Amplitudes and hidden symmetries in N=2 Chern-Simons Matter theory

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Based on

K.I, S.Jain, P.Nayak, V.Umesh, **arXiv :1710.04227** **(BCFW)**

K.I, S.Jain, S.Majumdar, P.Nayak, T.Neogi, T.Sharma, R.Sinha, V.Umesh, **arXiv : 1711.02672** **(Dual Superconformal symmetry)**

K.I, S.Jain, P.Nayak, T.Sharma, V.Umesh, **arXiv : 1712.nnppq** **(Yangian)**

References:

K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh, S.Yokoyama, **arXiv: 1505.06571**, JHEP 1510 (2015) 176.

S.Jain, M.Mandlik, S.Minwalla, T.Takimi, S.Wadia, S.Yokoyama, **arXiv: 1404.6373,** JHEP 1504 (2015) 129.

Part I

Introduction

N=2 Chern-Simons matter theory

- General renormalizable $\mathcal{N}=2$ theory with one fundamental multiplet

$$\mathcal{S}^L_{\mathcal{N}=2} = \int d^3x \bigg[-\frac{\kappa}{4\pi} \epsilon^{\mu\nu\rho} \mathrm{Tr} \left(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right) + \bar{\psi} i \not{\mathcal{D}} \psi - \mathcal{D}^\mu \bar{\phi} \mathcal{D}_\mu \phi + \frac{4\pi^2}{\kappa^2} (\bar{\phi}\phi)^3 + \frac{4\pi}{\kappa} (\bar{\phi}\phi)(\bar{\psi}\psi) + \frac{2\pi}{\kappa} (\bar{\psi}\phi)(\bar{\phi}\psi) \bigg]$$

- The theory exhibits a **strong-weak self duality** under the duality map

$$\kappa' = -\kappa \ , \ N' = |\kappa| - N + 1 \ , \ \lambda' = \lambda - \mathrm{Sgn}(\lambda)$$

- **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** $2 \to 2$ scattering amplitudes to all orders in the 't Hooft coupling.

- In the (**non-anyonic**) symmetric, anti-symmetric and adjoint channels of scattering the **amplitude is tree-level exact to all orders in λ**.

- In the **(anyonic)** singlet channel the coupling dependence is **extremely simple**.

2→2 scattering amplitude to all orders in λ in N=2 theory

- Tree level super amplitude

$$T_{\text{tree}} = \frac{4\pi}{\kappa} \frac{\langle 12 \rangle}{\langle 23 \rangle} \delta(\sum_{i=1}^{4} p_i) \delta^2 \left(\mathcal{Q}\right)$$

$$\delta^2(Q) = \sum_{i<j=1}^{n} \langle ij \rangle \eta_i \eta_j$$

- **All loop** super amplitude

$$T^{\text{non-anyonic}}_{\text{all-loop}} = T_{\text{tree}}$$

$$T^{\text{anyonic}}_{\text{all-loop}} = N \frac{\sin(\pi\lambda)}{\pi\lambda} T_{\text{tree}}$$

$$S^{\text{non-anyonic}} = I + i\, T^{\text{non-anyonic}}_{\text{all-loop}}$$

$$S^{\text{anyonic}} = \cos\left(\pi\lambda\right) I + i\, T^{\text{anyonic}}_{\text{all-loop}}$$

- Passes all consistency checks: **Unitarity and Duality**

Motivation

- Why is the 2 → 2 particle scattering in the non-anyonic channels **tree level exact**? and why does it have a very **simple coupling dependence in anyonic channel**?
- Maybe some **powerful symmetry** that protects the amplitude from renormalization.
- Is it possible to compute **all loop m → n scattering amplitudes** in the N=2 theory at least in the planar limit?
- Does the **non-renormalization** results of the 2 → 2 scattering continue to persist for arbitrary higher point amplitudes?
- What are the **generalization of the crossing rules** for the anyonic channels in an arbitrary m →n scattering.
- These computations would **test the duality** in regions un-probed by large N perturbation theory yet.

What we do

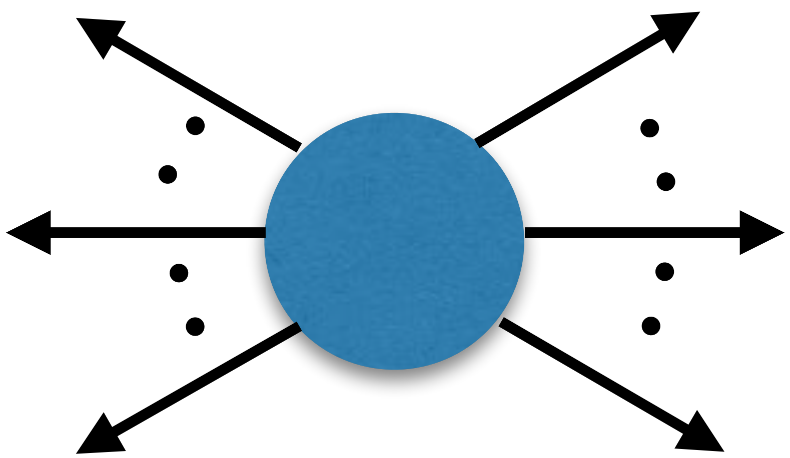
- As a first step towards the all loop m→n scattering, is it possible to write down **arbitrary m → n tree level amplitudes** ?
- We are able to achieve this via **BCFW recursions** **K.I, Jain, Nayak, Umesh**
- As a first step towards thinking about higher point loop amplitudes we identify a **hidden symmetry** in the 2 → 2 amplitude computed earlier that might explain the non-renormalization.
- This symmetry is known as **dual superconformal symmetry**.

  K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh
- The superconformal symmetry and dual superconformal symmetry together generate an infinite dimensional symmetry known as the **Yangian**.

  K.I, Jain, Nayak, Sharma, Umesh, to appear
- This suggests that the theory we are dealing with may be **integrable!**

Part II

All tree level amplitudes



- K.I, S.Jain, P.Nayak, V.Umesh, **arXiv :1710.04227**

BCFW recursions in 2+1 dimensions

- Recursion relations enable to construct **n point tree level scattering amplitudes from lower point tree level amplitudes.**

  Britto, Cachazo, Feng, Witten

- Central idea: **Dixon**
  - Tree level amplitudes are **continuously deformable** analytic functions of momenta.
  - Only type of singularities that can appear at tree level are **simple poles.**
  - One can **reconstruct amplitudes** for generic scattering kinematics knowing its behavior in **singular kinematics**.
  - In these singular regions **amplitudes factorize** into causally disconnected amplitudes with fewer legs, connected by an i**ntermediate onshell state**.

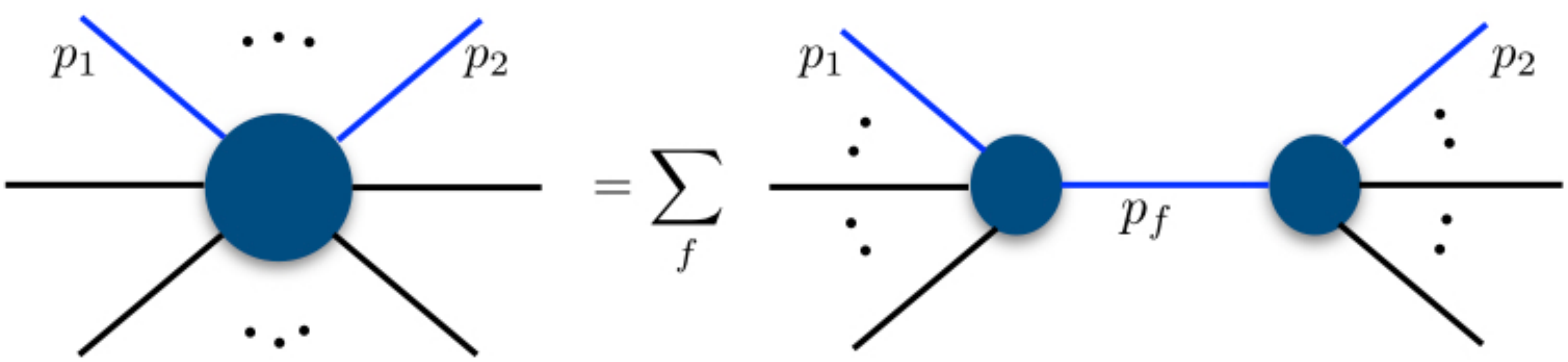
- We will focus on situation where the external particles are massless.

