EE5609 Matrix Theory

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Download the python code for QR decomposition from

https://github.com/kranthiakssy/AI20RESCH14002_PhD_IITH/tree/master/EE5609_Matrix_Theory/Assignment−8

Download the latex-file codes from

https://github.com/kranthiakssy/AI20RESCH14002_PhD_IITH/tree/master/EE5609_Matrix_Theory/Assignment−8

Assignment-8
Coordinate Geometry Exercises

Problem:

QR Decomposition:
Find the QR Decomposition of matrix,

$$\mathbf{A} = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} \tag{0.0.1}$$

Solution:

Let α and β be the column vectors of given matrix $\mathbf{A}$

$$\alpha = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{0.0.2}$$

$$\beta = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{0.0.3}$$

We can express these as,

$$\alpha = k_1 \mathbf{u_1} \tag{0.0.4}$$

$$\beta = r_1 \mathbf{u_1} + k_2 \mathbf{u_2} \tag{0.0.5}$$

Where,

$$k_1 = \|\alpha\| \tag{0.0.6}$$

$$\mathbf{u_1} = \frac{\alpha}{k_1} \tag{0.0.7}$$

$$r_1 = \frac{\mathbf{u_1}^T \beta}{\|\mathbf{u_1}\|^2} \tag{0.0.8}$$

$$\mathbf{u_2} = \frac{\beta - r_1 \mathbf{u_1}}{\|\beta - r_1 \mathbf{u_1}\|} \tag{0.0.9}$$

$$k_2 = \mathbf{u_2}^T \beta \tag{0.0.10}$$

From (0.0.4) and (0.0.5)

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{0.0.11}$$

$$\mathbf{A} = \mathbf{QR} \tag{0.0.12}$$

From the above equation we can see that $\mathbf{R}$ is an upper triangular matrix and $\mathbf{Q}$ is an orthogonal matrix

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \tag{0.0.13}$$

Now by using equations (0.0.2) to (0.0.10)

$$k_1 = \sqrt{9 + 16} = 5 \tag{0.0.14}$$

$$\mathbf{u_1} = \frac{1}{5} \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} \\ \frac{-4}{5} \end{pmatrix} \tag{0.0.15}$$

$$r_1 = \frac{\begin{pmatrix} \frac{3}{5} & \frac{-4}{5} \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \frac{-11}{5} \tag{0.0.16}$$

$$\mathbf{u_2} = \frac{\begin{pmatrix} -1 \\ 2 \end{pmatrix} - \frac{-11}{5} \begin{pmatrix} \frac{3}{5} \\ \frac{-4}{5} \end{pmatrix}}{\left\| \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \frac{-11}{5} \begin{pmatrix} \frac{3}{5} \\ \frac{-4}{5} \end{pmatrix} \right\|} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix} \tag{0.0.17}$$

$$k_2 = \begin{pmatrix} \frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \frac{2}{5} \tag{0.0.18}$$

From equations (0.0.11) and (0.0.12) the obtained $\mathbf{QR}$ decomposition is

$$\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{-4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 5 & \frac{-11}{5} \\ 0 & \frac{2}{5} \end{pmatrix} \tag{0.0.19}$$