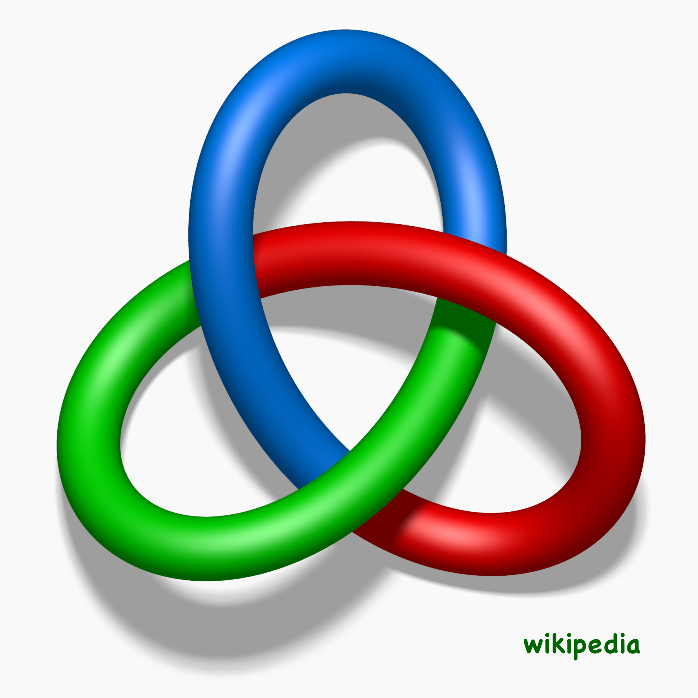
Bosonization duality in Chern-Simons matter theories

Karthik Inbasekar

אוניברסיטת בן-גוריון בנגב
Ben-Gurion University of the Negev

Jan 13, 2020

Pure Chern-Simons theories



"Anyone who knows anyons cannot describe them.
Anyone who can describe anyons does not know them."

Review article: Dunne

Physics in 2+1 dimensions

Physics in 2+1 dimensions has many **interesting features and intriguing surprises**.

The behavior of bosons/fermions and gauge fields differs significantly from the classical/quantum electrodynamics in 3+1 dimensions.

In addition to the usual Maxwell theory

Dimensions of mass

$$\frac{1}{g^2}\int d^3x F^{\mu\nu}F_{\mu\nu}$$

there exists a new type of gauge theory called Chern-Simons theory

First order in derivatives

$$\frac{\kappa}{2\pi}\int d^3x \epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$$

At first sight, does not appear to be manifestly gauge invariant.

Pure Chern-Simons theories

Under a gauge transformation, the action changes by a **total derivative**

$$\frac{\kappa}{2\pi}\partial_\mu\left(\epsilon^{\mu\nu\rho}\partial_\nu A_\rho\right)$$

The classical equations of motion better reflect the gauge invariance

$$\frac{\kappa}{2\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho} = 0$$

The solutions of this equation are **pure gauge**, unlike Maxwell theory where source free equations have plane wave solutions.

In other words, there are **no propagating degrees of freedom**.

The theory become extremely **interesting when coupled to matter**.

Chern-Simons matter theories

Coupling Chern-Simons theory with a source

$$\frac{\kappa}{2\pi}\int d^3x(\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - A_\mu J^\mu)$$

Changes the topological character significantly

$$\frac{\kappa}{2\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho} = J^\mu$$

$$\frac{\kappa}{2\pi}\epsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho} = 0 \Leftrightarrow \partial_\mu J^\mu = 0$$

Bianchi Identity

Current conservation

The physics is best illustrated in components $J = (\rho, J^i)$

$$\rho = \frac{\kappa}{2\pi}B \qquad J^i = \frac{\kappa}{2\pi}\epsilon^{ij}E_j$$

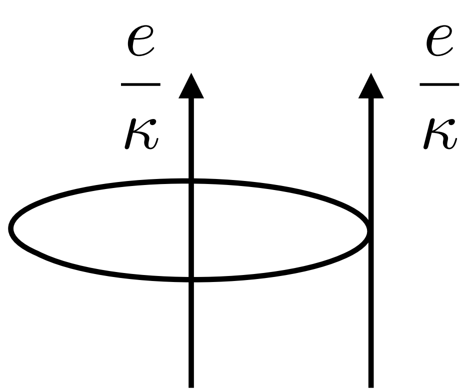
Chern-Simons matter theories : Physics

$$\rho = \frac{\kappa}{2\pi} B$$

Charge density $\propto$ magnetic field

The effect of Chern-Simons gauge field is to **attach local magnetic flux to interacting charged particles**!

Adiabatic excursion of such particles leads to the **Aharanov-Bohm effect**



$$\text{AB phase} = e^{ie\int A.dx} = e^{i\frac{e^2}{\kappa}}$$

The phase: interpreted as **point particle explanation of anyonic statistics**.

By choosing κ appropriately, the anyonic exchange phase can be chosen to behave either as bosons or fermions.

This **statistical transmutation is at the very heart of the bosonization duality** as we will see later.

Chern-Simons matter theories : Physics

$$J^i = \frac{\kappa}{2\pi}\epsilon^{ij}E_j$$

Identifying $\kappa = \dfrac{e^2\nu}{\hbar}$, the Chern-Simons action describes Hall-Conductivity of ν filled Landau levels.

In fact, Chern-Simons matter theories play a role as **effective field theories that describe Quantum Hall effect**.

ν describes the number of filled Landau levels, and this would make sense only if κ took integer values. Fractional QHE is another story..

In an abelian theory, the quantization of κ is subtle, and is best understood, by placing the theory in finite temperature ($S^1 \times S^2$) and studying its effect under large gauge transformations.

$$A_0 \to A_0 + \frac{2\pi\hbar}{e\beta} \qquad S_{CS} \to S_{CS} + \frac{2\pi\hbar^2\kappa}{e^2}$$

Tong

Non-abelian Chern-Simons theories

In non-abelian theories, **quantization of κ follows from gauge invariance**

$$S = \int d^3x \frac{\kappa}{4\pi} \epsilon^{\mu\nu\rho} \mathrm{Tr}(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho)$$

Gauge transformation $\quad A_\mu \to g^{-1} A_\mu g + g^{-1} \partial_\mu g$

Action changes by a boundary term proportional to the winding number

$$S \to S - 8\pi^2 \kappa N$$

eg for $g \in SU(2)$;

$$g = \exp\left(i\pi N \frac{\vec{x} \, . \, \vec{\sigma}}{\sqrt{\vec{x}^2 + R^2}} \right),$$

$$N \equiv w(g) = \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho} \mathrm{Tr} \left(g^{-1}\partial_\mu g g^{-1} \partial_\nu g g^{-1} \partial_\rho g \right)$$

In order for the action to be gauge invariant

$$\kappa = \frac{\text{Integer}}{2\pi}$$

The theory has no running parameters!

Non-abelian Chern-Simons theories coupled to matter

Non-abelian Chern-Simons theories coupled to matter in the fundamental representation in $U(N), SU(N), Sp(N)$ are interesting.

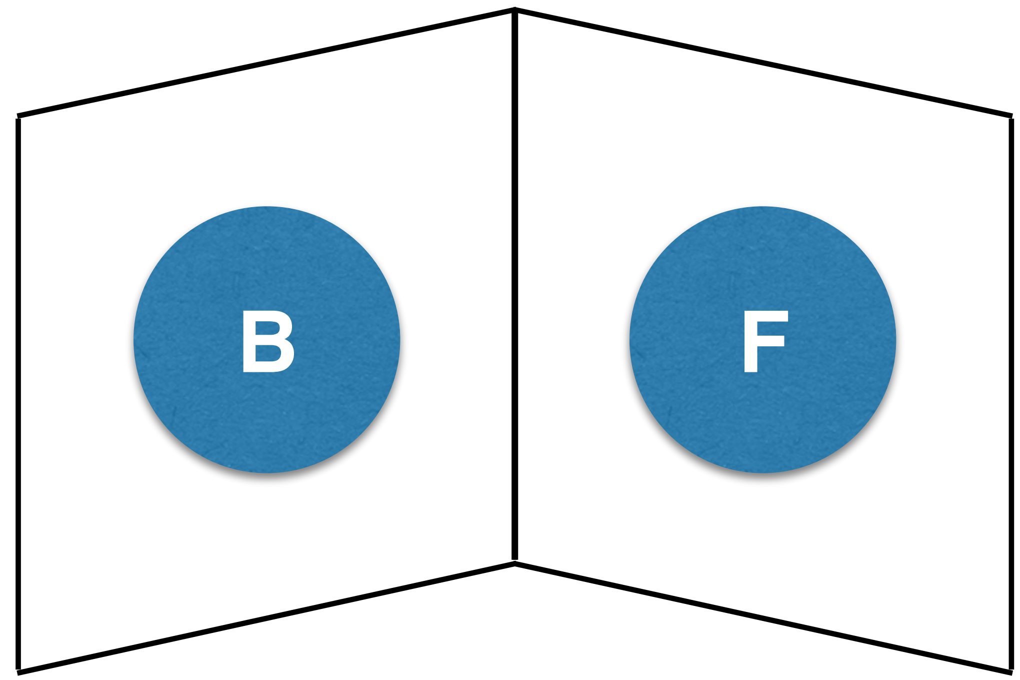
These theories are **exactly solvable in the large N limit**.

They are **holographically dual to classical higher spin gravity on** AdS_4

The finite N and k versions of these theories are relevant for condensed matter applications especially as effective field theories for integer and fractional quantum hall effect.

It is by now well established that these theories enjoy a **strong-weak type bozonization duality in Field theory**.

At the heart of this duality is the physical effect of the Aharanov-Bohm phase.

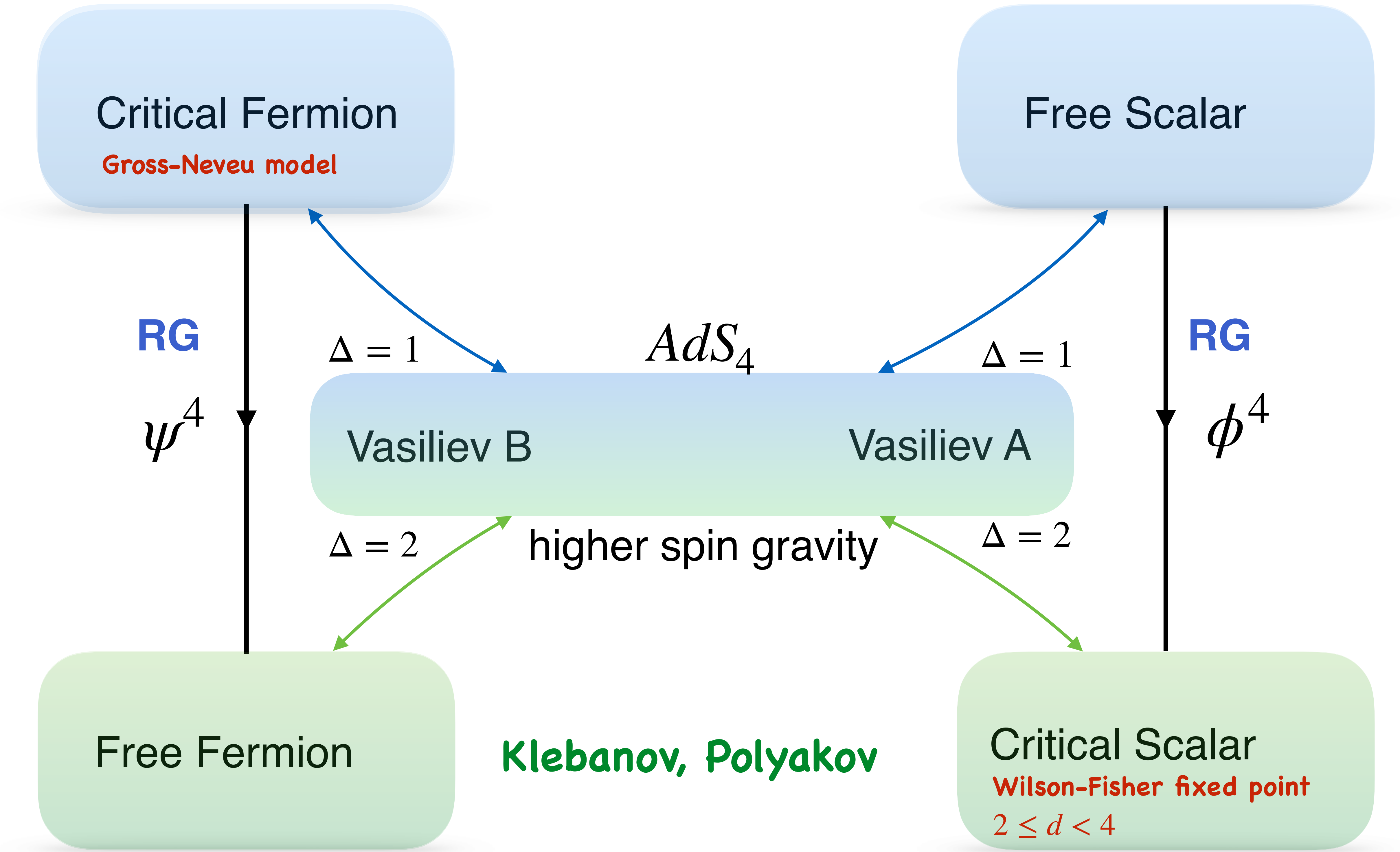


Bosonization duality

Aharony,Benini,Chang,Frishman,Giombi,Giveon,Gur-Ari,Gurucharan,Hsin,K.I, Jain
Karch,Kutasov,Maldacena,Mandlik,Minwalla,Moshe,Nayak,Prakash,
Radicevic,Sharma,Seiberg,Takimi,Trivedi,Tong,Yacoby,Yin,Yokoyama,Wadia,Witten,
Zhiboedov

Review article: Giombi

Vector models in 2+1 dimensions



What happens when you turn on Chern-Simons interactions?

