

Функции. геометрия в пространстве

09.02.26

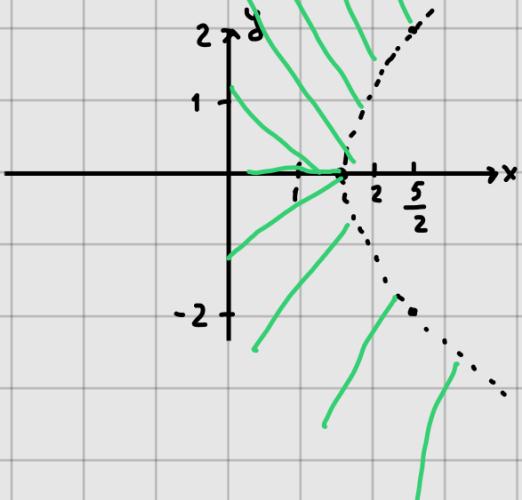
1. Построим область определения функции.

1.1. $Z = \ln(y^2 - 4x + 6)$

$$D(Z): y^2 - 4x + 6 > 0$$

$$y^2 > 4x - 6$$

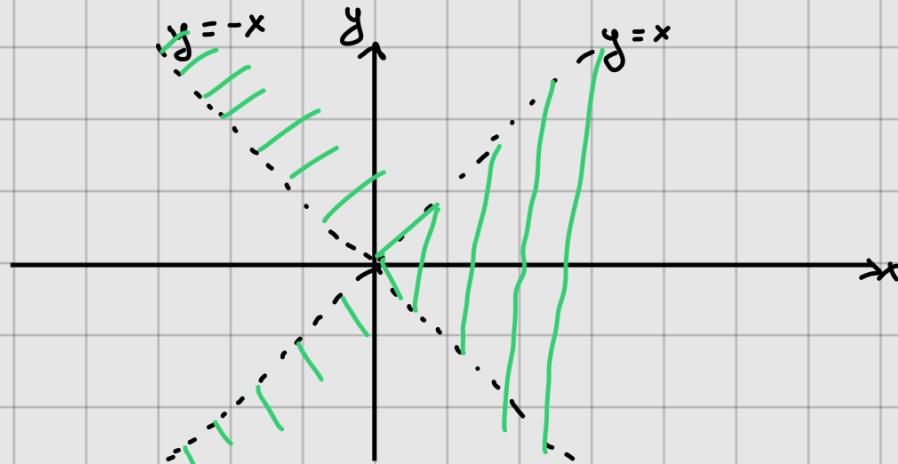
$$y^2 > 4\left(x - \frac{3}{2}\right) - \text{графика}$$



1.2.

$$Z = \frac{1}{\sqrt{x+y}} + \frac{1}{\sqrt{x-y}}$$

$$D(Z): \begin{cases} x+y > 0 \\ x-y > 0 \end{cases} \Leftrightarrow \begin{cases} y > -x \\ y < x \end{cases}$$



1.3.

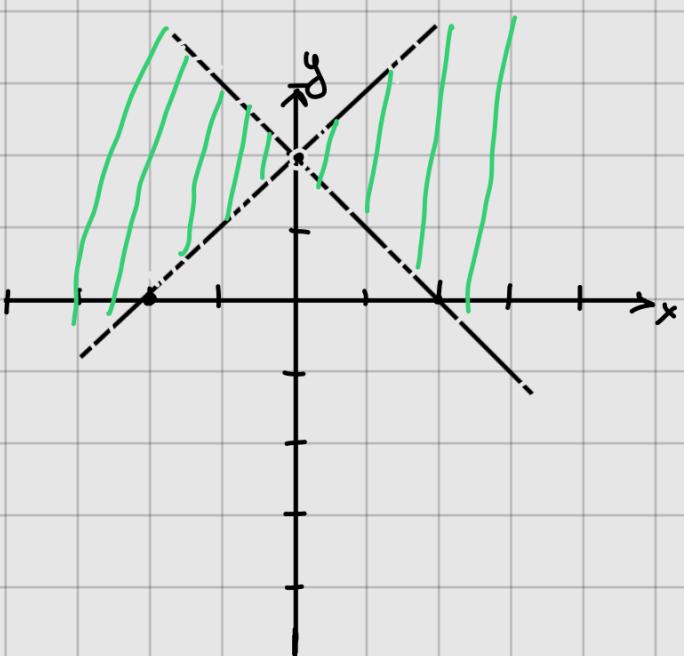
$$z = \arccos \frac{y-2}{x}$$

если $x < 0$, то знаки изменяются

$$\begin{cases} y \geq x+2 \\ y \leq 2-x \end{cases} \quad \begin{cases} y \geq 2-x \\ y \leq x+2 \end{cases}$$

$$-1 \leq \frac{y-2}{x} \leq 1$$

$$\begin{cases} \frac{y-2}{x} \geq -1 \\ \frac{y-2}{x} \leq 1 \end{cases} \quad \begin{array}{l} ① \frac{y-2}{x} = -1 \\ \frac{y-2+x}{x} = 0 \end{array}$$



$$x \neq 0$$

$$y-2+x=0$$

$$y = 2-x$$

$$② \frac{y-2}{x} = 1$$

$$\frac{y-2-x}{x} = 0$$

$$x \neq 0$$

$$y = 2+x$$

1.4.

