EE5609 Matrix Theory

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1 Assignment-2

1.1 Problem

Lines and Planes (81) : In each of the following cases, determine the normal to the plane and the distance from the origin.

$a)$ $(0\ 0\ 1)\mathbf{x} = 2$

$b)$ $(1\ 1\ 1)\mathbf{x} = 1$

$c)$ $(0\ 5\ 0)\mathbf{x} = -8$

$d)$ $(2\ 3\ -1)\mathbf{x} = 5$

1.2 Solution

If $ax + by + cz = k$ is a linear equation representing a plane, then the normal to that plane $\mathbf{n}$ is the coefficients of the linear equation.

$$\mathbf{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

The equation of plane can be written as

$$\mathbf{n}^T\mathbf{x} = c$$

The shortest distance between the plane given by $\mathbf{n}^T\mathbf{x} = c$ and origin is

$$\left| \frac{c}{\|\mathbf{n}\|} \right|$$

a)

normal vector $\quad \mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

c = 2

shortest distance from origin =

$$\left| \frac{2}{\sqrt{0^2 + 0^2 + 1^2}} \right| = 2$$

b)

normal vector $\quad \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

c = 1

shortest distance from origin =

$$\left| \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} \right| = \frac{1}{\sqrt{3}}$$

c)

normal vector $\quad \mathbf{n} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}$

c = -8

shortest distance from origin =

$$\left| \frac{-8}{\sqrt{0^2 + 5^2 + 0^2}} \right| = \frac{8}{5}$$

d)

normal vector $\quad \mathbf{n} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

c = 5
shortest distance from origin =

$$\left|\frac{5}{\sqrt{2^2+3^2+(-1)^2}}\right| = \frac{5}{\sqrt{14}}$$