Vector models coupled to Chern-Simons gauge theory

$SU(N_F)_{\kappa_F}$ Chern-Simons coupled to fundamental fermions (regular fermion)

$$S=\int d^3x\left(i\epsilon^{\mu\nu\rho}\frac{\kappa_F}{4\pi}\mathrm{Tr}(A_\mu\partial_\nu A_\rho-\frac{2i}{3}A_\mu A_\nu A_\rho)+\bar{\psi}^i\gamma^\mu D_\mu\psi_i+m_f\bar{\psi}^i\psi_i\right)$$

$SU(N_B)_{\kappa_B}$ Chern-Simons coupled to fundamental bosons

$$S=\int d^3x\left(i\epsilon^{\mu\nu\rho}\frac{\kappa_B}{4\pi}\mathrm{Tr}(A_\mu\partial_\nu A_\rho-\frac{2i}{3}A_\mu A_\nu A_\rho)+D_\mu\bar{\phi}D^\mu\phi\right.$$
$$\left.+\sigma\bar{\phi}\phi+N_B\frac{m_B^2}{b_4}\sigma-N_B\frac{\sigma^2}{2b_4}\right)$$

Wilson-Fisher limit (critical boson)

$$b_4\to\infty\ ,\ m_B\to\infty\ ,\ 4\pi\frac{m_B^2}{b_4}=\text{fixed}$$

Large N bosonization duality

t' Hooft large N limit $\lambda_B = \lim_{\substack{N_B \to \infty \\ \kappa_B \to \infty}} \frac{N_B}{\kappa_B}$ $\lambda_F = \lim_{\substack{N_F \to \infty \\ \kappa_F \to \infty}} \frac{N_F}{\kappa_F}$

Chern-Simons coupled to fundamental bosons at Wilson-Fisher limit

dual

Chern-Simons coupled to fundamental Fermions

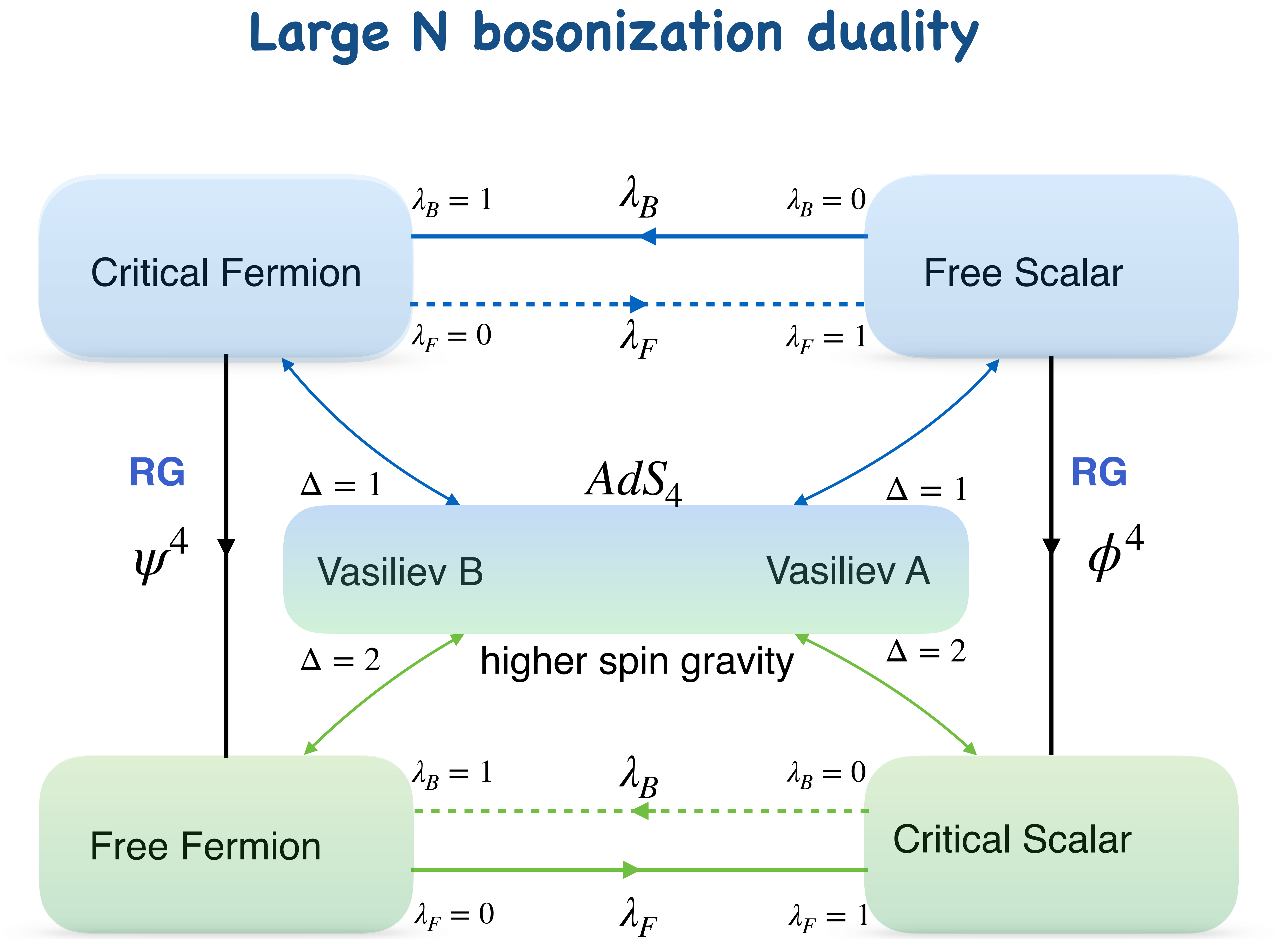
duality map
$$\begin{cases} \kappa_F = -\kappa_B \\ N_F = |\kappa_B| - N_B \\ \lambda_B = \lambda_F - \mathrm{Sign}(\lambda_F) \\ m_F = -m_B^{Crit}\lambda_B \end{cases}$$

There is a conjectured/untested finite N and k duality map

We will motivate this map from AB phases :)

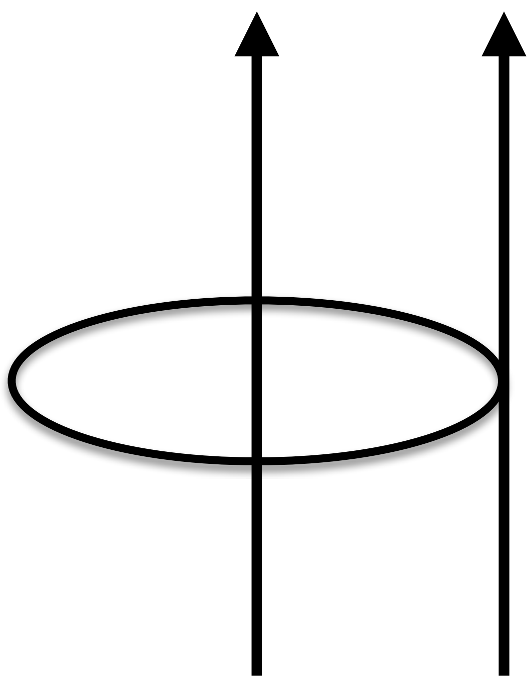
Physical observables computed on one side, match with observables on the other side under the duality map.

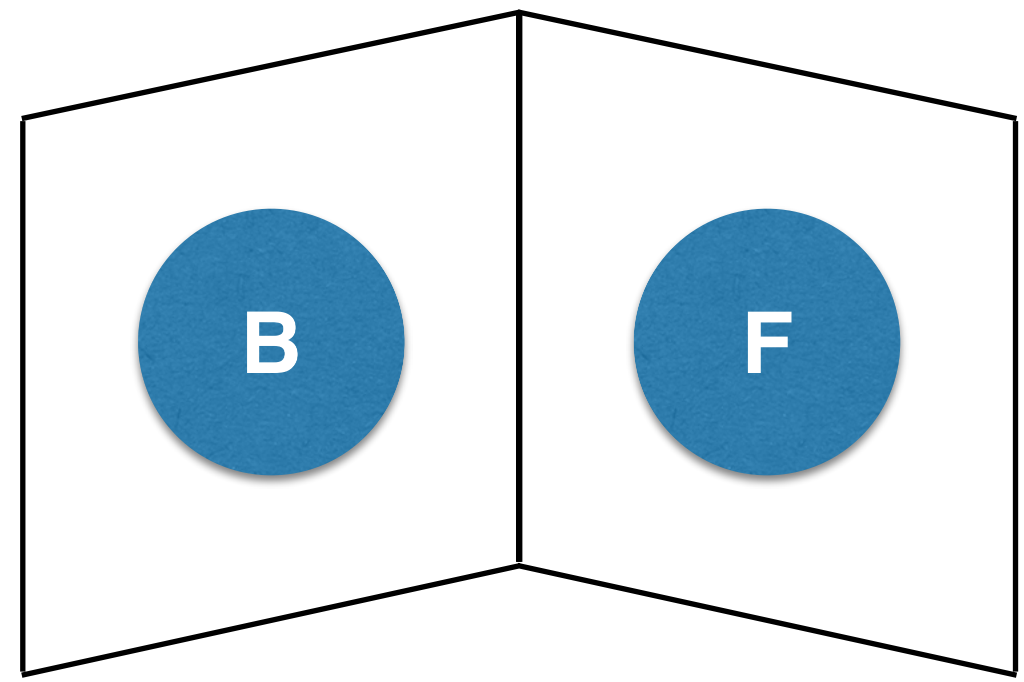
Strong-weak duality: Observables need to be computed to all loops!



Large N bosonization duality
Critical Fermion
Free Scalar
$\lambda_B = 1$
λ_B
$\lambda_B = 0$
$\lambda_F = 0$
λ_F
$\lambda_F = 1$
RG
ψ^4
RG
ϕ^4
$\Delta = 1$
AdS_4
$\Delta = 1$
Vasiliev B
Vasiliev A
$\Delta = 2$
higher spin gravity
$\Delta = 2$
Free Fermion
Critical Scalar
$\lambda_B = 1$
λ_B
$\lambda_B = 0$
$\lambda_F = 0$
λ_F
$\lambda_F = 1$

AB phase to duality





$\nu_B = \nu_F$

Not a derivation

$\lambda_B = \lambda_F - Sign(\lambda_F)$

AB phases in Chern-Simons matter theories at large N

Consider particles in representations R_1, R_2 of the gauge group

$$R_1 \otimes R_2 = \sum_m R_m$$

The Aharanov-Bohm phase of the particle R_1 as it circles around particle R_2 is $2\pi\nu_m$ in the mth channel of decomposition

$$\nu_m = \frac{4\pi}{\kappa}\mathrm{Tr}\left(T_{R_1}T_{R_2}\right) = \frac{2\pi}{\kappa}\left(C_2(R_1) + C_2(R_2) - C_m(R_m)\right)$$

Eg in SU(N)

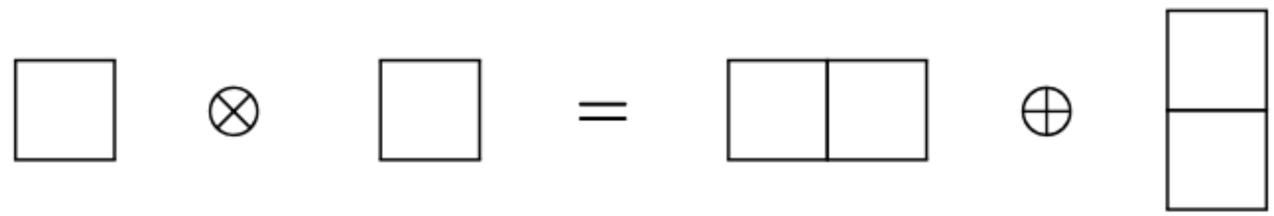
$$\text{Particles } (\mathbf{N}) \equiv \square\,, \quad \text{Anti-Particles } (\overline{\mathbf{N}}) \equiv \left.\begin{array}{c}\square\\ \square\\ \vdots\\ \square\end{array}\right\} N-1$$

Particles: Fundamental, Anti-Particles: Anti-Fundamental

AB phases in Chern-Simons matter theories at large N

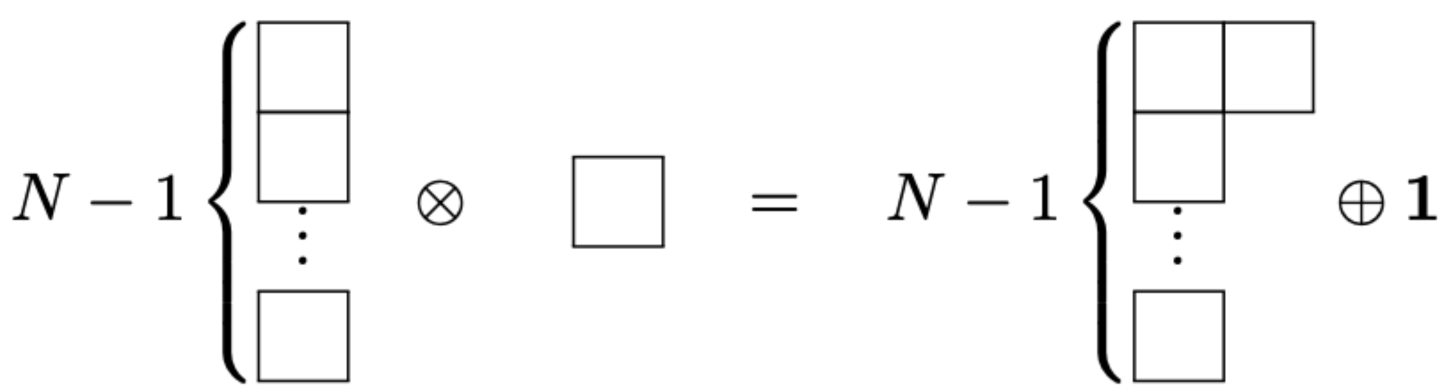
Eg in SU(N)

