Homework 7 Electronic

For Q1, IA RC method used (different from lecture slide)

logic

- Keep only ancestral graph
- Connect nodes with common child
- Make all edges undirect
- Read off properties

July 8th, 2020 at 11:59pm

1 Question 1: D-Separation

- marginal independent if there is no path between nodes
- conditional independent if all paths between nodes get across given nodes

You are given several graphical models below, and each graphical model is associated with an independence (or conditional independence) assertion. Please specify if the assertion is true or false.

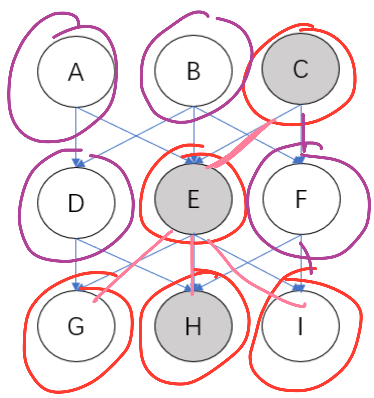
Please write your answer for each subpart in one row.

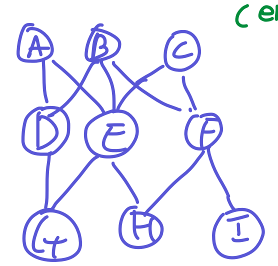
1. correspond to keep only ancestral graph

1) Assertion: It is guaranteed that **I** is independent of **G** given **H**, **E**, **C**

add only parents of given nodes (I, G, H, E, C here)

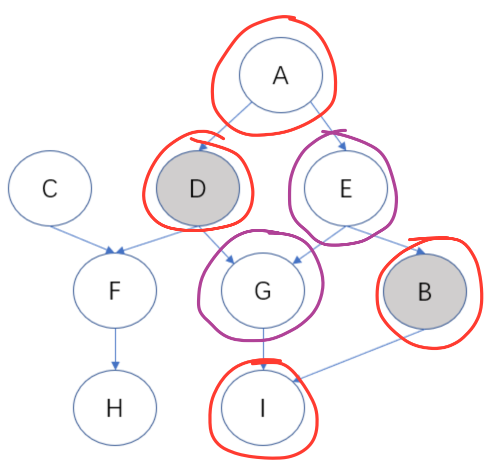
connect nodes with common child stripped (enough info.)

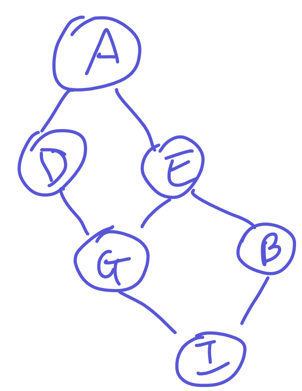




I → G has path through F, which is not given ⇒ $\boxed{\text{False}}$

2) Assertion: It is guaranteed that **A** is independent of **I** given **B**, **D**



