Variational Inference

Statistical Learning 3

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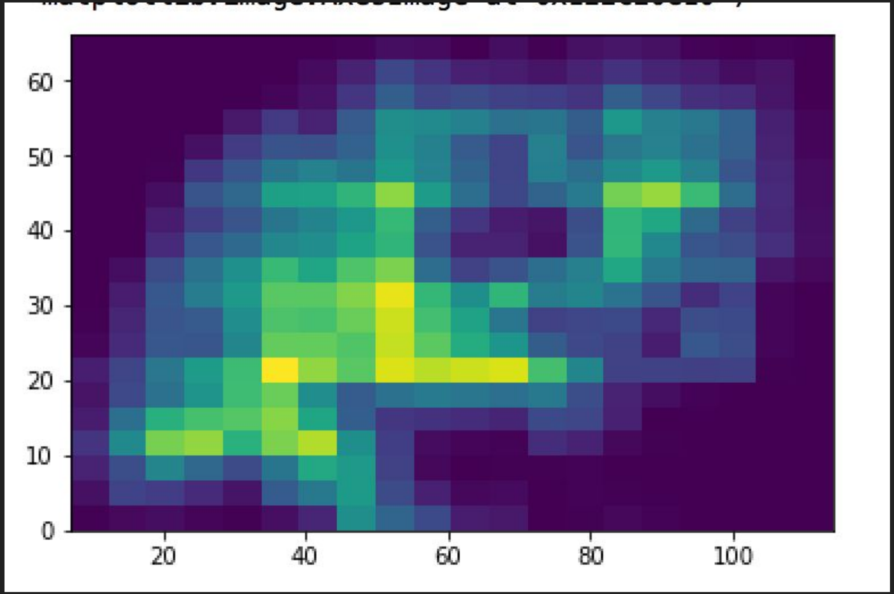
Classification of Brain regions

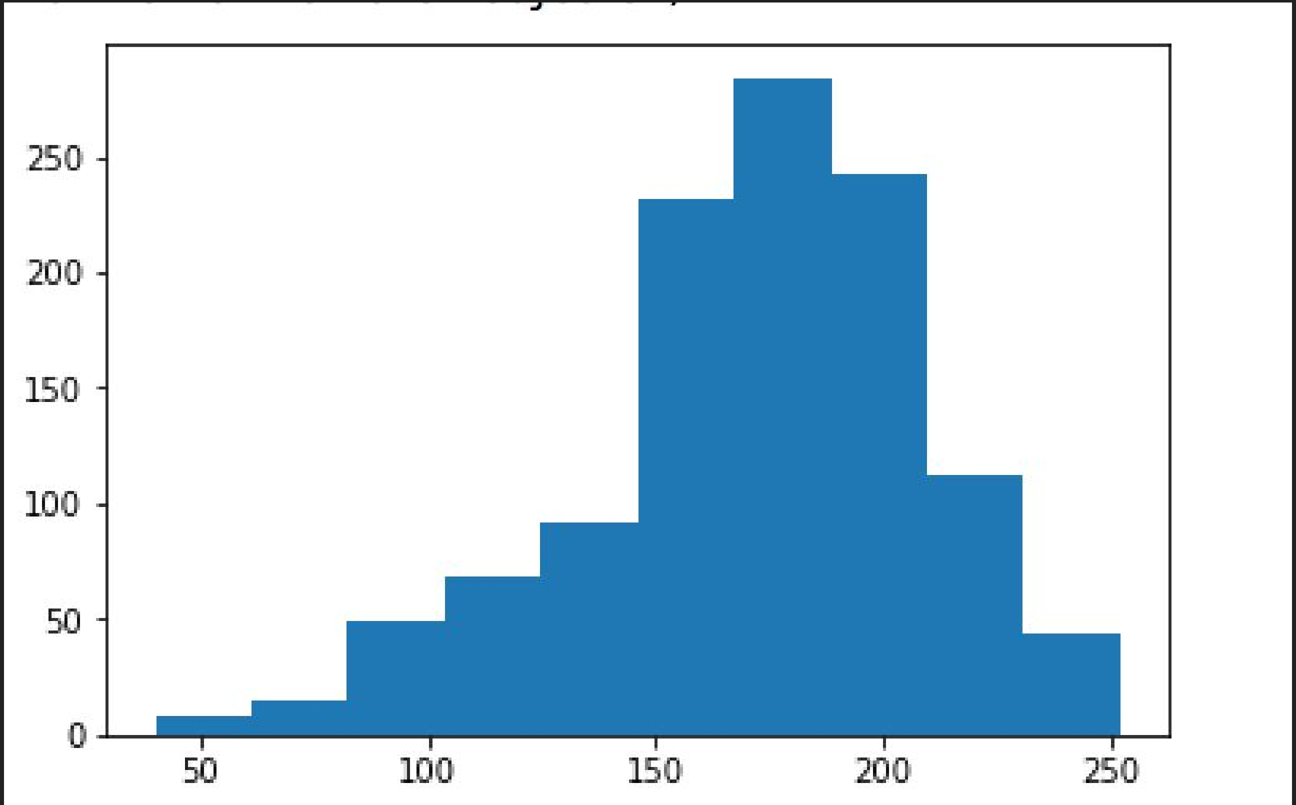
- Grey Matter
- White Matter
- Csf
- Diseased