For exchange of particles



Fundamental $\otimes$ Fundamental $=$ Symmetric $\oplus$ Anti-Symmetric

For exchange of particle and an anti-particle



Anti-Fundamental $\otimes$ Fundamental $=$ Adjoint $\oplus$ Singlet

AB phases in Chern-Simons matter theories at large N

$$C_2(F) = C_2(A) = \frac{N^2-1}{2N}\ , \qquad C_2(Sym) = \frac{N^2+N-2}{N}\ , \qquad C_2(ASym) = \frac{N^2-N-2}{N}\ ,$$

$$C_2(Adj) = N\ , \qquad C_2(Sing) = 0$$

AB phases

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa}\ , \ \nu_{ASym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}\ , \ \nu_{Adj} = \frac{1}{N\kappa}\ , \ \nu_{Sing} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

Let us simplify life a bit…

In the limit $N \to \infty\ , \kappa \to \infty$ the 't Hooft coupling $\lambda = \dfrac{N}{\kappa}$

$$\nu_{Sym} \sim \nu_{ASym} \sim \nu_{Adj} \sim \mathcal{O}\left(\frac{1}{N}\right)\ , \ \nu_{Sing} \sim -\lambda$$

AB phases in Chern-Simons matter theories at large N

$SU(N_B)_{\kappa_B}$ Chern-Simons coupled to fundamental bosons

AB exchange phase in singlet channel in the bosonic theory

$$e^{-i\pi\lambda_B}$$

$SU(N_F)_{\kappa_F}$ Chern-Simons coupled to fundamental fermions

AB exchange phase in singlet channel in the fermionic theory

$$(-1)\ e^{-i\pi\lambda_F} = e^{-i\pi\ (\lambda_F - Sign(\lambda_F))}$$

Matching the phases give the map

$$\lambda_B = \lambda_F - Sign(\lambda_F)$$

This cute but not at all rigorous "derivation" illustrates the physics behind the bosonization duality. 😑

How does one test a strong-weak duality?

In the strong coupling description (at critical points), there is no Lagrangian description. How do we compute anything at all?

In a weak coupling description (away from critical points), there is a Lagrangian description. We can compute physical observables. But how to get the results at strong coupling?

Compute observables to all orders in the 't Hooft large N limit.

- **Spectrum of single trace primary operators (matching operator dimensions).**
- **Thermal partition functions.**
- **Correlation functions of gauge invariant currents.**
- $2 \to 2$ **S matrices.**
- **……**

Rest of the talk.

Describe the duality in a specifically nice theory!

Perform precision computations in the planar large N limit.

- $2 \to 2$ **S matrices.**

- **Correlation functions of gauge invariant conserved currents.**

A key ingredient to compute these observables is the off-shell four point correlation function of fundamental fields

$$\langle \bar{\Phi}^i(p_1)\Phi_i(p_2)\bar{\Phi}^j(p_3)\Phi_j(p_4)\rangle$$

This can be computed **exactly to all loops** in the planar limit using Dyson-Schwinger methods.

A bit of shameless advertisement...

Based on:

Exact amplitude computations

K.I, Janagal, Shukla; arXiv: 2001.02363

K.I, Janagal, Shukla; Phys.Rev. D100 (2019) no.8, 085008

K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama JHEP 1510 (2015) 176

Exact correlation functions of conserved currents.

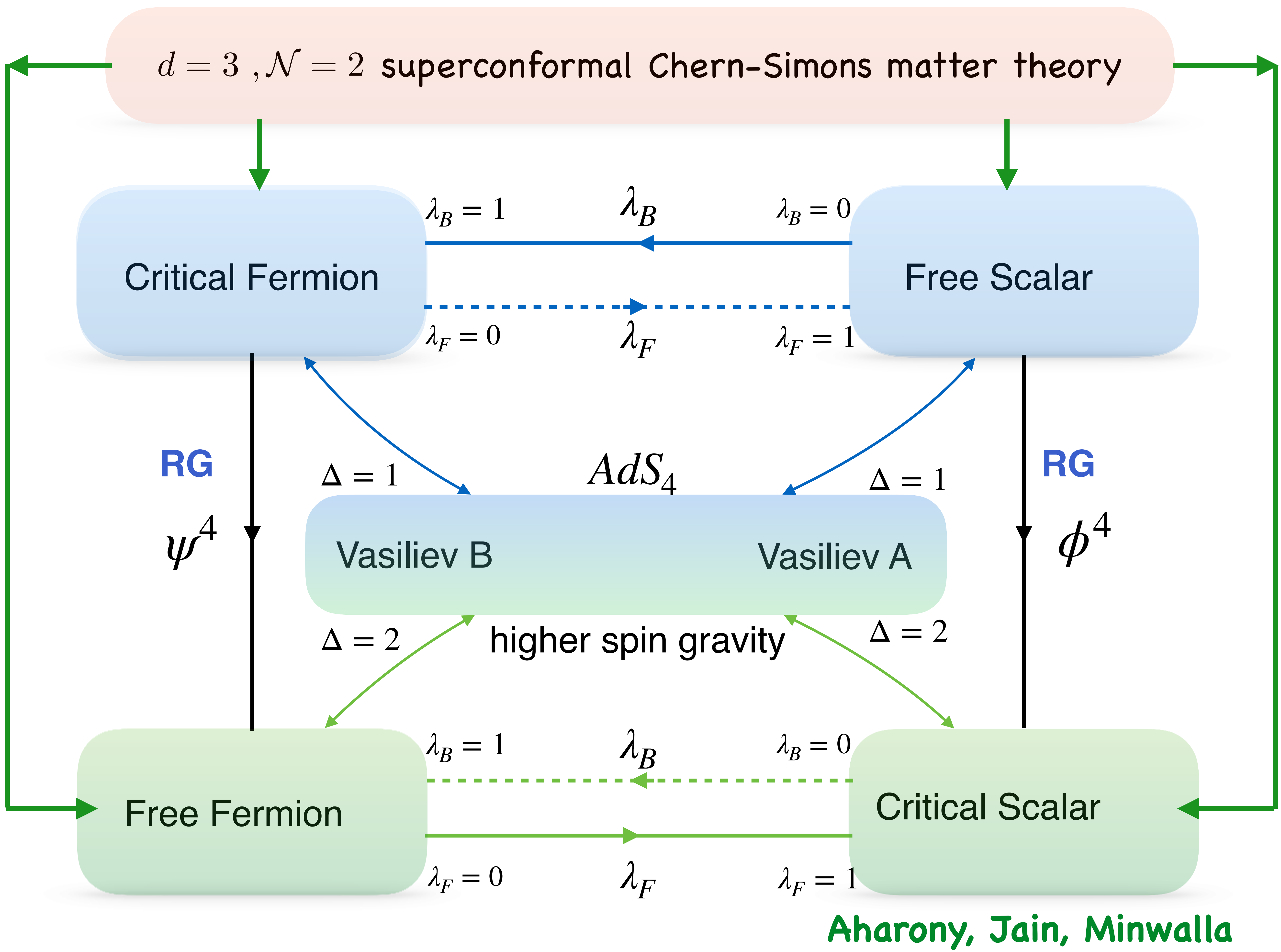
K.I, Jain, Malvimat, Mehta, Nayak, Sharma; arXiv 1907.11722

Other developments:

Dual superconformal symmetry, Yangian and BCFW recursions.

K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh; JHEP 1906 (2019) 016

K.I, Jain, Nayak, Umesh; Phys.Rev.Lett. 121 (2018) no.16, 161601


$d=3$, $\mathcal{N}=2$ superconformal Chern-Simons matter theory
Critical Fermion
Free Scalar
$\lambda_B = 1$
λ_B
$\lambda_B = 0$
$\lambda_F = 0$
λ_F
$\lambda_F = 1$
RG
ψ^4
RG
ϕ^4
$\Delta = 1$
AdS_4
$\Delta = 1$
Vasiliev B
Vasiliev A
$\Delta = 2$
higher spin gravity
$\Delta = 2$
$\lambda_B = 1$
λ_B
$\lambda_B = 0$
Free Fermion
Critical Scalar
$\lambda_F = 0$
λ_F
$\lambda_F = 1$
Aharony, Jain, Minwalla

Large N bosonization duality - SUSY version

$d = 3$, $\mathcal{N} = 2$ supersymmetric Chern-Simons theory coupled to matter in fundamental representation of SU(N)

$$\mathcal{S}_{\mathcal{N}=2} = -\int d^3x d^2\theta \Bigg(\frac{\kappa}{2\pi} \mathrm{Tr} \left(-\frac{1}{4} D_\alpha \Gamma^\beta D_\beta \Gamma^\alpha - \frac{1}{6} D^\alpha \Gamma^\beta \{\Gamma_\alpha, \Gamma_\beta\} - \frac{1}{24} \{\Gamma^\alpha, \Gamma^\beta\}\{\Gamma_\alpha, \Gamma_\beta\} \right)$$

$$-\frac{1}{2}(D^\alpha \bar{\Phi} + i\bar{\Phi}\Gamma^\alpha)(D_\alpha \Phi - i\Gamma_\alpha \Phi) + \frac{\pi}{\kappa}(\bar{\Phi}\Phi)^2 + m_0 \bar{\Phi}\Phi \Bigg)$$

$$\Phi = \phi + \theta\psi - \theta^2 F \ , \bar{\Phi} = \bar{\phi} + \theta\bar{\psi} - \theta^2 \bar{F} \ ,$$

$$\Gamma^\alpha = \chi^\alpha - \theta^\alpha B + i\theta^\beta A_\beta{}^\alpha - \theta^2(2\lambda^\alpha - i\partial^{\alpha\beta}\chi_\beta) \ . \qquad A_\beta{}^\alpha = A_\mu(\gamma^\mu)_\beta{}^\alpha$$

θ is a two component Grassmann spinor.

D_α is a supercovariant derivative in superspace $\quad D_\alpha = \dfrac{\partial}{\partial\theta^\alpha} + i\theta^\beta\partial_{\alpha\beta}$

Large N bosonization duality - SUSY version

The physical content of the theory is

$$S_{\mathcal{N}=2} = \int d^3x \Big[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right)$$
$$+\bar{\psi}(i\gamma^\mu\partial_\mu - m_0)\psi - \mathscr{D}^\mu\bar{\phi}\mathscr{D}_\mu\phi + m_0^2\bar{\phi}\phi$$
$$+\frac{4\pi m_0}{\kappa}(\bar{\phi}\phi)^2 + \frac{4\pi^2}{\kappa^2}(\bar{\phi}\phi)^3 + \frac{4\pi}{\kappa}(\bar{\phi})\phi(\bar{\psi}\psi) + \frac{2\pi}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi)$$

The theory exhibits a strong-weak self duality under the duality map

$$\kappa' = -\kappa\ ,\ N' = |\kappa| - N + 1\ ,\ \lambda' = \lambda - Sign(\lambda)\ ,\ m' = -m$$
$$\lambda = \frac{N}{\kappa}, N \to \infty, \kappa \to \infty$$

Admits a one parameter mass deformation that does not recieve quantum corrections. Avdeev, Grigoriev, Kondrashuk

K.I, Jain, Minwalla, Mazumdar, Umesh, Yokoyama

Bare propagator for matter = exact propagator

$$\langle \bar{\Phi}(\theta_1, p)\Phi(\theta_2, -p')\rangle = \frac{D^2_{\theta_1,p} - m_0}{p^2 + m_0^2}\delta^2(\theta_1 - \theta_2)(2\pi)^3\delta^3(p - p')$$

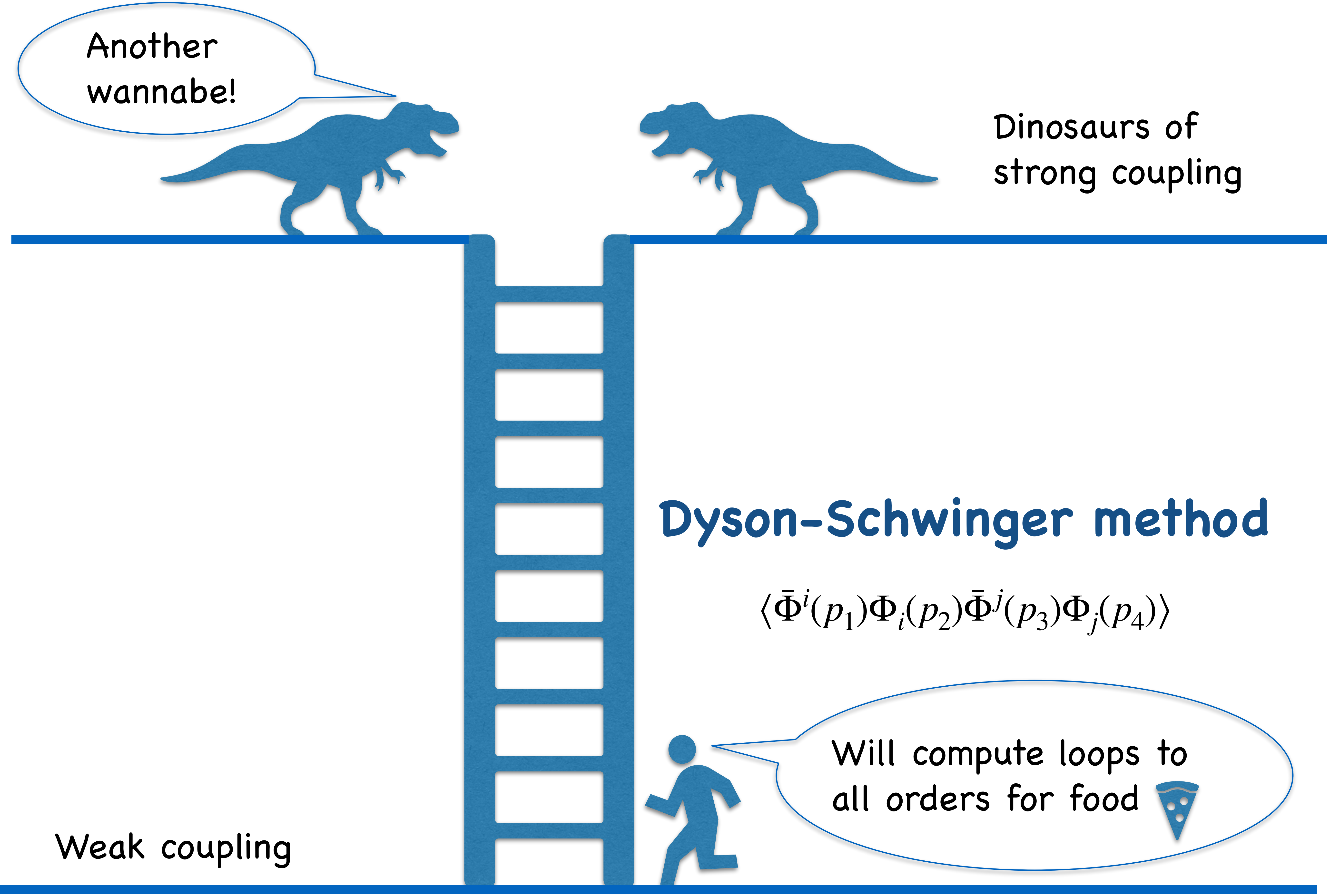
$$\langle \Gamma^-(\theta_1, p)\Gamma^-(\theta_2, -p')\rangle = -\frac{8\pi}{\kappa}\frac{\delta^2(\theta_1 - \theta_2)}{p_{--}}(2\pi)^3\delta^3(p - p')$$

K.I, Jain, Minwalla, Mazumdar, Umesh, Yokoyama

One might wonder why doesnt the gauge field propagator recieve corrections.

