Correlation functions of supersymmetric Chern-Simons matter theories at large N

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ISRAEL PHYSICAL SOCIETY

Based on

K.I, S.Jain, P.Nayak,T. Sharma (work in progress)

References:

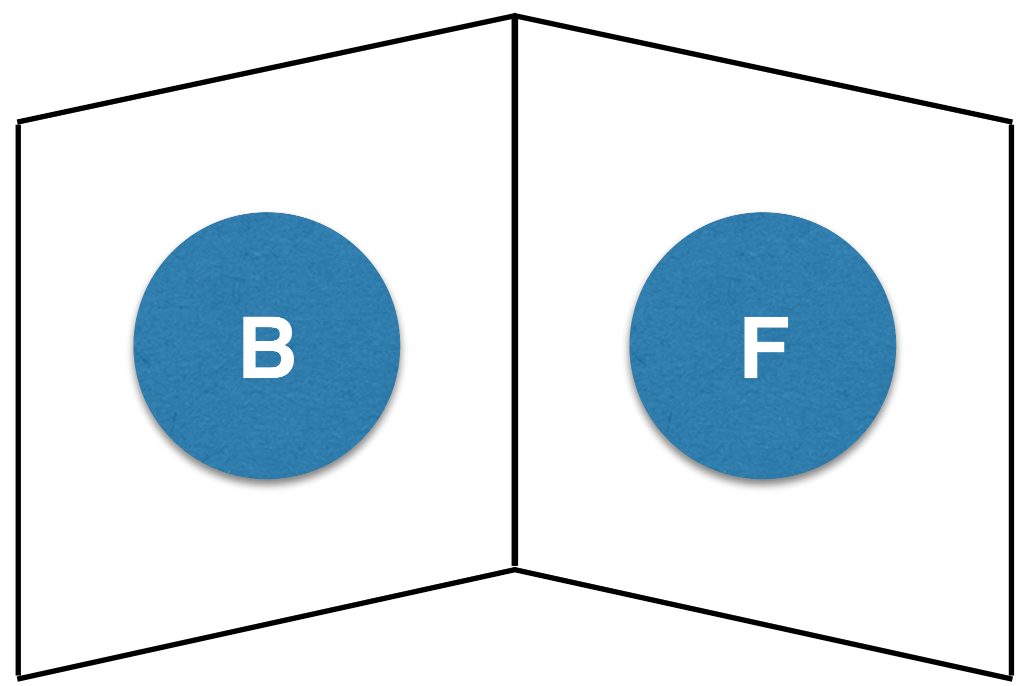
K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh, S.Yokoyama, **arXiv: 1505.06571**, **JHEP 1510 (2015) 176.**

O. Aharony, G. Gur-Ari, R. Yacoby, **arXiv: 1207.4593**, **JHEP 1212 (2012) 028**

Introduction

- Non-abelian Chern-Simons theories in three dimensions coupled to matter in fundamental representation of $SO(N), SU(N), U(N)$ are interesting conformal field theories.
- These theories are **effectively solvable in the large N limit**.
- They **enjoy a strong-weak bosonization duality**.
- They are **holographically dual to classical higher spin gravity** on AdS_4
- The finite N and k versions of these theories are **relevant for condensed matter applications in quantum hall effect**.

Introduction



Bosonization duality

Aharony,Bardeen,Benini,Chang,Frishman,Giombi,Giveon,Gur-Ari,Gurucharan,K.I,
Karch,Kutasov,Jain,Maldacena,Mandlik,Minwalla,Moshe,Sharma,Prakash,
Radicevic,Takimi,Trivedi,Seiberg,Sonnenschein,Tong,Yacoby,Yin,Yokoyama,Wadia,
Witten, Zhiboedov

Large N Bosonization duality

- $U(N_F)$ **Chern-Simons coupled to fundamental fermions (regular fermion)**

$$S = \int d^3x \left(i\epsilon^{\mu\nu\rho} \frac{\kappa_F}{4\pi} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) + \bar{\psi}^i \gamma^\mu D_\mu \psi_i + m_f \bar{\psi}^i \psi_i \right)$$

- $U(N_B)$ **Chern-Simons coupled to fundamental bosons**

$$S = \int d^3x \left(i\epsilon^{\mu\nu\rho} \frac{\kappa_B}{4\pi} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) + D_\mu \bar{\phi} D^\mu \phi \right.$$
$$\left. + \sigma \bar{\phi} \phi + N_B \frac{m_B^2}{b_4} \sigma - N_B \frac{\sigma^2}{2b_4} \right)$$

- **Wilson-Fisher limit (critical boson)**

$$b_4 \to \infty \ , \ m_B \to \infty \ , \ 4\pi \frac{m_B^2}{b_4} = \text{fixed}$$

