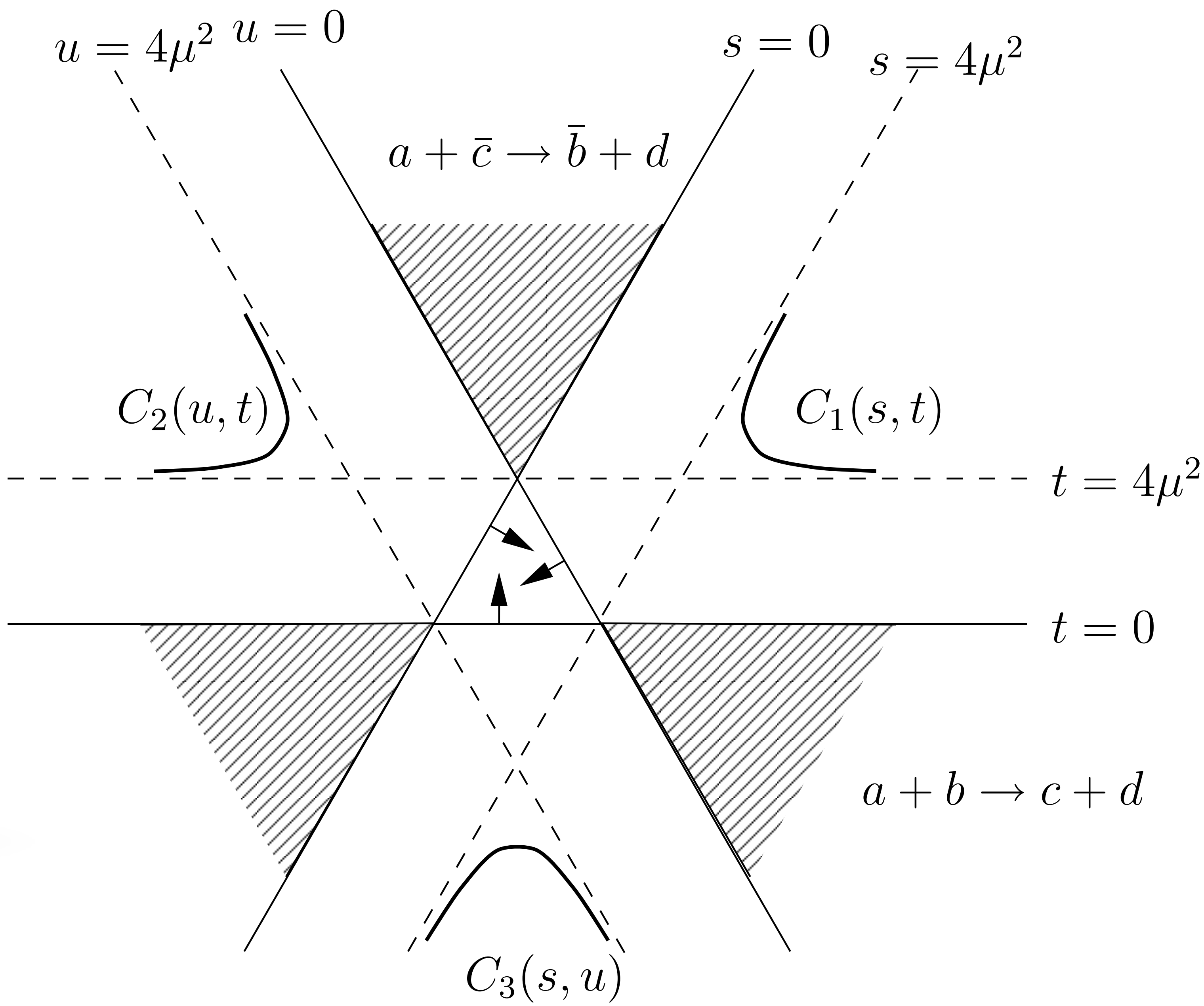
A related principle is **unitarity,** i.e the total probability of all possible processes at any given energy is unity.

$$SS^\dagger = 1$$

Crossing in the Mandelstam plane

$$S \propto \frac{1}{t - m_i}$$

t channel



$$S \propto \frac{1}{u - m_i}$$

u channel

$$S \propto \frac{1}{s - m_i}$$

s channel

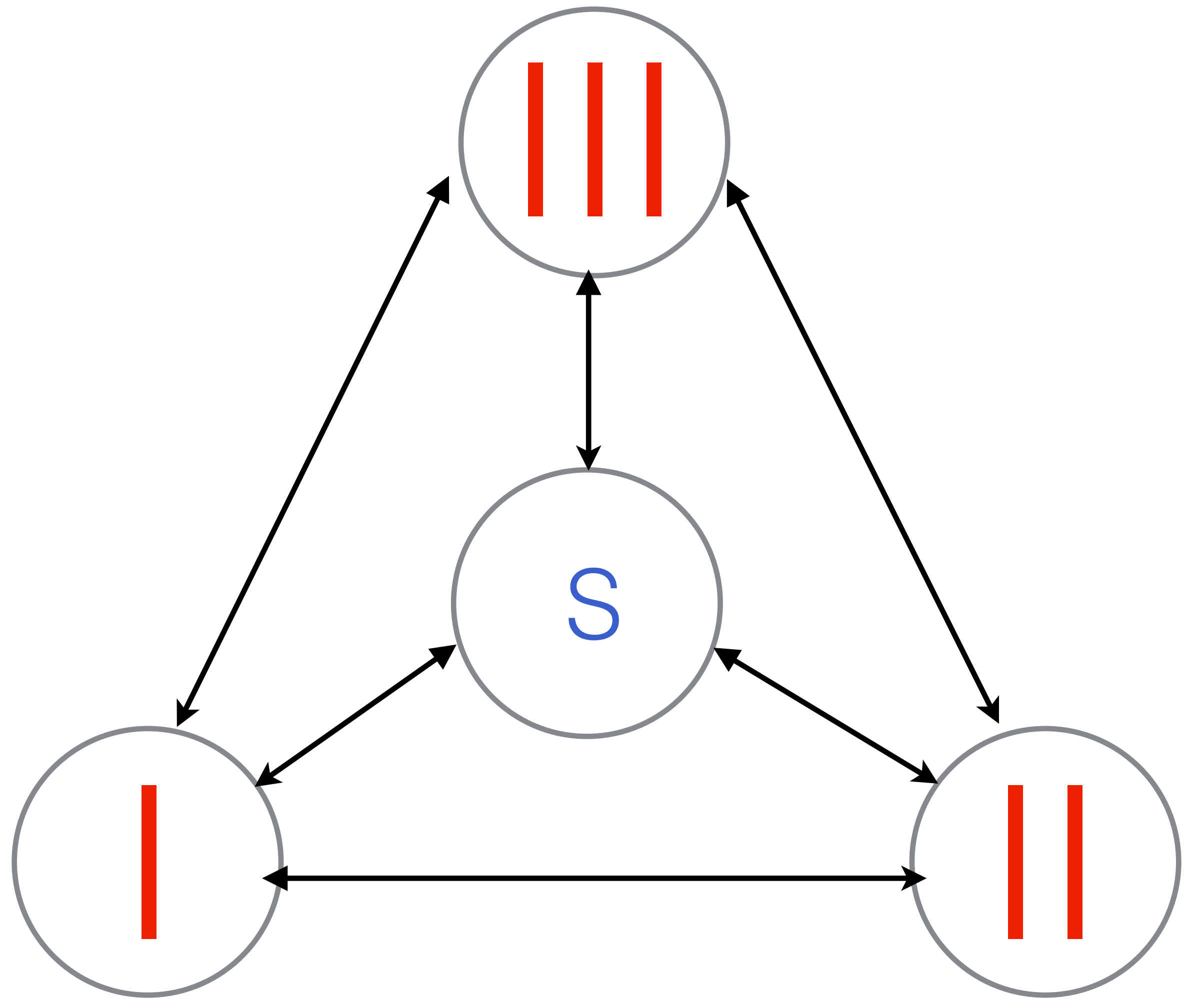
$$s + t + u = 4m^2$$

Fig. 1.2. Crossing reactions on the Mandelstam plane

Gribov

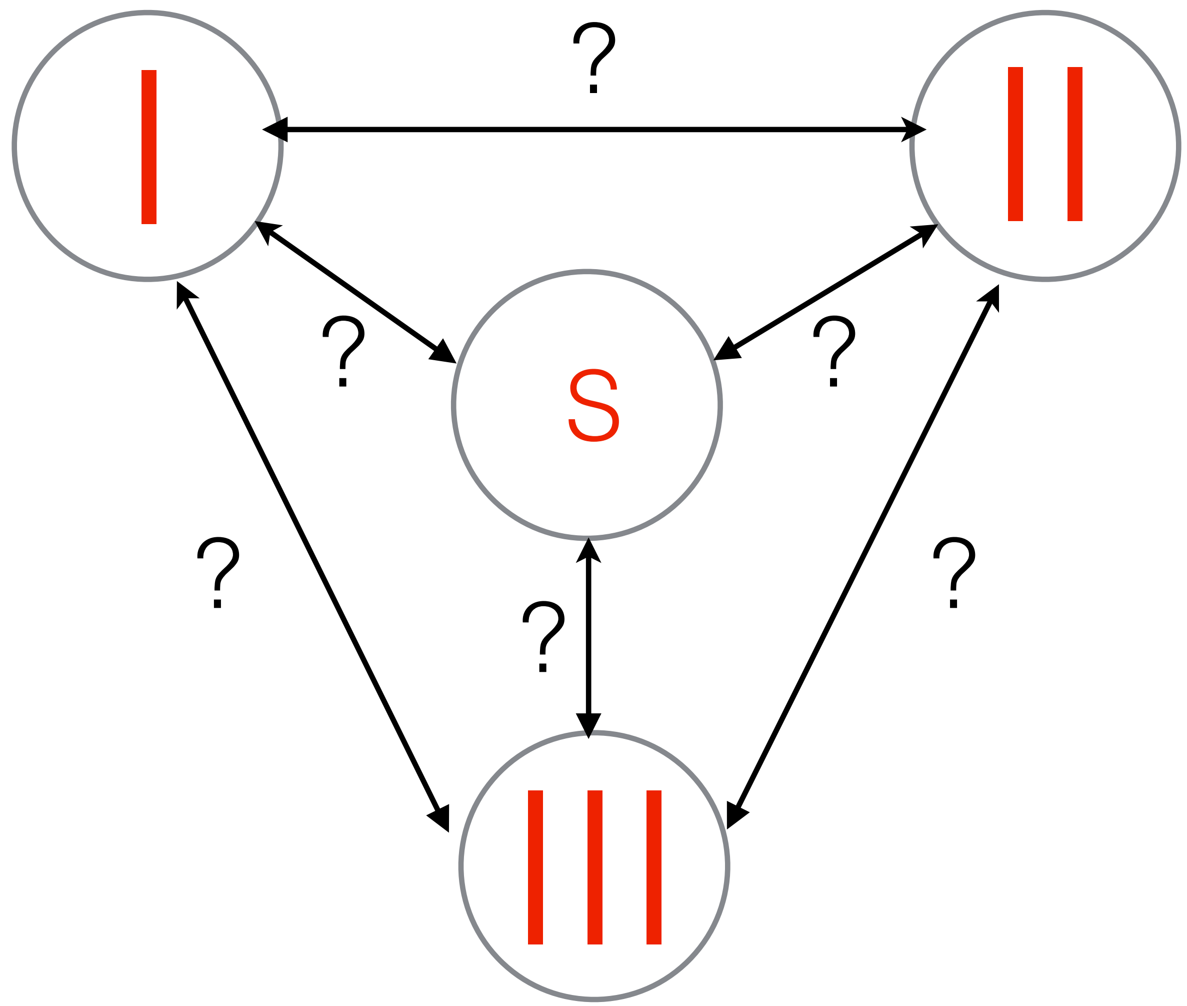
There exists **a single function that can be analytically continued to different channels of scattering**.