Solving spatial dependence with symmetry

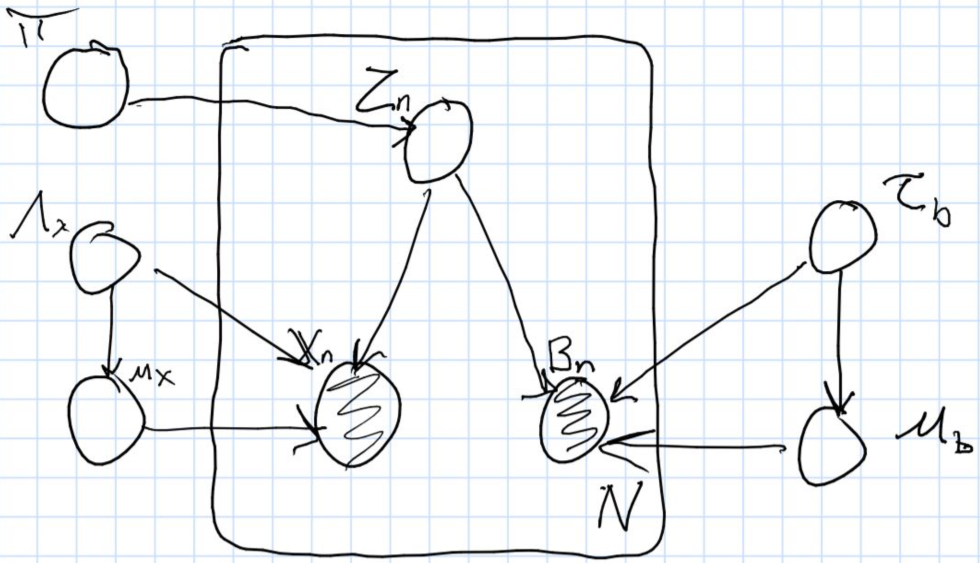
- Noting the brain images are symmetric around the y axis, the data approximates as a multivariate normal in R2



Pixel values basically normal



Proposed Model



$$P(\pi, z, x, B, \Lambda_x, \mu_x, \tau_b, \mu_b)$$

Combination of GMM and univariate Normal

$$p(\pi, Z, X, B, \Lambda_x, \mu_X, \tau_b, \mu_b)$$

$$= p(\pi)\, p(Z|\pi)\, p(X|Z, \Lambda_x, \mu_X)\, p(B|Z, \mu_b, \tau_b)$$

$$p(\mu_x|\Lambda_x)\, p(\Lambda_x)\, p(\mu_b|\tau_b)\, p(\tau_b)$$

assume $q(Z, \pi, \mu_x, \Lambda_x, \mu_b, \tau_x) = q(Z)(\pi, \mu_x, \Lambda_x, \mu_b, \tau_b)$

$$\ln q^*(Z) = E_{\pi, \mu_x, \mu_b, \Lambda_x, \tau_b}\left[p(X, B, \pi, \mu_x, \mu_b, \Lambda_x, \tau_b)\right]$$

assume factorized $\therefore$ updates are combination of

GMM and univariate gauss

Updates (From Bishop)