BCFW recursions in 2+1 dimensions

- Promote the amplitude into a one complex parameter family of amplitudes

$$A(z) = A(p_1, \ldots p_i(z), p_{i+1}, \ldots p_l(z), \ldots, p_{2n})$$

- The necessary and sufficient conditions are:
  - The momentum deformation should preserve on-shell conditions and momentum conservation.
  - The amplitude should be asymptotically well behaved under the deformation.



- A higher point amplitude factorizes into lower point amplitudes!

Preserving the onshell conditions

- In 3d the momentum shift is non-linear in z

$$\begin{pmatrix} \hat{\lambda}_1 \\ \hat{\lambda}_2 \end{pmatrix} = R \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \qquad R = \begin{pmatrix} \frac{z+z^{-1}}{2} & -\frac{z-z^{-1}}{2\ i} \\ \frac{z-z^{-1}}{2\ i} & \frac{z+z^{-1}}{2} \end{pmatrix}$$

$$p_i \to \frac{p_{ij}}{2} + q z^2 + \tilde{q} z^{-2} \qquad q^{\alpha\beta} = \frac{1}{4}(\lambda_2 + i\lambda_1)^\alpha (\lambda_2 + i\lambda_1)^\beta$$

$$p_j \to \frac{p_{ij}}{2} - q z^2 - \tilde{q} z^{-2} \qquad \tilde{q}^{\alpha\beta} = \frac{1}{4}(\lambda_2 - i\lambda_1)^\alpha (\lambda_2 - i\lambda_1)^\beta \ .$$

- The momentum deformations **preserve the onshell conditions**

$$p_i^2 = 0 \ , p_j^2 = 0$$

$$q.\tilde{q} = -\frac{1}{4} p_i.p_j \ , \ \ q + \tilde{q} = \frac{1}{2}(p_i - p_j) \ , q.p_{ij} = 0 \ , \ \ \tilde{q}.p_{ij} = 0$$

Gang, Huang, Koh, Lee, Lipstein

Asymptotic behavior

- Onshell susy methods, encode the **component amplitudes into a superamplitude**.
- **Susy ward identities** relate various component amplitudes and reduce the number of independent amplitudes.
- Susy also ensures that **if the independent component amplitudes are well behaved then the entire superamplitude is well behaved**.
- Using two independent methods we showed that the superamplitude is well behaved
  - **Background field expansion.** **Arkani-Hamed, Kaplan**
  - **Explicit Feynman diagram computation** of component amplitudes.
- The recursion formula then follows from **Cauchy residue theorem**.

The recursion formula for an arbitrary 2n point amplitude

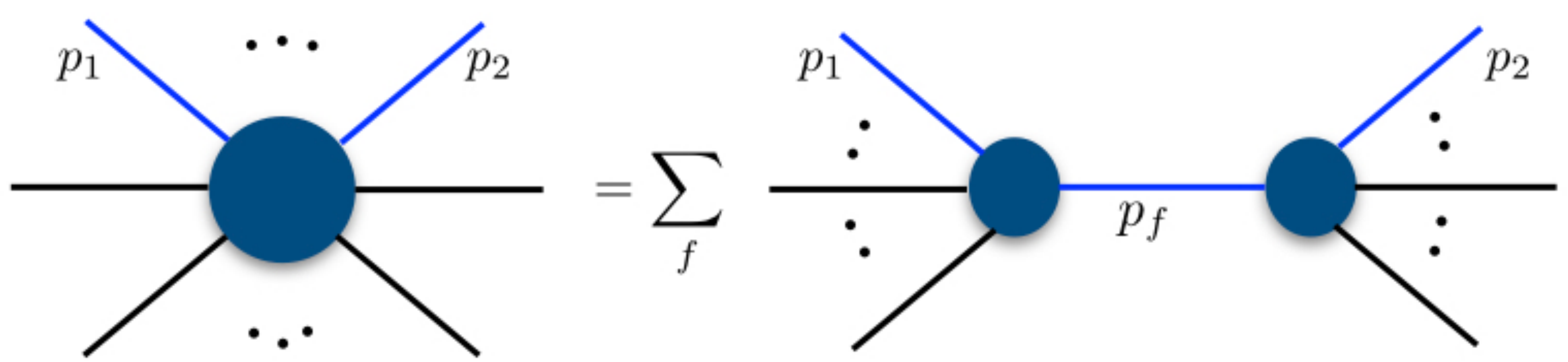
- Write a contour integral representation for the amplitude

$$\frac{1}{2\pi i}\oint_{C_{z=1}} \frac{dz}{z-1} A(z)$$

- Deform the contour to z→∞, If A(z) has no poles, the integral vanishes

$$A(z=1) = -\sum_{\text{poles}:z^i} \text{Res}_{z=z^i} \frac{A(z)}{z-1}$$

- remember that all the deformed momenta satisfy the onshell conditions!

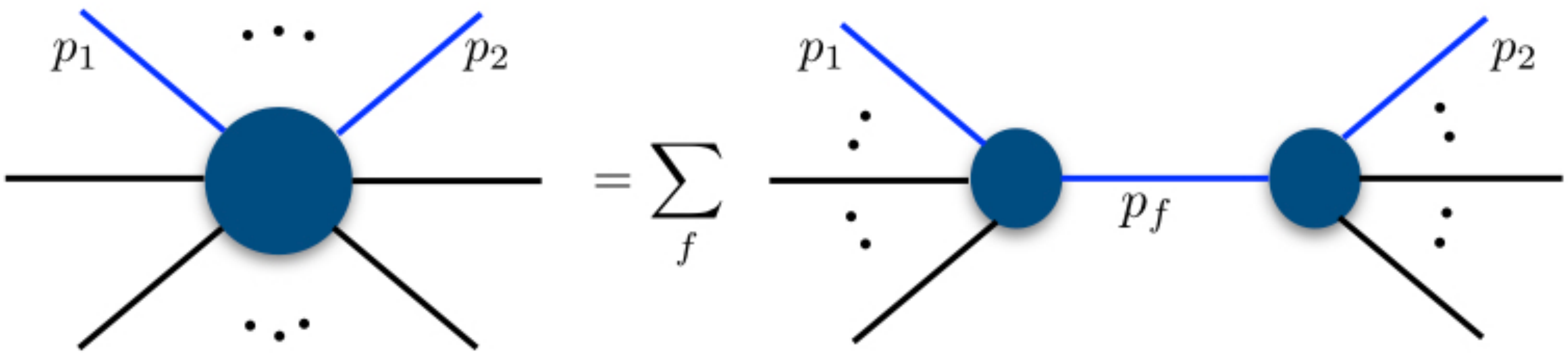


$$A(z=1) = -\sum_f \sum_{\text{poles}:z^i_f} \text{Res}_{z=z^i_f} \frac{1}{z-1} \frac{A_L(p_1 \dots p_i(z), \dots p_n) A_R(p_{n+1} \dots p_j(z), \dots p_{2n})}{\hat{p}_f^2(z)}$$

- We have used the fact that at **tree level the only possible singularities** are **simple poles**!

The recursion formula for arbitrary 2n point superamplitude

$$A_{2n}(z=1) = \sum_f \int \frac{d\theta}{p_f^2} \Big(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} A_L(z_{a;f}, \theta) A_R(z_{a;f}, i\theta)$$
$$+ (z_{a;f} \leftrightarrow z_{b;f}) \Big)$$