3d Large N Bosonization duality

t' Hooft large N limit

$$\lambda_B = \lim_{\substack{N_B \to \infty \\ \kappa_B \to \infty}} \frac{N_B}{\kappa_B} \qquad \lambda_F = \lim_{\substack{N_F \to \infty \\ \kappa_F \to \infty}} \frac{N_F}{\kappa_F}$$

- $U(N_B)$ Chern-Simons coupled to fundamental bosons at Wilson-Fisher limit

dual

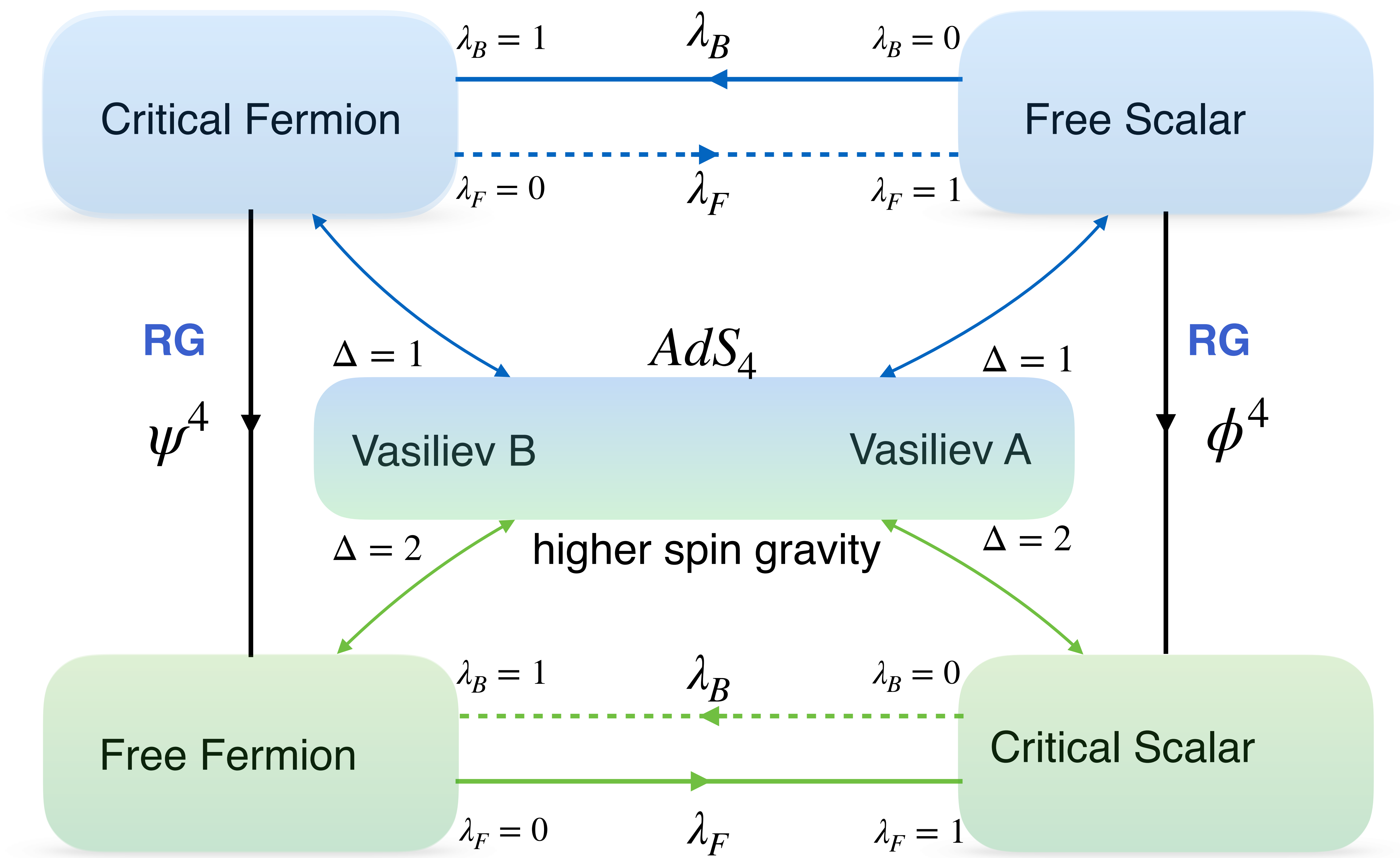
- $U(N_F)$ Chern-Simons coupled to fundamental Fermions

duality map

$$\begin{cases} \kappa_F = -\kappa_B \\ N_F = |\kappa_B| - N_B \\ \lambda_B = \lambda_F - \mathrm{Sign}(\lambda_F) \\ m_F = -m_B^{Crit} \lambda_B \end{cases}$$

- **Physical observables such as spectrum of single trace primaries, thermal partition functions, correlation functions of higher spin currents, S matrices, match on both sides under the map!**