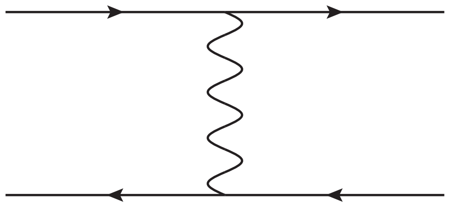
In the planar large N limit, diagrams that can contribute to the gauge field propagator are suppressed by $\mathcal{O}\left(\frac{1}{N}\right)$. Aharony, Gur-Ari, Yacoby

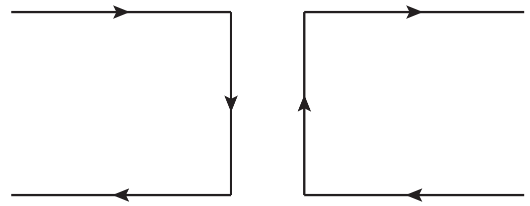
In light cone gauge, the triple vertex vanishes and ghosts are decoupled.


Another wannabe!
Dinosaurs of strong coupling
Dyson-Schwinger method
$\langle \bar{\Phi}^i(p_1)\Phi_i(p_2)\bar{\Phi}^j(p_3)\Phi_j(p_4)\rangle$
Will compute loops to all orders for food
Weak coupling

Comment 1: Large N counting - fewer diagrams

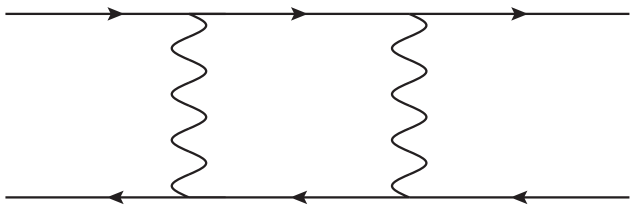
Tree

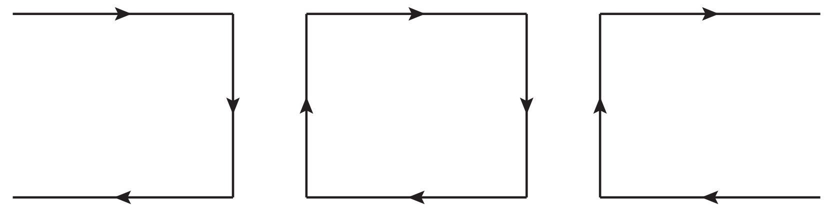




$$\frac{1}{\kappa} = \frac{\lambda}{N} \qquad \mathcal{O}\left(\frac{1}{N}\right)$$

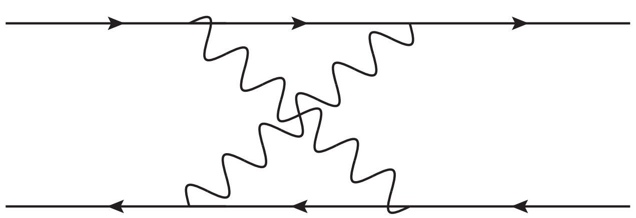
one loop (planar)

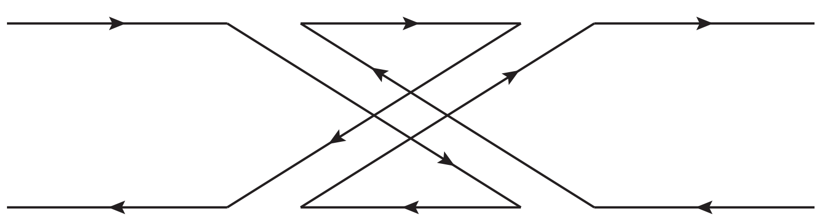




$$\frac{N}{\kappa^2} = \frac{\lambda^2}{N} \qquad \mathcal{O}\left(\frac{1}{N}\right)$$

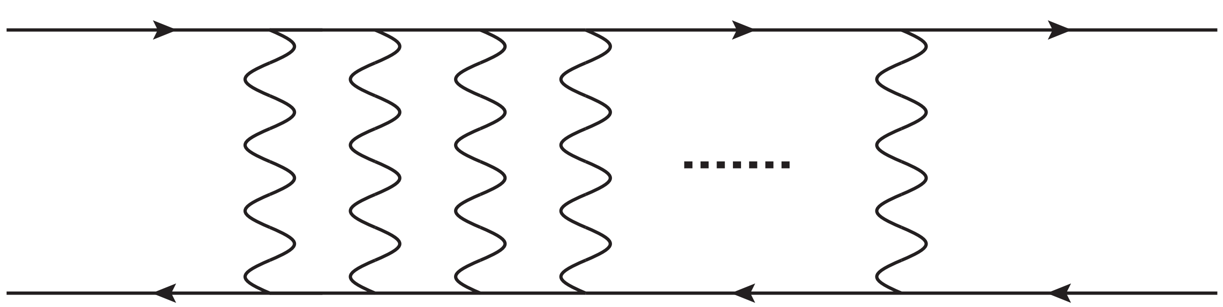
one loop (non planar)





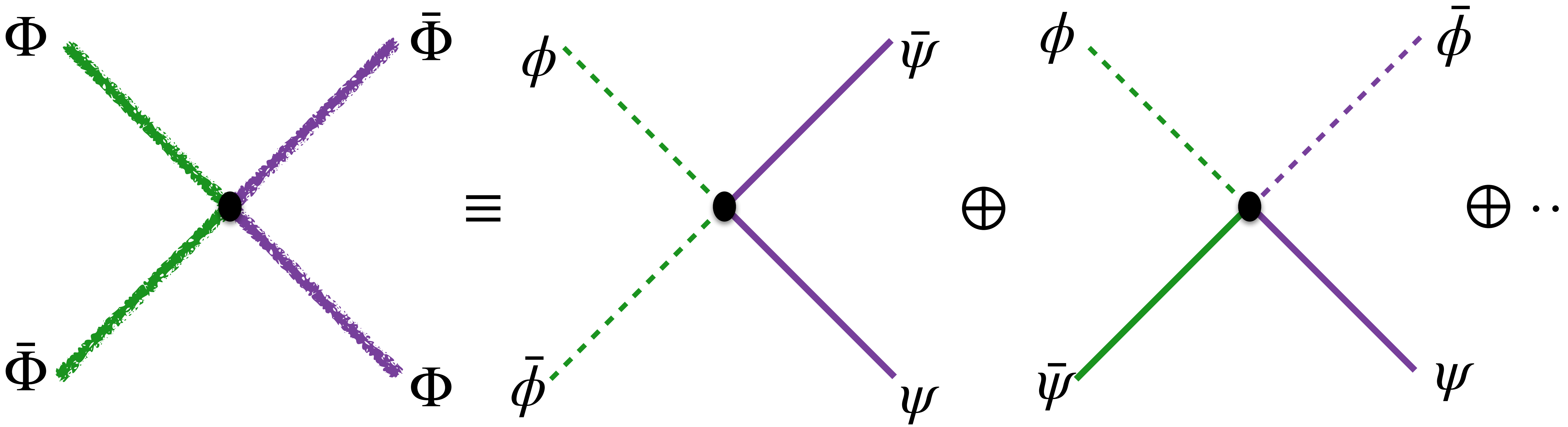
$$\frac{1}{\kappa^2} = \frac{\lambda^2}{N^2} \qquad \mathcal{O}\left(\frac{1}{N^2}\right)$$

All loops (planar)



$$\frac{N^l}{k^{l+1}} = \frac{\lambda^{l+1}}{N} \qquad \mathcal{O}\left(\frac{1}{N}\right)$$

Comment 2: Supergraphs - even fewer diagrams



A single supergraph is a **compact representation of several component graphs** expanded in a superspace basis (Taylor expansion in a grassmann space)

It follows that there are far few diagrams in the superspace basis than in components.

Comment 3: Susy light cone gauge

Susy light cone gauge in components (Euclidean)

$$\Gamma_- = 0 \;\Rightarrow\; A_- = A_1 + iA_2 = 0$$

Superspace Lagrangian becomes much simpler

$$\mathcal{S}_{\mathcal{N}=2} = -\int d^3x d^2\theta \left(-\frac{\kappa}{8\pi} \mathrm{Tr}\,(\Gamma^- \partial_{--} \Gamma^-) - \frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi - \frac{i}{2} \Gamma^- (\bar{\Phi} D_- \Phi - D_- \bar{\Phi} \Phi) + \frac{\pi}{\kappa} (\bar{\Phi}\Phi)^2 - m_0 \bar{\Phi}\Phi \right)$$

The price we pay is that correlation function we intend to compute will not be gauge invariant. (Off-shell functions are not gauge invariant in general)

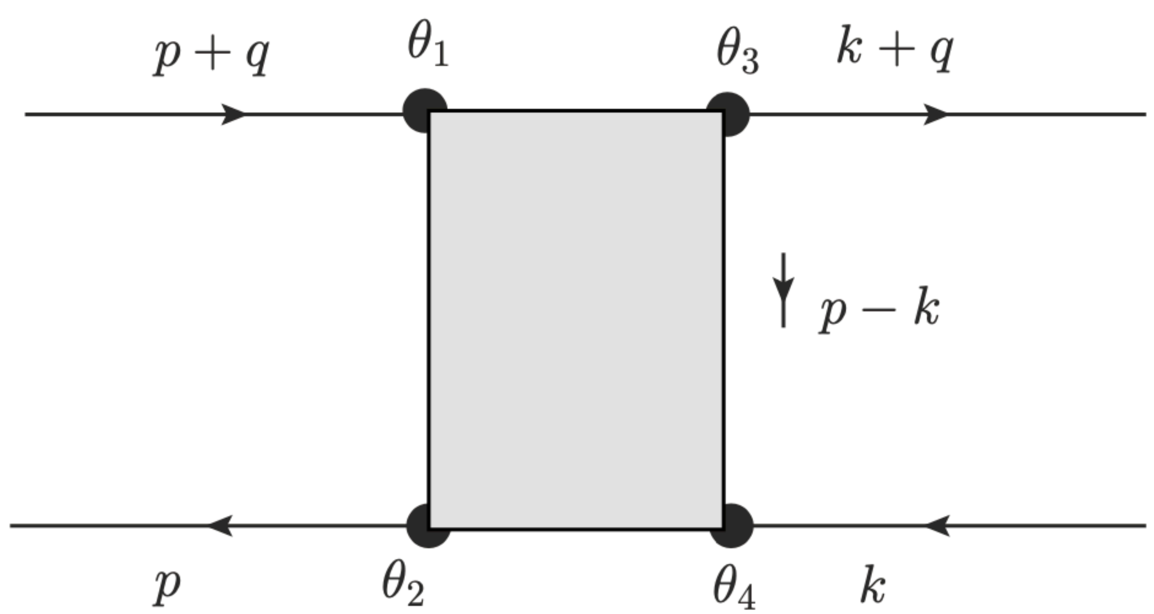
$$\langle \bar{\Phi}^i(\theta_1, p_1) \Phi_i(\theta_2, p_2) \bar{\Phi}^j(\theta_3, p_3) \Phi_j(\theta_4, p_4) \rangle$$

However, gauge invariant observables can be constructed out of it.

