EE5609 Matrix Theory

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Download the python code from

https://github.com/kranthiakssy/AI20RESCH14002_PhD_IITH/tree/master/EE5609_Matrix_Theory/Assignment−2

and latex-file codes from

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Assignment-2

Problem

Lines and Planes (81) : In each of the following cases, determine the normal to the plane and the distance from the origin.

(a) $\begin{pmatrix}0 & 0 & 1\end{pmatrix}\mathbf{x} = 2$
(b) $\begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} = 1$
(c) $\begin{pmatrix}0 & 5 & 0\end{pmatrix}\mathbf{x} = -8$
(d) $\begin{pmatrix}2 & 3 & -1\end{pmatrix}\mathbf{x} = 5$

Solution

If $ax+by+cz = k$ is a linear equation representing a plane, then the normal $\mathbf{n}$ to that plane is the coefficients of the linear equation

$$\mathbf{n} = \begin{pmatrix}a\\b\\c\end{pmatrix} \tag{0.0.1}$$

The equation of plane can be written as

$$\mathbf{n}^T\mathbf{x} = c \tag{0.0.2}$$

The shortest distance between the plane (0.0.2) and origin is

$$\frac{|c|}{||\mathbf{n}||} \tag{0.0.3}$$

$a)\begin{pmatrix}0 & 0 & 1\end{pmatrix}\mathbf{x} = 2$

$$\text{normal vector, } \mathbf{n} = \begin{pmatrix}0\\0\\1\end{pmatrix} \tag{0.0.4}$$

$$c = 2 \tag{0.0.5}$$

As per (0.0.3), shortest distance from origin =

$$\frac{|2|}{\sqrt{0^2+0^2+1^2}} = 2 \tag{0.0.6}$$

$b)\begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} = 1$

$$\text{normal vector, } \mathbf{n} = \begin{pmatrix}1\\1\\1\end{pmatrix} \tag{0.0.7}$$

$$c = 1 \tag{0.0.8}$$

As per (0.0.3), shortest distance from origin =

$$\frac{|1|}{\sqrt{1^2+1^2+1^1}} = \frac{1}{\sqrt{3}} \tag{0.0.9}$$

$c)\begin{pmatrix}0 & 5 & 0\end{pmatrix}\mathbf{x} = -8$

$$\text{normal vector, } \mathbf{n} = \begin{pmatrix}0\\5\\0\end{pmatrix} \tag{0.0.10}$$

$$c = -8 \tag{0.0.11}$$

As per (0.0.3), shortest distance from origin =

$$\frac{|-8|}{\sqrt{0^2+5^2+0^2}} = \frac{8}{5} \tag{0.0.12}$$

$d)\begin{pmatrix}2 & 3 & -1\end{pmatrix}\mathbf{x} = 5$

$$\text{normal vector, } \mathbf{n} = \begin{pmatrix}2\\3\\-1\end{pmatrix} \tag{0.0.13}$$

$$c = 5 \tag{0.0.14}$$

As per (0.0.3), shortest distance from origin =

$$\frac{|5|}{\sqrt{2^2+3^2+(-1)^2}} = \frac{5}{\sqrt{14}} \tag{0.0.15}$$