PANDEMIC SIMULATION

ISYE-6644 SIMULATION

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Introduction

For a pandemic or an epidemic, we can consider mainly three phases. In the very beginning, few people were infected by pandemics. Or there is a small proportion of unhealthy persons and many healthy persons. This phase is the early phase of the outbreak.

Now, the infected person causes infection to another healthy person and so on. The proportion of unhealthy people increases day by day. We can assume the increase in geometric proportion. At one point in time pandemic cases arise so rapidly and it's difficult to control within the given circumstances and available resources. That phase is the major outbreak of pandemics. When more and more people are infected, it’s difficult to test every person so it is determined to be infectious if those persons have similar symptoms to early detected person symptoms.

In the later time, people may have certain immunity so they can defend against disease naturally and recover. If by chance some vaccines are made, that may control the pandemic. That is about the end phase of the outbreak.

Compartment Modeling

Let S be the healthy persons, I be the infected persons and βSI be the rate of change of persons healthy to infected or infected to healthy then,

Rate of change of susceptible

$$\frac{dS}{dt} = -\beta S \frac{I}{N} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . . (1)$$

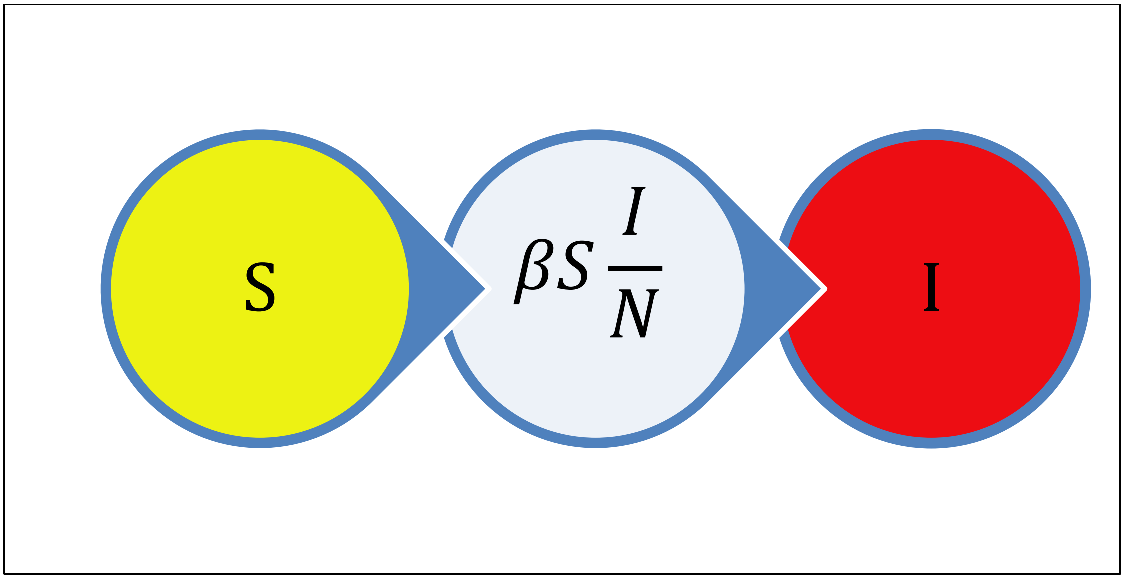
Here -ve sign indicates that susceptible person’s number decreases over time, t could be days, weeks, or months, and β is the transmission rate and N is the total population. Transmission rate is a function of **rate of contact** and **probability of transmission**.

