Optimizing Greed

The probability of the D-Grind

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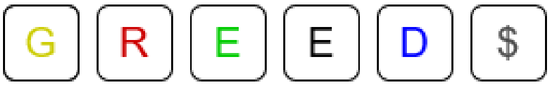

1 The Game

Greed is a dice game where you accumulate points by rolling specific combinations of dice. There are six dice, with their faces labelled the letters of the game: \$, $\mathbb{G}$, $\mathbb{R}$, $\mathbb{E}$, **E**, $\mathbb{D}$ (one E is green, one is black, but here they'll be distinguished by bold text). The scoring chart is shown below:

1 x $\mathbb{G}$	= 50
1 x $\mathbb{D}$	= 100
3 x $\mathbb{E}$	= 300
3 x **E**	= 300
3 x $\mathbb{R}$	= 400
3 x $\mathbb{G}$	= 500
3 x \$	= 600
4 x $\mathbb{D}$	= 1000
\$, $\mathbb{G}$, $\mathbb{R}$, $\mathbb{E}$, **E**, $\mathbb{D}$	= 1000
6 of a kind	= 5000

As you can see, only a few letters can score with only one occurrence; for many, you need to have 3 or more showing to be worth any points. You start your turn by rolling all 6 die, and you can re-roll dice and as much as you want, but after each roll you must set aside some amount of dice that increase your point total (and you cannot keep any that don't). If your roll has no scoring dice, you go bust and you lose all points you accumulated on the current turn. If you don't go bust you may choose to 'sit', ending your turn and adding the points you've set aside to your total score. Finally, if all 6 dice are scoring, you can re-roll all of them and continue adding points to those you already had set aside. We take this one step further and say that, if you would end your turn with 6 scoring dice, you MUST re-roll, and you cannot end your turn with 6 scoring dice showing but say you're only using 5 of them to get out of it.

There are two parts that make this tricky to optimize. The first is that players don't have to keep all scoring dice each roll, and can selectively keep dice to aim for a possible future state. The second is that the score of the dice you have set aside may be enhanced later in the turn. For instance, if I roll 1 $\mathbb{G}$ and 3 $\mathbb{E}$ I could have 350 points, or I could keep just the $\mathbb{G}$ for 50 and try and get another 2 to make it 500. The third and most challenging element is the ability to re-roll with 6 scoring dice. This essentially creates an infinite possible depth to each turn, and an infinite depth to any decision calculator.

2 The Theory

The decision to re-roll is relatively simply. Are the additional points I could get worth the risk of losing all the points I currently have. That is to say, is the sum of the probability of all future states multiplied by their point values greater than the points I have now.

Every combination of the 6 dice, each state, has a weighted score associated. This score is how many points you are likely to expect from being in this state. Unfortunately, the weighted score of a state is affected by the weighted score of future states