Crossing in the Mandelstam plane



Depending on whether the scattering quanta is bosonic or fermionic the **amplitude analytically continues up to an overall phase.**

Crossing Symmetry



Question:

What happens to "crossing symmetry" when the scattering particles have anyonic statistics?

Conjectured Crossing Symmetry with anyons

The S matrix in **ANY** Chern-Simons matter theory takes the following form

$$\mathcal{S}(s,\theta) = 8\pi\sqrt{s}\cos(\pi\nu)\delta(\theta) + i\frac{\sin(\pi\nu)}{\pi\nu}\mathcal{T}^{S;naive}(s,\theta)$$

phase dependence at forward scattering

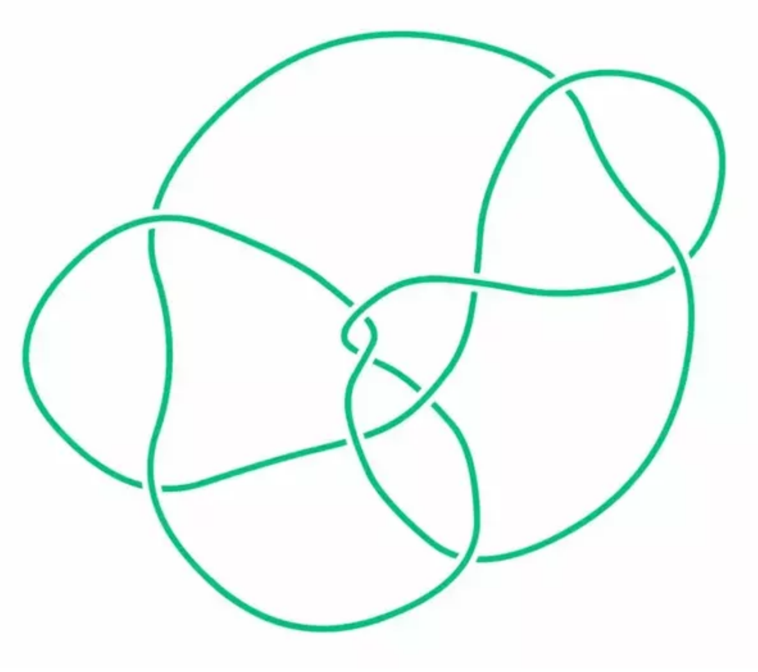
Crossing factor

Where ν is the **anyonic phase in the channel of scattering**.

$\mathcal{T}^{S;\mathrm{naive}}$ is obtained from **naive analytic continuation** from any of the non-anyonic channels.

S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

Non-relativistic Aharanov-Bohm scattering



"Anyone who knows anyons cannot describe them.
Anyone who can describe anyons does not know them."

Chern-Simons matter theories: Aharanov-Bohm phase

Pure Chern-Simons theory is **topological**. Solutions are pure gauge.

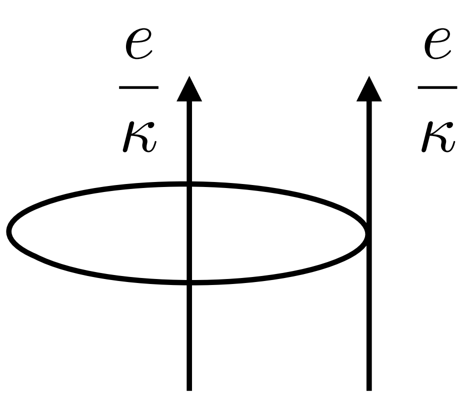
When **coupled to charged matter**

$$\frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - A_\mu J^\mu_{matter}$$

the **CS gauge field attaches magnetic fluxes to the particles**

$$\rho = \kappa B \ , \ J^i = \kappa\epsilon^{ij}E_j$$

Adiabatic excursion of such particles leads to the **Aharanov-Bohm effect**

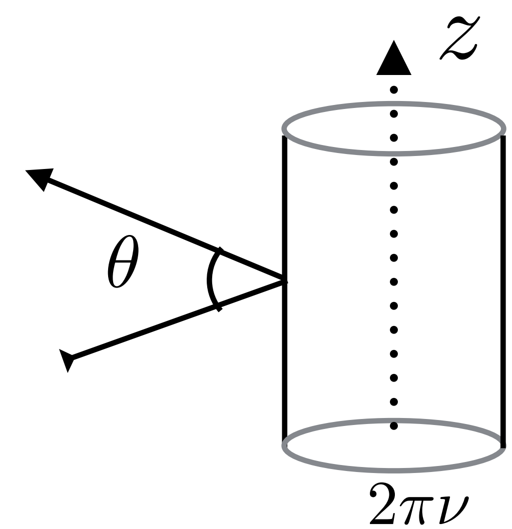


$$\text{AB phase} = e^{ie\int A.dx} = e^{i\frac{e^2}{\kappa}}$$

The phase: interpreted as **point particle explanation of anyonic statistics**.

Non-Relativistic Aharanov-Bohm scattering

Scattering of a unit charged particle of a flux tube



$$\left(-\frac{1}{2m}(\nabla + 2\pi i G\nu)^2 - \frac{\kappa^2}{2m}\right)\psi = 0$$

$$G_{ij} = \frac{\epsilon_{ij}}{2\pi}\partial_j \ln r$$

Boundary conditions

Wave function regularity at origin $\psi(r,\theta) \sim r^{|\nu|}$

At large r, reduces to incoming wave

Solution

$$\psi(r,\theta) \sim \frac{1}{\sqrt{2\pi k r}}\left(2\pi e^{-i(kr-\frac{\pi}{4})}\delta(\theta-\pi) + H(\theta)e^{i(kr-\frac{\pi}{4})}\right)$$

$$H(\theta) = 2\pi\cos(\pi\nu)\delta(\theta) + \sin(\pi\nu)\ \mathrm{Pv}\left(\frac{e^{-i\frac{\theta\ Sgn(\nu)}{2}}}{\sin(\frac{\theta}{2})}\right)$$

Note the dependence of phase in the forward scattering.

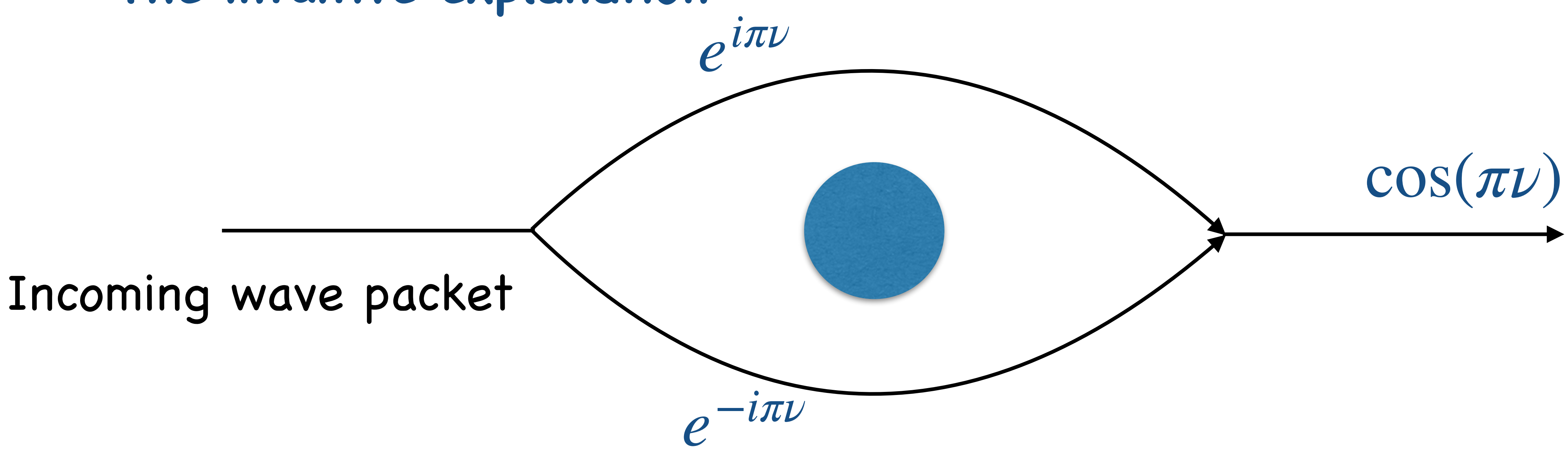
Aharanov-Bohm scattering and unitarity

This phase was **missed** in the original computation of Aharanov-Bohm scattering of anyonic particles.

Without this phase, the Aharanov-Bohm amplitude is not unitary!