3d Large N Bosonization duality



Large N duality - Supersymmetric version

- $d=3$, $\mathcal{N}=2$ superconformal Chern-Simons theory coupled to matter in fundamental representation of U(N)

$$\mathcal{S}_{\mathcal{N}=2} = -\int d^3x d^2\theta \Bigg(\frac{\kappa}{2\pi}\mathrm{Tr}\left(-\frac{1}{4}D_\alpha\Gamma^\beta D_\beta\Gamma^\alpha - \frac{1}{6}D^\alpha\Gamma^\beta\{\Gamma_\alpha,\Gamma_\beta\} - \frac{1}{24}\{\Gamma^\alpha,\Gamma^\beta\}\{\Gamma_\alpha,\Gamma_\beta\}\right)$$

$$-\frac{1}{2}(D^\alpha\bar{\Phi} + i\bar{\Phi}\Gamma^\alpha)(D_\alpha\Phi - i\Gamma_\alpha\Phi) + \frac{\pi}{\kappa}(\bar{\Phi}\Phi)^2\Bigg)$$

$$\Phi = \phi + \theta\psi - \theta^2 F \ , \bar{\Phi} = \bar{\phi} + \theta\bar{\psi} - \theta^2\bar{F} \ ,$$

$$\Gamma^\alpha = \chi^\alpha - \theta^\alpha B + i\theta^\beta A_\beta{}^\alpha - \theta^2(2\lambda^\alpha - i\partial^{\alpha\beta}\chi_\beta) \ .$$

- The theory exhibits a **strong-weak self duality** under the duality map

$$\kappa' = -\kappa \ , \ N' = |\kappa| - N + 1 \ , \ \lambda' = \lambda - \mathrm{Sgn}(\lambda)$$

$$\lambda = \frac{N}{\kappa} \ , N \to \infty, \kappa \to \infty$$

- Goal: **Compute correlation functions of conserved currents exactly to all orders in the coupling constant**.

Conserved currents

- A general **spin s current in the free field representation**

$$\mathcal{J}^{(s)} = \sum_{r=0}^{2s} (-1)^{\frac{r(r+1)}{2}} \binom{2s}{r} D^r \bar{\Phi} D^{2s-r} \Phi \qquad s = 0, \frac{1}{2}, 1, \ldots$$

$$D = \lambda^{\alpha_1} \lambda^{\alpha_2} \ldots \lambda^{\alpha_i} D_{\alpha_1} D_{\alpha_2} \ldots D_{\alpha_i}$$

$$\mathcal{J}^{(s)} = \lambda^{\alpha_1} \lambda^{\alpha_2} \ldots \lambda^{\alpha_{2s}} \mathcal{J}_{\alpha_1 \alpha_2 \ldots \alpha_{2s}}$$

- In the **free theory**, $\lambda = 0$, these **currents are exactly conserved**

$$D^\alpha \mathcal{J}_{\alpha \alpha_2 \ldots \alpha_{2s}} = 0$$

- In the interacting theory $\lambda \neq 0$, conservation is weakly broken by $\mathcal{O}\left(\frac{1}{N}\right)$

$$\Delta_s = s + 1 + \frac{\gamma_s(\lambda)}{N} + \ldots$$

- Replace $D \to \nabla$

$$\nabla_\alpha \Phi = D_\alpha \Phi - i \Gamma_\alpha \Phi$$

- Spin 0,1

$$\mathcal{J}_0 = \bar{\Phi}\Phi = \bar{\phi}\phi + \theta^\alpha(\bar{\phi}\psi_\alpha + \bar{\psi}_\alpha \phi) - \theta^2 \bar{\psi}\psi$$

$$\mathcal{J}_{\alpha\beta} = \bar{\Phi}\, \nabla_\alpha \nabla_\beta \Phi - 2\, \nabla_\alpha \bar{\Phi}\, \nabla_\beta \Phi + \nabla_\alpha \nabla_\beta \bar{\Phi} \Phi$$

Test of duality: Correlation functions of conserved currents

- **To compute these correlators exactly to all loops in large N**

$$\langle \mathcal{J}_0(q,\theta_1)\mathcal{J}_0(-q,\theta_2)\rangle$$
$$\langle \mathcal{J}_0(q,\theta_1)\mathcal{J}_0(q',\theta_2)\mathcal{J}_0(-q-q',\theta_3)\rangle$$
$$\langle \mathcal{J}_0(q,\theta_1)\mathcal{J}_0(q',\theta_2)\mathcal{J}_0(q'',\theta_3)\mathcal{J}_0(-q-q'-q'',\theta_4)\rangle$$

- In a CFT, the two point and three point functions are fixed by **conformal symmetry**, and in addition **higher spin symmetry, parity** further constrains the independent structures that appear in the correlators.

