EE5609 Matrix Theory

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1 Assignment-2

1.1 Problem

Lines and Planes (81) : In each of the following cases, determine the normal to the plane and the distance from the origin.

$a)$ $(0\ 0\ 1)\mathbf{x} = 2$

$b)$ $(1\ 1\ 1)\mathbf{x} = 1$

$c)$ $(0\ 5\ 0)\mathbf{x} = -8$

$d)$ $(2\ 3\ \ -1)\mathbf{x} = 5$

1.2 Solution

If $ax+by+cz=d$ is a linear equation representing a plane, then the normal to that plane $\vec{\mathbf{n}}$ is the coefficients of the linear equation.

$$\vec{\mathbf{n}} = (a\ b\ c)$$

The shortest distance between the plane and a point $\mathbf{P}$ out side the plane is

$$\left| \frac{\vec{\mathbf{PQ}}.\vec{\mathbf{n}}}{\|\vec{\mathbf{n}}\|} \right|$$

Where $\mathbf{Q}$ is any point on that plane

and $\vec{\mathbf{n}}$ is normal vector to that plane

a)
normal vector $\vec{\mathbf{n}} = (0\ 0\ 1)$
Origin $\mathbf{P} = (0\ 0\ 0)$
Point on the plane $\mathbf{Q} = (0\ 0\ 2)$
shortest distance from origin =

$$\left| \frac{\begin{pmatrix} 0 & 0 & 2 \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{0^2 + 0^2 + 1^2}} \right| = 2$$

b)
normal vector $\vec{\mathbf{n}} = (1\ 1\ 1)$
Origin $\mathbf{P} = (0\ 0\ 0)$
Point on the plane $\mathbf{Q} = (1\ 0\ 0)$
shortest distance from origin =

$$\left| \frac{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{1^2 + 1^2 + 1^2}} \right| = \frac{1}{\sqrt{3}}$$

c)
normal vector $\vec{\mathbf{n}} = (0\ 5\ 0)$
Origin $\mathbf{P} = (0\ 0\ 0)$
Point on the plane $\mathbf{Q} = (0\ -8/5\ 0)$

shortest distance from origin =

$$\left| \frac{\begin{pmatrix} 0 & -8/5 & 0 \end{pmatrix} . \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}}{\sqrt{0^2 + 5^2 + 0^2}} \right| = \frac{8}{5}$$

d)

normal vector $\vec{\mathbf{n}} = (2\ 3\ -1)$

Origin $\mathbf{P} = (0\ 0\ 0)$

Point on the plane $\mathbf{Q} = (5/2\ 0\ 0)$

shortest distance from origin =

$$\left| \frac{\begin{pmatrix} 5/2 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}}{\sqrt{2^2 + 3^2 + (-1)^2}} \right| = \frac{5}{\sqrt{14}}$$