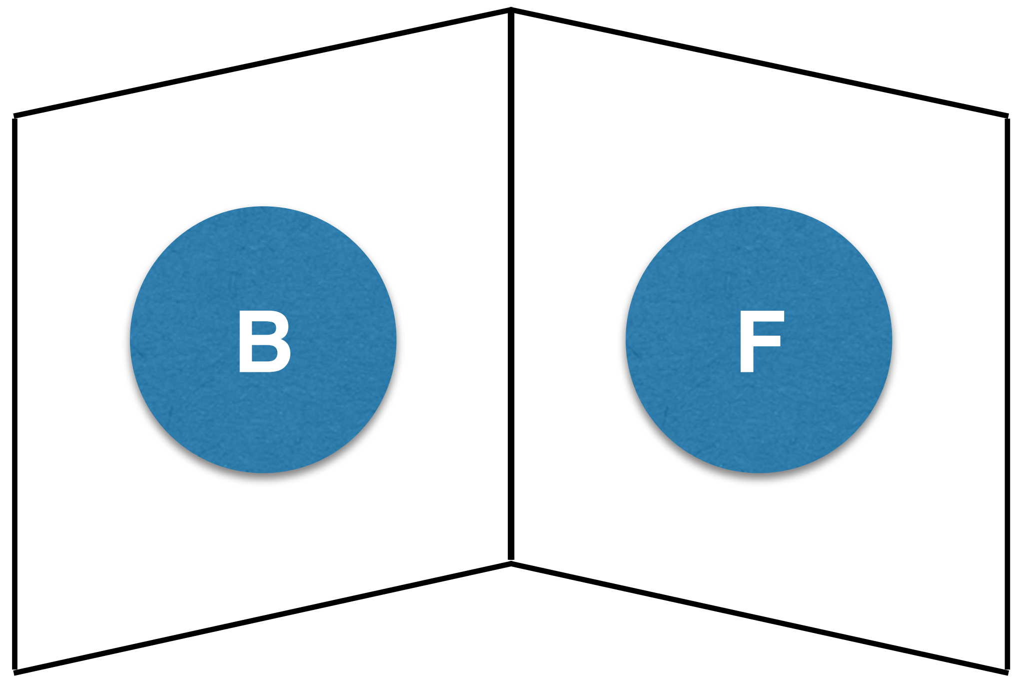
Ruijsennars; Bak, Jackiw, Pi

The intuitive explanation



Phase dependence arises due to interference of Aharanov-Bohm phases of the incoming wave packet.

The contribution to forward scattering should be a generic feature of scattering in any theory with anyonic excitations.



Bosonization duality

Aharony,Bardeen,Benini,Chang,Frishman,Giombi,Giveon,Gur-Ari,Gurucharan,K.I,
Karch,Kutasov,Jain,Maldacena,Mandlik,Minwalla,Moshe,Sharma,Prakash,
Radicevic,Takimi,Trivedi,Seiberg,Sonnenschein,Tong,Yacoby,Yin,Yokoyama,Wadia, Witten,
Zhiboedov

Chern-Simons matter theories at large N

Non-abelian Chern-Simons theories in three dimensions coupled to matter in fundamental representation of $SO(N), SU(N), U(N)$ are interesting conformal field theories.

These theories are **exactly solvable** in the large N limit.

They enjoy a remarkable **strong-weak bosonization duality.**

They are **holographically dual to classical higher spin gravity on** AdS_4

The finite N and k versions of these theories are relevant for condensed matter applications and in quantum hall effect.

Generic anyonic features are integral part of scattering in Chern-Simons matter theories.

Large N bosonization duality

$U(N_F)$ Chern-Simons coupled to fundamental fermions (regular fermion)

$$S = \int d^3x \left(i\epsilon^{\mu\nu\rho} \frac{\kappa_F}{4\pi} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) + \bar{\psi}^i \gamma^\mu D_\mu \psi_i + m_f \bar{\psi}^i \psi_i \right)$$

$U(N_B)$ Chern-Simons coupled to fundamental bosons

$$S = \int d^3x \left(i\epsilon^{\mu\nu\rho} \frac{\kappa_B}{4\pi} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) + D_\mu \bar{\phi} D^\mu \phi \right.$$
$$\left. + \sigma \bar{\phi} \phi + N_B \frac{m_B^2}{b_4} \sigma - N_B \frac{\sigma^2}{2b_4} \right)$$

Wilson-Fisher limit (critical boson)

$$b_4 \to \infty \ , \ m_B \to \infty \ , \ 4\pi \frac{m_B^2}{b_4} = \text{fixed}$$

Large N bosonization duality

t' Hooft large N limit

$$\lambda_B = \lim_{\substack{N_B \to \infty \\ \kappa_B \to \infty}} \frac{N_B}{\kappa_B} \qquad \lambda_F = \lim_{\substack{N_F \to \infty \\ \kappa_F \to \infty}} \frac{N_F}{\kappa_F}$$

$U(N_B)$ Chern-Simons coupled to fundamental bosons at Wilson-Fisher limit

dual

$U(N_F)$ Chern-Simons coupled to fundamental Fermions

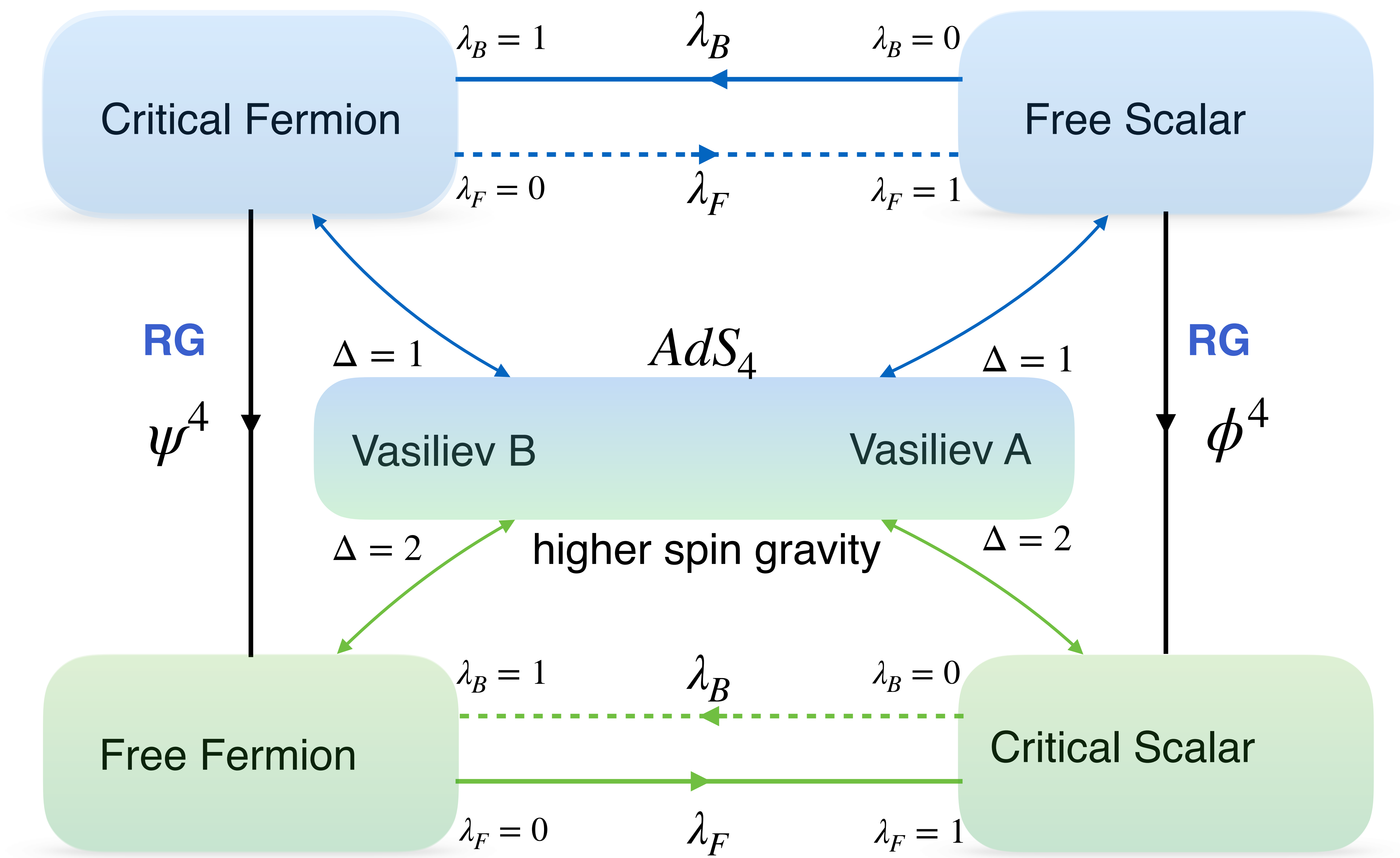
duality map
$$\begin{cases} \kappa_F = -\kappa_B \\ N_F = |\kappa_B| - N_B \\ \lambda_B = \lambda_F - \mathrm{Sign}(\lambda_F) \\ m_F = -m_B^{Crit} \lambda_B \end{cases}$$

There is a conjectured/untested finite N and k duality map

Physical observables computed on one side, match with observables on the other side under the duality map.

Strong-weak duality: Observables need to be computed to all loops!