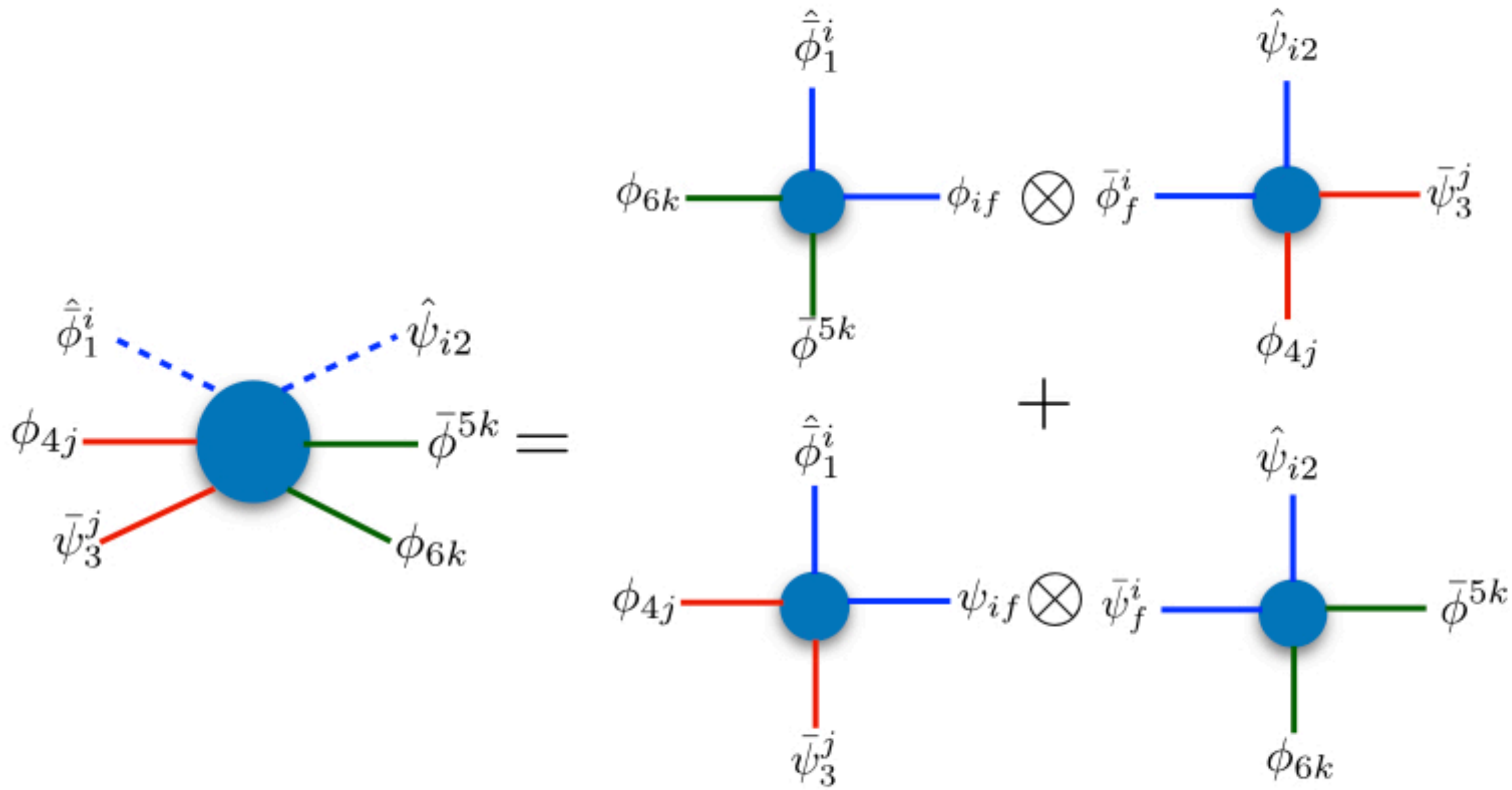
- $z_{a;f}, z_{b;f}$ are zeroes of $p_f^2(z) = 0$
- The formula can be recursively applied to write down any **higher point superamplitude in terms of products of the four point superamplitude**.

Eg: Six point amplitude as product of four point amplitudes

$$\langle\bar{\phi}_1\psi_2\bar{\psi}_3\phi_4\bar{\phi}_5\phi_6\rangle =$$

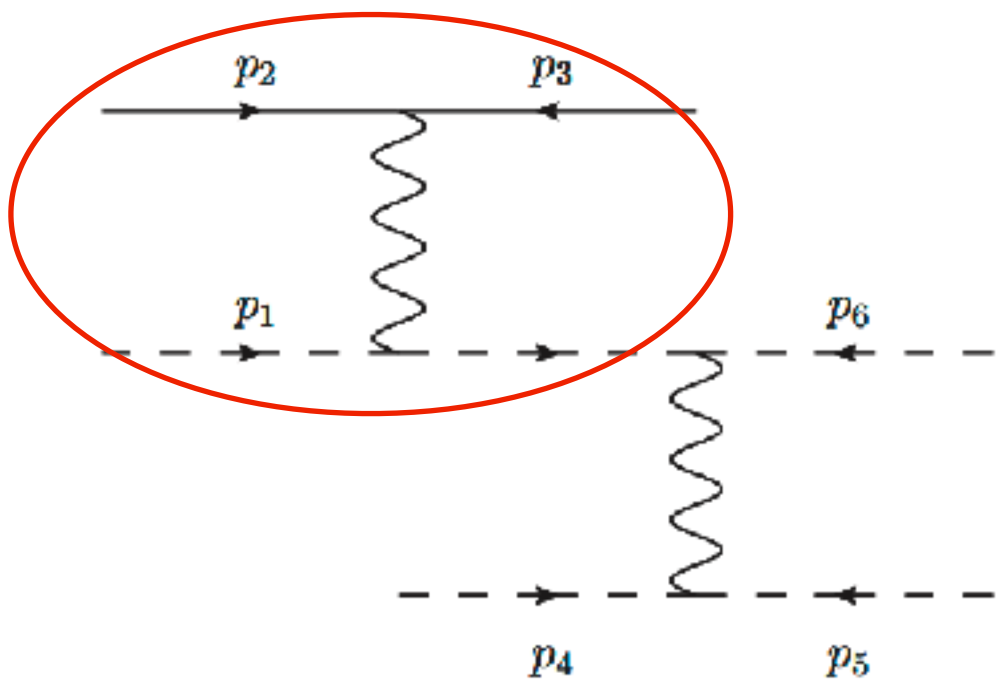
$$\left(z_{a;f}\,\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2}\langle\hat{\bar{\phi}}_1\hat{\phi}_f\bar{\phi}_5\phi_6\rangle_{z_{a;f}}\langle\hat{\bar{\phi}}_{(-f)}\hat{\psi}_2\bar{\psi}_3\phi_4\rangle_{z_{a;f}} + (z_{a;f}\leftrightarrow z_{b;f})\right)\frac{i}{p_f^2}\Big|_{p_f=p_{234}}$$

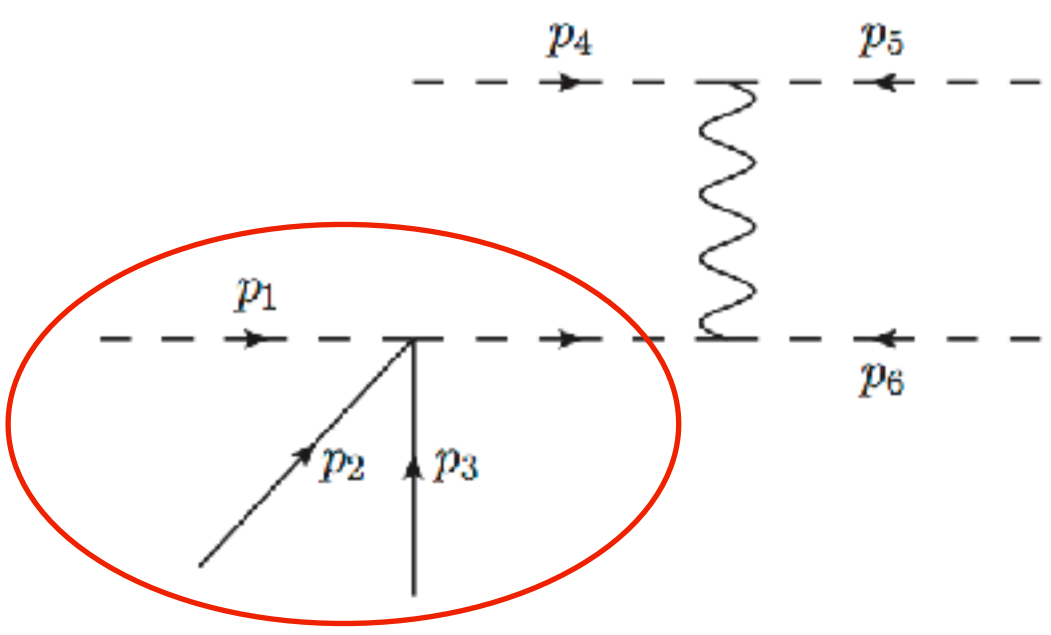
$$+\left(z_{a;f}\,\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2}\langle\hat{\bar{\phi}}_1\hat{\psi}_f\bar{\psi}_3\phi_4\rangle_{z_{a;f}}\langle\hat{\bar{\psi}}_{(-f)}\hat{\psi}_2\bar{\phi}_5\phi_6\rangle_{z_{a;f}} + (z_{a;f}\leftrightarrow z_{b;f})\right)\frac{i}{p_f^2}\Big|_{p_f=p_{256}}$$



Eg: Six point amplitude: Asymptotic behavior

- The **Asymptotic behavior involves very precise cancellations of divergences** in the Feynman diagram approach.
- For eg, the process $\langle\bar{\psi}_1\phi_2\bar{\phi}_3\psi_4\bar{\phi}_5\phi_6\rangle$ gets contribution from 15 diagrams.
- 5 of them are well behaved, the remaining 10 are **individually divergent**, However the **divergences cancel pair wise**.
- Typical cancellations are between





$$\sim -\frac{8\pi^2 iz}{\kappa^2}\frac{\langle q3\rangle\langle 45\rangle\langle 56\rangle\langle 46\rangle}{p_{45}^2 p_{123}^2} + \mathcal{O}(\frac{1}{z}) \ , \ \sim \frac{8\pi^2 iz}{\kappa^2}\frac{\langle q3\rangle\langle 45\rangle\langle 56\rangle\langle 46\rangle}{p_{45}^2 p_{123}^2} + \mathcal{O}(\frac{1}{z})$$

Recursion relations for non-supersymmetric theories!

