EE5609 Matrix Theory

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Download the python code for circle from

```
https://github.com/kranthiakssy/
    AI20RESCH14002_PhD_IITH/tree/master/
    EE5609_Matrix_Theory/Assignment−6
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Download the latex-file codes from

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https://github.com/kranthiakssy/
    AI20RESCH14002_PhD_IITH/tree/master/
    EE5609_Matrix_Theory/Assignment−6
```

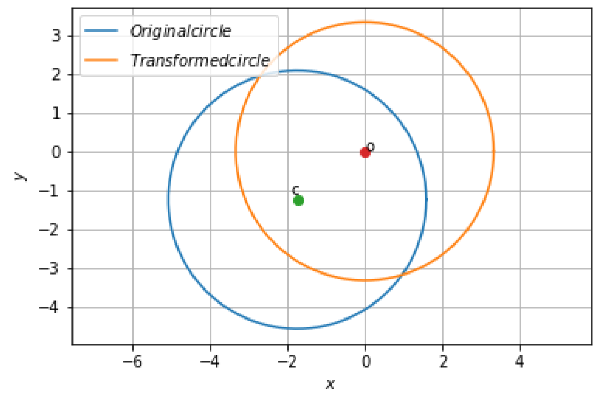

Fig. 0: Figure depicting transformation of circle

Assignment-6

RAMSEY

Problem:

Affine Transformation (3.4.8):
Show that, by changing the origin, the equation

$$2\mathbf{x}^T\mathbf{x} + \begin{pmatrix} 7 & 5 \end{pmatrix}\mathbf{x} - 13 = 0 \tag{0.0.1}$$

can be transformed to

$$8\mathbf{x}^T\mathbf{x} = 89 \tag{0.0.2}$$

Solution:

Eq (0.0.1) cab be written as

$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix} \frac{7}{2} & \frac{5}{2} \end{pmatrix}\mathbf{x} - \frac{13}{2} = 0 \tag{0.0.3}$$

$$\implies \mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} \frac{7}{4} & \frac{5}{4} \end{pmatrix}\mathbf{x} - \frac{13}{2} = 0 \tag{0.0.4}$$

The above eq (0.0.4) cab be compared with the circle equation gives as

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.0.5}$$

then

$$\mathbf{u} = \begin{pmatrix} \frac{7}{4} \\ \frac{5}{4} \end{pmatrix} \tag{0.0.6}$$

$$\implies centre, \mathbf{c} = \begin{pmatrix} \frac{-7}{4} \\ \frac{-5}{4} \end{pmatrix} \tag{0.0.7}$$

$$\|u\|^2 - r^2 = f \tag{0.0.8}$$

$$\implies r^2 = \|u\|^2 - f \tag{0.0.9}$$

$$\implies r^2 = \left(\frac{7}{4}\right)^2 + \left(\frac{5}{4}\right)^2 + \frac{13}{2} \tag{0.0.10}$$

$$\implies radius, r = \sqrt{\frac{89}{8}} \tag{0.0.11}$$

The eq (0.0.2) can be written by changing the origin as

$$(\mathbf{x} + \mathbf{c})^T(\mathbf{x} + \mathbf{c}) = \frac{89}{8} \tag{0.0.12}$$

$$\implies \mathbf{x}^T\mathbf{x} + \mathbf{x}^T\mathbf{c} + \mathbf{c}^T\mathbf{x} + \mathbf{c}^T\mathbf{c} = \frac{89}{8} \tag{0.0.13}$$

We know that

$$\mathbf{x}^T\mathbf{c} = \mathbf{c}^T\mathbf{x} \tag{0.0.14}$$

by substituting (0.0.14) in (0.0.13)

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{c}^T\mathbf{x} + \mathbf{c}^T\mathbf{c} = \frac{89}{8} \tag{0.0.15}$$

substituting the orgin of (0.0.1) in above eq (0.0.15)

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix}\frac{7}{4} & \frac{5}{4}\end{pmatrix}\mathbf{x} + \begin{pmatrix}\frac{-7}{4} & \frac{-5}{4}\end{pmatrix}\begin{pmatrix}\frac{-7}{4}\\ \frac{-5}{4}\end{pmatrix} = \frac{89}{8} \quad (0.0.16)$$

$$\implies \mathbf{x}^T\mathbf{x} + \begin{pmatrix}\frac{7}{2} & \frac{5}{2}\end{pmatrix}\mathbf{x} + \frac{74}{16} - \frac{89}{8} = 0 \quad (0.0.17)$$

$$\implies \mathbf{x}^T\mathbf{x} + \begin{pmatrix}\frac{7}{2} & \frac{5}{2}\end{pmatrix}\mathbf{x} - \frac{13}{2} = 0 \quad (0.0.18)$$

$$\implies 2\mathbf{x}^T\mathbf{x} + \begin{pmatrix}7 & 5\end{pmatrix}\mathbf{x} - 13 = 0 \quad (0.0.19)$$

$\therefore$ It is proved that by changing the origin in (0.0.2) we obtained (0.0.1).