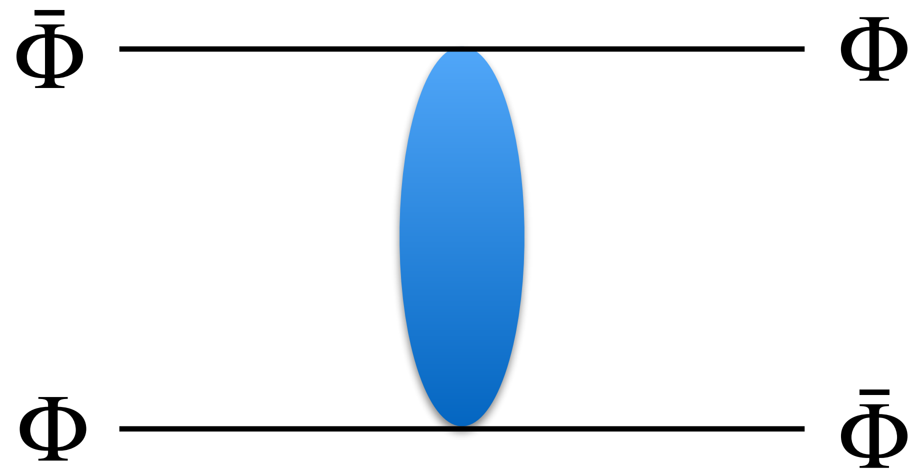
Maldacena, Zhiboedov

- While the **four point function is fixed only up to conformal cross ratios**.

- In addition, the correlation functions in superspace obey **supersymmetric ward identities**.

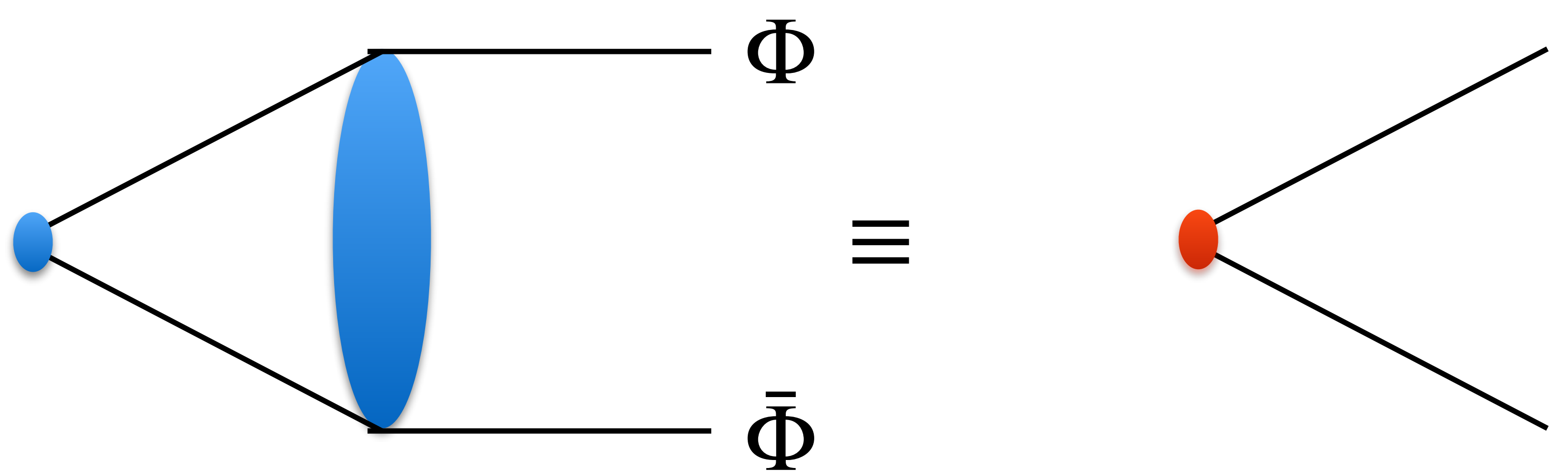
Test of duality: Correlation functions of conserved currents