$$\beta = r_c * p(T) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

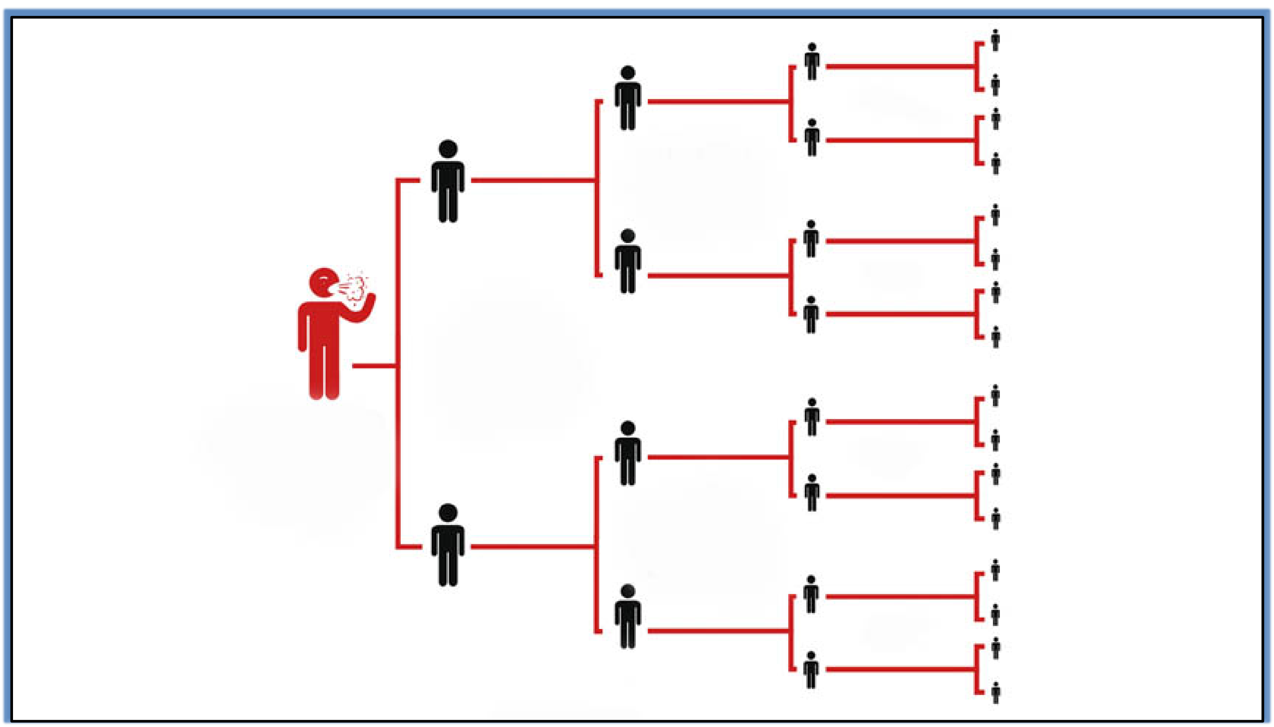
Similarly change in the infected people over time is

$$\frac{dI}{dt} = \beta S \frac{I}{N} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . . (3)$$

