

Let:

reserve a initial  $\Rightarrow r_{a0}$

reserve b initial  $\Rightarrow r_{b0}$

reserve b after execution  $\Rightarrow r_{b1}$

amount traded  $\Rightarrow a_{traded}$

fee  $\Rightarrow f$

amount in with fee  $\Rightarrow a_{in}$

constant product  $\Rightarrow k$

amount out  $\Rightarrow a_{out}$

market price  $\Rightarrow p_{market}$

mid price  $\Rightarrow p_{mid}$

price impact  $\Rightarrow p_{impact}$

## Calculations

$$a_{in} = a_{traded}(1 - f)$$

$$k = r_{a0} * r_{b0}$$

$$r_{b1} = \frac{k}{r_{a0} + a_{in}} = \frac{r_{a0} * r_{b0}}{r_{a0} + a_{in}}$$

$$a_{out} = r_{b0} - r_{b1} = r_{b0} - \frac{r_{a0} * r_{b0}}{r_{a0} + a_{in}} = \frac{r_{b0}(r_{a0} + a_{in}) - r_{a0} * r_{b0}}{r_{a0} + a_{in}}$$

$$p_{market} = \frac{a_{in}}{a_{out}} = a_{in} \div \frac{r_{b0} * a_{in}}{r_{a0} * a_{in}} = \frac{r_{a0} * a_{in}}{r_{b0}}$$

$$p_{mid} = \frac{r_{a0}}{r_{b0}}$$

$$p_{impact} = 1 - \frac{p_{mid}}{p_{market}} = 1 - \frac{r_{a0}}{r_{b0}} \div \frac{r_{a0} + a_{in}}{r_{b0}} = 1 - \frac{r_{a0}}{r_{a0} + a_{in}} = \frac{a_{in}}{r_{a0} + a_{in}}$$

## Calculating amount in (using amount out)

$$a_{in} = \frac{a_{out} * r_{a0}}{(a_{out} - r_{b0}) * (f - 1)}$$