- Step 1: **Exact four point correlator of fields to all loops**

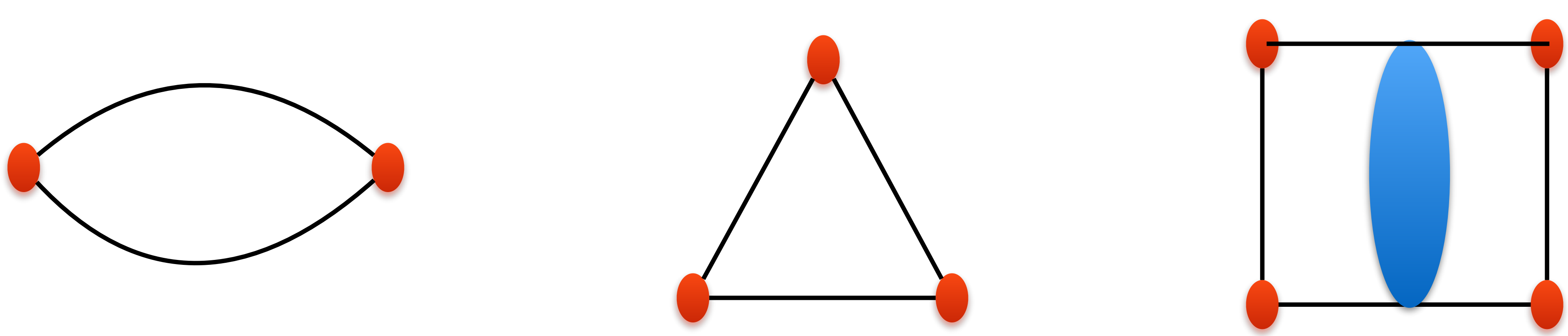


- Step 2: **Exact vertex/Form factor for the current insertion**

$\mathscr{J}_0(q) = \bar{\Phi}\Phi(q)$

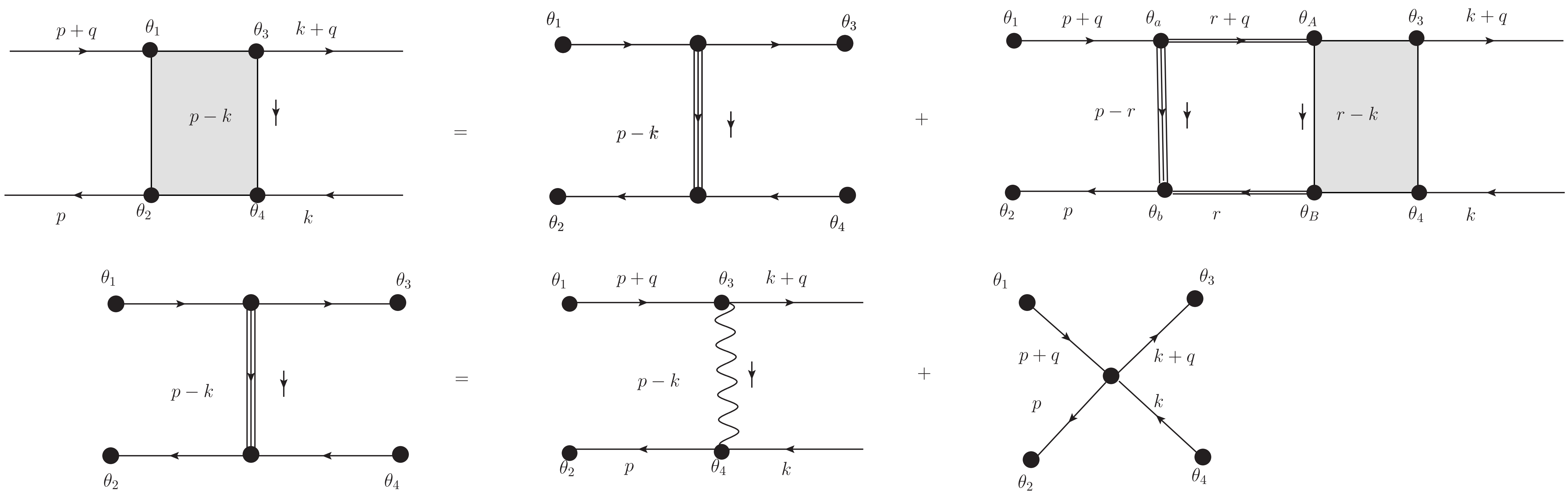


- Step 3: Compute the correlator



Spin zero correlators : Step 1

- Four point function of scalar super-fields computed to all orders in λ

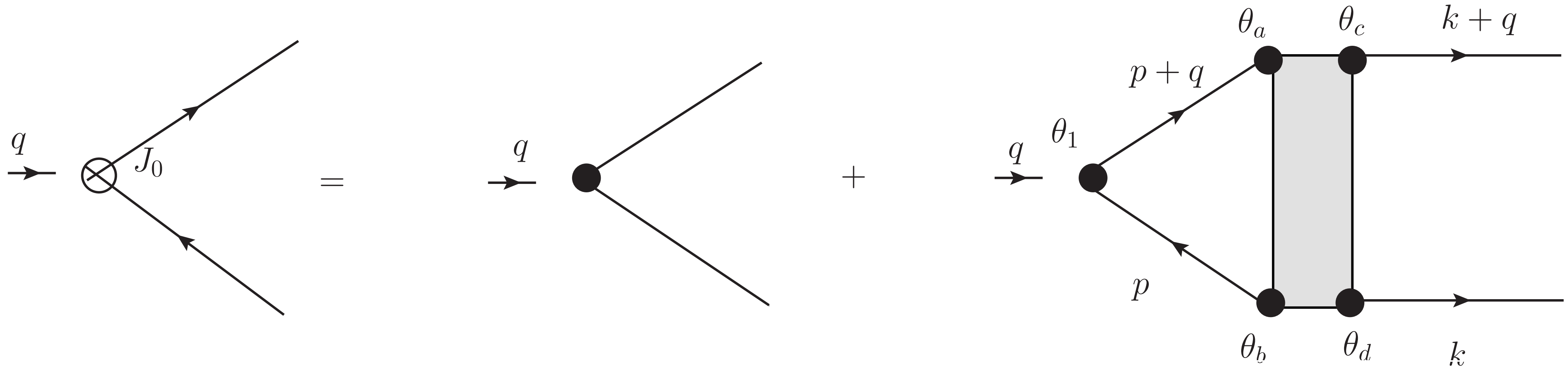


K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama

- Computed by Dyson-Schwinger methods, summing over planar diagrams, to all orders in the 't Hooft coupling λ

Form factors for Spin zero correlators : Step 2

- Form factors **K.I, S.Jain, P. Nayak, T.Sharma**

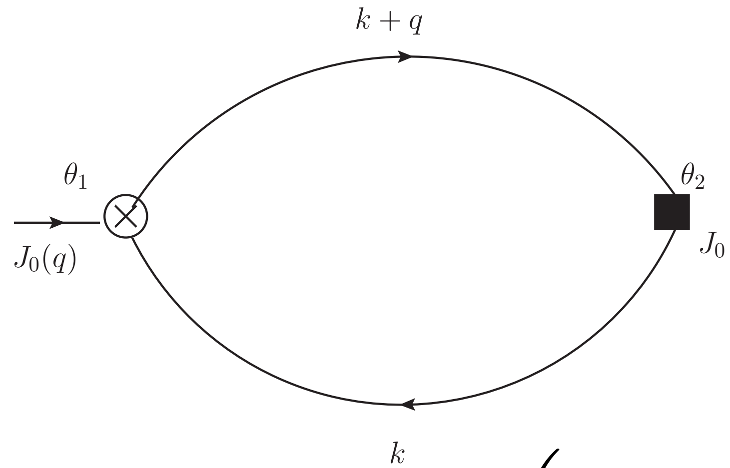


$$\mathcal{V}(X, X_{13}, X_{43}, q, k) = \exp\left(\frac{1}{3}X.(q.X_{13} + k.X_{43})\right)\mathcal{F}(X_{13}, X_{43}, q, k)$$