$$z = \sqrt{x - \sqrt{y}}$$

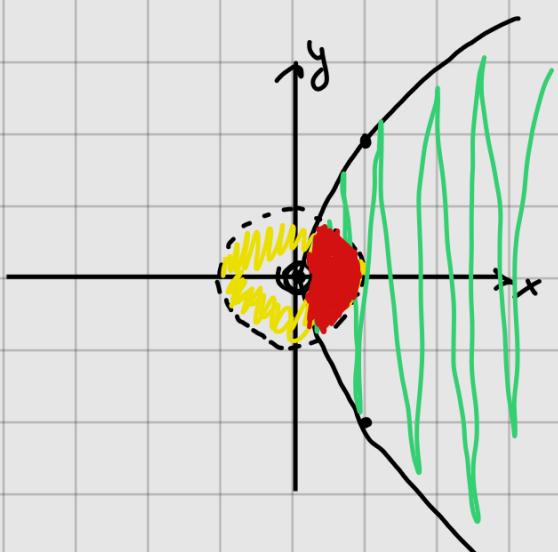
$$\mathbb{D}(z) : \begin{cases} x - \sqrt{y} \geq 0 \\ y \geq 0 \end{cases} \Leftrightarrow \begin{cases} x \geq \sqrt{y} \\ y \geq 0 \end{cases}$$



1.5.

$$z = \frac{\sqrt{4x-y^2}}{\ln(1-x^2-y^2)}$$

$$\mathcal{D}(z) : \begin{cases} 4x - y^2 \geq 0 \\ 1 - x^2 - y^2 > 0 \\ 1 - x^2 - y^2 \neq 1 \end{cases}$$



- ① $y^2 \leq 4x$
- ② $x^2 + y^2 < 1$
- ③ $x^2 + y^2 \neq 0 \Rightarrow x \neq 0; y \neq 0$

2. Определим фигуру поверхности 2-го порядка

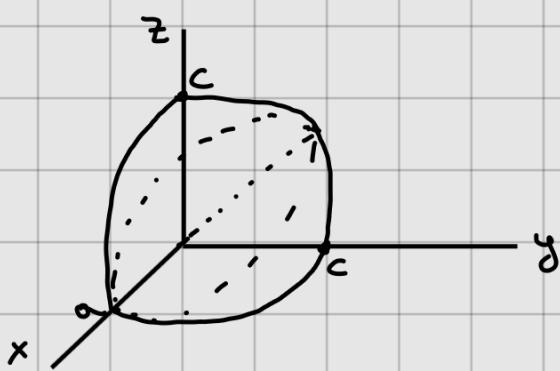
$$2.1. x^2 + 3y^2 + 4z^2 - 5 = 0$$

$$x^2 + 3y^2 + 4z^2 = 5 \mid :5$$

$$\frac{x^2}{5} + \frac{3}{5}y^2 + \frac{4}{5}z^2 = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad - \text{Эллипсоид}$$

$$a = \sqrt{5} \quad b = \sqrt{\frac{5}{3}} \quad c = \sqrt{\frac{5}{4}}$$

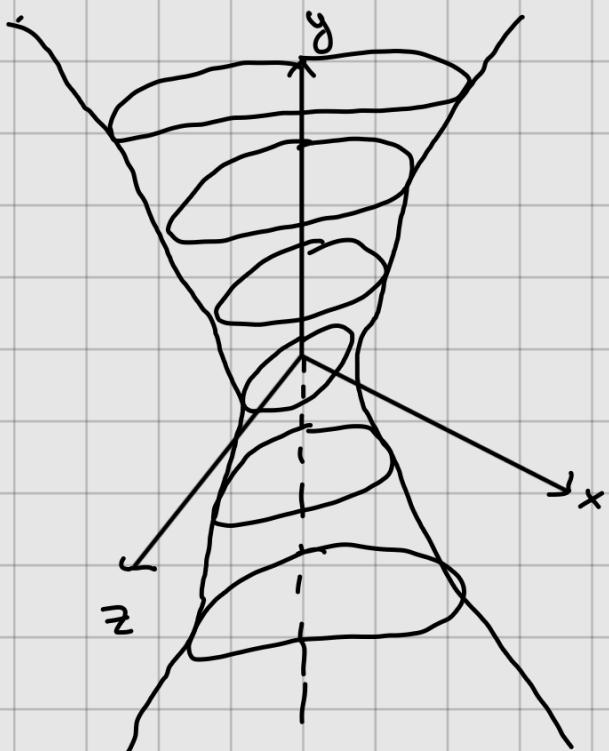


2.2.

