

```
% Will Kramlinger
% CE 3101
% Gonella
% 4/22/14
% HW7
```

```
%% PROBLEM 1
```

```
edit
```

```
function dydx = HW7(x,y)
dydx = .1 * y * (100 * (1 - x/100) - y);
end
```

```
function [x,y] = odeEULER(ODE,a,b,h,yINI)
```

```
% Will Kramlinger; 4/17/14
```

```
% Code appropriated from the textbook.
```

```
% odeEULER solves a first-order initial value ODE using Euler's explicit
% method.
```

```
% Input variables:
```

```
% ODE    Name for the function that calculates dy/dx.
```

```
% a      The first value of x.
```

```
% b      The last value of x.
```

```
% h      Step size.
```

```
% yINI   The value of the solution y at the first point (initial value).
```

```
% Output variables:
```

```
% x      A vector with the x coordinate of the solution points.
```

```
% y      A vector with the y coordinate of the solution points.
```

```
x(1) = a; y(1) = yINI;
```

```
N = (b-a)/h;
```

```
for i = 1:N
```

```
    x(i+1) = x(i) + h;
```

```
    y(i+1) = y(i) + h*ODE(x(i),y(i));
```

```
end
```

```
% plot(x,y);
```

```
end
```

```
function [x,y] = odeModEuler(ODE,a,b,h,yINI)
```

```
% Will Kramlinger; 4/17/14
```

```
% Code appropriated from textbook.
```

```
% odeModEuler solves a first order ODE using the modified Euler method.
```

```
% Input variables:
```

```
% ODE    Name for the function that calculates dy/dx.
```

```
% a      The first value of x.
```

```
% b      The last value of x.
```

```
% h      Step size.
```

```
% yINI   The value of the solution y at the first point (initial value).
```

```
% Output variables:
```

```
% x      A vector with the x coordinate of the solution points.
```

```
% y      A vector with the y coordinate of the solution points.
```

```
x(1) = a; y(1) = yINI;
```

```
N = (b - a)/h;
```

```

for i = 1:N
    x(i+1) = x(i) + h;
    SlopeEu = ODE(x(i),y(i));
    yEu = y(i) + SlopeEu*h;
    SlopeEnd = ODE(x(i+1),yEu);
    y(i+1) = y(i) + (SlopeEu + SlopeEnd) * h/2;
end

% plot(x,y);
end

```

```

function [x,y] = odeRK4(ODE,a,b,h,yINI)
% Will Kramlinger; 4/17/14
% Code appropriated from textbook.

% odeRK4 solves a first order initial value ODE using Runge-Kutta fourth
% order method.
% Input variables:
% ODE    Name for the function that calculates dy/dx.
% a      The first value of x.
% b      The last value of x.
% h      Step size.
% yINI   The value of the solution y at the first point (initial value).
% Output variables:
% x      A vector with the x coordinate of the solution points.
% y      A vector with the y coordinate of the solution points.

x(1) = a; y(1) = yINI;
N = (b - a)/h;
for i = 1:N
    x(i+1) = x(i) + h;
    K1 = ODE(x(i),y(i));
    xhalf = x(i) + h/2;
    yK1 = y(i) + K1*h/2;
    K2 = ODE(xhalf,yK1);
    yK2 = y(i) + K2*h/2;
    K3 = ODE(xhalf,yK2);
    yK3 = y(i) + K3*h;
    K4 = ODE(x(i+1),yK3);
    y(i+1) = y(i) + (K1 + 2*K2 + 2*K3 + K4)*h/6;
end

% plot(x,y);
end

```

```

function graph = superimpose(a,b,h,yINI)
% Will Kramlinger; 4/17/14
% The function plots the solutions of a differential equation, in this
% case the equation HW7, using previously written forward Euler,
% modified Euler, and RK4 functions.
% Input variables:
% a = The first value of x.
% b = The last value of x.
% h = Step size.
% yINI = The value of the solution y at the first point (initial value).
% Output variables:

```

```

% graph = instructs user that this variable is irrelevant and to look
%           at the graph

[xFE,yFE] = odeEULER(@HW7,a,b,h,yINI);
plot(xFE,yFE,'--');

hold on

[xMod,yMod] = odeModEuler(@HW7,a,b,h,yINI);
plot(xMod,yMod,'g:s');

[xRK4,yRK4] = odeRK4(@HW7,a,b,h,yINI);
plot(xRK4,yRK4,'r:p');

title('Comparison of ODE Solving Methods');
xlabel('t'); ylabel('N');
legend('ForwEuler','ModEuler','RK4','Location','best');

graph = 'Look at the graph, doggy.';

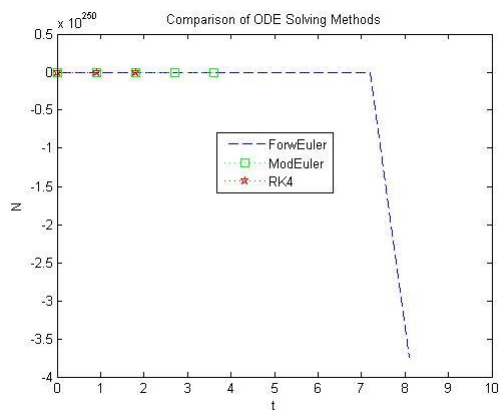
hold off
end

```

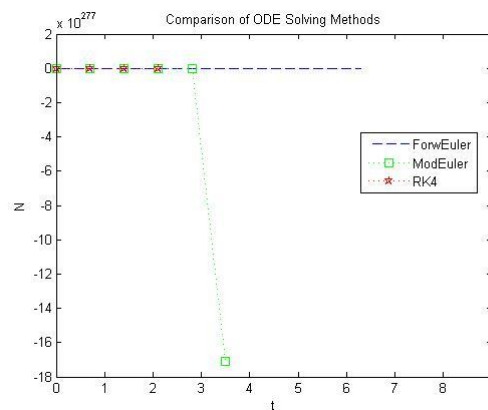
```

graph = superimpose(0,9,.9,90) % ...and repeated for smaller step sizes

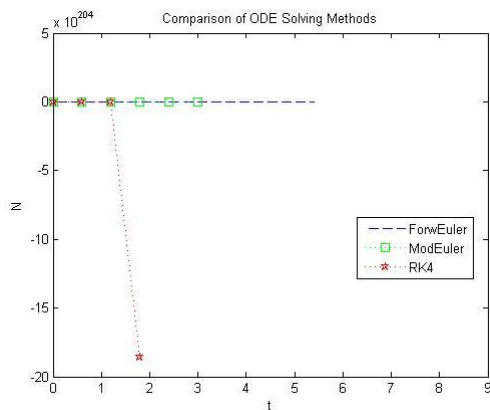
```



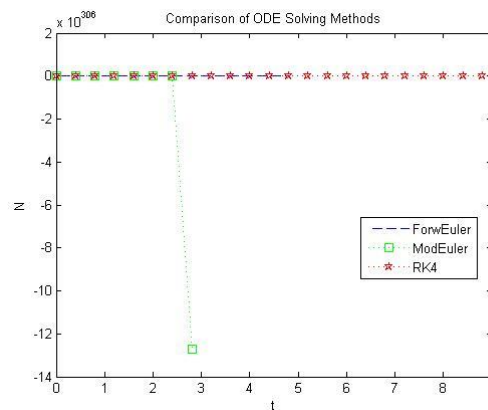
h = .9



h = .7

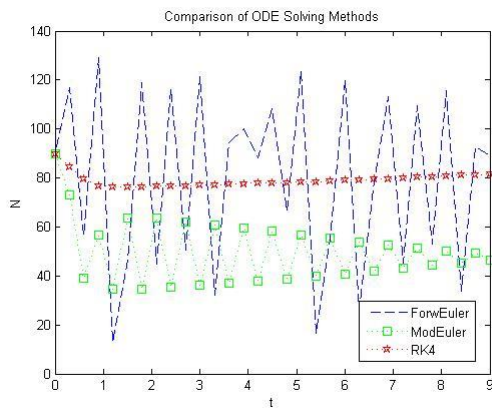


h = .6

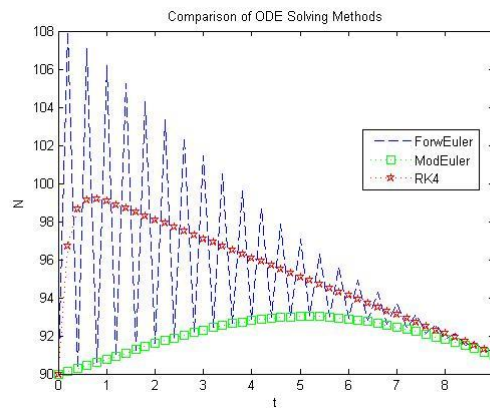


h = .4

% In the approximate range $.3 < h < .9$, all three methods tend to produce
 % solutions of $N \approx 0$ (or presumably some constant on the order of 10^1),
 % then often tend to $-\infty$; the step size is too large. It is
 % interesting to note that at about $h = .4$, the RK4 method is the first of
 % the methods not to tend to $-\infty$ for $0 < t < 9$. Then, Modified
 % Euler and Forward Euler follow suit, respectively.