Ward identity

$$\begin{aligned}\mathcal{F}(X_{13}, X_{43}, q, k) &= -X_{13}^{-}X_{13}^{+}X_{43}^{-}X_{43}^{+} \\ &+ \left(\frac{e^{2i\lambda\tan^{-1}\left(\frac{2k_s}{q_3}\right)} - 1}{2k_-}\right)X_{13}^{+}X_{43}^{+} \\ &+ \left(\frac{1 - e^{i\lambda\left(2\tan^{-1}\left(\frac{2k_s}{q_3}\right) - \pi\mathrm{sgn}(q_3)\right)}}{2q_3}\right)X_{43}^{-}X_{43}^{+} \\ &+ \left(\frac{2}{3} - \frac{1}{6}e^{2i\lambda\tan^{-1}\left(\frac{2k_s}{q_3}\right)} - \frac{1}{2}e^{i\lambda\left(2\tan^{-1}\left(\frac{2k_s}{q_3}\right) - \pi\mathrm{sgn}(q_3)\right)}\right)X_{13}^{-}X_{13}^{+}X_{43}^{-}X_{43}^{+}\end{aligned}$$

Two point function of spin zero super-currents



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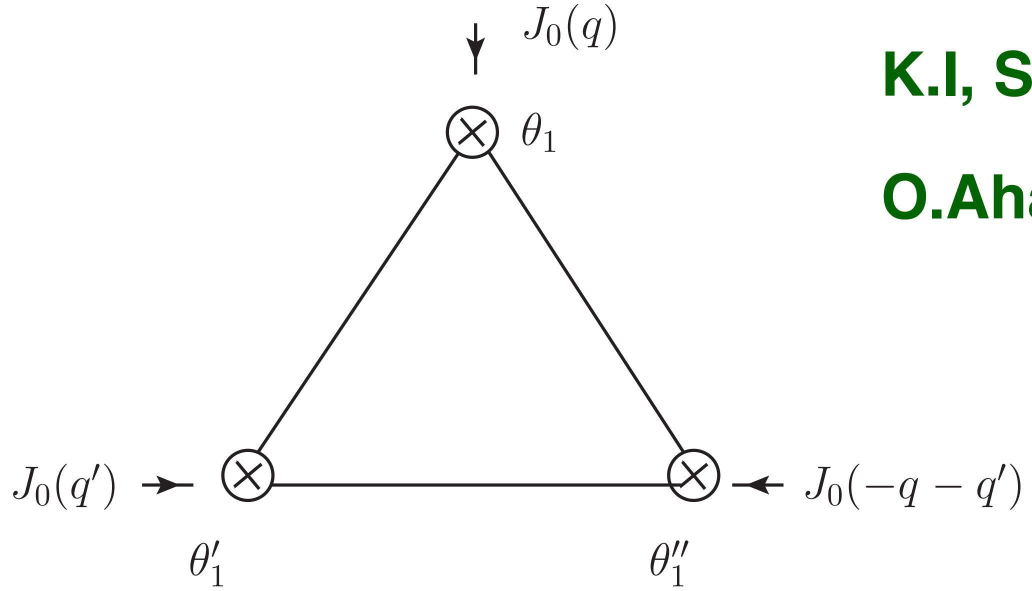
O.Aharony, A. Sharon

