EE5609 Matrix Theory

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1 Assignment-2

1.1 Problem

Lines and Planes (81) : In each of the following cases, determine the normal to the plane and the distance from the origin.

$a)$ $(0\ 0\ 1)\mathrm{x} = 2$ $\quad$ $c)$ $(0\ 5\ 0)\mathrm{x} = -8$

$b)$ $(1\ 1\ 1)\mathrm{x} = 1$ $\quad$ $d)$ $(2\ 3\ \ -1)\mathrm{x} = 5$

1.2 Solution

If $ax + by + cz = d$ is a linear equation representing a plane, then the normal to that plane $\vec{n}$ is the coefficients of the linear equation.

$$\vec{n} = (a\ b\ c)$$

The shortest distance between the plane and a point P out side the plane is

$$\left| \frac{\vec{PQ}.\vec{n}}{\|n\|} \right|$$

Where Q is any point on that plane

and $\vec{n}$ is normal vector to that plane

a)
normal vector $\vec{n} = (0\ 0\ 1)$
Origin P = (0 0 0)
Point on the plane Q = (0 0 2)
shortest distance from origin =

$$\left| \frac{\begin{pmatrix} 0 & 0 & 2 \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{0^2 + 0^2 + 1^2}} \right| = 2$$

b)
normal vector $\vec{n} = (1\ 1\ 1)$
Origin P = (0 0 0)
Point on the plane Q = (1 0 0)
shortest distance from origin =

$$\left| \frac{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{1^2 + 1^2 + 1^2}} \right| = \frac{1}{\sqrt{3}}$$

c)
normal vector $\vec{n} = (0\ 5\ 0)$
Origin P = (0 0 0)
Point on the plane Q = $(0\ -8/5\ 0)$

shortest distance from origin =

$$\left| \frac{\begin{pmatrix} 0 & -8/5 & 0 \end{pmatrix} . \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}}{\sqrt{0^2 + 5^2 + 0^2}} \right| = \frac{8}{5}$$

d)
normal vector $\quad \vec{n} = (2\ 3\ -1)$
Origin P = (0 0 0)
Point on the plane Q = (5/2 0 0)
shortest distance from origin =

$$\left| \frac{\begin{pmatrix} 5/2 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}}{\sqrt{2^2 + 3^2 + (-1)^2}} \right| = \frac{5}{\sqrt{14}}$$