VE 492 Homework6

Due: 23:59, July 1st

Question 1: Probability, Part I

Below is a table listing the probabilities of three binary random variables. Please write down the correct values for each marginal or conditional probability expression. And each value for one row. (3 significant figures)

$P(a|b) = \frac{P(a,b)}{P(b)}$

$P(X) = \sum_{y} P(X,y)$

Sample Answer:
0.160
0.200
0.333
…

X_0	X_1	X_2	$P(X_0,X_1,X_2)$
0	0	0	0.060
1	0	0	0.020
0	1	0	0.280
1	1	0	0.080
0	0	1	0.100
1	0	1	0.200
0	1	1	0.140
1	1	1	0.120

Expression:

$P(X_0 = 1, X_1 = 0, X_2 = 1)$ 0.200

$P(X_0 = 0, X_1 = 1)$ 0.28 + 0.14 = 0.420

$P(X_2 = 0)$ = 0.06 + 0.02 + 0.28 + 0.06 = 0.440

④ $P(X_1 = 0|X_0 = 1)$

⑤ $P(X_0 = 1, X_1 = 0\,|X_2 = 1)$

⑥ $P(X_0 = 1\,|X_1 = 0, X_2 = 1)$

④ $\frac{P(X_1=0,\,X_0=1)}{P(X_0=1)} = \frac{0.02+0.2}{0.02+0.2+0.28+0.12} = \frac{0.22}{0.42} = 0.524$

⑤ $\frac{P(X_0=1,X_1=0,X_2=1)}{P(X_2=1)} \quad \frac{0.2}{0.1+0.2+0.14+0.12} = \frac{0.2}{0.56} = 0.357$

⑥ $\frac{P(X_0=1,X_1=0,X_2=1)}{P(X_1=0,X_2=1)} = \frac{0.2}{0.1+0.2} = \frac{0.2}{0.3} = 0.667$

Question 2: Probability, Part II

You are given the prior distribution P(X), and two conditional distributions P(Y|X) and P(Z|Y) as below (you are also given the fact that Z is independent from X given Y). All variables are binary variables. Compute the table of their joint distribution based on the chain rule. And **write your answers for each blank in one row with the same format in Question 1.**

$Z \perp X \mid Y$

$P(y)\,P(x|y) = P(x,y)$

$P(x_1,x_2,x_3) = P(x_1)\,P(x_2|x_1)\,P(x_3|x_1,x_2)$

X	$P(X)$
0	0.300
1	0.700

Y	X	$P(Y\|X)$
0	0	0.500
1	0	0.500
0	1	0.900
1	1	0.100

Z	Y	$P(Z\|Y)$
0	0	0.200
1	0	0.800
0	1	0.400
1	1	0.600

$P(X,Y) = P(y)\,P(X|Y)$

$= P(x)\,P(Y|X)$

Table 1

X	Y	$P(X,Y)$	
0	0	0.150	0.3 · 0.5
1	0	0.630	0.7 · 0.9
0	1	0.150	0.3 · 0.5
1	1	0.070	0.7 · 0.1

$P(X,Y,Z)$

$= P(X)\,P(Y|X)\,P(Z|X,Y)$

$= P(x)\,P(Y|X)\,P(Z|Y)$

Table 2

	X	Y	Z	$P(X,Y,Z)$
④	0	0	0	0.030
	1	0	0	
	0	1	0	0.060
	1	1	0	0.028
	0	0	1	0.120
②	1	0	1	
③	0	1	1	
①	1	1	1	

① $0.7 \cdot 0.9 \cdot 0.2 = 0.126$

② $0.7 \cdot 0.9 \cdot 0.8 = 0.504$

③ $0.3 \cdot 0.5 \cdot 0.6 = 0.090$

④ $0.7 \cdot 0.1 \cdot 0.6 = 0.042$

Question 3: Probability, Part III

For the following four subparts, you are given three joint probability distribution tables. For each distribution, please identify if the given independence/conditional independence assumption is true or false (**write 'true' or 'false'**). For your convenience, we have also provided some marginal and conditional probability distribution tables that could assist you in solving this problem.

Please write your answer for each subpart in one row.

1) Assumption: X is independent from Y.

X	Y	$P(X,Y)$
0	0	0.240
1	0	0.120
0	1	0.460
1	1	0.180

X	$P(X)$
0	0.700
1	0.300

Y	$P(Y)$
0	0.360
1	0.640

$P(X,Y) = P(X)P(Y)$

$0.7 \cdot 0.36 = 0.252$

$P(X=0) \cdot P(Y=0) \neq P(X=0, Y=0) = 0.240$

false

2) Assumption: X is independent from Y.

X	Y	$P(X,Y)$
0	0	0.090
1	0	0.210
0	1	0.210
1	1	0.490

X	$P(X)$
0	0.300
1	0.700

X	Y	$P(X\|Y)$
0	0	0.300
1	0	0.700
0	1	0.300
1	1	0.700

$P(X|Y) = P(X)$

true

3) Assumption: X is independent from Y given Z.

$X \perp Y \mid Z$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.020
1	0	0	0.050
0	1	0	0.020
1	1	0	0.010
0	0	1	0.180
1	0	1	0.450
0	1	1	0.180
1	1	1	0.090

X	Y	Z	$P(X,Y\|Z)$
0	0	0	0.200
1	0	0	0.500
0	1	0	0.200
1	1	0	0.100
0	0	1	0.200
1	0	1	0.500
0	1	1	0.200
1	1	1	0.100

X	Z	$P(X\|Z)$
0	0	0.400
1	0	0.600
0	1	0.400
1	1	0.600

Y	Z	$P(Y\|Z)$
0	0	0.700
1	0	0.300
0	1	0.700
1	1	0.300

$P(X,Y|Z) = P(X|Z) \cdot P(Y|Z)$

false

4) Assumption: X is independent from Y given Z.

$X \perp Y \mid Z$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.060
1	0	0	0.010
0	1	0	0.020
1	1	0	0.010
0	0	1	0.540
1	0	1	0.180
0	1	1	0.090
1	1	1	0.090

X	Z	$P(X\|Z)$
0	0	0.800
1	0	0.200
0	1	0.700
1	1	0.300

X	Y	Z	$P(X\|Y,Z)$
0	0	0	0.857
1	0	0	0.143
0	1	0	0.667
1	1	0	0.333
0	0	1	0.750
1	0	1	0.250
0	1	1	0.500
1	1	1	0.500

$P(X \mid Z, Y) = P(X \mid Z)$

false