Univariate

$$\mu_N = \frac{\lambda_0 \mu_0 + N\overline{x}}{\lambda_0 + N} \quad (10.26)$$

$$\lambda_N = (\lambda_0 + N)\mathbb{E}[\tau]. \quad (10.27)$$

$$a_N = a_0 + \frac{N}{2}$$

$$b_N = b_0 + \frac{1}{2}\mathbb{E}_\mu\left[\sum_{n=1}^{N}(x_n - \mu)^2 + \lambda_0(\mu - \mu_0)^2\right].$$

GMM

$$N_k = \sum_{n=1}^{N} r_{nk} \quad (10.51)$$

$$\overline{\mathbf{x}}_k = \frac{1}{N_k}\sum_{n=1}^{N} r_{nk}\mathbf{x}_n \quad (10.52)$$

$$\mathbf{S}_k = \frac{1}{N_k}\sum_{n=1}^{N} r_{nk}(\mathbf{x}_n - \overline{\mathbf{x}}_k)(\mathbf{x}_n - \overline{\mathbf{x}}_k)^{\mathrm{T}}. \quad (10.53)$$

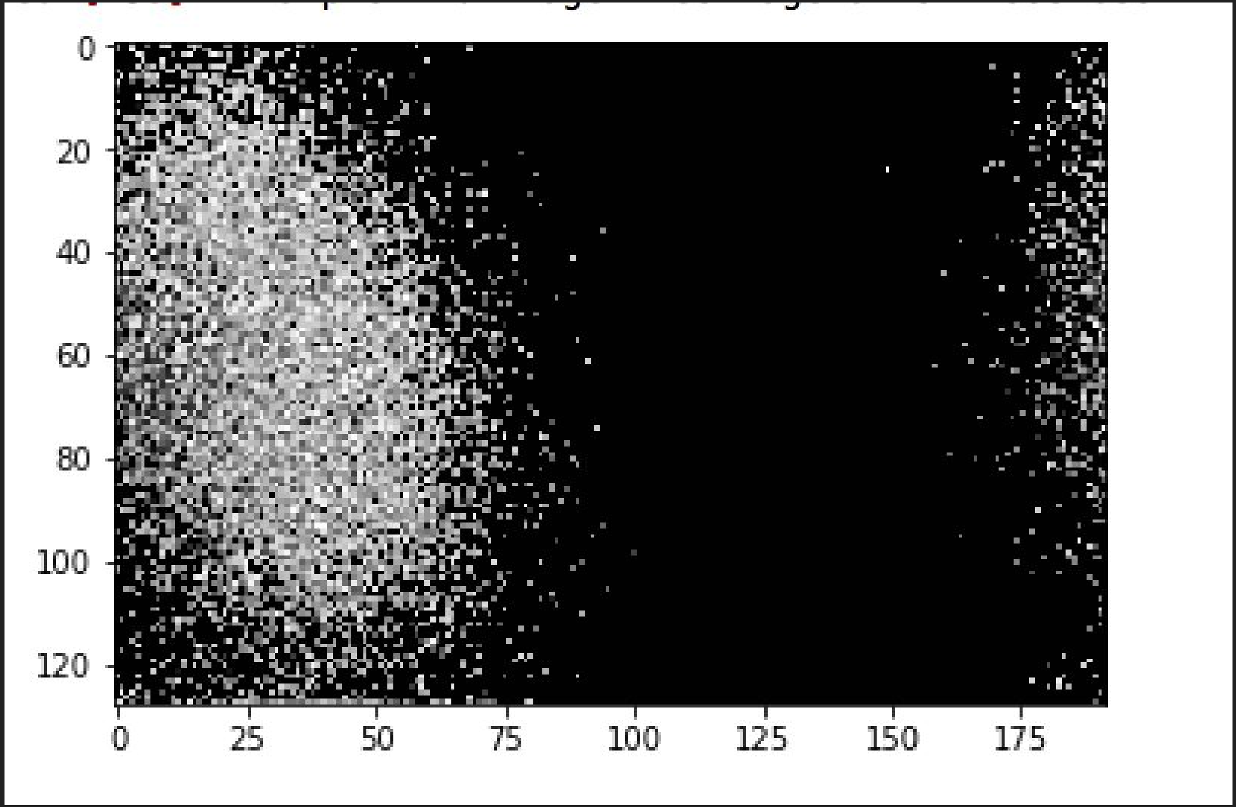
$$\beta_k = \beta_0 + N_k$$