Large N bosonization duality



Large N bosonization duality - SUSY version

$d=3\ ,\mathcal{N}=2$ superconformal Chern-Simons theory coupled to matter in fundamental representation of U(N)

$$\mathcal{S}_{\mathcal{N}=2}=-\int d^3x d^2\theta\Bigg(\frac{\kappa}{2\pi}\mathrm{Tr}\left(-\frac{1}{4}D_\alpha\Gamma^\beta D_\beta\Gamma^\alpha-\frac{1}{6}D^\alpha\Gamma^\beta\{\Gamma_\alpha,\Gamma_\beta\}-\frac{1}{24}\{\Gamma^\alpha,\Gamma^\beta\}\{\Gamma_\alpha,\Gamma_\beta\}\right)$$
$$-\frac{1}{2}(D^\alpha\bar{\Phi}+i\bar{\Phi}\Gamma^\alpha)(D_\alpha\Phi-i\Gamma_\alpha\Phi)+\frac{\pi}{\kappa}(\bar{\Phi}\Phi)^2\Bigg)$$

$$\Phi=\phi+\theta\psi-\theta^2F\ ,\bar{\Phi}=\bar{\phi}+\theta\bar{\psi}-\theta^2\bar{F}\ ,$$
$$\Gamma^\alpha=\chi^\alpha-\theta^\alpha B+i\theta^\beta A_\beta{}^\alpha-\theta^2(2\lambda^\alpha-i\partial^{\alpha\beta}\chi_\beta)\ .$$

The theory exhibits a strong-weak self duality under the duality map

$$\lambda=\frac{N}{\kappa}\ ,N\to\infty,\kappa\to\infty$$
$$\kappa'=-\kappa\ ,\ N'=|\kappa|-N+1\ ,\ \lambda'=\lambda-\mathrm{Sgn}(\lambda)$$

Scattering in Large N Chern-Simons theories

Scattering in U(N) Chern-Simons matter theories at large N

$2 \to 2$ Scattering, incoming particles in representations R_1, R_2

$$R_1 \otimes R_2 = \sum_m R_m$$

The S matrix can be decomposed into representations

$$S = \sum_m P_m S_m$$

S_m S matrix in the mth channel P_m Projector in the mth channel

The Aharanov-Bohm phase of the particle R_1 as it circles around particle R_2 is $2\pi\nu_m$ in the mth channel of scattering

$$\nu_m = \frac{4\pi}{\kappa}\mathrm{Tr}\left(T_{R_1}T_{R_2}\right) = \frac{2\pi}{\kappa}\left(C_2(R_1) + C_2(R_2) - C_m(R_m)\right)$$

Scattering in U(N) Chern-Simons matter theories at large N

Particles: Fundamental, Anti-Particles: Anti-Fundamental

Fundamental $\otimes$ Fundamental = Symmetric $\oplus$ Anti-Symmetric

Fundamental $\otimes$ Anti-Fundamental = Adjoint $\oplus$ Singlet

$$C_2(F) = C_2(A) = \frac{N^2-1}{2N}\,, \quad C_2(Sym) = \frac{N^2+N-2}{N}\,, \quad C_2(ASym) = \frac{N^2-N-2}{N}\,, \quad C_2(Adj) = N\,, \quad C_2(Sing) = 0$$

Anyonic phases

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa}\,, \quad \nu_{ASym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}\,, \quad \nu_{Adj} = \frac{1}{N\kappa}\,, \quad \nu_{Sing} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

In the limit $N \to \infty\,, \kappa \to \infty$ the 't Hooft coupling $\lambda = \frac{N}{\kappa}$

$$\nu_{Sym} \sim \nu_{ASym} \sim \nu_{Adj} \sim \mathcal{O}\left(\frac{1}{N}\right)\,, \quad \nu_{Sing} \sim \mathcal{O}(\lambda)$$

The singlet channel is effectively anyonic in the large N ,large κ limit

Scattering in U(N) Chern-Simons matter theories at large N

The T matrices themselves have the large N behavior

$$T_{Sym} \sim T_{ASym} \sim T_{Adj} \sim \mathcal{O}\left(\frac{1}{N}\right) \ , T_{Sing} \sim \mathcal{O}(1)$$

Unitarity is a non-trivial check only for the singlet channel. In other channels it follows from hermiticity at leading order in large N

$$i(T - T^{\dagger}) = TT^{\dagger}$$

Observation: **Naive crossing symmetry** rules from any of the **non-anyonic channels to the singlet channel** leads to a **non unitary** S matrix.

S.Jain, M. Mandlik, S.Minwalla , T.Takimi S.Wadia, S.Yokoyama

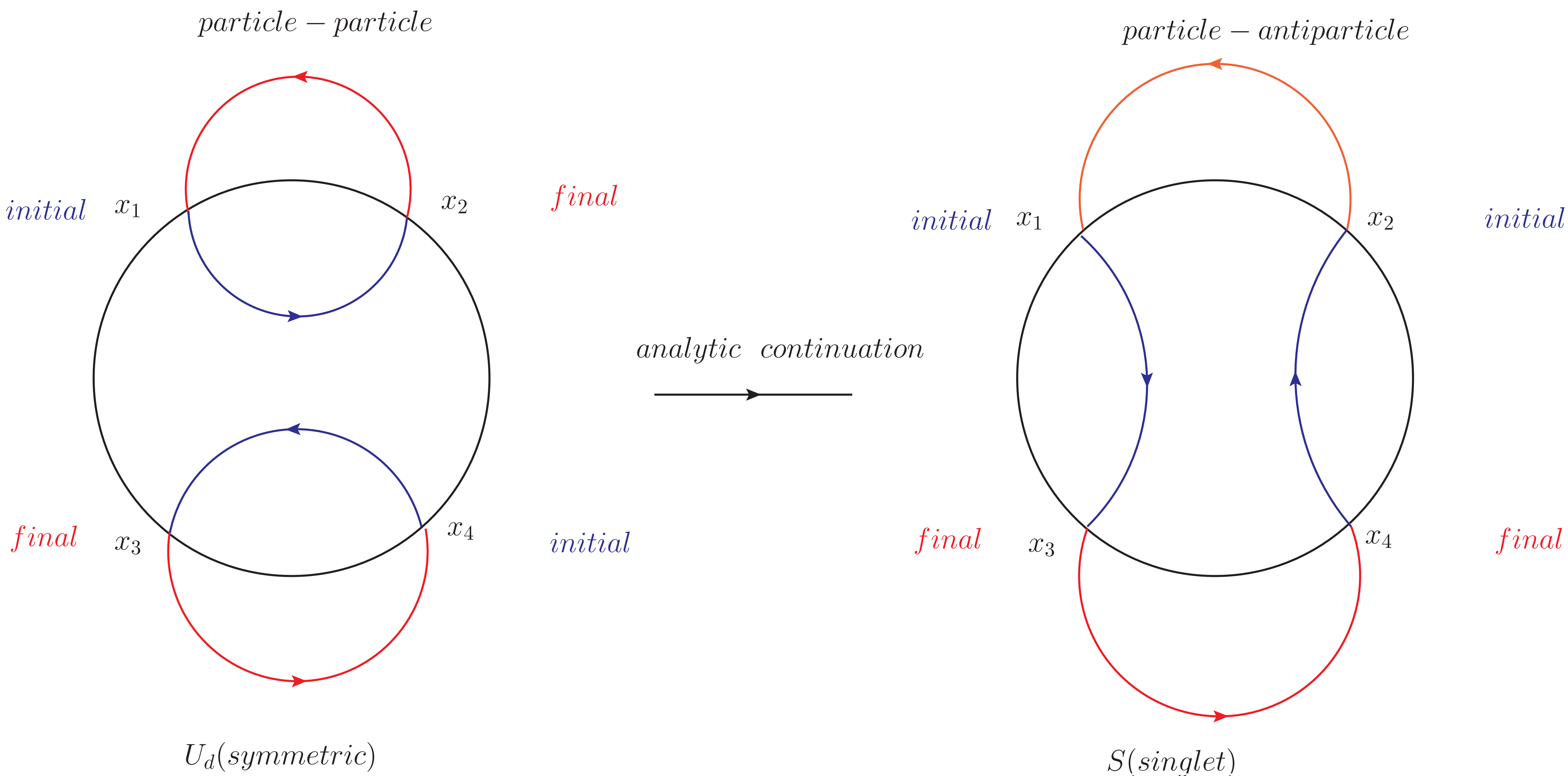
Conjectured S matrix for the anyonic channel

$$S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\nu)\ \delta(\theta) + i\frac{\sin(\pi\nu)}{\pi\nu}T^{S;naive}(s,\theta)$$

$$\nu_{Sing} = -\lambda$$

$$S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\ \delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}T^{S;naive}(s,\theta)$$

Modified crossing rules: Heuristic explanation



$$T_{U_d} W_{U_d} \rightarrow T_{Sing} W_{Sing}$$

$$\frac{W_{U_d}}{W_{Sing}} = \frac{\oint \text{with 2 circular Wilson lines}}{\oint \text{with 1 circular Wilson line}} = \frac{\sin(\pi\lambda)}{\pi\lambda}$$

Witten

Tests of the conjecture

Modulation of forward scattering and modified crossing rules appear to be **universal and generic** to Chern-Simons matter theories.

Tests

Unitarity of the S matrix

3d Bosonization duality Aharony,Bardeen,Benini,Chang,Frishman,Giombi,Giveon, GurAri,Gurucharan,Kutasov,Jain,Maldacena,Mandlik,Minwalla,Moshe,Sharma,Prakash,Takimi, Trivedi,Seiberg,Sonnenschein,Yacoby,Yin,Yokoyama,Wadia,Witten,Zhiboedov

Non-relativistic limit should be consistent with Aharanov-Bohm result.

Verifications

U(N) Chern-Simons coupled to fundamental bosons.
U(N) Chern-Simons coupled to fundamental fermions.

S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

$\mathcal{N} = 1{,}2$ supersymmetric Chern-Simons matter theories.

K.I, S.Jain, S.Mazumdar, S.Minwalla, V. Umesh, S.Yokoyama

Scattering in higher supersymmetric theories

Scattering in higher supersymmetric Chern-Simons matter theories

Amplitudes in $\mathcal{N}=2$ supersymmetric Chern-Simons matter theories have remarkably simple properties

The all loop result

$$T_{symm}^{all\ loop} = T_{Asymm}^{all\ loop} = T_{Adj}^{all\ loop} = T_{tree}$$

$$T_{singlet}^{all\ loop} = N\frac{\sin\pi\lambda}{\pi\lambda}T_{tree}$$

$$T_{\rm tree} = \frac{4\pi}{\kappa}\frac{\langle 12\rangle}{\langle 23\rangle}\delta\Big(\sum_{i=1}^{4}p_i\Big)\delta^2(\mathcal{Q})$$

$$\delta^2(Q) = \sum_{i<j=1}^{n}\langle ij\rangle\eta_i\eta_j$$

$$A_i = a_i + \eta_i\alpha_i$$

$$A_i^\dagger = a_i^\dagger\eta_i + \alpha_i^\dagger$$

$$S = \langle 0|A_4A_3A_2^\dagger A_1^\dagger|0\rangle$$

exhibits dual superconformal symmetry!

K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh

Yangian symmetry K.I, Jain, Nayak, Sharma (in progress)

Perturbative amplitudes in N=6 ABJM theory

Four point one loop amplitude in ABJM theory vanishes up to $\mathcal{O}(\epsilon)$ at one loop in dimensional regularization $3-2\epsilon$

Bianchi, Leoni, Mauri, Penati, Santambrogio

At **two loops**

$$\mathcal{A}_4^{\text{2-loop}} = \left(\frac{N}{k}\right)^2 \mathcal{A}_4^{\text{tree}} \left[-\frac{(-\mu^{-2}y_{13}^2)^{-\epsilon} + (-\mu^{-2}y_{24}^2)^{-\epsilon}}{(2\epsilon)^2} + \frac{1}{2}\ln^2\left(\frac{y_{13}^2}{y_{24}^2}\right) + 4\zeta_2 - 3\ln^2 2 + \mathcal{O}(\epsilon) \right]$$

Elvang & Huang

This is **inconsistent with perturbative unitarity**, the cuts in the two loop amplitudes cannot be saturated by a vanishing one loop result.