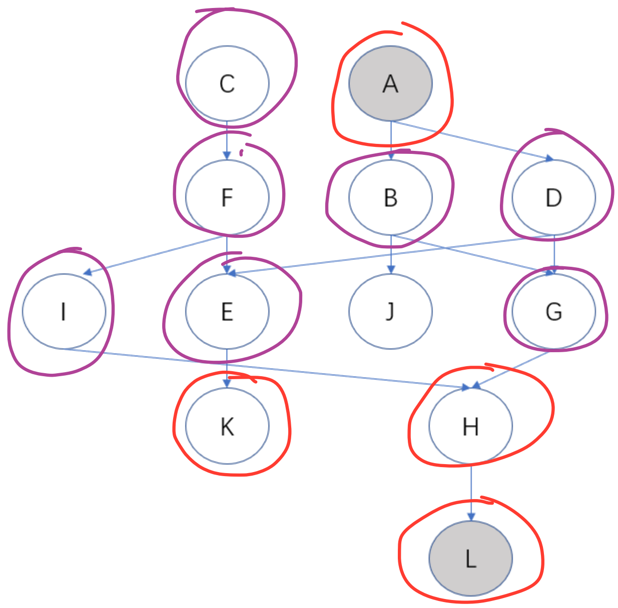


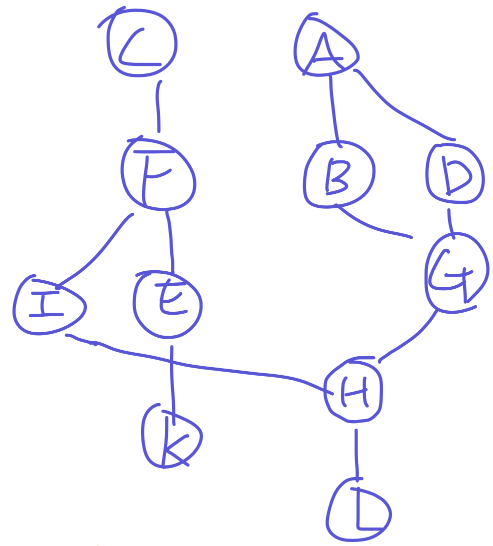
A → I

path through E, G (not given)

⇒ $\boxed{\text{False}}$

3) Assertion: It is guaranteed that **K** is independent of **H** given **A**, **L**

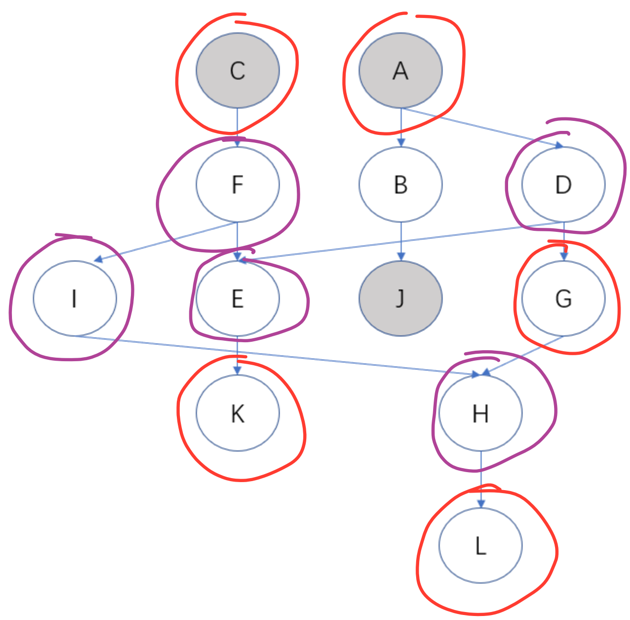


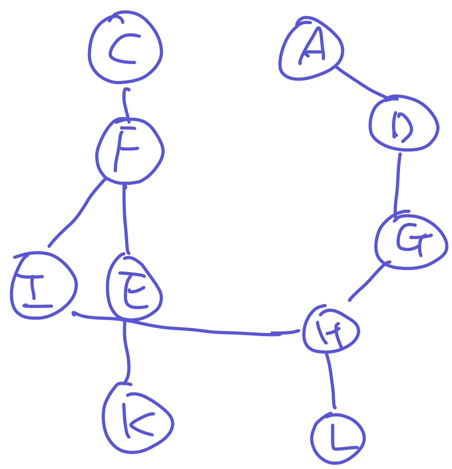


K→H path without A, L exist

⇒ $\boxed{\text{False}}$

4) Assertion: It is guaranteed that **L** is independent of **K** given **A**, **C**, **J**

