

EE5609 Matrix Theory

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Download the latex-file codes from

https://github.com/kranthiakssy/AI20RESCH14002_PhD_IITH/tree/master/EE5609_Matrix_Theory/Assignment-5

ASSIGNMENT-5 GEOLIN

Problem:

Triangle Exercises (1.19):

D is a point on side BC of $\triangle ABC$ such that $AD = AC$. Show that $AB > AD$

Solution:

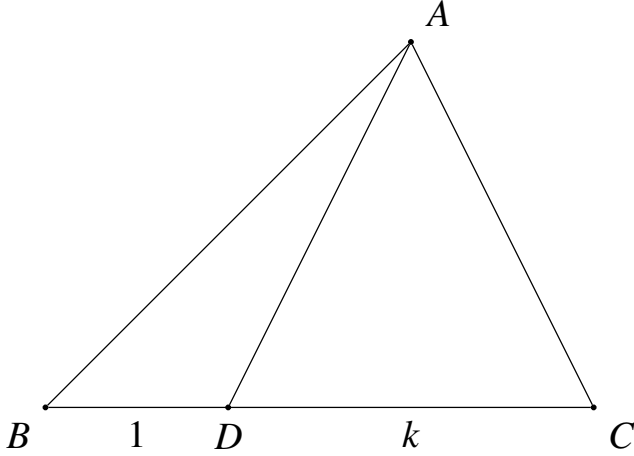


Fig. 0: Triangle generated using LaTeX-Tikz

The above Fig. 0 shows that, point D placed on side BC of $\triangle ABC$ such that

$$\|\mathbf{D} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{A}\| \quad (0.0.1)$$

Let Point D bisecting the side BC at 1:k ratio and Direction vectors of AB, AD & AC are $\mathbf{B} - \mathbf{A}$, $\mathbf{D} - \mathbf{A}$ & $\mathbf{C} - \mathbf{A}$ respectively.

By applying the section formula for bisecting the line internally

$$(\mathbf{D} - \mathbf{A}) = \frac{k(\mathbf{B} - \mathbf{A}) + 1(\mathbf{C} - \mathbf{A})}{1 + k} \quad (0.0.2)$$

$$\Rightarrow \|\mathbf{D} - \mathbf{A}\|^2 = \frac{k^2 \|\mathbf{B} - \mathbf{A}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2}{(1 + k)^2} \quad (0.0.3)$$

substituting (0.0.1)

$$\|\mathbf{D} - \mathbf{A}\|^2 = \frac{k^2 \|\mathbf{B} - \mathbf{A}\|^2}{(1 + k)^2} + \frac{\|\mathbf{D} - \mathbf{A}\|^2}{(1 + k)^2} \quad (0.0.4)$$

$$\Rightarrow \|\mathbf{D} - \mathbf{A}\|^2 \left(1 - \frac{1}{(1 + k)^2}\right) = \frac{k^2 \|\mathbf{B} - \mathbf{A}\|^2}{(1 + k)^2} \quad (0.0.5)$$

$$\Rightarrow \|\mathbf{D} - \mathbf{A}\|^2 \left(1 + \frac{2}{k}\right) = \|\mathbf{B} - \mathbf{A}\|^2 \quad (0.0.6)$$

$$\therefore AB > AD \quad (0.0.7)$$

for $k > 0$
Hence Proved.