

EE5609 Matrix Theory

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1 Assignment-2

1.1 Problem

Lines and Planes (81) : In each of the following cases, determine the normal to the plane and the distance from the origin.

- $a) (0 \ 0 \ 1)\mathbf{x} = 2$ $c) (0 \ 5 \ 0)\mathbf{x} = -8$
 $b) (1 \ 1 \ 1)\mathbf{x} = 1$ $d) (2 \ 3 \ -1)\mathbf{x} = 5$

1.2 Solution

If $ax + by + cz = d$ is a linear equation representing a plane, then the normal to that plane $\vec{\mathbf{n}}$ is the coefficients of the linear equation.

$$\vec{\mathbf{n}} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

The equation of plane can be written as

$$\mathbf{n}^T \mathbf{x} = d$$

The shortest distance between the plane given by $\mathbf{n}^T \mathbf{x} = d$ and origin is

$$\left| \frac{d}{\|\vec{\mathbf{n}}\|} \right|$$

a)

normal vector $\vec{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$d = 2$

shortest distance from origin =

$$\left| \frac{2}{\sqrt{0^2 + 0^2 + 1^2}} \right| = 2$$

b)

normal vector $\vec{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$d = 1$

shortest distance from origin =

$$\left| \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} \right| = \frac{1}{\sqrt{3}}$$

c)

normal vector $\vec{n} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}$

$d = -8$

shortest distance from origin =

$$\left| \frac{-8}{\sqrt{0^2 + 5^2 + 0^2}} \right| = \frac{8}{5}$$

d)

normal vector $\vec{n} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

$$d = 5$$

shortest distance from origin =

$$\left| \frac{5}{\sqrt{2^2 + 3^2 + (-1)^2}} \right| = \frac{5}{\sqrt{14}}$$