EE5609 Matrix Theory

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Download the latex-file codes from

https://github.com/kranthiakssy/AI20RESCH14002_PhD_IITH/tree/master/EE5609_Matrix_Theory/Assignment−10

Assignment-10

Problem:

Let $\mathbf{B} = \begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\end{pmatrix}$ be the ordered basis for R^3 consisting of

$$\alpha_{\mathbf{1}} = \begin{pmatrix}1 & 0 & -1\end{pmatrix}, \alpha_{\mathbf{2}} = \begin{pmatrix}1 & 1 & 1\end{pmatrix}, \alpha_{\mathbf{3}} = \begin{pmatrix}1 & 0 & 0\end{pmatrix}.$$

What are the coordinates of vector $\begin{pmatrix}a & b & c\end{pmatrix}$ in the ordered basis $\mathbf{B}$?

Solution:

Given

$$\mathbf{B} = \begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 0\\ -1 & 1 & 0\end{pmatrix} \tag{0.0.1}$$

be the ordered basis for R^3,then the coordinates of vector,

$$\alpha = \begin{pmatrix}a\\ b\\ c\end{pmatrix} \tag{0.0.2}$$

in the ordered basis R^3 is the vector,

$$[\alpha]_{\mathbf{B}} = \begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} \tag{0.0.3}$$

hence

$$x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3 = \alpha \tag{0.0.4}$$

substituting (0.0.1) and (0.0.2) in (0.0.4)

$$\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 0\\ -1 & 1 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}a\\ b\\ c\end{pmatrix} \tag{0.0.5}$$

augmented matrix form

$$\begin{pmatrix}1 & 1 & 1 & a\\ 0 & 1 & 0 & b\\ -1 & 1 & 0 & c\end{pmatrix} \tag{0.0.6}$$

converting above matrix into row reduced echelon form

$$\begin{pmatrix}1 & 1 & 1 & a\\ 0 & 1 & 0 & b\\ -1 & 1 & 0 & c\end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3+R_1} \begin{pmatrix}1 & 1 & 1 & a\\ 0 & 1 & 0 & b\\ 0 & 2 & 1 & c+a\end{pmatrix} \tag{0.0.7}$$

$$\xleftrightarrow{R_3 \leftarrow R_3-2R_2} \begin{pmatrix}1 & 1 & 1 & a\\ 0 & 1 & 0 & b\\ 0 & 0 & 1 & a-2b+c\end{pmatrix} \tag{0.0.8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1-R_2} \begin{pmatrix}1 & 0 & 1 & a-b\\ 0 & 1 & 0 & b\\ 0 & 0 & 1 & a-2b+c\end{pmatrix} \tag{0.0.9}$$

$$\xleftrightarrow{R_1 \leftarrow R_1-R_3} \begin{pmatrix}1 & 0 & 0 & b-c\\ 0 & 1 & 0 & b\\ 0 & 0 & 1 & a-2b+c\end{pmatrix} \tag{0.0.10}$$

$\therefore$ The coordinates of α w.r.t $\mathbf{B}$ is

$$[\alpha]_{\mathbf{B}} = \begin{pmatrix}b-c\\ b\\ a-2b+c\end{pmatrix} \tag{0.0.11}$$