$$\langle J_0(\theta_1,q)J_0(\theta_2,-q)\rangle = \frac{N}{8\pi|q|\lambda}\exp(-\theta_1^\alpha\theta_2^\beta q_{\alpha\beta})\Big(\sin(\pi\lambda)+|q|(1-\cos(\pi\lambda))\delta^2(\theta_1-\theta_2)\Big)$$

$$\begin{aligned}
\langle J^{BB}(q)J^{BB}\rangle &= \frac{N}{|q|}\frac{\sin(\pi\lambda)}{8\pi\lambda}\\
\langle J^{FF}(q)J^{FF}\rangle &= -N|q|\frac{\sin(\pi\lambda)}{8\pi\lambda}\\
\langle J^{BF}_\alpha(q)J^{BB}\rangle &= 0\\
\langle J^{BF}_\alpha(q)J^{FF}\rangle &= 0\\
\langle J^{BF}_\alpha(q)J^{BF}_\beta\rangle &= \frac{N}{8}\left(\frac{q_{\alpha\beta}}{|q|}\,\frac{\sin(\pi\lambda)}{\pi\lambda}+C_{\alpha\beta}\frac{1-\cos(\pi\lambda)}{\pi\lambda}\right)\\
\langle J^{FF}(q)J^{BB}\rangle &= \frac{N}{8\pi\lambda}\big(\cos(\pi\lambda)-1\big)
\end{aligned}$$

$$J^{BB}(q)=\bar\phi\phi(q)\ ,\ \ J^{BF}_\alpha(q)=\bar\phi\psi_\alpha(q)+\bar\psi_\alpha\phi(q)\ ,\ \ J^{FF}=\bar\psi\psi(q)$$

Three point function of spin zero super-currents



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$$
\begin{aligned}
\langle J_0(\theta_1,q)J_0(\theta_1',q')J_0(\theta_1'',-q-q')\rangle =& \left(\frac{N}{72\ q_3 q_3'(q_3+q_3')}\frac{\sin(\pi\lambda)}{\pi\lambda}\right)\Big[-9\cos(\pi\lambda) \\
&+9i\sin(\pi\lambda)\left(q_3 X^-_{11''}X^+_{11''}+q_3' X^-_{1'1''}X^+_{1'1''}\right) \\
&+3\cos(\pi\lambda)\ (q_3'-q_3)\left(X^-_{11''}X^+_{1'1''}-X^-_{1'1''}X^+_{11''}\right) \\
&-\cos(\pi\lambda)\ (q_3^2+7q_3q_3'+q_3'^2)\ X^-_{11''}X^+_{11''}X^-_{1'1''}X^+_{1'1''}\Big] \\
&\times e^{\frac{1}{3}X\cdot\left(q\cdot X_{11''}+q'\cdot X_{1'1''}\right)}
\end{aligned}
$$

Three point function of spin zero super-currents

$$\langle J^{BB}(q) J^{BB}(q') J^{BB} \rangle = -\frac{N}{8\, q_3 q'_3 (q_3 + q'_3)} \frac{\sin(2\pi\lambda)}{2\pi\lambda}$$

$$\langle J^{FF}(q) J^{FF}(q') J^{FF} \rangle = -\frac{N}{8} \frac{\sin^2(\pi\lambda)}{\pi\lambda}$$

$$\langle J^{FF}(q) J^{BB}(q') J^{BB} \rangle = \frac{N}{8\, q'_3 (q_3 + q'_3)} \frac{\sin^2(\pi\lambda)}{\pi\lambda}$$

$$\langle J_{+}^{BF}(q) J_{-}^{BF}(q') J^{BB} \rangle = \frac{N(q_3 - q'_3)}{24\, q_3 q'_3 (q_3 + q'_3)} \frac{\sin(2\pi\lambda)}{\pi\lambda}$$

$$\langle J^{FF}(q) J_{+}^{BF}(q') J_{-}^{BF} \rangle = -\frac{N}{144\, q_3 q'_3 (q_3 + q'_3)} \left(18 q_3 q'_3 \frac{\sin^2(\pi\lambda)}{\pi\lambda} - i \left(2q_3^2 - 7 q_3 q'_3 - 4 q'^2_3\right) \frac{\sin(2\pi\lambda)}{\pi\lambda} \right)$$

$$\langle J^{FF}(q) J^{FF}(q') J^{BB} \rangle = \frac{N\ \left(4q_3^2 + q_3 q'_3 + 4 q'^2_3\right)}{144\, q_3 q'_3 (q_3 + q'_3)} \frac{\sin(2\pi\lambda)}{\pi\lambda}$$

