

# Cooperative Air-Ground Robot Localization

## - final project description

ASEN 5044

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## 1 Introduction

Robust and reliable outdoor localization and inertial navigation remains challenging for autonomous robotic vehicles. Although Global Positioning System (GPS) devices are now ubiquitous, GPS may not operate reliably in certain locations (e.g. urban environments, forest canopies, mountain valleys, etc.) and can be subject to spoofing, jamming, or cyberattacks. With the expected deployment of multi-vehicle robot teams for many applications, one way to address this challenge is for robots to augment their existing localization solutions with peer-to-peer tracking information derived from other nearby robots. This approach is called *cooperative localization*.

## 2 Physical system

The system depicted in Figure 1 is a simplified 2D variation of an uncrewed air-ground robotic team performing cooperative navigation. The system consists of a 4-wheeled unmanned ground vehicle (UGV) that moves along the ground while cooperatively tracking and communicating with an unmanned aerial vehicle (UAV) that flies a 2D path at a constant altitude above the ground vehicle. The UAV and UGV are controlled by independent inputs, but obtain relative measurements to one another through their encounter. The UGV cannot obtain reliable GPS measurements and effectively switches its receiver off during its encounter with the UAV, while the UAV maintains reliable GPS throughout the encounter.

Although the system is designed for decentralized cooperative localization (i.e. where the UAV and UGV can independently estimate their own and each other's states), a centralized base station can also be used to collect measurements from both vehicles to estimate their combined states and thus provide a 'gold standard' for assessing decentralized algorithms. This assignment will focus on the development of the centralized estimator only, using simple non-linear vehicle motion models.

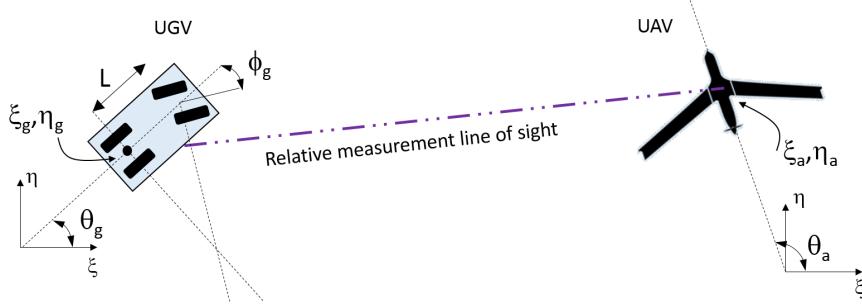


Figure 1: UAV-UGV cooperative localization problem setup.

### 3 Dynamical system

The UGV's motion is modeled kinematically here as a simple 4-wheeled steerable Dubin's car with front-rear wheel separation length  $L$ , with East position  $\xi_g$  and North position  $\eta_g$  in an inertial frame, and with heading angle  $\theta_g$ . The UGV's control inputs are  $\mathbf{u}_g = [v_g, \phi_g]^T$ , the linear velocity (m/s) and steering angle (rad), respectively. This leads to the non-linear equations of motion

$$\dot{\xi}_g = v_g \cos \theta_g + \tilde{w}_{x,g} \quad (1)$$

$$\dot{\eta}_g = v_g \sin \theta_g + \tilde{w}_{y,g} \quad (2)$$

$$\dot{\theta}_g = \frac{v_g}{L} \tan \phi_g + \tilde{w}_{\omega,g}, \quad (3)$$

where  $\tilde{\mathbf{w}}_g(t) = [\tilde{w}_{x,g}, \tilde{w}_{y,g}, \tilde{w}_{\omega,g}]^T$  is the process noise on the UGV states.

The fixed-wing UAV motion is likewise modeled kinematically as a simple Dubin's unicycle, with East position  $\xi_a$  and North position  $\eta_a$  in an inertial frame, with heading angle  $\theta_a$ . The UAV's control inputs are  $\mathbf{u}_a = [v_a, \omega_a]^T$ , the linear velocity (m/s) and angular rate (rad/s), respectively. This leads to the non-linear equations of motion

$$\dot{\xi}_a = v_a \cos \theta_a + \tilde{w}_{x,a} \quad (4)$$

$$\dot{\eta}_a = v_a \sin \theta_a + \tilde{w}_{y,a} \quad (5)$$

$$\dot{\theta}_a = \omega_a + \tilde{w}_{\omega,a} \quad (6)$$

where  $\tilde{\mathbf{w}}_a(t) = [\tilde{w}_{x,a}, \tilde{w}_{y,a}, \tilde{w}_{\omega,a}]^T$  is the process noise on the UAV states.

Hence, the combined system state, control inputs, and disturbance inputs are

$$\begin{aligned}\mathbf{x}(t) &= [\xi_g \quad \eta_g \quad \theta_g \quad \xi_a \quad \eta_a \quad \theta_a]^T, \\ \mathbf{u}(t) &= [\mathbf{u}_g \quad \mathbf{u}_a]^T, \\ \tilde{\mathbf{w}}(t) &= [\tilde{\mathbf{w}}_g \quad \tilde{\mathbf{w}}_a]^T\end{aligned}$$

The sensing model for this system is given by a combination of noisy ranges and azimuth angles of the UGV relative to the UAV, noisy azimuth angles of the UAV relative to the UGV, and noisy UAV GPS measurements,

$$\mathbf{y}(t) = \begin{bmatrix} \tan^{-1} \left( \frac{\eta_a - \eta_g}{\xi_a - \xi_g} \right) - \theta_g \\ \sqrt{(\xi_g - \xi_a)^2 + (\eta_g - \eta_a)^2} \\ \tan^{-1} \left( \frac{\eta_g - \eta_a}{\xi_g - \xi_a} \right) - \theta_a \\ \xi_a \\ \eta_a \end{bmatrix} + \tilde{\mathbf{v}}(t),$$

where  $\tilde{\mathbf{v}}(t) \in \mathbb{R}^5$  is the sensor error vector (which can be modeled by AWGN).

## 4 Nominal system parameters

For this project, assume  $L = 0.5$  m for the UGV and that the UGV has a limited range of steering angles  $\phi_g \in [-\frac{5\pi}{12}, \frac{5\pi}{12}]$  rads, with a top ground speed of  $v_{g,max} = 3$  m/s. Further assume that the the UAV's turn rate is limited to  $\omega_g \in [-\frac{\pi}{6}, \frac{\pi}{6}]$  rad/s and flies with velocity  $v_a \in [10, 20]$  m/s.

As a nominal trajectory for the UGV, assume that the UGV starts at  $(\xi_g, \eta_g) = (10, 0)$  with  $\theta_g = \pi/2$  rad, and initiates a steady continuous turning maneuver with  $v_g = 2$  m/s and  $\phi = -\pi/18$  rad. As a nominal trajectory for the UGV, assume the UAV starts at  $(\xi_a, \eta_a) = (-60, 0)$  with  $\theta_a = -\pi/2$  rad, and initiates a steady continuous turning maneuver with  $v_a = 12$  m/s and  $\omega_a = \pi/25$  rad/s.