Будови поверхні (y)

$$3x^2 - 4y^2 + 6z^2 - 6 = 0$$

$$3x^2 - 4y^2 + 6z^2 = 6 \quad | :6 \quad \frac{1}{2}x^2 - \frac{2}{3}y^2 + z^2 = 1$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$a = \sqrt{2}, \quad b = \sqrt{\frac{3}{2}}, \quad c = 1$$

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1 + \frac{y^2}{b^2}$$

Однополостний
циліндричний
будови осі Oy

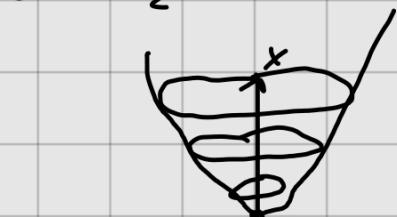
2.3.

$$4y^2 + 3z^2 = 6x^2 - 2 = 0$$

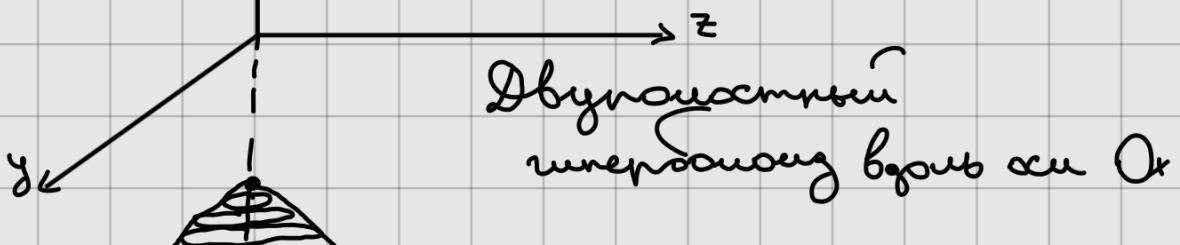
Будови поверхні (x)