Comment 4: Supersymmetry constraints

Even before beginning the computation, SUSY constrains the general form of the four point correlation function.

$$\langle \bar{\Phi}(\theta_1, p+q)\Phi(\theta_2, -p)\Phi(\theta_3, k)\bar{\Phi}(\theta_4, -k-q)\rangle = V(\theta_1, \theta_2, \theta_3, \theta_4, p, q, k)$$



$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - \theta^\beta p_{\beta\alpha}$$

$$\{Q_\alpha, Q_\beta\} = 2p_{\alpha\beta}$$

The ward identity is a simple statement of translation invariance in superspace.

$$(Q_{\theta_1, p+q} + Q_{\theta_2, -p} + Q_{\theta_3, -k-q} + Q_{\theta_4, k})V(\theta_1, \theta_2, \theta_3, \theta_4, p, q, k) = 0\,.$$

Comment 4: Supersymmetry constraints

Susy ward identity fixes the exponential form, multiplied by a unknown function of $X_{ij} = \theta_i - \theta_j$ and $X = \sum_{i=1}^{4} \theta_i$

$$V(\theta_1, \theta_2, \theta_3, \theta_4, p, q, k) = \exp\left(\frac{1}{4} X.(p.X_{12} + q.X_{13} + k.X_{43})\right) F(X_{12}, X_{13}, X_{43}, p, q, k)$$

$$F(X_{12}, X_{13}, X_{43}, p, q, k) = \begin{array}{l} X_{12}^{+} X_{43}^{+} \Big(A(p,k,q) X_{12}^{-} X_{43}^{-} X_{13}^{+} X_{13}^{-} + B(p,k,q) X_{12}^{-} X_{43}^{-} \\ \qquad + C(p,k,q) X_{12}^{-} X_{13}^{+} + D(p,k,q) X_{13}^{+} X_{43}^{-} \Big) \end{array},$$

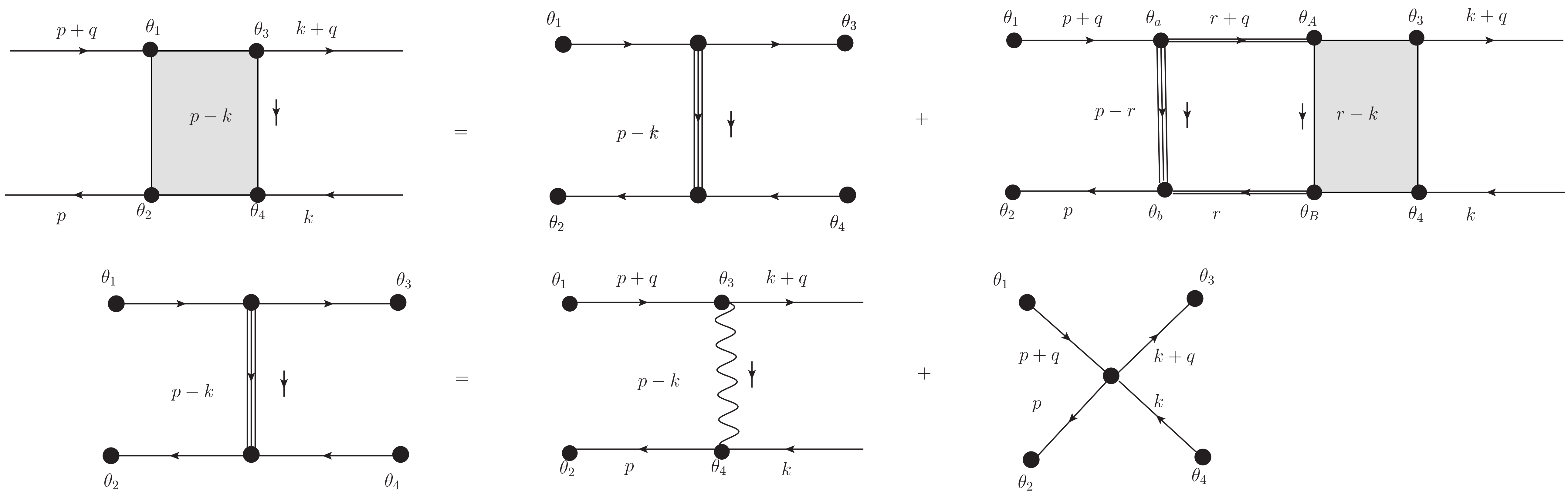
The form of the function F is an ansatz based on

- Perturbative calculations
- Closure of the structure V under multiplication
- Associativity of the structure V under multiplication.

$$(V_1 \star V_2) \star V_3 = V_1 \star (V_2 \star V_3)$$

This closure property is absolutely crucial.

Step 1: Dyson-Schwinger equation in superspace



$$
\begin{aligned}
V(\theta_1,\theta_2,\theta_3,\theta_4,p,q,k) &= V_0(\theta_1,\theta_2,\theta_3,\theta_4,p,q,k) \\
&\quad + \int \frac{d^3r}{(2\pi)^3} d^2\theta_a d^2\theta_b d^2\theta_A d^2\theta_B \Big(N V_0(\theta_1,\theta_2,\theta_a,\theta_b,p,q,r) \\
&\qquad P(r+q,\theta_a,\theta_A) P(r,\theta_B,\theta_b) V(\theta_A,\theta_B,\theta_3,\theta_4,r,q,k) \Big)
\end{aligned}
$$

Step 2: Integral equations in components

$$A(p,k,q)+\frac{2\pi i}{\kappa}+i\pi\lambda\int\frac{d^3r}{(2\pi)^3}\frac{q_3\left(p_-(2A-2Bq_3-Ck_-)+(4A+2Ck_--3Dp_-+4Bq_3)r_-+2Dr_-^2\right)}{(r^2+m^2)((r+q)^2+m^2)(p-r)_-}=0$$

$$B(p,k,q)+2\pi i\lambda\int\frac{d^3r}{(2\pi)^3}\frac{r_-(2A-Ck_-+2Bq_3+Dr_-)}{(r^2+m^2)((r+q)^2+m^2)(p-r)_-}=0$$

$$C(p,k,q)+\frac{4\pi i}{\kappa(k-p)_-}+8\pi i\lambda\int\frac{d^3r}{(2\pi)^3}\frac{Cq_3r_-}{(r^2+m^2)((r+q)^2+m^2)(p-r)_-}=0$$

Decoupled equation!!

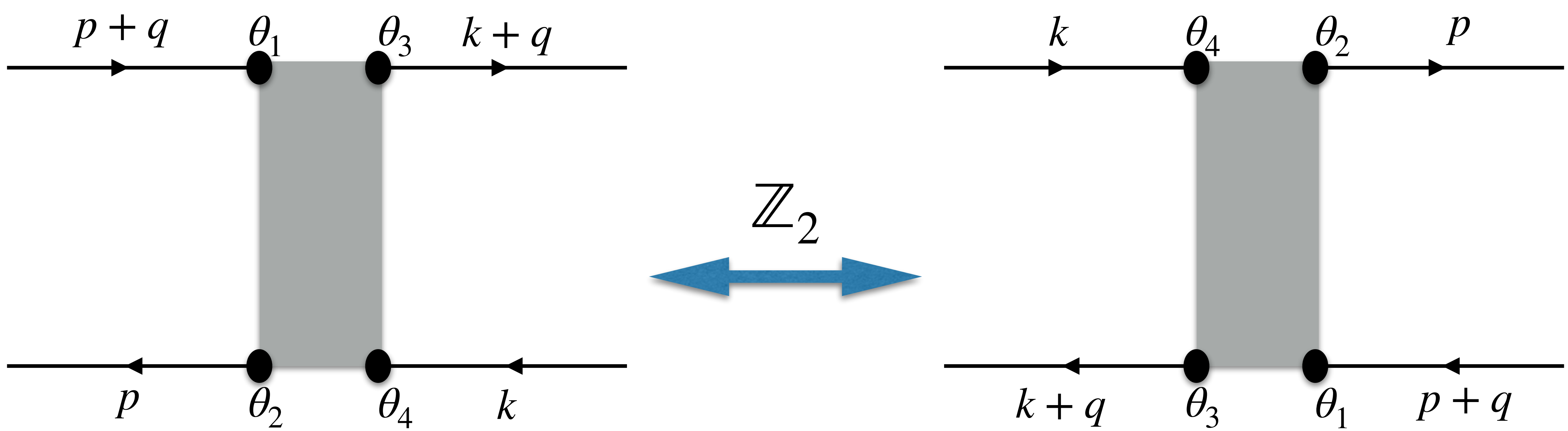
$$D(p,k,q)+\frac{4\pi i}{\kappa(k-p)_-}+2\pi i\lambda\int\frac{d^3r}{(2\pi)^3}\frac{q_3(-2A+2Bq_3+Ck_-+3Dr_-)}{(r^2+m^2)((r+q)^2+m^2)(p-r)_-}=0$$

There are four coupled integral equation for four unknown functions.

We have one decoupled equation that can be solved.

It is still impossible to solve the rest of the coupled integral equations if one **crucial symmetry** is not observed.

Step 3: Integrability and $\mathbb{Z}_2$



$$p \to k+q, \quad k \to p+q, \quad q \to -q\,,$$
$$\theta_1 \to \theta_4, \quad \theta_2 \to \theta_3, \quad \theta_3 \to \theta_2, \quad \theta_4 \to \theta_1\;.$$

This leads to

$$A(p,k,q) = A(k,p,-q)$$
$$B(p,k,q) = B(k,p,-q)$$
$$C(p,k,q) = -\,D(k,p,-q)$$
$$D(p,k,q) = -\,C(k,p,-q)$$

Under the dual description the **D equation becomes decoupled and can be solved**.

Voila: Exact four point correlator to all loops!!

$$V(\theta_1,\theta_2,\theta_3,\theta_4,p,q,k) = \exp\left(\frac{1}{4}X.(p.X_{12}+q.X_{13}+k.X_{43})\right)F(X_{12},X_{13},X_{43},p,q,k)$$

$$F(X_{12},X_{13},X_{43},p,q,k) = \begin{array}{l} X_{12}^{+}X_{43}^{+}\Big(A(p,k,q)X_{12}^{-}X_{43}^{-}X_{13}^{+}X_{13}^{-} + B(p,k,q)X_{12}^{-}X_{43}^{-} \\ \qquad + C(p,k,q)X_{12}^{-}X_{13}^{+} + D(p,k,q)X_{13}^{+}X_{43}^{-}\Big) \end{array},$$

$$A(p,k,q) = -\frac{2i\pi e^{2i\lambda\left(\tan^{-1}\frac{2\sqrt{k_s^2+m^2}}{q_3}-\tan^{-1}\frac{2\sqrt{m^2+p_s^2}}{q_3}\right)}}{\kappa}$$

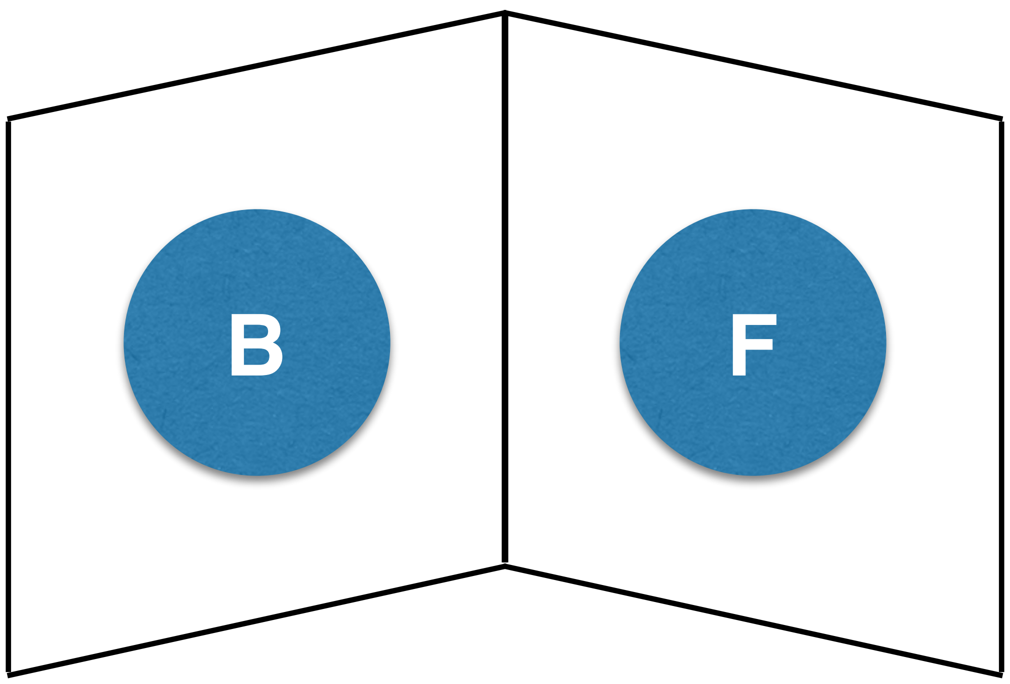
$$B(p,k,q) = 0\ ,$$

$$C(p,k,q) = -\frac{4i\pi e^{2i\lambda\left(\tan^{-1}\frac{2\sqrt{k_s^2+m^2}}{q_3}-\tan^{-1}\frac{2\sqrt{m^2+p_s^2}}{q_3}\right)}}{\kappa(k-p)_-}$$

$$D(p,k,q) = -\frac{4i\pi e^{2i\lambda\left(\tan^{-1}\frac{2\sqrt{k_s^2+m^2}}{q_3}-\tan^{-1}\frac{2\sqrt{m^2+p_s^2}}{q_3}\right)}}{\kappa(k-p)_-}$$

We have solved the **integral equations exactly to all orders in the 't Hooft coupling** λ in the planar large N limit.

Gauge invariant observables and test of duality



Observable 1: $2 \to 2$ S matrix

$2 \to 2$ amplitudes in $\mathcal{N}=2$ theory

The S matrix is obtained from the off-shell correlation function by sending the external legs to be onshell (symmetric,anti-symmetric, adjoint channels)

$$
\begin{aligned}
\mathcal{T}_B^{\mathcal{N}=2} &= \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} - \frac{8\pi m}{\kappa}\,, \\
\mathcal{T}_F^{\mathcal{N}=2} &= \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} + \frac{8\pi m}{\kappa}
\end{aligned}
$$

We obtain the remarkable result that the **$2 \to 2$ amplitude computed exactly to all orders in the 't Hooft coupling λ is tree level exact!**

K.I, Jain, Minwalla, Mazumdar, Umesh, Yokoyama

Under the duality map $\kappa' \to -\kappa, m' \to -m$, the bosonic and fermionic S matrices map to one another under an overall unobservable phase.

