Aalto University
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Zero-shot classifier based on robust ellipsoid optimization

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0 What is zero-shot learning?

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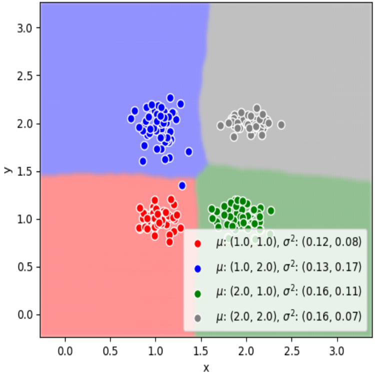
- Zero-shot classifier = Classifier (machine learning model) capable of predicting labels it hasn't seen in training
- Potentially useful when number of labels is very large or acquiring training data is very expensive
- Used e.g. in neuroimaging studies of language, where the datapoints (grand averaged evoked responses to stimuli) are expensive to gather in large quantities and the number of labels (words or concepts) is vast

0 Why aren't most classifiers capable of zero-shot?

- Classifiers (generally) find a mapping:

$$\mathcal{F} : F^d \to L \qquad (f : X^d \to Y)$$



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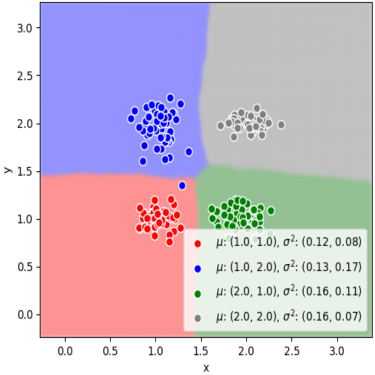

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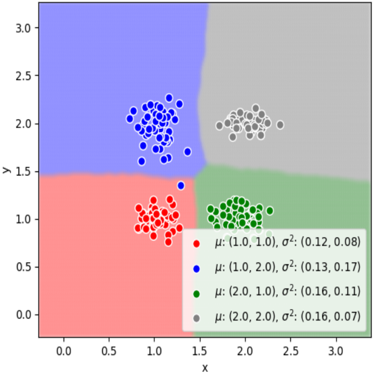
- i.e. divide the feature space into regions for each label with *decision boundaries*
- Not well suited for classifying labels not seen in training



0 Approach for zero-shot learning

- (Palatucci et al. 2009)[1] propose a *semantic output code classifier* as a composition of two different mapping functions:

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\mathcal{H} &= (\mathcal{S} \circ \mathcal{L})(\cdot) \\
\mathcal{S} &: F^d \rightarrow S^p \\
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- One-hot encodings make the mapping $\mathcal{L}$ trivial
- Real effort is in finding the mapping $\mathcal{S}$

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0 Outlined approach for finding mapping $\mathcal{S}$

- The mapping $\mathcal{S}$ can be considered as a conditional expectation:

$$\mathbb{E}[\mathbf{s}|\mathbf{f}] = \mathcal{S}(\mathbf{f}), \quad \text{where } \mathbf{s} \in S^p \text{ and } \mathbf{f} \in F^d$$

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- Function K is the kernel function
- Function d_i gives the distance from label i to point $\mathbf{f}$

0 Properties of the semantic space

- Good choice of semantic vectors maintain the semantic and syntactic relationships between the underlying concepts

[2] Mikolov, T., Sutskever, I., Chen, K., Corrado, G., Dean, J., (2013). *Distributed representations of words and phrases and their compositionality*. Advances in Neural Information Processing Systems

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- Works very intuitively: ”brother” $-$ ”man” $+$ ”woman” $=$ ”sister”

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- Assume that $\mathbf{X}$ follows Gaussian distribution or something very close to it
- Then $\mathbf{X}$ is defined by mean $\mathbf{m}$ and covariance matrix $K_{\mathbf{XX}}$

0 Mahalanobis distance

- Mean and covariance matrix define an ellipsoid

$$\mathcal{E}_A(\mathbf{m}) = \{\mathbf{x} \in \mathbb{R}^d : (\mathbf{x} - \mathbf{m})^T A(\mathbf{x} - \mathbf{m}) \leq 1\}, \quad \text{where } A = K_{\mathbf{XX}}^{-1}$$

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- Main goal finding good *robust* approximations of the covariance matrix

0 Visualizations