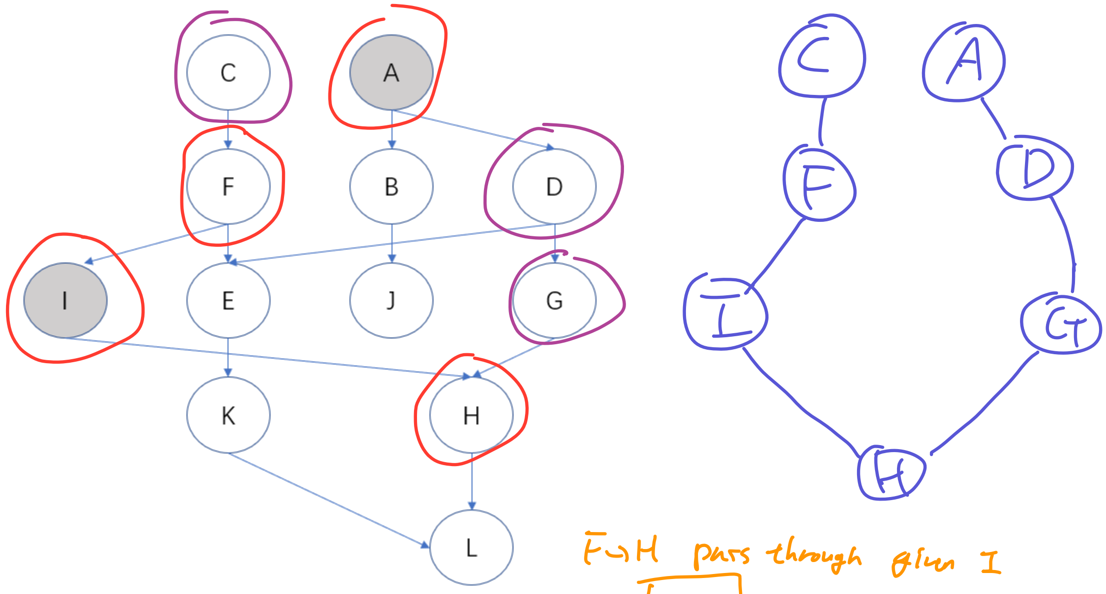




L→K other than A, C, J exists

⇒ $\boxed{\text{False}}$

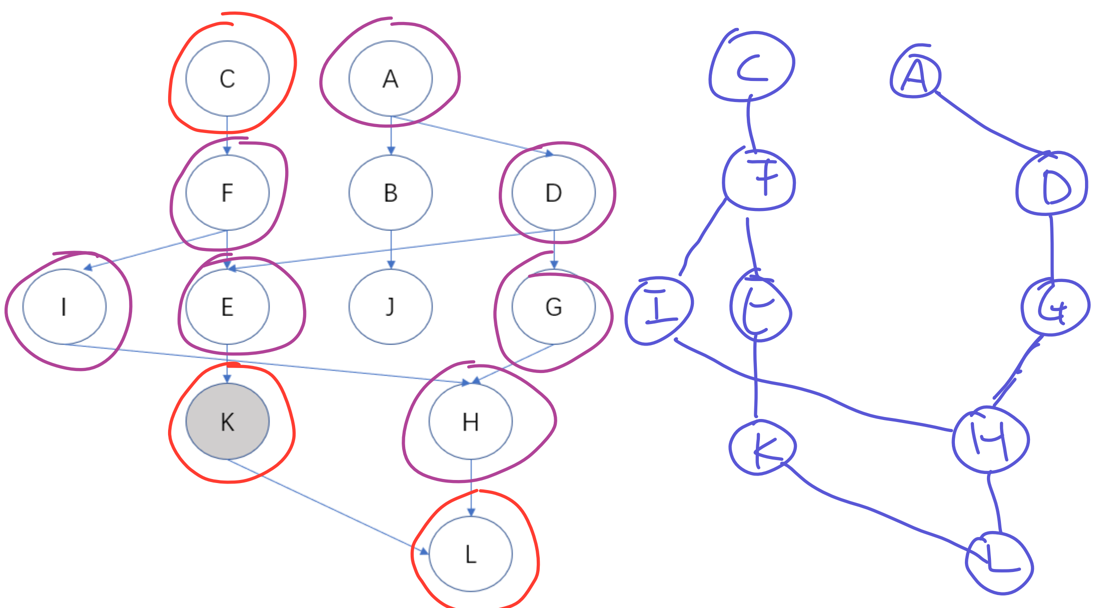
5) Assertion: It is guaranteed that **F** is independent of **H** given **A**, **I**



F→H pass through given I

→ true

6) Assertion: It is guaranteed that **C** is independent of **L** given **K**.