The non-renormalization is a consequence of exact dual superconformal symmetry to all loops. K.I, Jain, Majumdar, Nayak, Neogi, Sinha,Sharma, Umesh

A very similar result was obtained recently in $\mathcal{N}=3$ theories.

K.I, Janagal, Shukla

$2 \to 2$ amplitude in singlet channel

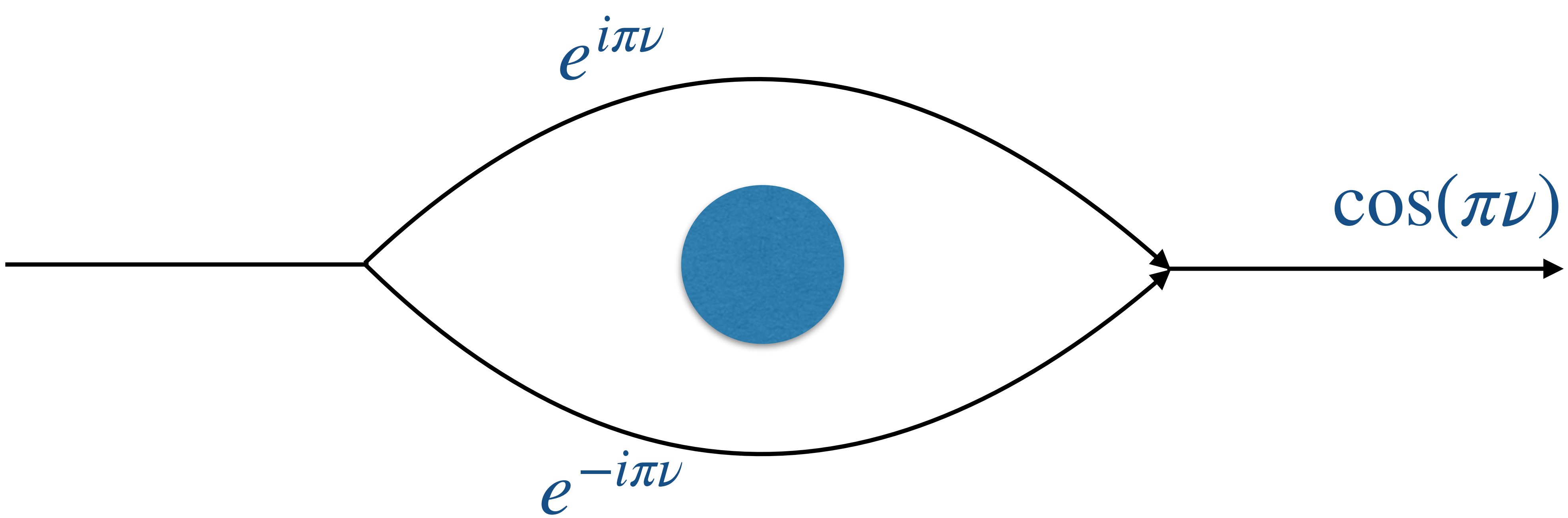
Since the singlet channel is anyonic, a direct computation of the S matrix in this channel is not possible.

Observation: **Naive crossing symmetry** rules from any of the **non-anyonic channels to the singlet channel** leads to a **non unitary** S matrix.

Conjecture:

$$S(s,\theta) = 8\pi\sqrt{s}\cos(\pi\nu)\ \delta(\theta) + i\frac{\sin(\pi\nu)}{\pi\nu}T^{S;naive}(s,\theta)$$

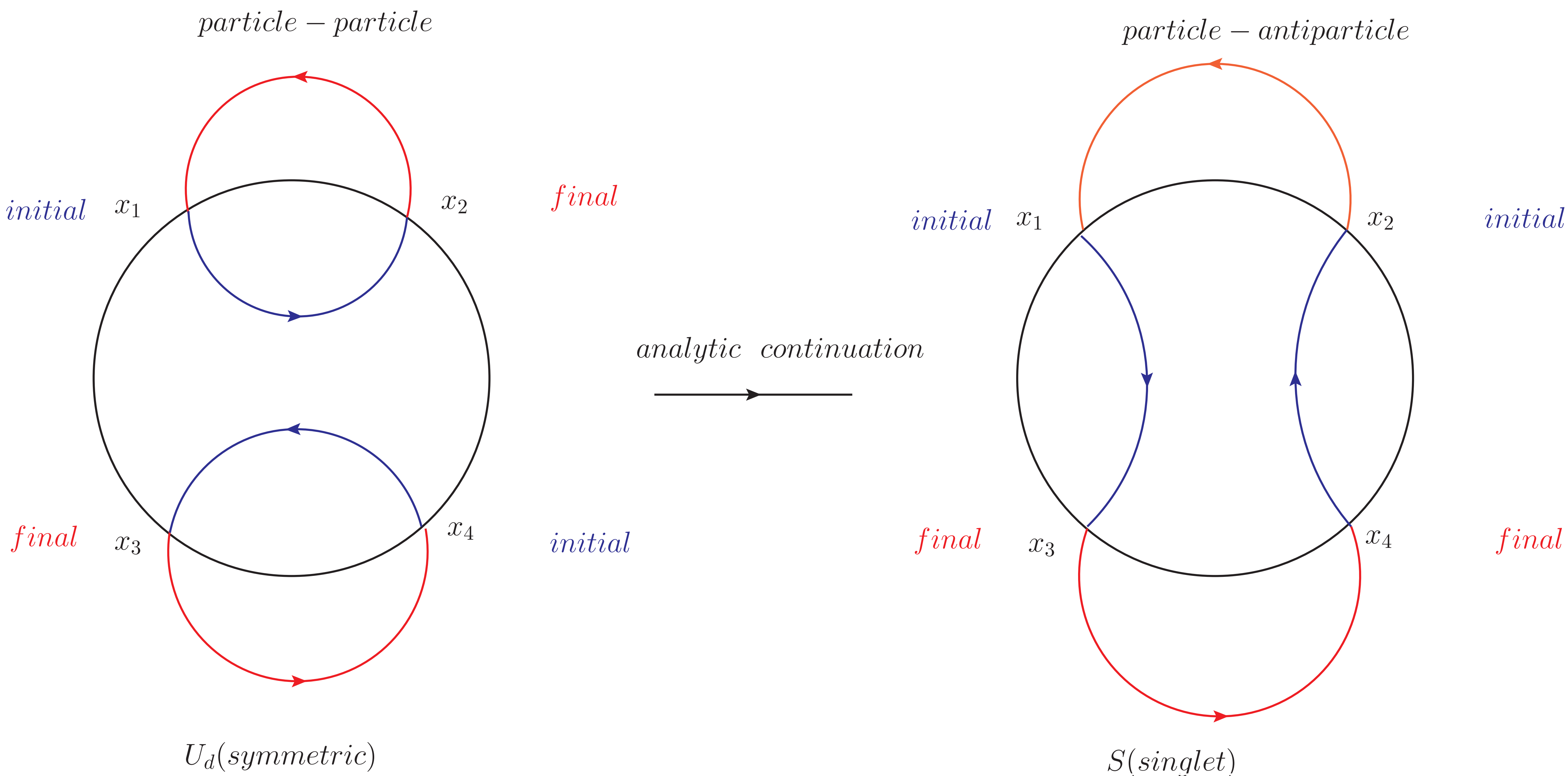
S.Jain, M. Mandlik, S.Minwalla , T.Takimi S.Wadia, S.Yokoyama



Phase dependence in forward scattering arises due to interference of Aharanov-Bohm phases of the incoming wave packet.

The contribution to forward scattering should be a generic feature of scattering in any theory with anyonic excitations.

Modified crossing rules: Heuristic explanation



$$T_{U_d} W_{U_d} \to T_{Sing} W_{Sing}$$

$$\frac{W_{U_d}}{W_{Sing}} = \frac{\oint \text{with 2 circular Wilson lines}}{\oint \text{with 1 circular Wilson line}} = \frac{\sin(\pi\lambda)}{\pi\lambda}$$

Witten

Tests of the conjecture

Modulation of forward scattering and modified crossing rules appear to be **universal and generic** to Chern-Simons matter theories.

Tests

Unitarity of the S matrix

3d Bosonization duality Aharony,Bardeen,Benini,Chang,Frishman,Giombi,Giveon, GurAri,Gurucharan,Kutasov,Jain,Maldacena,Mandlik,Minwalla,Moshe,Sharma,Prakash,Takimi, Trivedi,Seiberg,Sonnenschein,Yacoby,Yin,Yokoyama,Wadia,Witten,Zhiboedov

Non-relativistic limit should be consistent with Aharanov-Bohm result.

Verifications

S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

U(N) Chern-Simons coupled to fundamental bosons.

U(N) Chern-Simons coupled to fundamental fermions.

$\mathcal{N} = 1{,}2$ supersymmetric Chern-Simons matter theories.

K.I, S.Jain, S.Mazumdar, S.Minwalla, V. Umesh, S.Yokoyama

$\mathcal{N} = 3$ supersymmetric Chern-Simons matter theories.

K.I, Janagal, Shukla

$2 \to 2$ amplitude in singlet channel for $\mathcal{N} = 2$sa

$$\mathcal{T}_B^{S;\mathcal{N}=2}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2) - 8m) \ ,$$
$$\mathcal{T}_F^{S;\mathcal{N}=2}(s,\theta) = -8\pi i\sqrt{s}(\cos(\pi\lambda)-1)\delta(\theta) + \sin(\pi\lambda)(4i\sqrt{s}\cot(\theta/2) + 8m).$$

Under the duality map $\kappa' \to -\kappa, m' \to -m, \lambda' \to \lambda - Sign(\lambda)$, the bosonic and fermionic S matrices map to one another under an overall unobservable phase.

The S matrix in the singlet channel continues to be simple, but not tree level exact. **This is consistent with the unitarity requirements in the singlet channel**.

$$T_{Sym} \sim T_{ASym} \sim T_{Adj} \sim \mathcal{O}\left(\frac{1}{N}\right) \ , T_{Sing} \sim \mathcal{O}(1)$$

Unitarity is a non-trivial check only for the singlet channel. In other channels it follows from hermiticity at leading order in large N

$$i(T - T^{\dagger}) = TT^{\dagger}$$

Observable 2: Correlation functions of current operators.

Conserved higher spin currents

It is well known that in a free field theory

$$S = \int d^3x \frac{1}{2}(\partial_\mu \phi)^2$$

there exists an infinite tower of higher spin symmetries.

$$J_{\mu_1\mu_2\ldots\mu_s} = \sum_{k=0}^{s} c_{sk}\partial_{\{\mu_1\ldots\mu_k}\phi\partial_{\mu_{k+1}\ldots\mu_s\}}\phi$$

These are symmetric, traceless irreducible reprenstations of $SO(3)$ of spin s.

$$\partial^\mu J_{\mu\mu_2\ldots\mu_s} = 0$$

The currents are exactly conserved in the free theory.

In interacting theories especially with CS, the current conservation is weakly broken by $\mathcal{O}\left(\frac{1}{N}\right)$ effects.

Conserved currents in $\mathcal{N}=2$ theory

$$J^{(s)} = \sum_{r=0}^{2s} (-1)^{\frac{r(r+1)}{2}} \binom{2s}{r} \nabla^r \bar{\Phi} \nabla^{2s-r} \Phi \qquad s = 0, \frac{1}{2}, 1, \ldots$$

Some examples

$$J_0 = \bar{\Phi}\Phi$$

$$J_\alpha = \bar{\Phi}\nabla_\alpha\Phi - \nabla_\alpha\bar{\Phi}\Phi = \bar{\Phi}D_\alpha\Phi - D_\alpha\bar{\Phi}\Phi - 2i\bar{\Phi}\Gamma_\alpha\Phi$$

$$J_{\alpha\beta} = \bar{\Phi}\nabla_\alpha\nabla_\beta\Phi - 2\nabla_\alpha\bar{\Phi}\nabla_\beta\Phi + \nabla_\alpha\nabla_\beta\bar{\Phi}\Phi$$

$$J_{\alpha\beta\gamma} = \bar{\Phi}\nabla_\alpha\nabla_\beta\nabla_\gamma\Phi - 3\nabla_\alpha\bar{\Phi}\nabla_\beta\nabla_\gamma\Phi - 3\nabla_\alpha\nabla_\beta\bar{\Phi}\nabla_\gamma\Phi + \nabla_\alpha\nabla_\beta\nabla_\gamma\bar{\Phi}\Phi$$

In components for eg the spin zero current

$$J^{(0)}(\theta, x) = J_0^b(x) + \theta^\alpha\Psi_\alpha(x) - \theta^2 J_0^f(x)$$

$$J_0^b(x) = \bar{\phi}\phi(x), \quad \Psi_\alpha(x) = (\bar{\phi}\psi_\alpha + \bar{\psi}_\alpha\phi)(x), \quad J_0^f(x) = \bar{\psi}\psi(x)$$

Conserved currents in $\mathcal{N}=2$ theory

In the free limit $\lambda \to 0$, the currents are exactly conserved

$$D^\alpha J_{\alpha\alpha_2\ldots\alpha_{2s}} = 0$$

For non zero λ, the symmetries are weakly broken by $\mathcal{O}\left(\frac{1}{N}\right)$ effects.

We are interested in computing the gauge invariant correlation functions

$$\langle J_0(q,\theta_1)J_0(-q,\theta_2)\rangle$$
$$\langle J_0(q,\theta_1)J_0(q',\theta_2)J_0(-q-q',\theta_3)\rangle$$
$$\langle J_0 q,\theta_1)J_0(q',\theta_2)J_0(q'',\theta_3)J_0(-q-q'-q'',\theta_4)\rangle$$

and to test their invariance under duality.