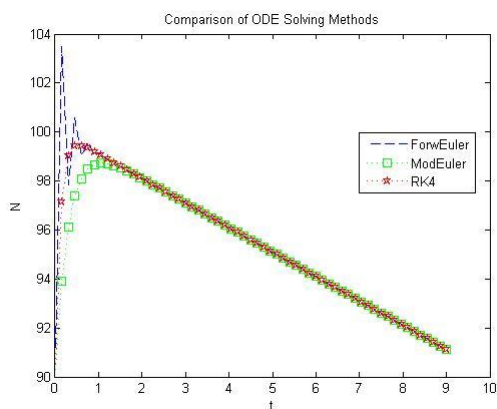
$h = .3$



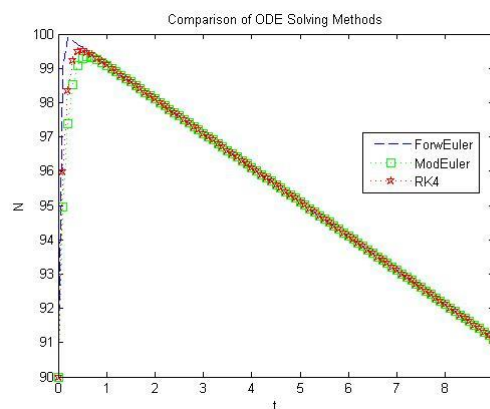
$h = .2$

% At $h \approx .3$, the solutions begin to take on distinctive characters. The
 % RK4 solution can be approximated by $N = 80$ (which is why I assume N is
 % slightly greater than 0 in the preceding paragraph). The other 2
 % solutions suggest N oscillates for $0 < t < 9$.

% At around $h = .2$, the RK4 method begins to resemble the assumed actual
 % solution. Forward Euler still gives an oscillating function (though
 % less erratic) while Modified Euler starts to resemble a curve. For all
 % 3, the range of N values get closer to those of the actual solution.



$h = .15$



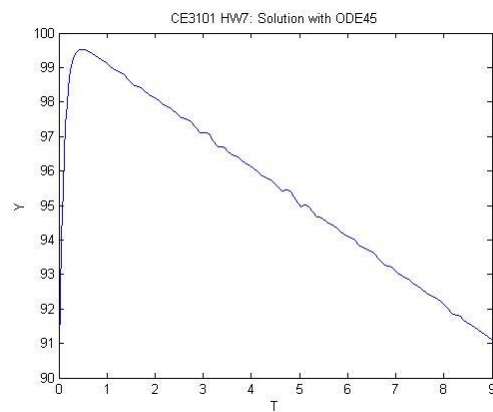
$h = .1$

% As h decreases from $.2$ to $.1$, Modified Euler and Forward Euler,
 % respectively, both begin to converge to the solution given by RK4.
 % Slight oscillatory character is still visible for the Forward Euler
 % solution at $h = .15$. At $h = .1$, all three solutions resemble the actual
 % solution.

```
% As the step size is decreased past h = .1, there is very little  
% discernable change in the solutions and the differences between them.
```

```
% The trends seen in the solutions due to decreasing step size agree with  
% those which can be intuitively predicted. In terms of expected global  
% error, RK4 < Modified Euler < Forward Euler. Thus, the solutions given  
% by the three methods should begin to converge (if they do) to the actual  
% solution in that order, which is seen above. The lack of any predictor-  
% corrector characteristics for Forward Euler results in it providing a  
% misleading oscillating solution for a relatively wide range of h values.  
% None of the methods are immune to a step size which is too large.
```

```
[T,Y] = ode45(@HW7,[0 9],90);  
plot(T,Y);
```



```
% The solution obtained using ode45 seems to be a reasonable  
% representation of the solutions obtained with the other methods, once  
% their step sizes are ~.1 or lower. Given the ease of use of ode45, as  
% well as the lack of mandatory step size specification, it appears to be  
% a very convenient but still relatively accurate function.
```