Curiously, the three loop amplitude is non-vanishing. Bianchi, Leoni

Perhaps a careful study of amplitudes **including the anyonic effects** could clarify these results.

Scattering in higher supersymmetric Chern-Simons matter theories

We have a well defined and robust techniques to compute amplitudes in N=1 superspace. K.I, S.Jain, S.Mazumdar, S.Minwalla, V. Umesh, S.Yokoyama

Higher supersymmetric theories can be formulated in N=1 superspace by **adding additional flavor matter and interactions**.

Our long term goal is to set up a Dyson-Schwinger method to compute exact amplitudes in ABJM.

We initiate baby steps towards generalizing the methods for higher supersymmetric theories formulated in N=1 superspace.

In the rest of the talk we discuss the N=3 Chern-Simons matter theory and computation of exact four point amplitudes in this theory.

As we will see, the integral equations are complicated, yet the final results are incredibly simple.

Scattering in N=3 Supersymmetric Chern-Simons matter theory

N=3 theory in N=1 superspace

The N=3 Superconformal theory is usually formulated in N=2 superspace consists of a **pair of chiral multiplets transforming in conjugate representations of the gauge group.** **D. Gaiotto, X. Yin**

$$(Q_i, \tilde{Q}^i)$$

We re-formulate the theory in N=1 superspace following **Gaiotto, Witten**

The reason is **purely technical** (no intuitive or easy way to implement supersymmetric light cone gauge in N=2 superspace)

The **price we pay is loss of manifest SU(2) R symmetry in superspace**, However this **can be easily recovered** by going to components.

SO(2) R charges **SU(2) doublets**

$$\begin{pmatrix} Q_i \\ \bar{\tilde{Q}}_i \end{pmatrix} \rightarrow \begin{pmatrix} \Phi_i^+ \\ \Phi_i^- \end{pmatrix} \quad \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \qquad \phi_i^A = \begin{pmatrix} \phi_i^+ \\ \phi_i^- \end{pmatrix}, \bar{\phi}_A^i = \begin{pmatrix} (\bar{\phi}^+)^i \\ (\bar{\phi}^-)^i \end{pmatrix}$$

Chang, Minwalla, Yin, Sharma

N=3 theory in N=1 superspace

SO(2) R Charge association to components in SU(2) doublets

$$\phi^A = \begin{pmatrix} \phi^+ \\ \phi^- \end{pmatrix} \begin{matrix} + \\ - \end{matrix} \;,\; \phi_A = \begin{pmatrix} -\phi^- \\ \phi^+ \end{pmatrix} \begin{matrix} - \\ + \end{matrix} \;,\; \psi^A = \begin{pmatrix} \psi^+ \\ -\psi^- \end{pmatrix} \begin{matrix} + \\ - \end{matrix} \;,\; \psi_A = \begin{pmatrix} \psi^- \\ \psi^+ \end{pmatrix} \begin{matrix} - \\ + \end{matrix}$$

$$\bar\phi_A = \begin{pmatrix} \bar\phi^+ \\ \bar\phi^- \end{pmatrix} \begin{matrix} - \\ + \end{matrix} \;,\; \bar\phi^A = \begin{pmatrix} -\bar\phi^- \\ \bar\phi^+ \end{pmatrix} \begin{matrix} + \\ - \end{matrix} \;,\; \bar\psi_A = \begin{pmatrix} \bar\psi^+ \\ -\bar\psi^- \end{pmatrix} \begin{matrix} - \\ + \end{matrix} \;,\; \bar\psi^A = \begin{pmatrix} \bar\psi^- \\ \bar\psi^+ \end{pmatrix} \begin{matrix} + \\ - \end{matrix}$$

The theory has a **unique mass deformation**. It is a **triplet** under the SU(2) R symmetry.

C. Cardova, T. Dumitrescu, K. Intriligator

$$-\bar\phi^A M_A{}^D M_D{}^E \phi_E + \bar\psi^{\beta A} M_A{}^D \psi_{\beta D}$$

$$M_A{}^B = \sum_{i=1}^{3} m_i (\sigma^i)_A{}^B$$

Without loss of generality one can rotate to a convenient frame.

Mass deformed N=3 theory in N=1 superspace

In N=1 Superspace, this appears similar to the **twisted/untwisted mass deformations** (with opposite signs)

$$\mathcal{S}^E_{\mathcal{N}=3} = -\int d^3x d^2\theta \Big[\frac{\kappa}{4\pi} Tr\Big(-\frac{1}{4} D^\alpha \Gamma^\beta D_\beta \Gamma_\alpha + \frac{i}{6} D^\alpha \Gamma^\beta \{\Gamma_\alpha, \Gamma_\beta\} + \frac{1}{24}\{\Gamma^\alpha, \Gamma^\beta\}\{\Gamma_\alpha, \Gamma_\beta\}\Big)$$
$$- \frac{1}{2}(D^\alpha \bar{\Phi}^+ + i\bar{\Phi}^+\Gamma^\alpha)(D_\alpha \Phi^+ - i\Gamma_\alpha \Phi^+) - \frac{1}{2}(D^\alpha \bar{\Phi}^- + i\bar{\Phi}^-\Gamma^\alpha)(D_\alpha \Phi^- - i\Gamma_\alpha \Phi^-)$$
$$- \frac{\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^+\Phi^+\right) - \frac{\pi}{\kappa}\left(\bar{\Phi}^-\Phi^-\right)\left(\bar{\Phi}^-\Phi^-\right) + \frac{4\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^-\Phi^-\right)$$
$$+ \frac{2\pi}{\kappa}\left(\bar{\Phi}^+\Phi^-\right)\left(\bar{\Phi}^-\Phi^+\right) - (m_0 \bar{\Phi}^+\Phi^+ - m_0\bar{\Phi}^-\Phi^-)\Big]$$

In components (Wess-Zumino gauge)

$$S^E_{\mathcal{N}=3} = \int d^3x \Big[\mathrm{Tr}\left(\frac{i\kappa}{4\pi}\epsilon^{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right)\right)$$
$$- i\bar{\psi}^A \not{D}\psi_A - m_0\bar{\psi}^A(\sigma^3)_A^{\ B}\psi_B + \mathcal{D}^\mu\bar{\phi}_A\mathcal{D}_\mu\phi^A + m_0^2\bar{\phi}_A\phi^A$$
$$+ \frac{4\pi^2}{\kappa^2}(\bar{\phi}_A\phi^B)(\bar{\phi}_B\phi^C)(\bar{\phi}_C\phi^A) - \frac{4\pi}{\kappa}(\bar{\phi}_A\phi^B)(\bar{\psi}^A\psi_B) - \frac{2\pi}{\kappa}(\bar{\psi}^A\phi_B)(\bar{\phi}^B\psi_A)$$
$$+ \frac{4\pi}{\kappa}(\bar{\psi}^A\phi_A)(\bar{\phi}^B\psi_B) + \frac{2\pi}{\kappa}(\bar{\psi}^A\phi_A)(\bar{\psi}^B\phi_B) + \frac{2\pi}{\kappa}(\bar{\phi}^A\psi_A)(\bar{\phi}^B\psi_B)$$
$$- \frac{4\pi m_0}{\kappa}(\bar{\phi}^A\phi_A)(\bar{\phi}^C(\sigma_3)_C^{\ D}\phi_D)\Big]$$

Mass deformed N=3 theory in N=1 superspace

It is straightforward to check that the action is manifestly invariant under the N=3 susy transformations.

$$
\begin{aligned}
Q_{BC\alpha}\phi_A =& \psi_{\alpha(B}\,\epsilon_{C)A},\\
Q_{BC\alpha}\bar{\phi}^A =& -\bar{\psi}_{\alpha(B}\,\delta_{C)}{}^A,\\
Q_{BC\alpha}\psi_{\beta A} =& -i\mathcal{D}_{\alpha\beta}\phi_{(B}\,\epsilon_{C)A} + m_0 C_{\alpha\beta}\phi_{(B}(\sigma^3)_{C)A}\\
& + \frac{2\pi}{\kappa}C_{\alpha\beta}(\bar{\phi}_A\phi_{(B})\phi_{C)} + \frac{2\pi}{\kappa}C_{\alpha\beta}(\bar{\phi}_{(B}\phi_{C)})\phi_A,\\
Q_{BC\alpha}\bar{\psi}^{\beta A} =& i\mathcal{D}_\alpha{}^\beta\bar{\phi}_{(B}\,\delta_{C)}{}^A + m_0\delta_\alpha{}^\beta\bar{\phi}_{(B}(\sigma_3)_{C)}^A\\
& + \frac{2\pi}{\kappa}\delta_\alpha{}^\beta(\bar{\phi}_{(B}\phi^A)\bar{\phi}_{C)} - \frac{2\pi}{\kappa}\delta_\alpha{}^\beta(\bar{\phi}_{(B}\phi_{C)})\bar{\phi}^A,\\
Q_{BC\alpha}A_\mu^a =& -\frac{4\pi}{\kappa}(\gamma_\mu)_\alpha{}^\beta\bar{\phi}^i_{(B}(T^a)_i{}^j\psi_{C)\beta j} - \frac{4\pi}{\kappa}(\gamma_\mu)_\alpha{}^\beta\bar{\psi}^i_{\beta(B}(T^a)_i{}^j\phi_{C)j}
\end{aligned}
$$

The **mass term** explicitly appears in the susy algebra and hence **is protected against loop corrections**.