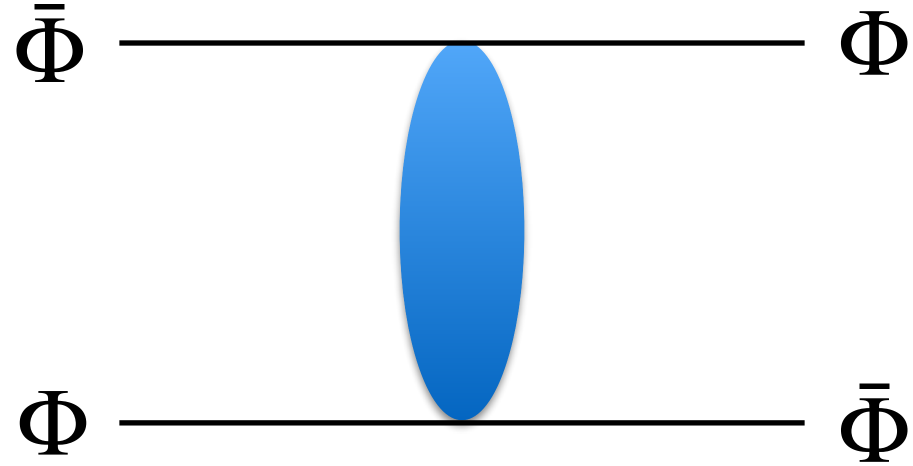
In a CFT, the two point and three point functions are fixed by conformal symmetry, While the four point function is fixed only up to conformal cross ratios.

In addition, parity, higher spin symmetry constrain the structures that can appear in a correlator.

Maldacena, Zhiboedov

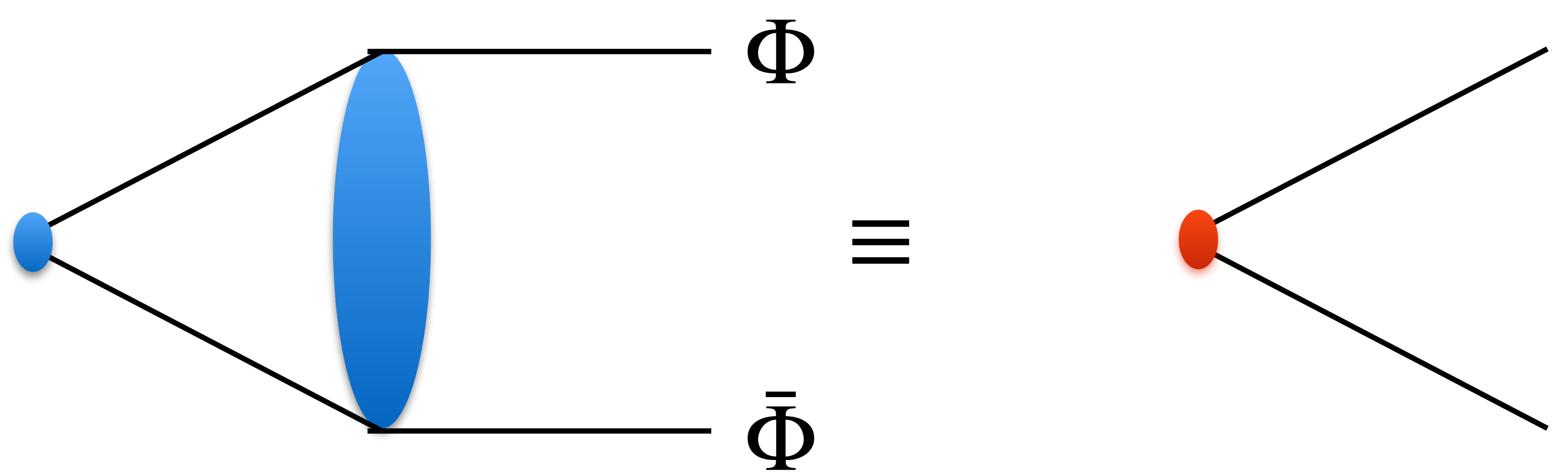
Spin zero correlators in $\mathcal{N} = 2$ theory

Step 1: Exact four point correlator of fields to all loops

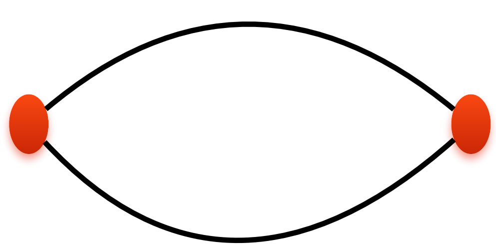


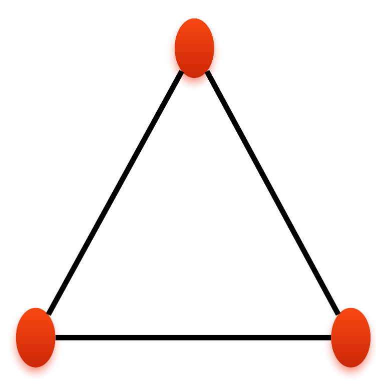
Step 2: Exact vertex/Form factor for the current insertion

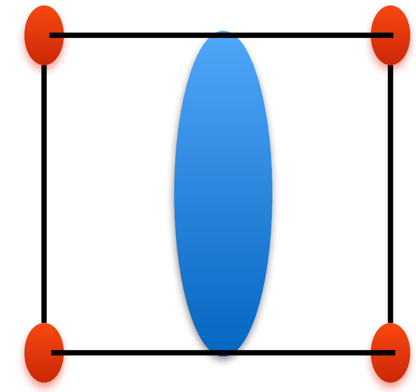
$J_0(q) = \bar{\Phi}\Phi(q)$



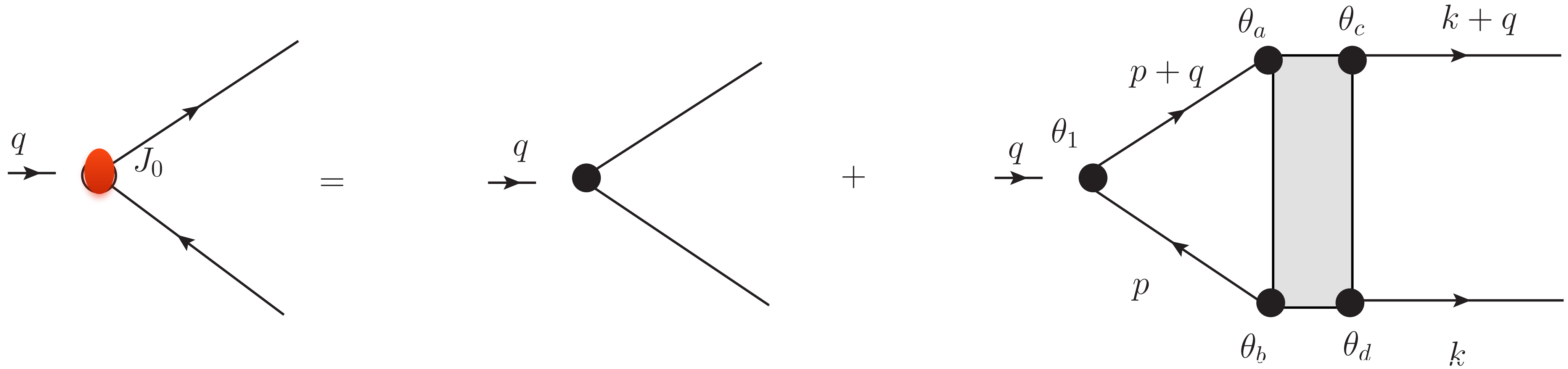
Step 3: Compute the correlator







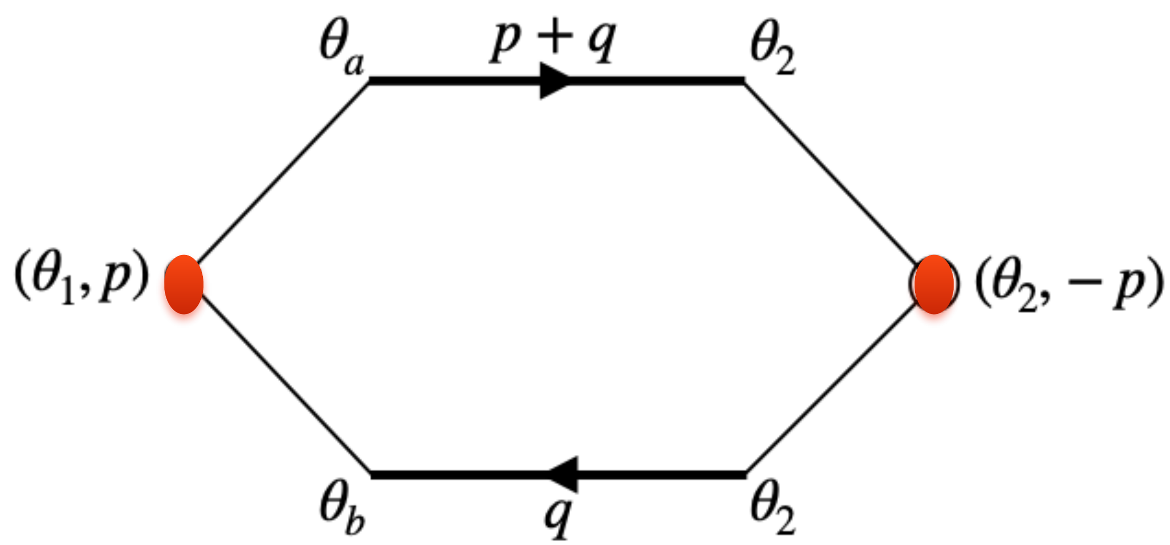
Exact vertex



$$\mathcal{V}(X, X_{13}, X_{43}, q, k) = \exp\left(\frac{1}{3}X.(q.X_{13} + k.X_{43})\right)\mathcal{F}(X_{13}, X_{43}, q, k)$$

$$\begin{aligned}
\mathcal{F}(X_{13}, X_{43}, q, k) = & -X_{13}^- X_{13}^+ X_{43}^- X_{43}^+ \\
& + \left(\frac{e^{2i\lambda \tan^{-1}\left(\frac{2k_s}{q_3}\right)} - 1}{2k_-}\right) X_{13}^+ X_{43}^+ \\
& + \left(\frac{1 - e^{i\lambda\left(2\tan^{-1}\left(\frac{2k_s}{q_3}\right) - \pi \mathrm{sgn}(q_3)\right)}}{2q_3}\right) X_{43}^- X_{43}^+ \\
& + \left(\frac{2}{3} - \frac{1}{6}e^{2i\lambda \tan^{-1}\left(\frac{2k_s}{q_3}\right)} - \frac{1}{2}e^{i\lambda\left(2\tan^{-1}\left(\frac{2k_s}{q_3}\right) - \pi \mathrm{sgn}(q_3)\right)}\right) X_{13}^- X_{13}^+ X_{43}^- X_{43}^+
\end{aligned}$$

Two point correlator



$$\langle J_0(\theta_1,p)J_0(\theta_2,r)\rangle$$
$$= (2\pi)^3\delta^3(p+r)N\frac{e^{-\theta_1.p.\theta_2}}{8|p|}\left(\frac{\sin(\pi\lambda)}{\pi\lambda}+|p|\delta^2(\theta_{12})\frac{1-\cos(\pi\lambda)}{\pi\lambda}\right)$$

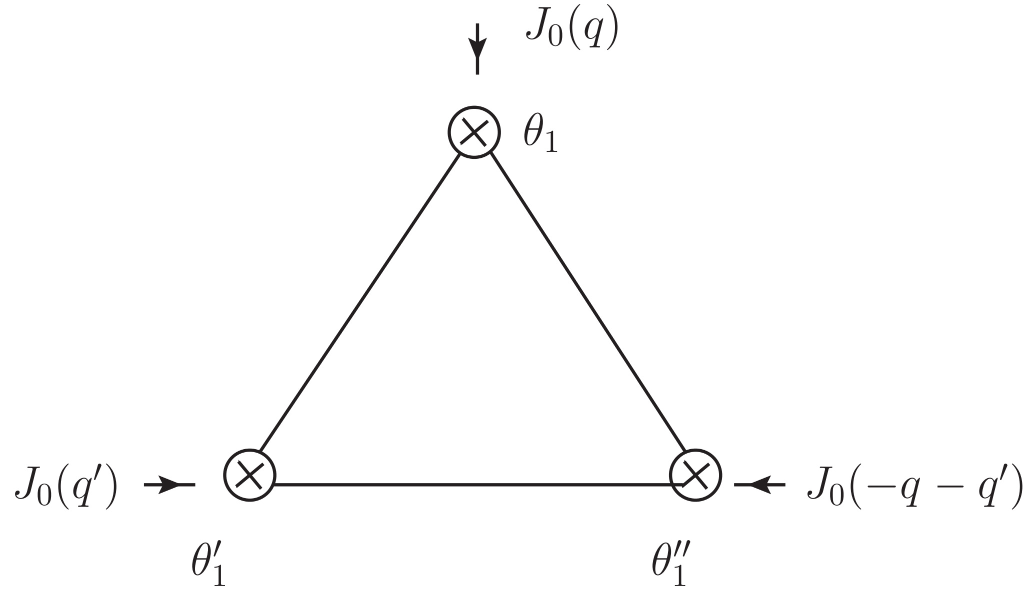
bosonic/fermionic two point correlators map under duality upto a phase

$$\langle J_0^b(p)J_0^b(-p)\rangle = \frac{N}{8|p|}\frac{\sin(\pi\lambda)}{\pi\lambda}$$
$$\langle J_0^f(p)J_0^f(-p)\rangle = -\frac{N|p|}{8}\frac{\sin(\pi\lambda)}{\pi\lambda}$$

Momentum deoendence is fixed by conformal invariance.