Two independent structures

$$J^{BB}(q) = \bar{\phi}\phi(q)\ ,\ \ J_{\alpha}^{BF}(q) = \bar{\phi}\psi_{\alpha}(q) + \bar{\psi}_{\alpha}\phi(q)\ ,\ \ J^{FF} = \bar{\psi}\psi(q)$$

Four point function of spin zero super-currents

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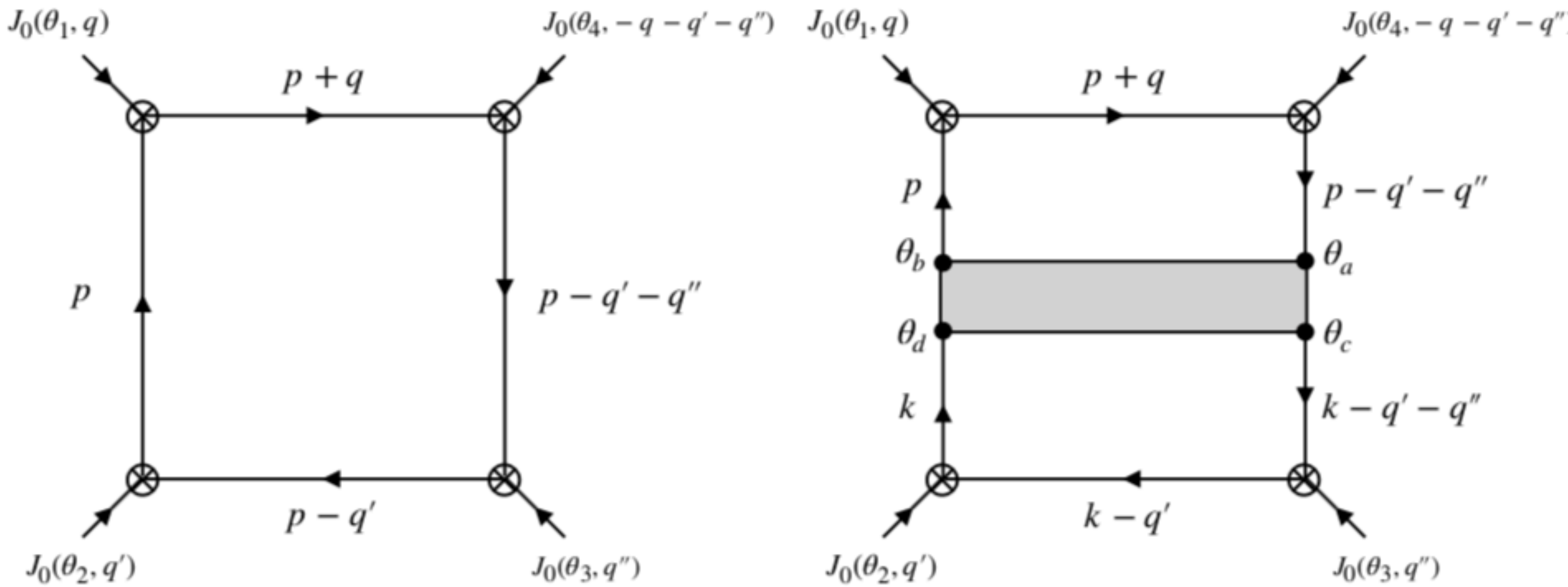

- For the non-supersymmetric fermion+CS theory the result was computed in the **double soft limit**.
- The result for general kinematics is argued to be proportional to that of the "**free theory**" using the known **three point functions, Lorentzian OPE inversion formula and crossing symmetry**. Zhiboedov, Turlaci
- We **expect a similar result for the supersymmetric theory** as well.

Summary

- We computed **2 point and 3 point functions of spin zero currents exactly to all orders in the ’t Hooft coupling in N=2 susy Chern-Simons matter theory**.

- The results are **consistent with the conjectured strong-weak duality** in this theory.

- From general arguments: we expect that the **three point function of any higher spin current is fixed in terms of two independent structures**.

- We also expect that the **four point function of spin zero super-currents will be proportional to that of the free theory**.

- It would be interesting to find arguments to **generalise the Maldacena-Zhiboedov theorem to supersymmetric theories**.

Thank you!!