- BCFW does not apply to the **non-susy CS coupled to fermions/bosons** since the amplitudes **do not have good asymptotic behavior**.

- It is possible to extract the recursion relations for non-susy fermionic/bosonic CS matter theories from the N=2 results!! Eg:

  - At **tree level,** the Feynman diagrams for an **all fermion amplitude are same** for susy/non-susy theory.

  - **Susy ward identity**: The four point super amplitude is completely specified by one function, choose it to be the four fermion amplitude.

  - Use this information recursively in the BCFW formula!

- An arbitrary higher point tree level amplitude in the fermionic CS matter theory can be entirely written in terms of **4 fermion amplitude**.

Recursion relations for non-supersymmetric theories!

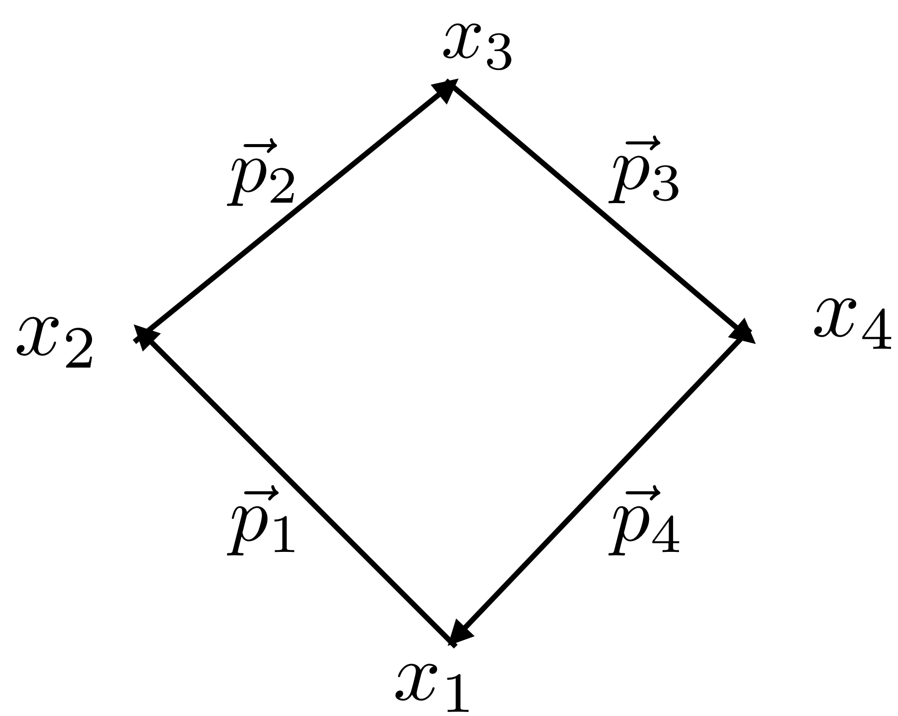
$$
\begin{aligned}
&\langle\bar\psi_1\psi_2\bar\psi_3\psi_4\bar\psi_5\psi_6\rangle = \\
&\Bigg(z_{a;f}\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2}\left[-\frac{z_{a;f}^2+1}{2z_{a;f}}+i\,\frac{z_{a;f}^2-1}{2z_{a;f}}\,\frac{\langle\hat 14\rangle}{i\langle\hat f4\rangle}\,\frac{\langle\hat f6\rangle}{\langle\hat 26\rangle}\right] \\
&\qquad\qquad\qquad\times\langle\hat{\bar\psi}_1\hat\psi_f\bar\psi_3\psi_4\rangle\langle\hat{\bar\psi}_{(-f)}\hat\psi_2\bar\psi_5\psi_6\rangle_{z_{a;f}} \\
&+(z_{a;f}\leftrightarrow z_{b;f})\Bigg)\frac{i}{p_f^2}\bigg|_{p_f=p_{234}} \\
&-\Bigg(z_{a;f}\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2}\left[-\frac{z_{a;f}^2+1}{2z_{a;f}}+i\,\frac{z_{a;f}^2-1}{2z_{a;f}}\,\frac{\langle\hat 16\rangle}{i\langle\hat f6\rangle}\,\frac{\langle\hat f4\rangle}{\langle\hat 24\rangle}\right] \\
&\qquad\qquad\qquad\times\langle\hat{\bar\psi}_1\hat\psi_f\bar\psi_5\psi_6\rangle\langle\hat{\bar\psi}_{(-f)}\hat\psi_2\bar\psi_3\psi_4\rangle_{z_{a;f}} \\
&+(z_{a;f}\leftrightarrow z_{b;f})\Bigg)\frac{i}{p_f^2}\bigg|_{p_f=p_{256}}
\end{aligned}
$$

Main highlights

- We obtained **BCFW recursion relations for arbitrary m→n tree level scattering amplitudes** in N=2 Chern-Simons matter theory.
- We were also able to extract the **recursions for non-supersymmetric Chern-Simons theory coupled to fundamental fermions**.
- Similar exercise can also be done for the bosonic theory.
- We saw an explicit example of the recursions for a **six point amplitude as a product of four point amplitudes**.
- The recursions can be iteratively applied to write **all higher point amplitudes in terms of products of four point amplitudes**.

Part III

Hidden symmetry: Dual superconformal invariance



- K.I, S.Jain, S.Majumdar, P.Nayak, T.Neogi, T.Sharma, R.Sinha, V.Umesh, **arXiv : 1711.02672**

Dual variables

- The dual variables realize momentum conservation linearly in the x variables

$$x^{\alpha\beta}_{i,i+1} = x^{\alpha\beta}_{i} - x^{\alpha\beta}_{i+1} = p^{\alpha\beta}_{i} = \lambda^{\alpha}_{i}\lambda^{\beta}_{i}$$
$$\theta^{\alpha}_{i,i+1} = \theta^{\alpha}_{i} - \theta^{\alpha}_{i+1} = q^{\alpha}_{i} = \lambda^{\alpha}_{i}\eta_{i}$$

- momentum and supermomentum conservation imply

$$P^{\alpha\beta} = \sum_{i} p^{\alpha\beta}_{i} = x^{\alpha\beta}_{n+1} - x^{\alpha\beta}_{1} = 0,$$
$$\mathcal{Q}^{\alpha} = \sum_{i} q^{\alpha}_{i} = \theta^{\alpha}_{n+1} - \theta^{\alpha}_{1} = 0.$$

- The **four point super amplitude in dual space**

$$\mathcal{A}_4 = \frac{\langle 12\rangle}{\langle 23\rangle}\delta(\sum_{i=1}^{4} p_i)\delta^2(\mathcal{Q}) \xrightarrow[\text{dual space}]{} \mathcal{A}_4 = \sqrt{\frac{x^2_{1,3}}{x^2_{2,4}}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- Goal is to show that this is **invariant under the superconformal symmetry in the dual variables**.