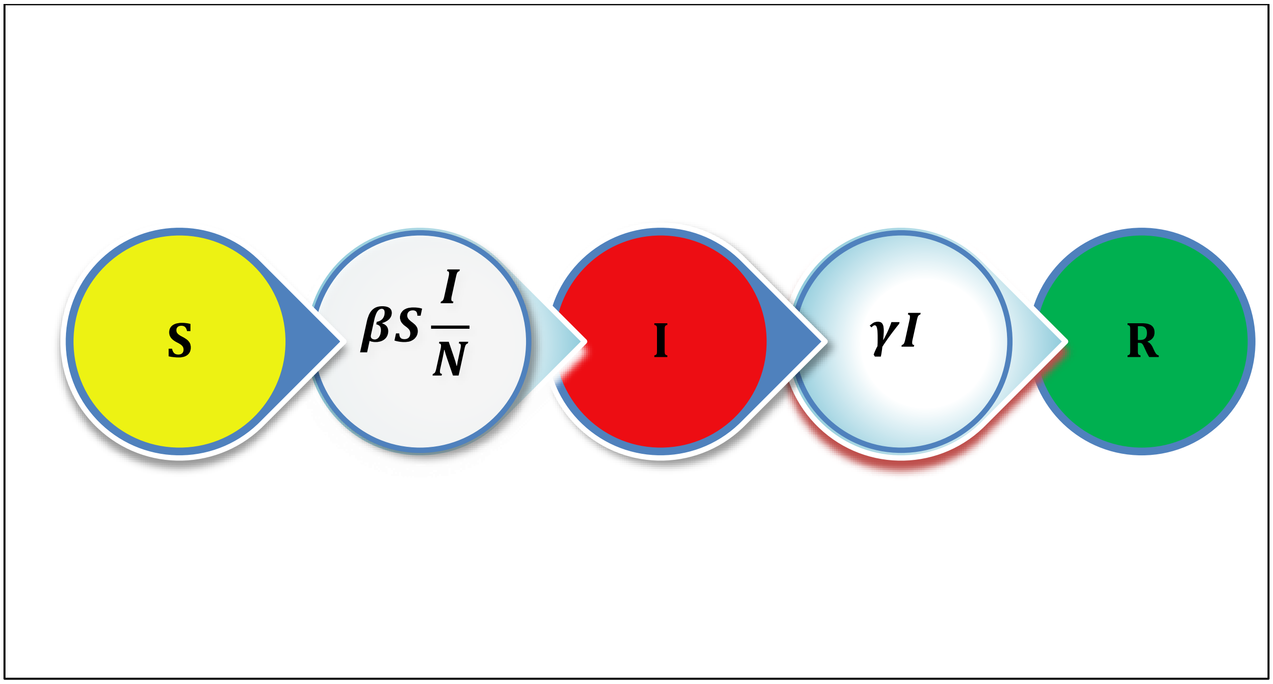
Here the positive sign indicates infection rate increases while the pandemic goes on. In that sense infected person ratio almost increases.



For simplicity let's choose 1 person infected and can transfer disease to 2 person and so on. In this pattern we can see geometric pattern as shown in figure 2.



Pandemic outbreak doesn't increase all the time. Some people recovered in the meantime. If we include recovered person in the model, it becomes SIR model.

