EE5609 Matrix Theory

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Download the python code for ellipse from

```
https://github.com/kranthiakssy/
    AI20RESCH14002_PhD_IITH/tree/master/
    EE5609_Matrix_Theory/Assignment-7
```

Download the latex-file codes from

```
https://github.com/kranthiakssy/
    AI20RESCH14002_PhD_IITH/tree/master/
    EE5609_Matrix_Theory/Assignment-7
```

Assignment-6
Coordinate Geometry Exercises

Problem:

Conics (1.16):
Find points on the curve

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} \mathbf{x} = 1 \tag{0.0.1}$$

at which tangents are

(a) parallel to x-axis
(b) parallel to y-axis

Solution:

The standard ellipse equation can be given by

$$\mathbf{x}^T \mathbf{D} \mathbf{x} = 1, \mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \lambda_1, \lambda_2 > 0 \tag{0.0.2}$$

By comparing (0.0.1) with (0.0.2)

$$\mathbf{D} = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} \tag{0.0.3}$$

$$\lambda_1 = \frac{1}{9}, \lambda_2 = \frac{1}{16} \tag{0.0.4}$$

$$\mathbf{V} = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -1 \tag{0.0.5}$$

The Point(s) of contact for ellipse can be given by

$$\mathbf{q} = \mathbf{V}^{-1}(k\mathbf{n} - \mathbf{u}) \tag{0.0.6}$$

$$k = \pm\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \tag{0.0.7}$$

(a) parallel to x-axis
The tangents are parallel to x-axis, their direction and normal vectors are respectively,

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.0.8}$$

substituting (0.0.4), (0.0.5) and (0.0.8) in (0.0.7)

$$k = \pm\sqrt{\frac{1}{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 9 & 0 \\ 0 & 16 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}} \tag{0.0.9}$$

$$\implies k = \pm\sqrt{\frac{1}{16}} \tag{0.0.10}$$

$$\implies k = \pm\frac{1}{4} \tag{0.0.11}$$

substituting (0.0.11) and (0.0.5) in (0.0.6)

$$\mathbf{q} = \begin{pmatrix} 9 & 0 \\ 0 & 16 \end{pmatrix} \left(\pm\frac{1}{4} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \tag{0.0.12}$$

$$\implies \mathbf{q} = \begin{pmatrix} 0 \\ \pm 4 \end{pmatrix} \tag{0.0.13}$$

$\therefore$ the points of contacts with tangents parallel to x-axis are

$$\mathbf{q_{1x}} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{q_{2x}} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{0.0.14}$$

(b) parallel to y-axis
The tangents are parallel to y-axis, their direction and normal vectors are respectively,

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.0.15}$$

substituting (0.0.4), (0.0.5) and (0.0.15) in

(0.0.7)

$$k = \pm\sqrt{\frac{1}{\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}9 & 0\\0 & 16\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}}} \tag{0.0.16}$$

$$\implies k = \pm\sqrt{\frac{1}{9}} \tag{0.0.17}$$

$$\implies k = \pm\frac{1}{3} \tag{0.0.18}$$

substituting (0.0.18) and (0.0.5) in (0.0.6)

$$\mathbf{q} = \begin{pmatrix}9 & 0\\0 & 16\end{pmatrix}\left(\pm\frac{1}{3}\begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right) \tag{0.0.19}$$

$$\implies \mathbf{q} = \begin{pmatrix}\pm 3\\0\end{pmatrix} \tag{0.0.20}$$

$\therefore$ the points of contacts with tangents parallel to y-axis are

$$\mathbf{q_{1y}} = \begin{pmatrix}3\\0\end{pmatrix}, \mathbf{q_{2y}} = \begin{pmatrix}-3\\0\end{pmatrix} \tag{0.0.21}$$

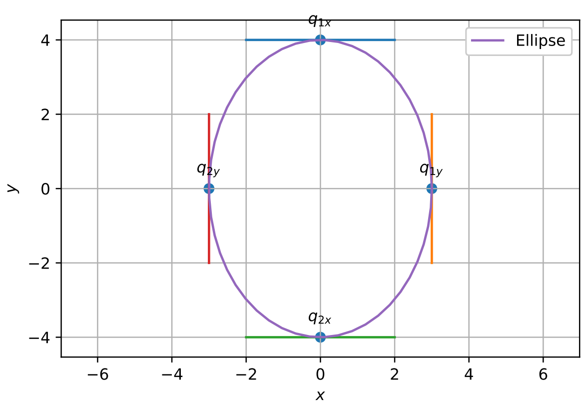

Figure depicting ellipse with tangents