Superconformal algebra in dual space

- The N=2 superconformal algebra in **dual space** is generated by

$$\{P_{\alpha\beta},\ M_{\alpha\beta},\ D,\ K_{\alpha\beta},\ R,\ Q_\alpha,\ \bar{Q}_\alpha,\ S_\alpha,\ \bar{S}_\alpha\}$$

$$P_{\alpha\beta} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i^{\alpha\beta}}, \quad D = -\sum_{i=1}^{n} \left(x_i^{\alpha\beta} \frac{\partial}{\partial x_i^{\alpha\beta}} + \frac{1}{2}\theta_i^\alpha \frac{\partial}{\partial \theta_i^\alpha} \right),$$

$$Q_\alpha = \sum_{i=1}^{n} \frac{\partial}{\partial \theta_i^\alpha}, \quad \bar{Q}_\alpha = \sum_{i=1}^{n} \theta_i^\beta \frac{\partial}{\partial x_i^{\beta\alpha}},$$

$$M_{\alpha\beta} = \sum_{i=1}^{n} \left(x_{i\alpha}{}^{\gamma} \frac{\partial}{\partial x_i^{\gamma\beta}} + \frac{1}{2}\theta_{i\alpha} \frac{\partial}{\partial \theta_i^\beta} \right), \quad R = \sum_{i=1}^{n} \theta_i^\alpha \frac{\partial}{\partial \theta_i^\alpha}$$

- The remaining generators can be expressed using the inversion operator

$$I\left[x_i^{\alpha\beta}\right] = \frac{x_i^{\alpha\beta}}{x_i^2}, \quad I\left[\theta_i^\alpha\right] = \frac{x_i^{\alpha\beta}\theta_{i\beta}}{x_i^2}$$

$$K_{\alpha\beta} = I P_{\alpha\beta} I, \quad S_\alpha = I Q_\alpha I, \quad \bar{S}_\alpha = I \bar{Q}_\alpha I.$$

Dual superconformal invariance N=2 vs ABJM

- Note that the **delta functions for N=2 transform under the inversion** as

$$I\left[\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)\right] = x_1^4\, \delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- Whereas in the **N=4 and ABJM case, the corresponding delta function is invariant under the inversion**!

$$A^{(4)}_{ABJM} = \frac{1}{\sqrt{x_{1,3}^2 x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(6)}(\theta_1 - \theta_5)$$

$$\tilde{K}^{\alpha\beta}\mathcal{A}^{(4)}_{ABJM} = \left(K^{\alpha\beta} + \frac{1}{2}\sum_{j=1}^{4}\Delta_j x_j^{\alpha\beta}\right)\mathcal{A}^{(4)}_{ABJM} = 0\ ,\ \ \Delta_i = \{1,1,1,1\}$$

Gang, Huang, Koh, Lee, Lipstein

- So **it was expected that the superamplitude in the N=2 theory would not have any dual superconformal invariance** at all.

- However, in the N=2 case, **dual superconformal invariance, still works but the weights become non-homogeneous.**

Dual superconformal invariance of the superamplitude

- The four point amplitude in the N=2 theory is

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- In this form the **translation, Lorentz invariance and supersymmetry invariance of the amplitude is manifest.**

- The amplitude is just a function of the square differences in the x variable.

- Under Dilatations it transforms as a eigenfunction of weight 4.

- Under R symmetry it transforms as eigenfunction of weight 2.

- To show the dual superconformal invariance it is sufficient to show the invariance under $K_{\alpha\beta}, S_\alpha, \bar{S}_\alpha$

Dual superconformal invariance of the superamplitude

$$
\begin{aligned}
K_{\alpha\beta}\left[\mathcal{A}_4\right] &= I P_{\alpha\beta} I \left[\mathcal{A}_4\right] \\
&= I \sum_{i=1}^{4} \partial_{i\alpha\beta} \left[x_1^4 \sqrt{\frac{x_2^2 x_4^2}{x_1^2 x_3^2}}\ \mathcal{A}_4 \right] \\
&= I \left[-\frac{1}{2} x_1^4 \sqrt{\frac{x_2^2 x_4^2}{x_1^2 x_3^2}} \left(3\frac{x_1^{\alpha\beta}}{x_1^2} + \frac{x_2^{\alpha\beta}}{x_2^2} + \frac{x_4^{\alpha\beta}}{x_4^2} - \frac{x_3^{\alpha\beta}}{x_3^2} \right) \mathcal{A}_4 \right] \\
&= -\frac{1}{2} \left(3x_1^{\alpha\beta} + x_2^{\alpha\beta} + x_4^{\alpha\beta} - x_3^{\alpha\beta} \right) \mathcal{A}_4 \\
&= -\frac{1}{2} \left(\sum_{j=1}^{4} \Delta_j x_j^{\alpha\beta} \right) \mathcal{A}_4 \qquad \text{w/} \quad \{\Delta_j\} = \{3, 1, -1, 1\}
\end{aligned}
$$

- So the invariance under $K_{\alpha\beta}, S_\alpha, \bar{S}_\alpha$

$$
\tilde{K}^{\alpha\beta} \mathcal{A}^{(4)} = \left(K^{\alpha\beta} + \frac{1}{2} \sum_{j=1}^{4} \Delta_j x_j^{\alpha\beta} \right) \mathcal{A}^{(4)} = 0 \qquad \tilde{\bar{S}}_\alpha \left[\mathcal{A}_4\right] = \left(\bar{S}_\alpha + \frac{1}{2} \left(\sum_{j=1}^{4} \Delta_j \theta_{j\alpha} \right) \right) \left[\mathcal{A}_4\right] = 0
$$

$$
S_\alpha \left[\mathcal{A}_4\right] = I Q_\alpha I \left[\mathcal{A}_4\right] = I Q_\alpha \left[x_1^4 \sqrt{\frac{x_2^2 x_4^2}{x_1^2 x_3^2}}\ \mathcal{A}_4 \right] = 0.
$$

Dual superconformal invariance at all loops

- We showed that the function A4 is dual superconformal invariant!

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- The **tree level superamplitude is dual superconformal invariant**.

$$T_{tree} = \frac{4\pi}{\kappa}\mathcal{A}_4$$

- The **all loop results** computed in **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** are **also dual superconformal invariant.**

$$T_{sym}^{all\ loop} = T_{Asym}^{all\ loop} = T_{Adj}^{all\ loop} = T_{tree}$$

$$T_{sing}^{all\ loop} = N\frac{\sin\pi\lambda}{\pi\lambda}T_{tree}$$

- Now that we know this symmetry exists, **can we reverse the argument and do an S matrix bootstrap to fix the general structure of the amplitude**?

Constraining amplitudes from dual superconformal symmetry

- The **four point amplitude in momentum space** can be interpreted as a **four point correlator in dual space**, then dual conformal invariance fixes

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$

$$= \frac{1}{x_{12}^{\Delta_1+\Delta_2} x_{34}^{\Delta_3+\Delta_4}} \left(\frac{x_{24}}{x_{14}}\right)^{\Delta_1-\Delta_2} \left(\frac{x_{14}}{x_{13}}\right)^{\Delta_3-\Delta_4} f(u,v,\kappa,\lambda)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}.$$

- Since $x_{ij}^2 = p_i^2 = 0$, the correlator is understood in the limit

$$\frac{u}{v}\bigg|_{onshell} = \frac{x_{12}^2 x_{34}^2}{x_{14}^2 x_{23}^2}\bigg|_{onshell} = \frac{p_1^2 p_3^2}{p_2^2 p_4^2}\bigg|_{onshell} = constant$$

- **If dual superconformal symmetry is exact it fixes the momentum (x) dependence completely***

$$f(u,v,\kappa,\lambda) = g(\kappa,\lambda)$$

Constraining amplitudes from dual superconformal symmetry

- In general the S matrix could get complicated functions with poles and branch cuts.

- If dual conformal invariance is an exact symmetry at loop level then no such behavior appears.

- Non trivial momentum dependence could still appear from $x_{i,j}^{\Delta}$ when Δ gets correction from loops.

- This can give rise to log dependence for instance, However these do not appear if there are no IR divergences.

- **If we assume that there are no IR divergences (none seen in the calculation), and that the dual conformal invariance is an exact symmetry, then the momentum dependence is fixed**.

4 point amplitude as a free field correlator in dual space

- Recall that $\{\Delta_1, \Delta_2, \Delta_3, \Delta_4\} = \frac{1}{2}\{4-1, 1, -1, 1\}$
- The factor of 4 is due to momentum+supermomentum conservation and can be removed. $I\left[\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)\right] = x_1^4\, \delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$
- With this identification the operator dimensions are

$$\tilde{\Delta}_1 = \Delta_3 = -\frac{1}{2}$$

$$\Delta_2 = \Delta_4 = \frac{1}{2}$$

- The four point correlator in dual space gets fixed to (cancellations in limiting sense)

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = g(\kappa, \lambda)\sqrt{\frac{x_{13}^2}{x_{24}^2}}$$

- Same as the amplitude without the delta functions!

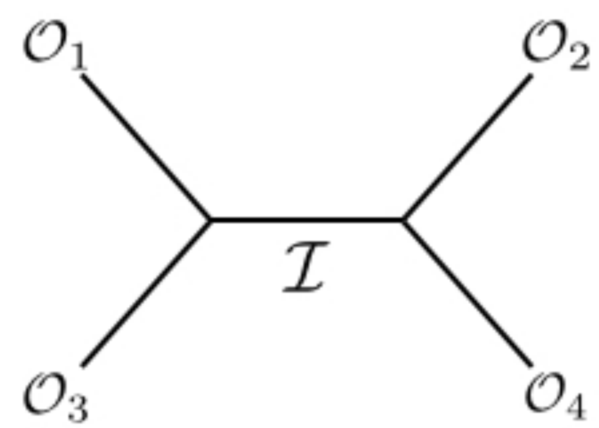
$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

4 point amplitude as a free field correlator in dual space

- In a general CFT, in the **double light cone limit, only Identity operators are expected to contribute**!