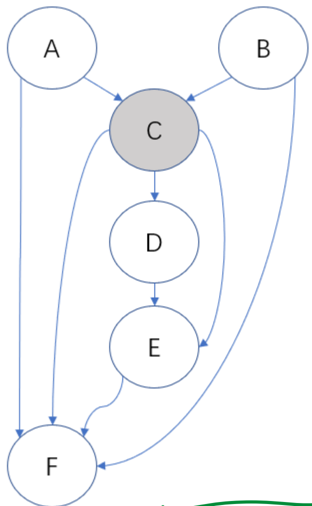


C→L without K exists

⇒ False

2 Question 2: Variable Elimination

Consider the graphical model shown below, where all variables have binary domains. We are given the query $P(F|+c)$. Assume that we run variable elimination with the following ordering: B, D, E, A.



$f_1(A, +c, E, F) = \sum_b P(b)\, P(+c|A,b)\, P(F|A,+c,b,E)$

After introducing evidence, we have the following factors:

$$P(A), \quad P(\cancel{B}), \quad P(+c \mid \cancel{A, B}), \quad P(D \mid +c), \quad P(E \mid +c, D), \quad P(F \mid A, \cancel{+c}, B, E)$$

Step 1: After joining on B and summing out over B, we have generated a new factor f_1 over the following variables and/or evidence (note B is not included after summing out over B):

A	B	+c	D	E	F
x		x		x	x

$f_2(+c, E) = \sum_d P(d|+c)\, P(E|+c,d)$

After this step, the remaining factors are:

P(A)	P(B)	P(+c|A,B)	P(D|+c)	P(E|+c,D)	P(F|A,+c,B,E)	f_1
x			x	x		x

Step 2: After joining on D and summing out over D, we have generated a new factor f_2 over the following variables and/or evidence (note D is not included after summing out over D):

7. Sample answer:
A~~B~~ + c~~D~~EF +c E $f_2(+c, E)$

After this step, the remaining factors are:
8 Sample answer:
P(A)~~P(B)P(+c | A, B)P(D | +c)P(E | +c, D)P(F | A, +c, B, E)~~f1f2

Step 3: After joining on E and summing out over E, we have generated a new factor f_3 over the following variables and/or evidence (note E is not included after summing out over E):
Sample answer:

9. ~~**AB**~~ **+ c**~~**DE**~~**F** A+cF

$$\boxed{f_3(A,+c,F) = \sum_e f_1(A,+c,e,E)\, f_2(+c,e)}$$

After this step, the remaining factors are:
Sample answer:

10. **P(A)P**~~**(B)P(+c | A, B)P(D | +c)P(E | +c, D)P(F | A, +c, B, E)f1f2**~~**f3**

Step 4: After joining on A and summing out over A, we have generated a new factor f_4 over the following variables and/or evidence (note A is not included after summing out over A):
Sample answer:

11. **AB + cDEF** +cF

$$\boxed{f_4(+c,F) = \sum_a P(a)\, f_3(a,+c,F)}$$

After this step, the remaining factors are:
Sample answer:

12. ~~**P(A)P(B)P(+c | A, B)P(D | +c)P(E | +c, D)P(F | A, +c, B, E)f1f2**~~**f3f4**

Reminder: The final factor (or the product of final factors if there are more than one) is guaranteed to be equal to selected joint $P(F, +c)$. To answer the original query, we need to renormalize to obtain $P(F|+c)$.