Supersymmetric light cone gauge

$$\Gamma_- = 0 \;\Rightarrow\; A_- = A_1 + iA_2 = 0$$

In this gauge, the action simplifies to

$$\begin{aligned}
\mathcal{S}^E_{\mathcal{N}=3} = -\int d^3x d^2\theta \Big[& \frac{\kappa}{16\pi} Tr(\Gamma^- i\partial_{--}\Gamma^-) - \sum_{a=\pm} \frac{1}{2} D^\alpha \bar{\Phi}^a D_\alpha \Phi^a - \frac{i}{2}\Gamma^-(\bar{\Phi}^a D_- \Phi^a - D_- \bar{\Phi}^a \Phi^a) \\
& - \frac{\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^+\Phi^+\right) - \frac{\pi}{\kappa}\left(\bar{\Phi}^-\Phi^-\right)\left(\bar{\Phi}^-\Phi^-\right) + \frac{4\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^-\Phi^-\right) + \frac{2\pi}{\kappa}\left(\bar{\Phi}^+\Phi^-\right)\left(\bar{\Phi}^-\Phi^+\right) \\
& - \left(m_0 \bar{\Phi}^+\Phi^+ - m_0 \bar{\Phi}^-\Phi^-\right) \Big]
\end{aligned}$$

The exact propagators are

$$P_{\bar{\Phi}^\pm\Phi^\pm}(\theta_1,\theta_2,p) \equiv \langle \bar{\Phi}^\pm(\theta_1,p)\Phi^\pm(\theta_2,-p')\rangle = \frac{D^2_{\theta_1,p} \pm m}{p^2+m^2}\delta^2(\theta_1-\theta_2)(2\pi)^3\delta^3(p-p') \quad \text{exact:}$$

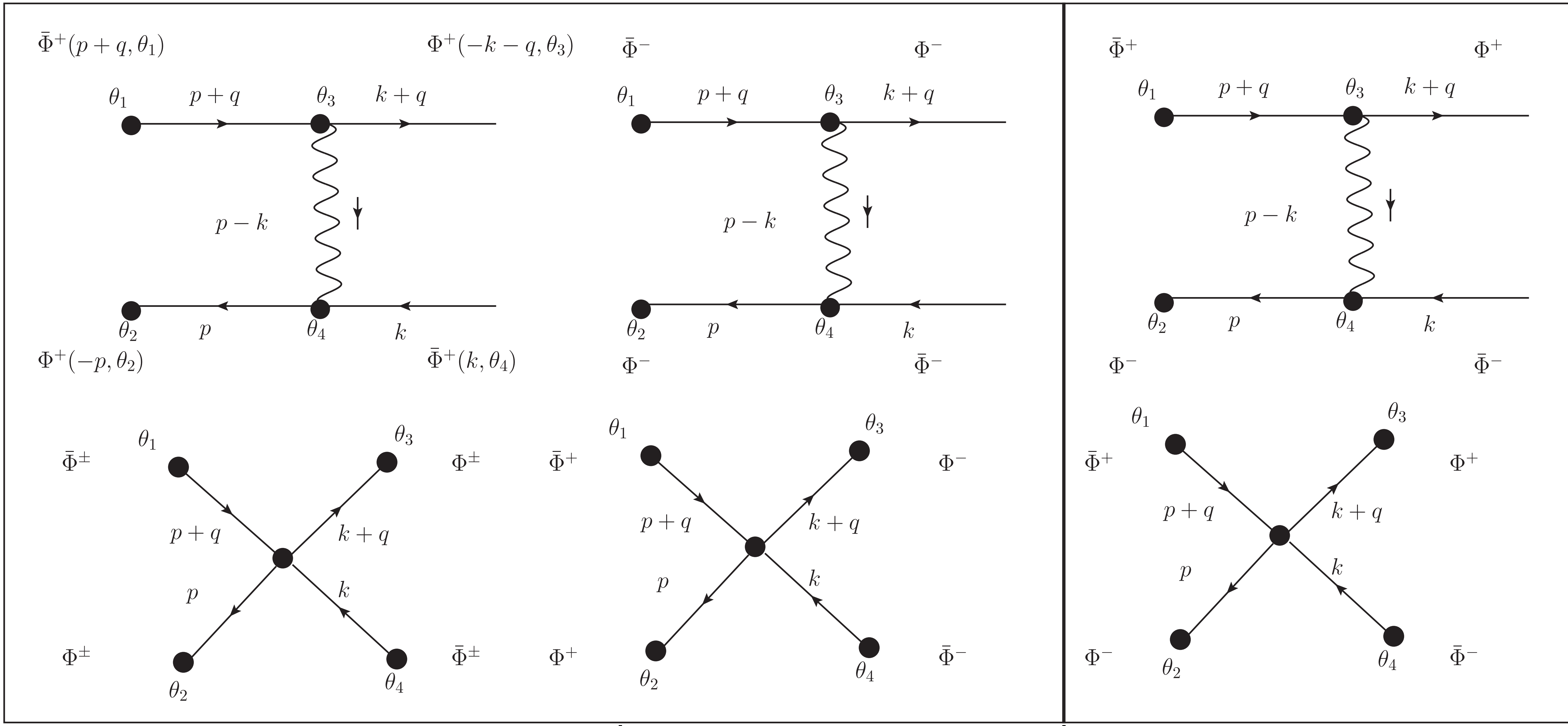
SUSY

$$P_\Gamma(\theta_1,\theta_2,p) \equiv \langle \Gamma^-(\theta_1,p)\Gamma^-(\theta_2,-p')\rangle = -\frac{8\pi}{\kappa}\frac{\delta^2(\theta_1-\theta_2)}{p_{--}}(2\pi)^3\delta^3(p-p')$$

Large N

To compute the four point amplitude: We first compute the off-shell four point correlator to all orders, then take the onshell limit to read off the S matrices.

Tree level amplitudes



$$\bar{\Phi}^\pm\Phi^\pm \to \bar{\Phi}^\pm\Phi^\pm$$

$$T_B^{tree} = \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} - \frac{8m\pi}{\kappa}$$

$$T_F^{tree} = \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} + \frac{8m\pi}{\kappa}$$

$$\bar{\Phi}^+\Phi^+ \to \bar{\Phi}^-\Phi^-$$

$$T_B^{tree} = T_F^{tree} = 0$$

$$\bar{\Phi}^+\Phi^- \to \bar{\Phi}^-\Phi^+$$

$$T_B^{tree} = T_F^{tree} = \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-}$$

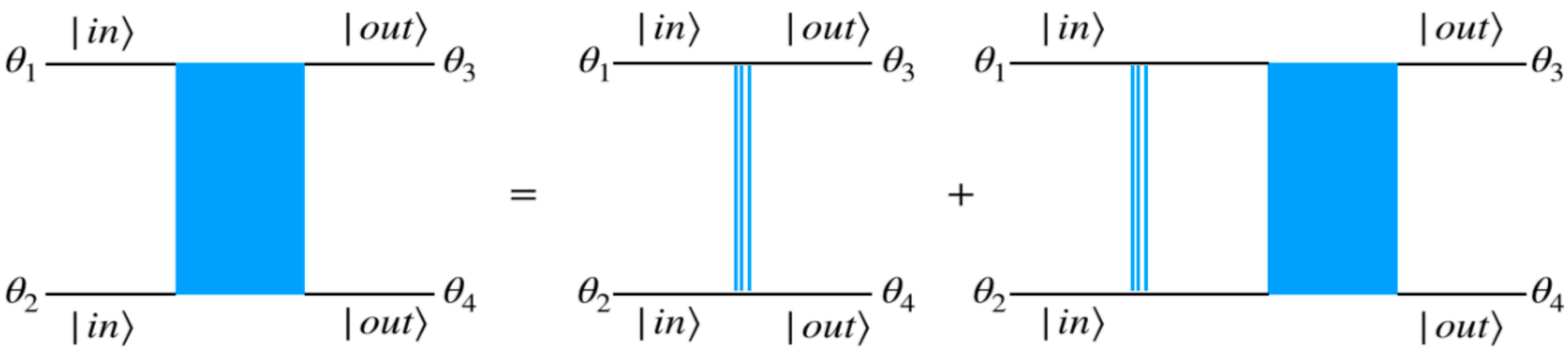
Neutral Sector

Charged sector

Organising the Dyson-Schwinger equations

Color contracted ()

$$-\frac{\pi}{\kappa}\left(\bar{\Phi}^{+}\Phi^{+}\right)\left(\bar{\Phi}^{+}\Phi^{+}\right)-\frac{\pi}{\kappa}\left(\bar{\Phi}^{-}\Phi^{-}\right)\left(\bar{\Phi}^{-}\Phi^{-}\right)+\frac{4\pi}{\kappa}\left(\bar{\Phi}^{+}\Phi^{+}\right)\left(\bar{\Phi}^{-}\Phi^{-}\right)+\frac{2\pi}{\kappa}\left(\bar{\Phi}^{+}\Phi^{-}\right)\left(\bar{\Phi}^{-}\Phi^{+}\right)$$



Neutral Sector Total SO(2) R charge vanishes at in/out of DS

$$M=\begin{pmatrix}\langle\bar{\Phi}^{+}(\theta_1)\Phi^{+}(\theta_2)\Phi^{+}(\theta_3)\bar{\Phi}^{+}(\theta_4)\rangle & \langle\bar{\Phi}^{+}(\theta_1)\Phi^{+}(\theta_2)\Phi^{-}(\theta_3)\bar{\Phi}^{-}(\theta_4)\rangle\\ \langle\bar{\Phi}^{-}(\theta_1)\Phi^{-}(\theta_2)\Phi^{+}(\theta_3)\bar{\Phi}^{+}(\theta_4)\rangle & \langle\bar{\Phi}^{-}(\theta_1)\Phi^{-}(\theta_2)\Phi^{-}(\theta_3)\bar{\Phi}^{-}(\theta_4)\rangle\end{pmatrix}$$

Charged sector Non-vanishing SO(2) R charge at in/out of DS

$$\langle\bar{\Phi}^{-}(\theta_1)\Phi^{+}(\theta_2)\Phi^{-}(\theta_3)\bar{\Phi}^{+}(\theta_4)\rangle$$

The integral equations for these two sectors do not mix!