- In the channel where $(\mathcal{O}_1, \mathcal{O}_3)$ and $(\mathcal{O}_2, \mathcal{O}_4)$ are brought together

$$\tilde{\Delta}_1 = \Delta_3 = -\frac{1}{2}$$
$$\Delta_2 = \Delta_4 = \frac{1}{2}$$



- The four point amplitude can be accounted for by an **identity exchange**.

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \langle \mathcal{O}_1(x_1)\mathcal{O}_3(x_3)\rangle\langle \mathcal{O}_2(x_2)\mathcal{O}_4(x_4)\rangle$$
$$= c_1 c_2 \sqrt{\frac{x_{13}^2}{x_{24}^2}}$$

- This suggests $g(\kappa, \lambda) = c_1 c_2$.

- It would be interesting to understand the CFT interpretation of these operators, and also to see what happens in the cross channel.

Part III
Summary

Summary

- We started with a goal of computing **arbitrary m→n tree level scattering amplitudes** in U(N) $\mathcal{N} = 2$ Chern-Simons matter theories with fundamental matter.
- We achieved this via **BCFW recursion relations**, this enabled us to express arbitrary n point amplitudes as products of four point amplitudes!
- We saw an explicit example where the six point amplitude is expressed as a product of two four point amplitudes via two factorization channels.
- The non-susy amplitudes do not satisfy the BCFW requirements.
- However we were able to use the fact that the four point superamplitude in the $\mathcal{N} = 2$ theory is specified by one function and that the tree level amplitudes are identical to the non-susy case to write recursions for the non-susy theory as well.
- We saw an explicit example for the six fermion amplitude in the fermion coupled Chern-Simons theory.

Summary

- We showed that the **all loop 2→2 scattering amplitude** computed in **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** is **dual superconformal invariant**.
- Thus **dual superconformal symmetry is all loop exact**, at least for the 4 point amplitude.
- The presence of dual conformal symmetry then allows us to interpret the **amplitude in momentum space as a correlator in dual space**.
- We argued that **if dual conformal symmetry was an exact symmetry it fixed the momentum dependence of the amplitude completely**.
- We interpreted the four point amplitude in dual space as a free field correlator where the identity operator exchange accounted for it.
- However, **general principles such as unitarity, duality and dual conformal symmetry are insufficient to fix the overall coupling dependence**.

Yangian Symmetry

- The presence of the **superconformal and dual superconformal symmetries indicate a Yangian symmetry** in the amplitude.

- A Yangian algebra is an associative Hopf Algebra generated by

$$[J^A, J^B] = f_C^{AB} J^C \ , \ [J^A, Q^B] = f_C^{AB} Q^C$$

- J^A take values in a Lie group G, both J^A and Q^A are constrained to obey the Serre relations in addition to Jacobi Identity.

$$[Q^A, [Q^B, J^C]] + [Q^B, [Q^C, J^A]] + [Q^C, [Q^A, J^B]] = \frac{1}{24} f^{ADK} f^{BEL} f^{CFM} f_{KLM} \{J_D, J_E, J_F\}$$

$$[[Q^A, Q^B], [J^C, Q^D]] + [[Q^C, Q^D], [J^A, Q^B]] = \frac{1}{24}(f^{AGL} f^{BEM} f_K^{CD} + f^{CGL} f^{DEM} f_K^{AB}) \times f^{KFN} f_{LMN} \{J_G, J_E, J_F\}$$

- Under repeated commutations the Q's generate an infinite dimensional Symmetry algebra.

Yangian Symmetry in N=2 Chern-Simons theory

- The Yangian has a basis (labelled by levels)

$$\mathcal{J}_0^A = J^A \ , \ \mathcal{J}_1^A = Q^A \ , \ldots , \ \mathcal{J}_n^A$$

$$[\mathcal{J}_{(0)}^A, \mathcal{J}_{(0)}^B\} = f^{AB}{}_C \mathcal{J}_{(0)}^C \qquad\qquad [\mathcal{J}_{(0)}^A, \mathcal{J}_{(1)}^B\} = f_C^{AB} \mathcal{J}_{(1)}^C$$

- The generators $\mathcal{J}_n^A$ are "n local" operators. The infinite dimensional symmetry is generated by commutators of $\mathcal{J}_n^A$.
- For the N=2 theory the spacetime superconformal symmetry is $Osp(2|4)$

$$\mathcal{J}_{(0)}^A = \{p_{\alpha\beta}, m_{\alpha\beta}, d, k_{\alpha\beta}, r, q_\alpha, \bar{q}_\alpha, s_\alpha, \bar{s}_\alpha\}$$

- A general ansatz for the Level 1 generators **Kazakov et al.**

$$\mathcal{J}_{(1)}^A = \frac{1}{2} f_{BC}^A \sum_{j<k} \mathcal{J}_{j,(0)}^C \mathcal{J}_{k,(0)}^B + \sum_k v^l \mathcal{J}_{l,(0)}^A$$

- The dual generators K and S when restricted to onshell superspace map to Level 1 Yangian generators of the spacetime superconformal algebra.

$$K_{\alpha\beta}(\lambda_\alpha, \eta) \equiv p_{\alpha\beta}^{(1)}$$

$$\bar{S}_\alpha(\lambda_\alpha, \eta) \equiv \bar{q}_\alpha^{(1)}$$

- The remaining restricted dual generators have a trivial automorphism to Level 0 generators of the spacetime superconformal algebra.

K.I, Jain, Nayak, Sharma, Umesh, to appear

Yangian Symmetry in N=2 Chern-Simons theory

- Note that K, S commute!, so how does the infinite dimensional algebra appear?

$$K_{\alpha\beta}(\lambda_\alpha, \eta) \equiv p^{(1)}_{\alpha\beta}$$
$$\bar{S}_\alpha(\lambda_\alpha, \eta) \equiv \bar{q}^{(1)}_\alpha$$

- The remaining Level 1 generators are obtained by commuting K with the $\mathcal{J}^A_{(0)}$
- These remaining Level 1 (other than [K,S]) generate the Level 2 upon commutation and so on.
- The Yangian works exactly the same way as it did for N=4 SYM and ABJM.
- The Yangian invariance of the amplitude then boils down to the statement

$$\mathcal{J}^A_{(0)} \mathcal{A}_4 = 0\ ,\ \mathcal{J}^A_{(1)} \mathcal{A}_4 = 0\ ,\ \Longrightarrow\ \ \mathcal{Y} \mathcal{A}_4 = 0,$$

- Thus **superconformal and dual superconformal symmetries generate a Yangian symmetry!**

K.I, Jain, Nayak, Sharma, Umesh, to appear

Things to do: higher point amplitudes

- As an extension of the above story, **dual conformal invariance for higher point amplitudes and Yangian invariance**.
- A general n point correlator has n(n-3)/2 independent cross ratios, for eg six point function out of 9 crossratios, three survive onshell

$$u = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2}, \quad v = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{14}^2}, \quad , w = \frac{x_{35}^2 x_{26}^2}{x_{36}^2 x_{25}^2}.$$

- The six point amplitude can be determined upto a function of 3 variables. in principle this can allow for much more complicated dependence on the momentum.
- In general **higher point loop amplitudes** need not be proportional to the tree level answer as it was the case in the four point amplitude, and they **can also get non-trivial renormalization**.
- If the Yangian story goes through for the six point amplitude, it would be interesting to see if the **symmetry can be seen at the level of the Lagrangian**!