$$4y^2 + 3z^2 - 6x^2 = -2 \quad | :(-2)$$

$$3x^2 - 2y^2 - \frac{3}{2}z^2 = 1$$

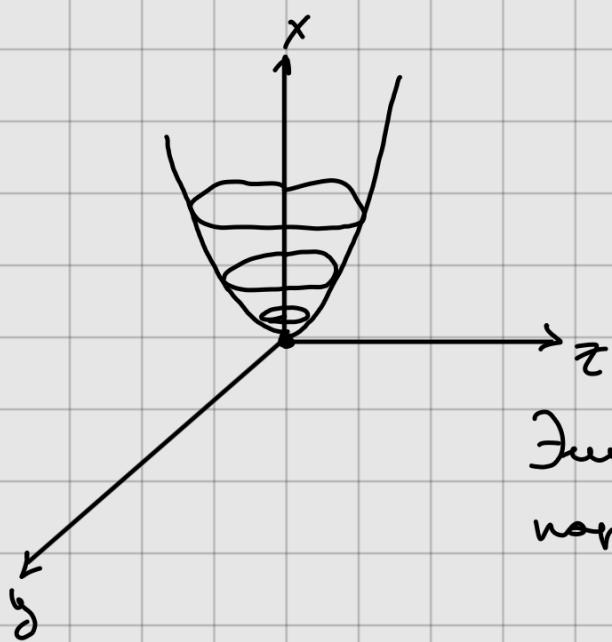


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



Двуполостний
циліндричний будови осі Ox

2.4. $3z^2 + y^2 - 4x = 0$



$$3z^2 + y^2 = 4x$$

$$\frac{3}{4}z^2 + \frac{y^2}{4} = x$$

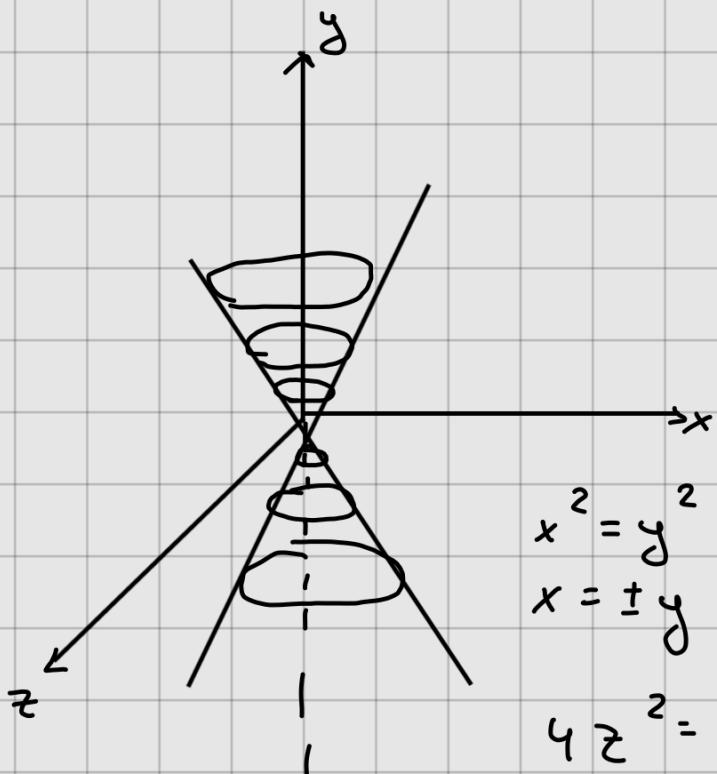
$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = x$$

$$b=2, c=\sqrt{\frac{4}{3}}$$

Эллиптический
парaboloid

2.5.

$$x^2 + 4z^2 - y^2 = 0$$



$$x^2 + 4z^2 = y^2$$

$$x^2 = y^2$$

$$x = \pm y$$

$$4z^2 = y^2$$

$$2z = \pm y$$

корень

3. Численные поверхности в трёхмерном пространстве.

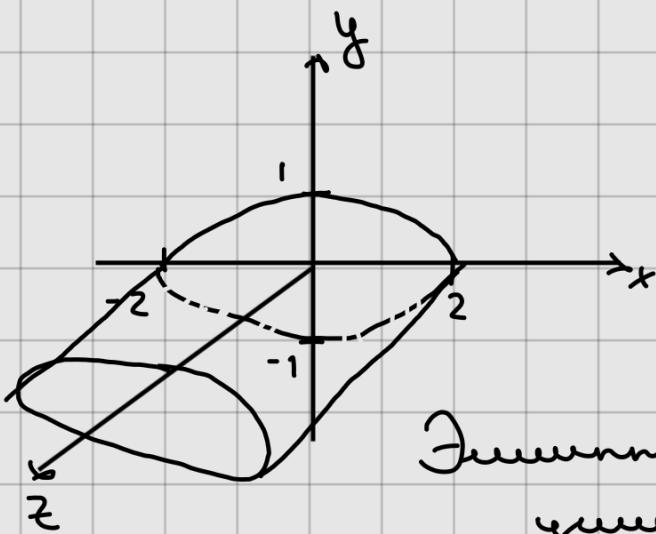
3.1. $x^2 + 4y^2 - 4 = 0$

$$x^2 + 4y^2 = 4$$

$$\frac{x^2}{4} + y^2 = 1$$

$$a=2, b=1$$

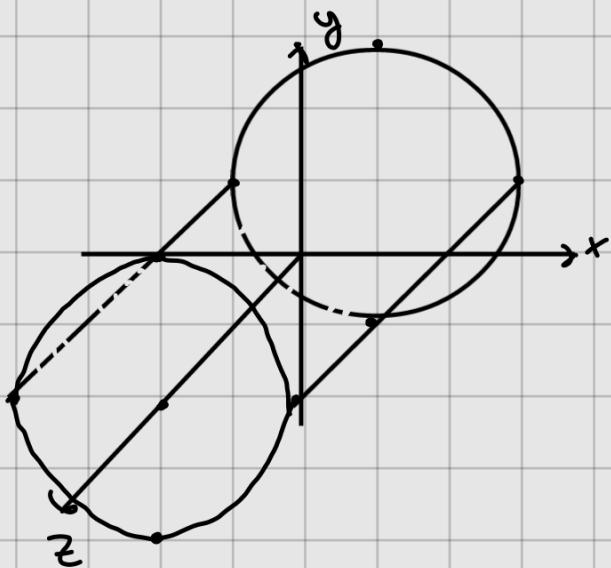
Эллипс в Oxy



Эллиптический
цилиндр.

3.2. $x^2 - 2x + y^2 - 2y - 2 = 0$

$$(x-1)^2 + (y-1)^2 = 4$$



3.3. $z^2 - y^2 - 2y - 2 = 0$

$$z^2 - y^2 - 2y - 1 - 1 = 0$$

$$z^2 - (y+1)^2 = 1$$

$$\frac{z^2}{a^2} - \frac{(y+1)^2}{b^2} = 1$$