N=1 Constraints on correlators

The N=1 ward identity fixes the general four point correlator to be of the form

$$\langle\bar{\Phi}(\theta_1, p+q)\Phi(\theta_2, -p)\Phi(\theta_3, k)\bar{\Phi}(\theta_4, -k-q)\rangle = V(\theta_1, \theta_2, \theta_3, \theta_4, p, q, k)$$

$$V(\theta_1, \theta_2, \theta_3, \theta_4, p, q, k) = \exp\left(\frac{1}{4}X.(p.X_{12} + q.X_{13} + k.X_{43})\right)F(X_{12}, X_{13}, X_{43}, p, q, k)$$

$$F(X_{12}, X_{13}, X_{43}, p, q, k) = \begin{array}{c} X_{12}^+ X_{43}^+ \Big(A(p,k,q) X_{12}^- X_{43}^- X_{13}^+ X_{13}^- + B(p,k,q) X_{12}^- X_{43}^- \\ + C(p,k,q) X_{12}^- X_{13}^+ + D(p,k,q) X_{13}^+ X_{43}^- \Big) \end{array} ,$$

Neutral Sector **Naively 16 unknowns!**

$$M = \begin{pmatrix} \langle\bar{\Phi}^+(\theta_1)\Phi^+(\theta_2)\Phi^+(\theta_3)\bar{\Phi}^+(\theta_4)\rangle & \langle\bar{\Phi}^+(\theta_1)\Phi^+(\theta_2)\Phi^-(\theta_3)\bar{\Phi}^-(\theta_4)\rangle \\ \langle\bar{\Phi}^-(\theta_1)\Phi^-(\theta_2)\Phi^+(\theta_3)\bar{\Phi}^+(\theta_4)\rangle & \langle\bar{\Phi}^-(\theta_1)\Phi^-(\theta_2)\Phi^-(\theta_3)\bar{\Phi}^-(\theta_4)\rangle \end{pmatrix}$$

Charged sector **4 unknown functions**

$$\langle\bar{\Phi}^-(\theta_1)\Phi^+(\theta_2)\Phi^-(\theta_3)\bar{\Phi}^+(\theta_4)\rangle$$

Integral equations in the charged sector

$$V_{\bar{\Phi}^+\Phi^-;\Phi^+\bar{\Phi}^-}(\theta_1,\theta_2,\theta_3,\theta_4,p,q,k) = V_{0:\bar{\Phi}^+\Phi^-;\Phi^+\bar{\Phi}^-}(\theta_1,\theta_2,\theta_3,\theta_4,p,q,k)$$
$$+\int \frac{d^3r}{(2\pi)^3} d^2\theta_a d^2\theta_b d^2\theta_A d^2\theta_B \Big(N V_{0:\bar{\Phi}^+\Phi^-;\Phi^+\bar{\Phi}^-}(\theta_1,\theta_2,\theta_a,\theta_b,p,q,r)$$
$$P_{\bar{\Phi}^+\Phi^+}(r+q,\theta_a,\theta_A) P_{\bar{\Phi}^-\Phi^-}(r,\theta_B,\theta_b) V_{\bar{\Phi}^+\Phi^-;\Phi^+\bar{\Phi}^-}(\theta_A,\theta_B,\theta_3,\theta_4,r,q,k)\Big)$$

Without doing any calculation one can argue that the solution should be identical to that of the N=2 theory.

$$\frac{2\pi}{\kappa}\left(\bar{\Phi}^+\Phi^-\right)\left(\bar{\Phi}^-\Phi^+\right) \qquad\qquad \frac{\pi}{\kappa}(\bar{\Phi}\Phi)(\bar{\Phi}\Phi)$$

The effective quartic coupling in the charged sector is identical to that of the N=2 theory.

However the propagators for $\Phi^\pm$ appear with **different signs for the masses** and one has to be a bit careful.

Integral equations in the charged sector

$$A(p,k,q) + \frac{2\pi i}{\kappa} + i\pi\lambda \int \frac{d^3r}{(2\pi)^3} \frac{q_3\left(p_-(2A - 2Bq_3 - Ck_-) + (4A + 2Ck_- - 3Dp_- + 4Bq_3)r_- + 2Dr_-^2\right)}{(r^2+m^2)((r+q)^2+m^2)(p-r)_-} = 0$$

$$B(p,k,q) + 2\pi i\lambda \int \frac{d^3r}{(2\pi)^3} \frac{r_-(2A - Ck_- + 2Bq_3 + Dr_-)}{(r^2+m^2)((r+q)^2+m^2)(p-r)_-} = 0$$

Integrable system

$$C(p,k,q) + \frac{4\pi i}{\kappa(k-p)_-} + 8\pi i\lambda \int \frac{d^3r}{(2\pi)^3} \frac{Cq_3r_-}{(r^2+m^2)((r+q)^2+m^2)(p-r)_-} = 0$$

$$D(p,k,q) + \frac{4\pi i}{\kappa(k-p)_-} + 2\pi i\lambda \int \frac{d^3r}{(2\pi)^3} \frac{q_3(-2A + 2Bq_3 + Ck_- + 3Dr_-)}{(r^2+m^2)((r+q)^2+m^2)(p-r)_-} = 0$$

Solution is identical to that of the N=2 theory

$$A(p,k,q) = -\frac{2i\pi e^{2i\lambda\left(\tan^{-1}\frac{2\sqrt{k_s^2+m^2}}{q_3} - \tan^{-1}\frac{2\sqrt{m^2+p_s^2}}{q_3}\right)}}{\kappa}$$

$$B(p,k,q) = 0\ ,$$

$$C(p,k,q) = -\frac{4i\pi e^{2i\lambda\left(\tan^{-1}\frac{2\sqrt{k_s^2+m^2}}{q_3} - \tan^{-1}\frac{2\sqrt{m^2+p_s^2}}{q_3}\right)}}{\kappa(k-p)_-}$$

$$D(p,k,q) = -\frac{4i\pi e^{2i\lambda\left(\tan^{-1}\frac{2\sqrt{k_s^2+m^2}}{q_3} - \tan^{-1}\frac{2\sqrt{m^2+p_s^2}}{q_3}\right)}}{\kappa(k-p)_-}$$

Integral equations in the neutral sector

More involved set of matrix integral equations!

$$
\begin{aligned}
M(\theta_1,\theta_2,\theta_3,\theta_4,p,q,k) =& M_0(\theta_1,\theta_2,\theta_3,\theta_4,p,q,k) \\
&+ \int \frac{d^3r}{(2\pi)^3} d^2\theta_a d^2\theta_b d^2\theta_A d^2\theta_B N M_0(\theta_1,\theta_2,\theta_a,\theta_b,p,q,r) \\
&PP(\theta_a,\theta_b,\theta_A,\theta_B,r,q) M(\theta_A,\theta_B,\theta_3,\theta_4,r,q,k)
\end{aligned}
$$

$$
M(\theta_A,\theta_B,\theta_3,\theta_4,p,q,k) = \begin{pmatrix} V_{\bar{\Phi}^+\Phi^+;\Phi^+\bar{\Phi}^+}(\theta_A,\theta_B,\theta_3,\theta_4,r,q,k) & V_{\bar{\Phi}^+\Phi^+;\Phi^-\bar{\Phi}^-}(\theta_A,\theta_B,\theta_3,\theta_4,r,q,k) \\ V_{\bar{\Phi}^-\Phi^-;\Phi^+\bar{\Phi}^+}(\theta_A,\theta_B,\theta_3,\theta_4,r,q,k) & V_{\bar{\Phi}^-\Phi^-;\Phi^-\bar{\Phi}^-}(\theta_A,\theta_B,\theta_3,\theta_4,r,q,k) \end{pmatrix}
$$

$$
PP(\theta_a,\theta_b,\theta_A,\theta_B,r,q) = \begin{pmatrix} P_{\bar{\Phi}^+\Phi^+}(r+q,\theta_a,\theta_A) P_{\bar{\Phi}^+\Phi^+}(r,\theta_B,\theta_b) & 0 \\ 0 & P_{\bar{\Phi}^-\Phi^-}(r+q,\theta_a,\theta_A) P_{\bar{\Phi}^-\Phi^-}(r,\theta_B,\theta_b) \end{pmatrix}
$$

This leads to **sixteen coupled equations for sixteen unknown functions** and seems to be a hopeless task...

Integral equations in the neutral sector

However, there is a residual SO(2) symmetry in the theory

$$\mathcal{S}^E_{\mathcal{N}=3} = -\int d^3x d^2\theta \Big[\frac{\kappa}{16\pi} Tr(\Gamma^- i\partial_{--}\Gamma^-) - \sum_{a=\pm} \frac{1}{2} D^\alpha \bar{\Phi}^a D_\alpha \Phi^a - \frac{i}{2}\Gamma^-(\bar{\Phi}^a D_- \Phi^a - D_-\bar{\Phi}^a \Phi^a)$$
$$- \frac{\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^+\Phi^+\right) - \frac{\pi}{\kappa}\left(\bar{\Phi}^-\Phi^-\right)\left(\bar{\Phi}^-\Phi^-\right) + \frac{4\pi}{\kappa}\left(\bar{\Phi}^+\Phi^+\right)\left(\bar{\Phi}^-\Phi^-\right) + \frac{2\pi}{\kappa}\left(\bar{\Phi}^+\Phi^-\right)\left(\bar{\Phi}^-\Phi^+\right.$$
$$- \left(m_0\bar{\Phi}^+\Phi^+ - m_0\bar{\Phi}^-\Phi^-\right)\Big]$$

$$\Phi^+ \leftrightarrow \Phi^- \qquad\qquad m_0 \leftrightarrow -m_0$$

This is essentially N=3 supersymmetry in disguise and is a symmetry of all the correlators.

This reduces the number of independent variables to eight, and using integrability of the resultant equations can be solved exactly.

The solution is unfortunately too horrendous to present here :)

Integral equations - summary

We set up the integral equations in the charged and neutral sectors.

We obtained all the four point correlators in these sectors in the kinematic regime $q_{\pm} = 0$

This allows easy extraction of the S matrix in the symmetric, anti-symmetric and adjoint channels of scattering by taking an onshell limit of the correlators. (q_{μ} is momentum transfer)

However, it is impossible to extract the singlet channel directly, since q_{μ} is center of mass energy and cannot be spacelike.

Any attempts to use naive crossing symmetry would ignore the anyonic effects in the singlet channel and lead to non-unitary amplitude.

We use the conjectured crossing rules to obtain the singlet channel S matrices.