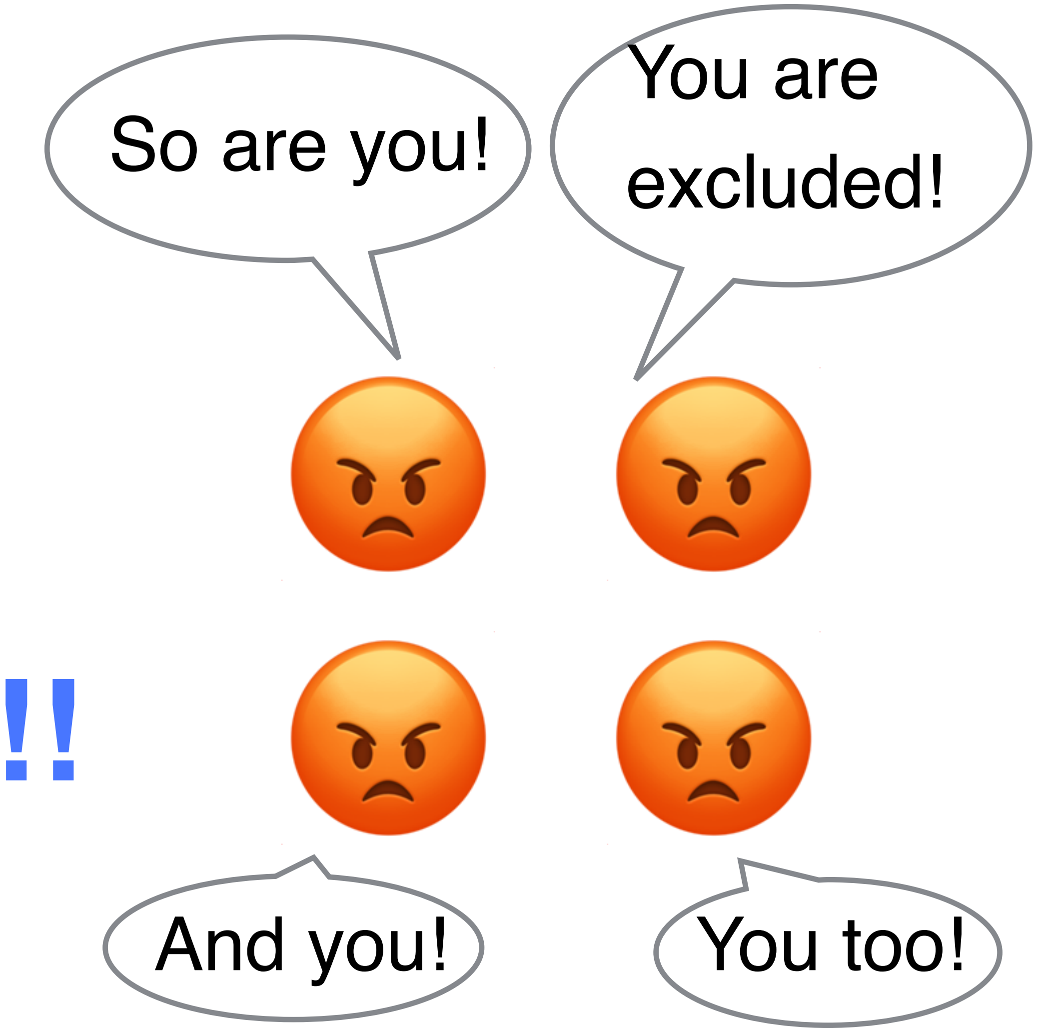
Beisert,Garrus, Russo

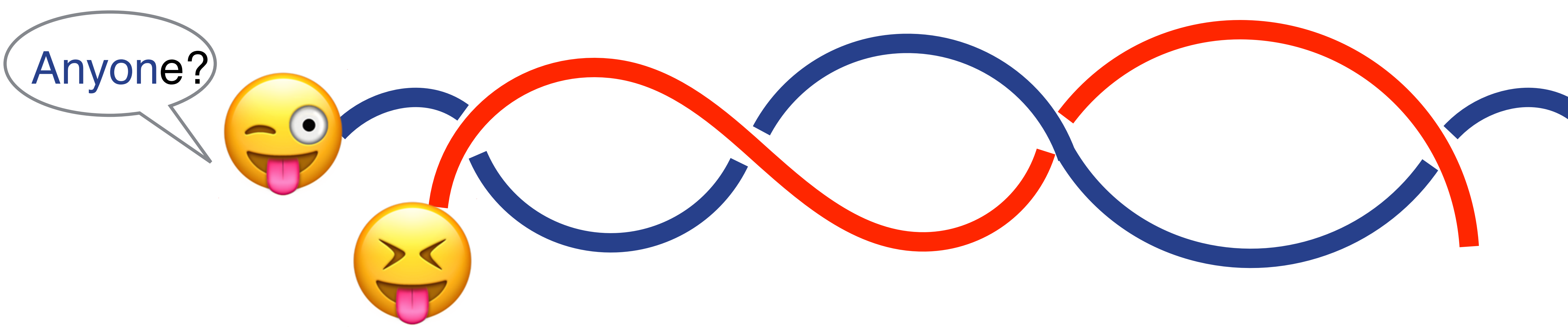
Even more things to do…

- **Amplitude-Wilson loop duality, Orthogonal grassmanian** representation of the amplitudes.
- More **stringent tests of duality**, computing arbitrary higher point correlators, alternative to Feynman diagrams?
- What are the c**rossing rules for anyonic channels in arbitrary higher point amplitudes**?
- What is the anyonic phase structure for an **n-particle Aharonov-Bohm scattering**?
- Does some anomalous form of dual conformal invariance survive for non-supersymmetric amplitudes, if so can it constrain the form of the amplitude? (In the massless limit, these amplitudes are also quite simple)


We are happy together!

Thank you!!


So are you!
You are excluded!
And you!
You too!


Anyone?

Aside: Spinor helicity basis

- Spinor helicity representation

$$p_i^{\alpha\beta} = p_i^{\mu}\sigma_{\mu}^{\alpha\beta} = \lambda_i^{\alpha}\lambda_i^{\beta}$$

$$(p_i + p_j)^2 = 2p_i.p_j = -\langle\lambda_i^{\alpha}\lambda_{j,\alpha}\rangle^2 = \langle ij\rangle^2$$

- Onshell Supersymmetry generators

$$\mathcal{Q} = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} \lambda_i \eta_i,$$

$$\bar{\mathcal{Q}} = \sum_{i=1}^{n} \bar{q}_i = \sum_{i=1}^{n} \lambda_i \partial_{\eta_i}$$

Scattering in U(N) Chern-Simons matter theories

- Consider $2 \to 2$ scattering of particles in representations R_1 and R_2 of U(N)

$$R_1 \times R_2 = \sum_m R_m$$

- The S matrix takes the schematic form

$$S = \sum_m P_m S_m$$

$$P_m : \text{ projector in } m^{th} \text{ rep, } S_m \text{ is scattering in } m^{th} \text{ channel.}$$

- The Aharanov-Bohm phase of the particle R_1 as it circles around particle R_2 is $2\pi\nu_m$ where

$$\nu_m = \frac{4\pi}{\kappa} T_1^a T_2^a = \frac{2\pi}{\kappa} \left(C_2(R_1) + C_2(R_2) - C_2(R_m) \right)$$

- Scattering amplitude in the m'th exchange channel: Aharanov-Bohm scattering of a unit charge particle off a flux tube of flux $2\pi\nu_m$

Scattering in U(N) Chern-Simons matter theories

- Channels of scattering

$$\text{Fundamental} \otimes \text{Fundamental} \rightarrow \text{Symm}(U_d) \oplus \text{Asymm}(U_e)$$

$$\text{Fundamental} \otimes \text{Antifundamental} \rightarrow \text{Adjoint}(T) \oplus \text{Singlet}(S)$$

- The quadratic Casimirs

$$C_2(F) = C_2(A) = \frac{N^2-1}{2N} \; , \; C_2(Sym) = \frac{N^2+N-2}{N}$$

$$C_2(ASym) = \frac{N^2-N-2}{N} \; , \; C_2(Adj) = N \; , \; C_2(Sing) = 0$$

- Anyonic phase

$$\nu_{Sym} = \frac{1}{\kappa} - \frac{1}{N\kappa} \; , \; \nu_{Asym} = -\frac{1}{\kappa} - \frac{1}{N\kappa}$$

$$\nu_{Adj} = \frac{1}{N\kappa} \; , \; \nu_{Sin} = -\frac{N}{\kappa} + \frac{1}{N\kappa}$$

- In the large N, large κ limit, define 't Hooft coupling $\lambda = \frac{N}{\kappa}$

$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right) \; , \nu_{Sing} \sim O(\lambda)$$

Scattering in U(N) Chern-Simons matter theories

- Anyonic phases in the large N, large κ limit

$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right) \;, \nu_{Sing} \sim O(\lambda)$$

- The T matrices themselves have the large N behavior

$$T_{Asym} \sim T_{Sym} \sim T_{Adj} \sim O\left(\frac{1}{N}\right) \;, \; T_{sing} \sim O(1)$$

- The singlet channel is effectively anyonic in the large N ,large κ limit.

- Unitarity $i(T^\dagger - T) = TT^\dagger$ is a non-trivial check only for the singlet channel. In other channels it follows from hermiticity.

- Observation: Naive crossing symmetry rules from any of the non-anyonic channels to the singlet channel leads to a non unitary S matrix.