$$\mathbf{m}_k = \frac{1}{\beta_k}(\beta_0\mathbf{m}_0 + N_k\overline{\mathbf{x}}_k)$$

$$\mathbf{W}_k^{-1} = \mathbf{W}_0^{-1} + N_k\mathbf{S}_k + \frac{\beta_0 N_k}{\beta_0 + N_k}(\overline{\mathbf{x}}_k - \mathbf{m}_0)(\overline{\mathbf{x}}_k - \mathbf{m}_0)^{\mathrm{T}}$$

$$\nu_k = \nu_0 + N_k.$$

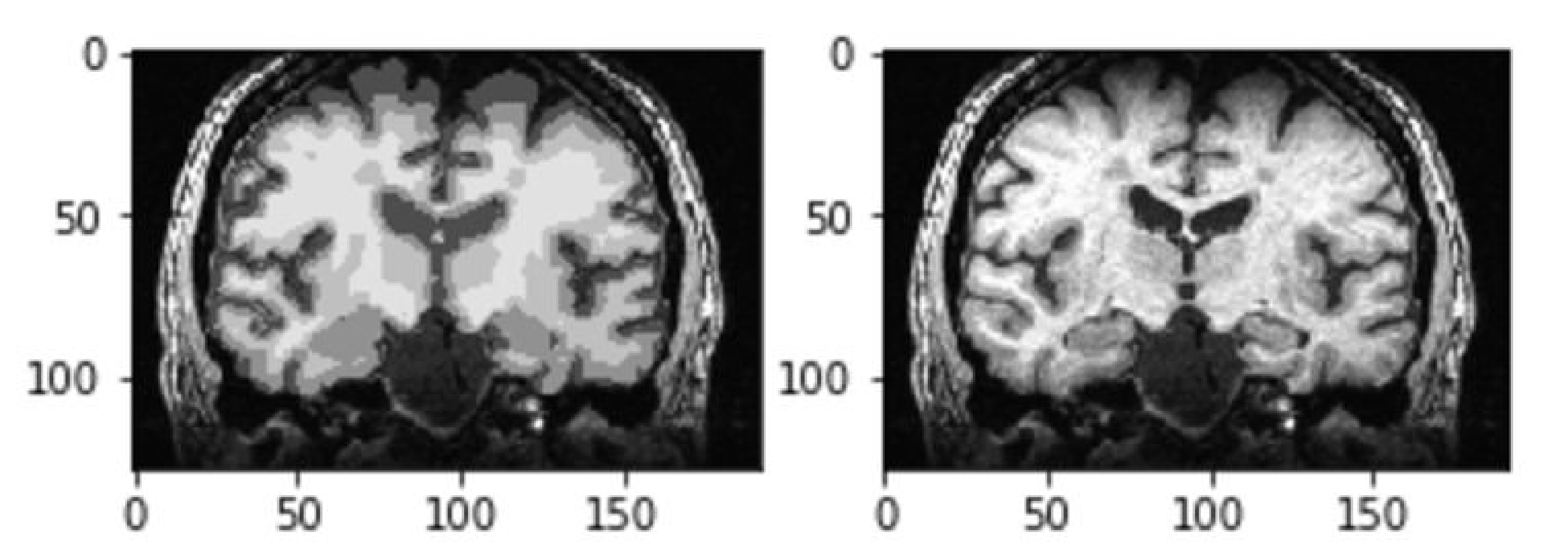
Simulated data for half of brain using model



Doesn’t quite get the spatial representation.

Standard Model

Used EM developed last term. Adapted K-means and used scatter matrix for covariance (See Jupyter Notebook)



Variational inference mixture of mixtures

- Did not have time to implement model (see model_test.py for progress)