EE5609 Matrix Theory

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Download the python code from

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https://github.com/kranthiakssy/AI20RESCH14002_PhD_IITH/tree/master/EE5609_Matrix_Theory/Assignment−4
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and latex-file codes from

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Assignment-4

Problem:

Determinants (79):
Using properties of determinants, prove that:

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

Solution:

$$LHS = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} \tag{0.0.1}$$

By expanding using sum property

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix} \tag{0.0.2}$$

By using switching of rows(or columns) property

$$= (-1)\begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix} \tag{0.0.3}$$

$$= (-1)^2\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix} \tag{0.0.4}$$

By using scalar multiplication property

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + (pxyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \tag{0.0.5}$$

$$= (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \tag{0.0.6}$$

By applying row reduction

$$= (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \tag{0.0.7}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_3} (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} \tag{0.0.8}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_1} (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 0 & y-z & y^2-z^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} \tag{0.0.9}$$

By using scalar multiplication property

$$= (1+pxyz)(y-z)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+z \\ 0 & 1 & z+x \end{vmatrix} \tag{0.0.10}$$

By applying the determinant formula

$$= (1+pxyz)(y-z)(z-x)(z+x-y-z) \tag{0.0.11}$$

$$= (1+pxyz)(x-y)(y-z)(z-x) \tag{0.0.12}$$

$$= RHS \tag{0.0.13}$$

Hence Proved.