S.Jain, M. Mandlik, S.Minwalla,T.Takimi S.Wadia, S.Yokoyama

- Conjecture: Singlet channel S matrices have the form

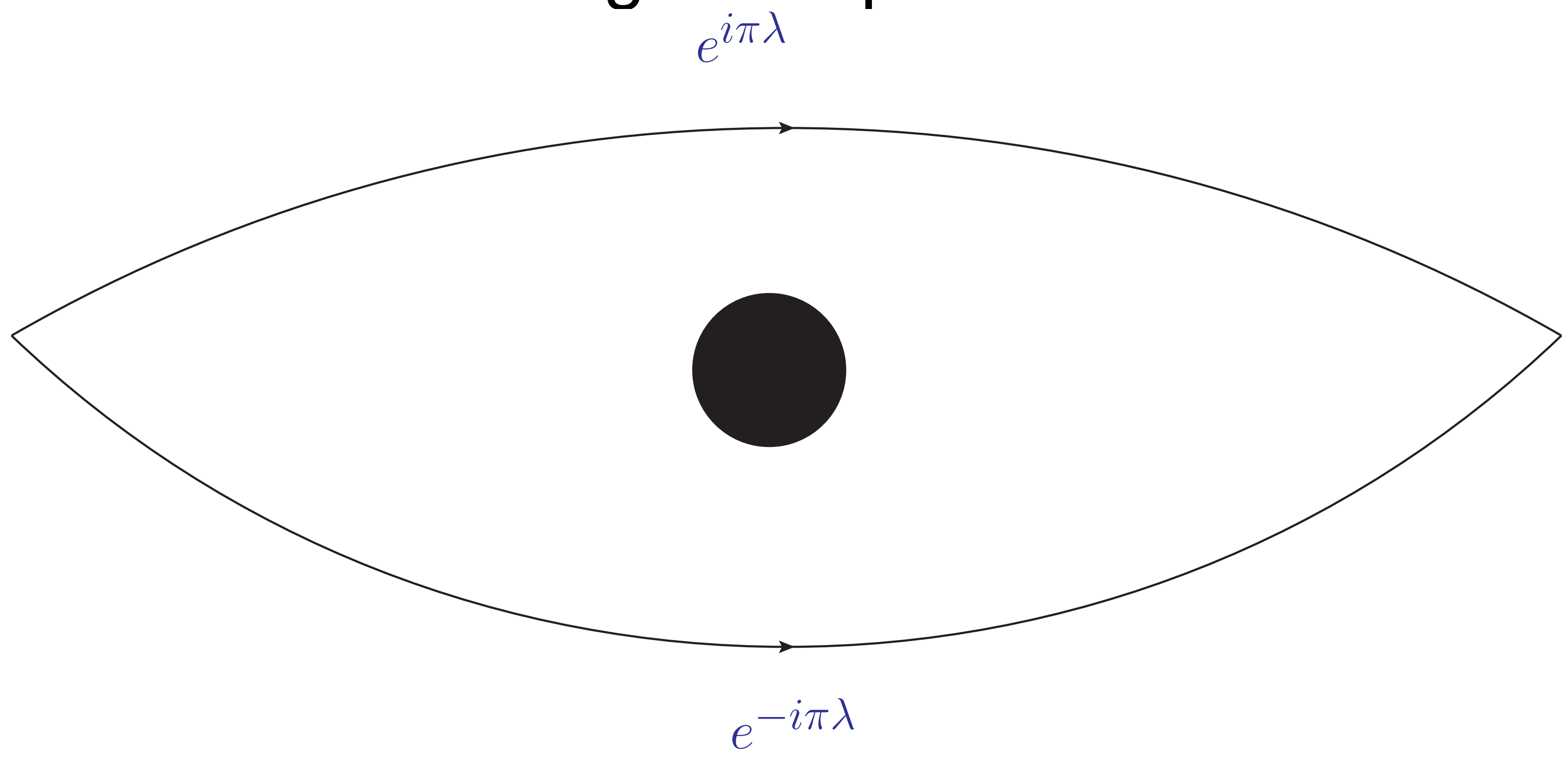
$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\mathrm{naive}}(s,\theta)$$

- $\mathcal{T}^{S;\mathrm{naive}}$ is the matrix obtained from naive analytic continuation of particle-particle scattering.

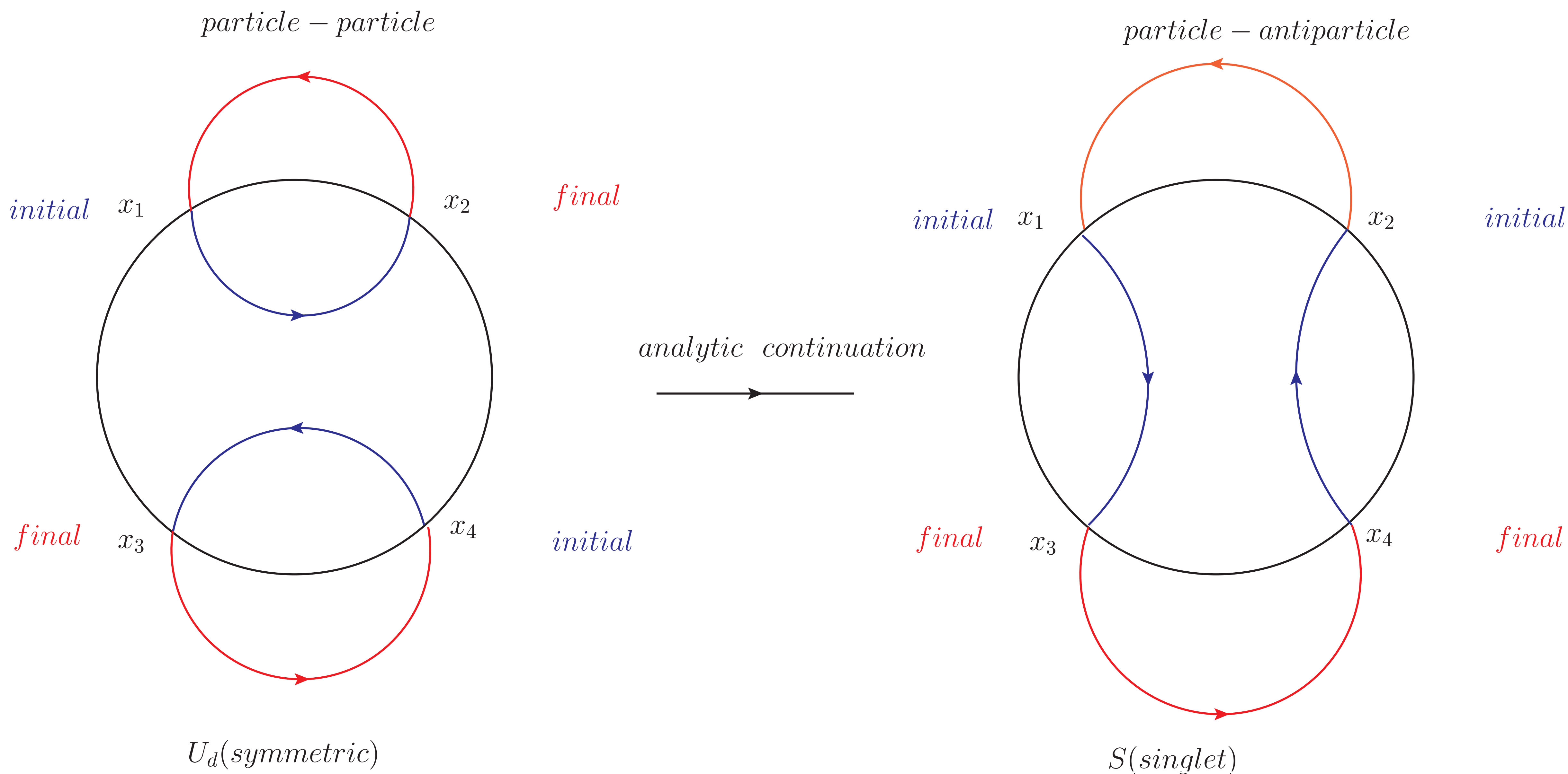
Nature of the conjecture: Delta function

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$

- The conjectured S matrix has a non-analytic delta function piece.
- The delta function modulated by anyonic phase is already known to be necessary to unitarize Aharonov-Bohm scattering.
- $\cos\pi\lambda$ in the identity term is due to the interference of the Aharanov-Bohm phases of the incoming wave packets.



Modified crossing rules: Heuristic explanation



- Attach Wilson lines to make correlators gauge invariant

$$T_{U_d} W_{U_d} \to T_S W_S$$

$$\frac{W_{U_d}}{W_S} = \frac{\oint \text{with 2 circular Wilson lines}}{\oint \text{with 1 circular Wilson line}} = \frac{\sin(\pi\lambda)}{\pi\lambda}$$ Witten