S matrices in non-anyonic channels computed to all orders in 't Hooft coupling

Charged sector

$$\mathcal{T}_B(\bar{\phi}^+\phi^- \to \bar{\phi}^-\phi^+) = \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-}$$

$$\mathcal{T}_F(\bar{\psi}^+\psi^- \to \bar{\psi}^-\psi^+) = \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-}$$

Neutral sector

Tree level exact to all orders!!

$$\begin{pmatrix} \mathcal{T}_B(\bar{\phi}^+\phi^+ \to \bar{\phi}^+\phi^+) & \mathcal{T}_B(\bar{\phi}^+\phi^+ \to \bar{\phi}^-\phi^-) \\ \mathcal{T}_B(\bar{\phi}^-\phi^- \to \bar{\phi}^+\phi^+) & \mathcal{T}_B(\bar{\phi}^-\phi^- \to \bar{\phi}^-\phi^-) \end{pmatrix} = \begin{pmatrix} \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} - \frac{8m\pi}{\kappa} & 0 \\ 0 & \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} + \frac{8m\pi}{\kappa} \end{pmatrix} \tag{F.2}$$

neutralfinal

$$\begin{pmatrix} \mathcal{T}_F(\bar{\psi}^+\psi^+ \to \bar{\psi}^+\psi^+) & \mathcal{T}_F(\bar{\psi}^+\psi^+ \to \bar{\psi}^-\psi^-) \\ \mathcal{T}_F(\bar{\psi}^-\psi^- \to \bar{\psi}^+\psi^+) & \mathcal{T}_F(\bar{\psi}^-\psi^- \to \bar{\psi}^-\psi^-) \end{pmatrix} = \begin{pmatrix} \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} - \frac{8m\pi}{\kappa} & 0 \\ 0 & \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-} + \frac{8m\pi}{\kappa} \end{pmatrix} \tag{F.3}$$

Despite the complicated correlators, the S matrices continue to enjoy remarkable simplicity!

Restoration of SU(2) R symmetry and duality invariance

$$\begin{aligned}\langle\bar{\phi}^A\phi_A\bar{\phi}^B\phi_B\rangle|_{onshell} =& \frac{1}{2}\left(\langle\bar{\phi}^+\phi^+\bar{\phi}^+\phi^+\rangle + \langle\bar{\phi}^+\phi^+\bar{\phi}^-\phi^-\rangle + \langle\bar{\phi}^-\phi^-\bar{\phi}^+\phi^+\rangle + \langle\bar{\phi}^-\phi^-\bar{\phi}^-\phi^-\rangle\right)|_{onshell}\\ =& \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-}\end{aligned}$$

$$\begin{aligned}\langle\bar{\psi}^A\psi_A\bar{\psi}^B\psi_B\rangle|_{onshell} =& \frac{1}{2}\left(\langle\bar{\psi}^+\psi^+\bar{\psi}^+\psi^+\rangle + \langle\bar{\psi}^+\psi^+\bar{\psi}^-\psi^-\rangle + \langle\bar{\psi}^-\psi^-\bar{\psi}^+\psi^+\rangle + \langle\bar{\psi}^-\psi^-\bar{\psi}^-\psi^-\rangle\right)|_{onshell}\\ =& \frac{4\pi i q_3}{\kappa}\frac{(k+p)_-}{(k-p)_-}\end{aligned}$$

The N=3 covariant form of the S matrices continue to enjoy the full SU(2) R symmetry, eventhough the mass deformation explicitly breaks it to SO(2)!

Under the duality map the bosonic and fermionic amplitudes trivially map to one another upto a phase.

S matrices in non-anyonic channels

In the symmetric, anti-symmetric and adjoint channels of scattering

$$\mathcal{T}_B^{\mathcal{N}=3} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2}$$
$$\mathcal{T}_F^{\mathcal{N}=3} = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2}$$

N=2

The S matrices are tree level exact!

Since $$T_{Sym} \sim T_{ASym} \sim T_{Adj} \sim \mathcal{O}\left(\frac{1}{N}\right)$$

Unitarity is guaranteed by Hermiticity in these channels.

S matrix in the singlet channel

Conjectured form S.Jain, M. Mandlik, S.Minwalla , T.Takimi S.Wadia, S.Yokoyama

$$S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\ \delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}T^{S;naive}(s,\theta)$$

Singlet channel in N=3 theory

$$S_B^{\mathcal{N}=3}(s,\theta) = S_F^{\mathcal{N}=3}(s,\theta) = 8\pi\sqrt{s}\cos(\pi\lambda)\ \delta(\theta) - 4\sqrt{s}\sin(\pi\lambda)\ \cot\left(\frac{\theta}{2}\right)$$

The S matrix continues to be simple but not tree level exact!!

Naive crossing => tree level exactness in the singlet that would violate unitarity! $i(T - T^\dagger) = TT^\dagger$

Unitarity! $T_{Sing} \sim \mathcal{O}(1)$

$$T_B^{\mathcal{N}=3}(s,\theta) = T_F^{\mathcal{N}=3}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda) - 1)\ \delta(\theta) + 4i\sqrt{s}\sin(\pi\lambda)\ \cot\left(\frac{\theta}{2}\right)$$

The result above is identical to the N=2 result and is unitary.

in progress...

What about the rest of the components in the superamplitude?

In the massless limit, and the entire superamplitude is fixed in terms of one independent function.

The mass deformations break the R symmetry to SO(2), and it is not straightforward to work with manifest N=3 with less R symmetry.

With the mass deformations the entire N=3 superamplitude is expected to be fixed by two independent functions.

Then a manifestly N=3 covariant unitarity equation would give the unitarity conditions for the other component amplitudes.

Unitarity of the independent components (that we have shown) would then ensure the unitarity of the superamplitude.

Summary

Summary

In QFT **crossing symmetry** is the statement that there exists a **single function** from which all the channels of scattering can be obtained by **analytic continuation**.

Naive crossing symmetry works when the scattering particles are of definite (bosonic/fermionic) statistics.

When the scattering quanta are **anyonic, the usual crossing rules lead to non-unitary S matrices**.

General structure of S matrices in the anyonic channels is of the form

$$\mathcal{S}(s,\theta) = 8\pi\sqrt{s}\underbrace{\cos(\pi\nu)}_{\text{phase dependence at forward scattering}}\delta(\theta) + i\underbrace{\frac{\sin(\pi\nu)}{\pi\nu}}_{\text{Crossing factor}}\mathcal{T}^{S;naive}(s,\theta)$$

modifications appear to be **universal for all CS matter theories**.

Summary

We presented some explicit computations of exact four point amplitude in the N=3 Chern-Simons matter theory.

Although the integral equations were quite complicated we solved them using the symmetries of the problem.

We solved the Dyson-Schwinger equations to all orders in the 't Hooft coupling and presented exact solutions.

The bosonic/fermionic S matrices in the symmetric/antisymmetric/adjoint channels are tree level exact (vanishing loop corrections to all orders in the 't Hooft coupling)

The singlet channel S matrices continue to be simple and are modified in a minimal way by the conjectured rules, and satisfy unitarity and duality.

Summary

The bosonic and fermionic S matrices of the N=3 theory are identical to that of the N=2 theory.

We expect that the following results for the N=2 theory generalise to the N=3 theory as well.

dual superconformal symmetry of the four point amplitude

K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh

Yangian symmetry **K.I, Jain, Nayak, Sharma (in progress)**

We expect that the following results for the N=2 theory generalise to the N=3 theory as well.

However, we do not expect such results to generalise for $\mathcal{N} \geq 4$ theories, since the gauge group has to be bi-fundamental

We do expect some simplicity in the all loop amplitudes for $\mathcal{N} \geq 4$

Open questions

At finite N and k, all the channels are anyonic!

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa} \ , \ \nu_{Asym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}$$
$$\nu_{Adj} = \frac{1}{N\kappa} \ , \ \nu_{Sin} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

It would be valuable to have a first principle derivation of the crossing rules.

Anyonic phases in ABJ theory

$$U(N)_k \times U(M)_{-k}$$

Bi-fundamental/Anti-Bifundamental scattering

$$(R_{F_N}, R_{AF_M}) \otimes (R_{AF_N}, R_{F_M}) = (R_{\mathrm{Adj}_N}, R_{\mathrm{Adj}_M}) \oplus (R_{\mathrm{Adj}_N}, R_{\mathrm{Sing}_M}) \oplus (R_{\mathrm{Sing}_N}, R_{\mathrm{Adj}_M}) \oplus (R_{\mathrm{Sing}_N}, R_{\mathrm{Sing}_M})$$

Anyonic phases (looks complicated)

$$\begin{aligned}\nu_{n,m} &= \frac{2}{\kappa}(T_{U(N)})^a_1 (T_{U(N)})^a_2 - \frac{2}{\kappa}(T_{U(M)})^a_1 (T_{U(M)})^a_2 \\ &= \frac{C_2(R_n) - C_2(R_{n_1}) - C_2(R_{n_2})}{\kappa} - \frac{C_2(R_m) - C_2(R_{m_1}) - C_2(R_{m_2})}{\kappa}\end{aligned}$$

$$\begin{aligned}\nu_{\mathrm{Adj}_N,\mathrm{Adj}_M} &= \left(\frac{1}{N\kappa}\right) - \left(\frac{1}{M\kappa}\right) = -\left(\frac{1}{M} - \frac{1}{N}\right)\frac{1}{\kappa} \\ \nu_{\mathrm{Adj}_N,\mathrm{Sing}_M} &= \left(\frac{1}{N\kappa}\right) - \left(-\frac{M}{\kappa} + \frac{1}{M\kappa}\right) = \frac{M}{\kappa} - \left(\frac{1}{M} - \frac{1}{N}\right)\frac{1}{\kappa} \\ \nu_{\mathrm{Sing}_N,\mathrm{Adj}_M} &= \left(-\frac{N}{\kappa} + \frac{1}{N\kappa}\right) - \left(\frac{1}{M\kappa}\right) = -\frac{N}{\kappa} - \left(\frac{1}{M} - \frac{1}{N}\right)\frac{1}{\kappa} \\ \nu_{\mathrm{Sing}_N,\mathrm{Sing}_M} &= \left(-\frac{N}{\kappa} + \frac{1}{N\kappa}\right) - \left(-\frac{M}{\kappa} + \frac{1}{M\kappa}\right) = \frac{M-N}{\kappa} - \left(\frac{1}{M} - \frac{1}{N}\right)\frac{1}{\kappa}\end{aligned}$$

Anyonic phases in ABJM theory

$$U(N)_k \times U(N)_{-k}$$

Remarkable simplicity even at finite N and k

$$\nu_{\mathrm{Adj}_N,\mathrm{Adj}_N} = 0$$
$$\nu_{\mathrm{Adj}_N,\mathrm{Sing}_N} = \frac{N}{\kappa}$$
$$\nu_{\mathrm{Sing}_N,\mathrm{Adj}_N} = -\frac{N}{\kappa}$$
$$\nu_{\mathrm{Sing}_N,\mathrm{Sing}_N} = 0$$

At large N reduces to those of vector like models.

It appears that the crossing rules should be simpler to derive in the ABJM theory.