Three point correlator

Three point correlator is quite complicated, once again the bosonic and fermionic components match under duality.



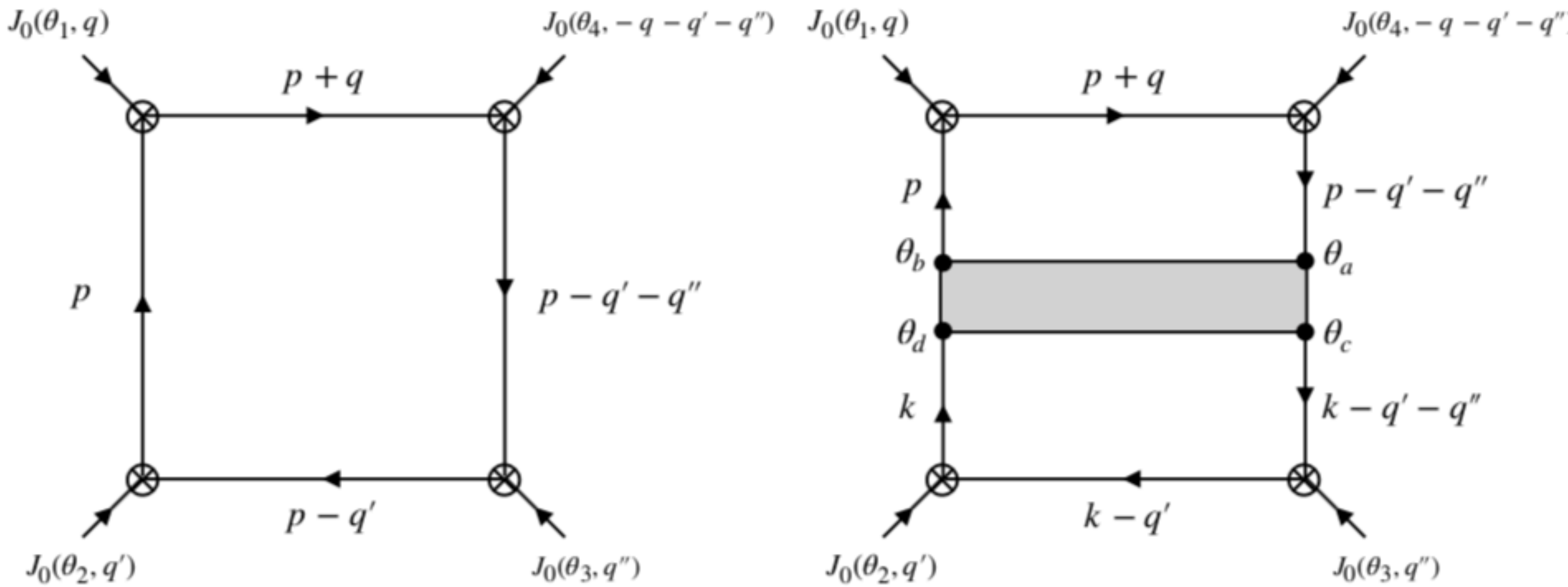
O.Aharony, A. Sharon

K.I, Jain, Malvimat, Mehta, Nayak, Sharma;

$$
\begin{aligned}
\langle J_0(\theta_1,q)J_0(\theta'_1,q')J_0(\theta''_1,-q-q')\rangle = &\left(\frac{N}{72\; q_3 q'_3(q_3+q'_3)}\frac{\sin(\pi\lambda)}{\pi\lambda}\right)\Bigg[-9\cos(\pi\lambda) \\
&+9i\sin(\pi\lambda)\left(q_3 X^-_{11''}X^+_{11''}+q'_3 X^-_{1'1''}X^+_{1'1''}\right) \\
&+3\cos(\pi\lambda)\;(q'_3-q_3)\left(X^-_{11''}X^+_{1'1''}-X^-_{1'1''}X^+_{11''}\right) \\
&-\cos(\pi\lambda)\;(q_3^2+7q_3q'_3+q'^2_3)\;X^-_{11''}X^+_{11''}X^-_{1'1''}X^+_{1'1''}\Bigg] \\
&\times e^{\frac{1}{3}X\cdot\left(q\cdot X_{11''}+q'\cdot X_{1'1''}\right)}
\end{aligned}
$$

four point correlator (open question)

K.I, Jain, Malvimat, Mehta, Nayak, Sharma;

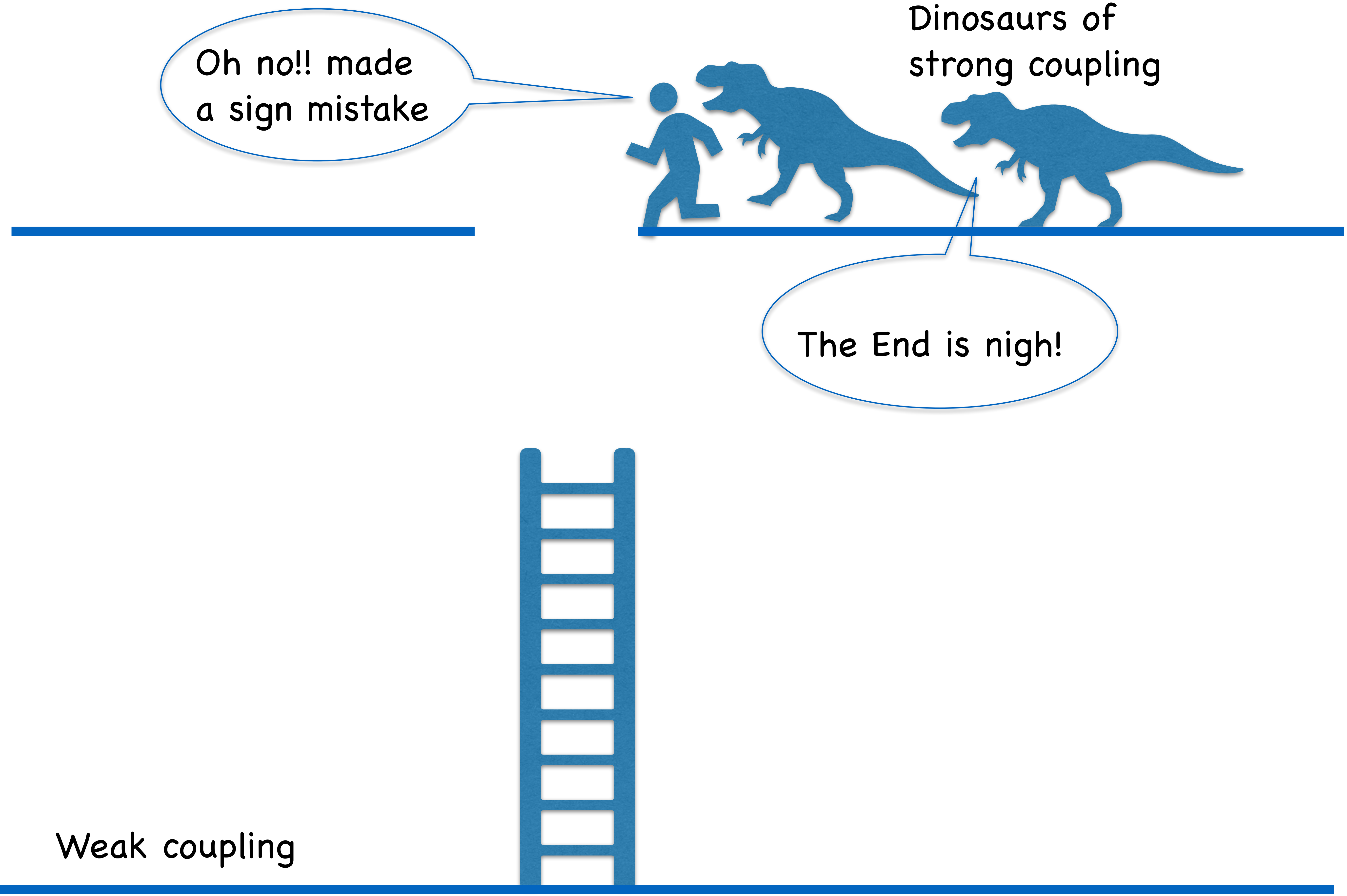


For the non-supersymmetric fermion+CS theory the result was computed in the double soft limit.

The result for general kinematics is argued to be proportional to that of the "free theory" using the known three point functions, Lorentzian OPE inversion formula and crossing symmetry.

Zhiboedov, Turlaci

This is the most important correlator: since double trace operators can be exchanged. (Mixing of objects of different order in N)


Dinosaurs of strong coupling
Oh no!! made a sign mistake
The End is nigh!
Weak coupling

Discussion

In this talk, we discussed 2+1 dimensional Chern-Simons matter theories.

The effect of **Chern-Simons gauge field is to attach magnetic fluxes to interacting charged particles**.

This leads to the **Aharanov-Bohm phase and statistical transmutation**.

Non-abelian Chern-Simons matter theories enjoy a **strong-weak bosonization duality at large N**.

We motivated the bosonization duality from the perspective of AB phases, higher spin symmetries and RG flows.

Any test of the duality would **require computation of observables to all loops**.

Discussion

We took the **specific example of $\mathcal{N}=2$ supersymmetric Chern-Simons matter theories and studied the duality.**

We illustrated the **Dyson-Schwinger procedure** for computing the exact four point function of scalar superfields in the large N limit.

We took the onshell limit and obtained the **S matrices which were manifestly invariant under duality**.

We also computed several **gauge invariant correlation functions of conserved currents and find consistency with duality in every step**.

The four point function where double trace operators can flow and cause mixing between operators of different order in N is hard to compute using our methods and is still a open question.

Outlook

Recently there has been a finite N,κ conjecture of the bosonization duality

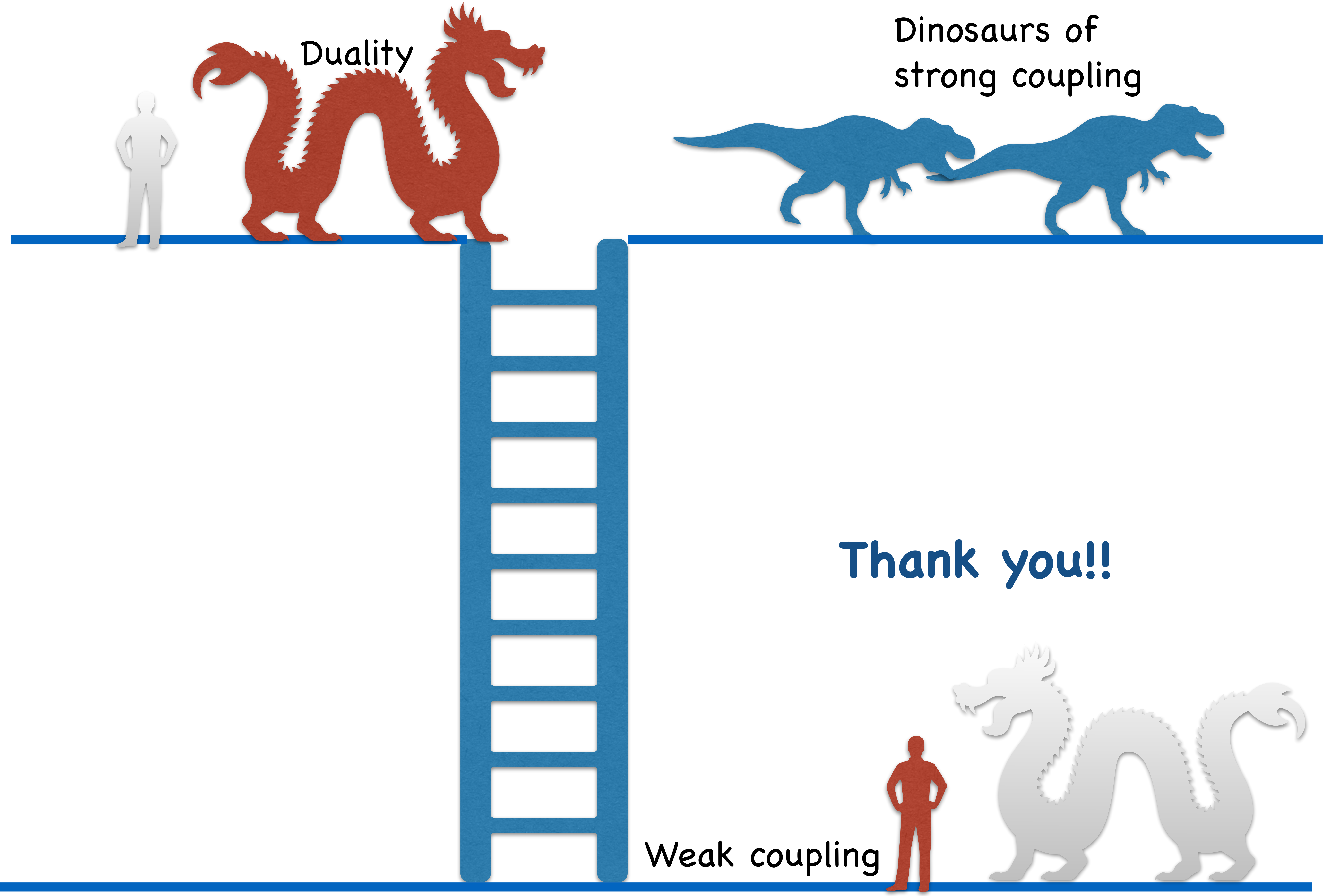
Aharony

Applications of this duality have led to the first analytic derivation of the well known particle-vortex duality in condensed matter physics and also led to the web of dualities.

Melitsky,Senthil,Seiberg, Wang,Witten

There has not been any verifications of the finite N,κ duality via any precision tests.

This duality is most likely to be relevant in applications in Condensed matter physics, especially in QHE.


Duality
Dinosaurs of
strong coupling
Thank you!!
Weak coupling