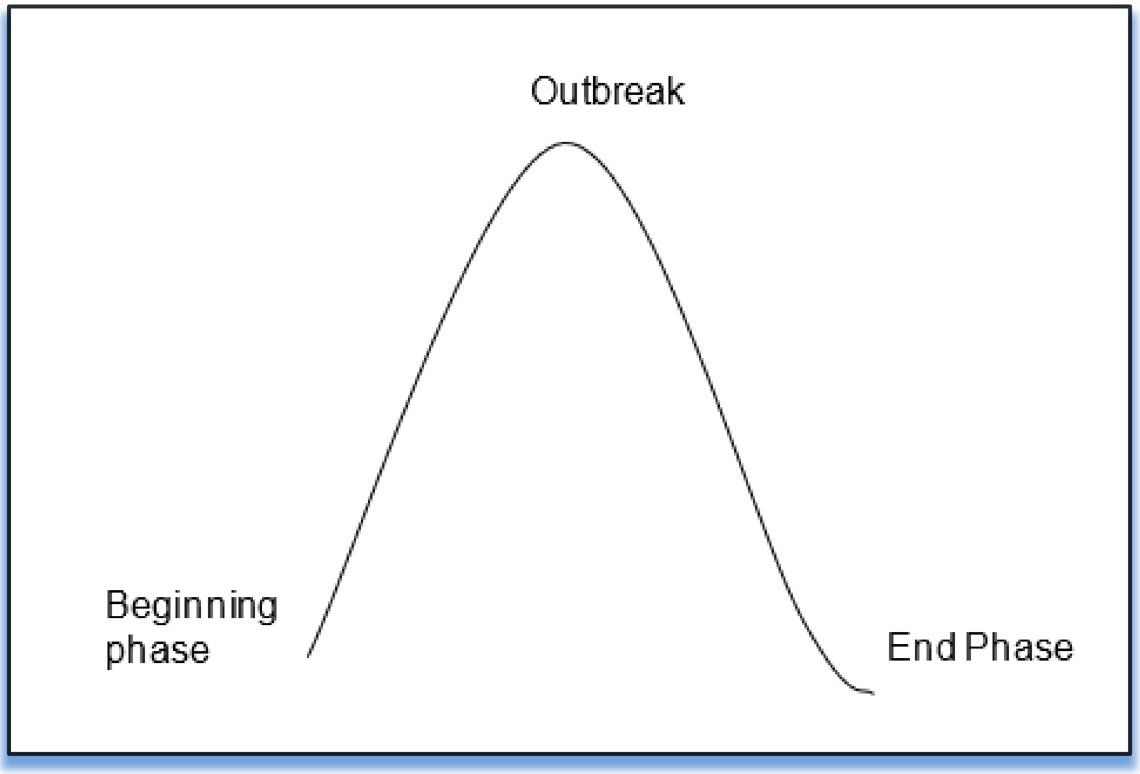


$$\frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

$$\frac{dR}{dt} = \gamma I \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \text{(5)}$$

Here γI is rate of change of recovery. Basic SIR model can capture the “shape” of an outbreak. It involves many simplifying assumptions e.g., individuals are immediately infectious after infection, everyone mixes with everyone else etc. Even this simple model can be used to explore key epidemiological concepts. In this simple model, death state is not stated explicitly. Death person number will be some portion of infected people. At any point in time, Total number of people (N) must add up:

$$N = I + S + R \ldots\ldots\ldots.\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$



To convert the above equations in more simplified version,

$$\frac{dS}{dt} = \frac{S_{t+1} - S_t}{(t+1) - t} = \frac{S_{t+1} - S_t}{1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(7)}$$

Here time change is 1 day considered and $\frac{I}{N}$ is just the proportion so our equations become

$$S_{t+1} - S_t = -\beta S_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(8)}$$

$$I_{t+1} - I_t = \beta S_t - \gamma I_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(9)}$$

$$R_{t+1} - R_t = \gamma I_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(10)}$$

$$N_t = S_t + I_t + R_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(11)}$$

A simple model using above equation, Excel gives the following approximation, and its corresponding plot is shown

SIR model			
Total Population(N)=	1000	Recovery rate(γ)=	0.10
Infected(I)=	1		
Transmission rate(β)=	0.50		

Time	Susceptible	Infected	Recovered
0	1000.00	1.00	0.00
1	500.00	500.90	0.10
2	250.00	700.81	50.19
3	125.00	755.73	120.27
4	62.50	742.66	195.84
5	31.25	699.64	270.11
6	15.63	645.30	340.07
7	7.81	588.58	404.60
8	3.91	533.63	463.46
9	1.95	482.22	516.83
10	0.98	434.98	565.05
11	0.49	391.97	608.55
12	0.24	353.01	647.74
13	0.12	317.83	683.04
14	0.06	286.11	714.83
15	0.03	257.53	743.44
16	0.02	231.79	769.19
17	0.01	208.62	792.37
18	0.00	187.76	813.23
19	0.00	168.99	832.01
20	0.00	152.09	848.91
21	0.00	136.88	864.12
22	0.00	123.19	877.81
23	0.00	110.88	890.12
24	0.00	99.79	901.21
25	0.00	89.81	911.19
26	0.00	80.83	920.17

Plot shows susceptible number of people decreases over time. Recovered persons also increases and it becomes constant about 75 days (but I have shown only up to 36 days). Infected person also decreases over time.

Consider a classroom of 21 elementary school kids. 20 of the kids are healthy (and susceptible to flu) on Day 1. Tommy (the 21st kid) walks in with the flu and starts interacting with his potential victims. To keep things simple, let's suppose that Tommy comes to school every day (whether he's sick) and will be infectious for 3 days. Thus, there are 3 chances for Tommy to infect the other kids — Days 1, 2, and 3. Suppose that the probability that he infects any individual susceptible kid on any of the three days is p = 0.02; and suppose that all kids and days are independent (so that you have i.i.d. Bern(p) trials). If a kid gets infected by Tommy, he will then become infectious for 3 days as well, starting on the next day.

(a) What is the distribution of the number of kids that Tommy infects on Day 1?

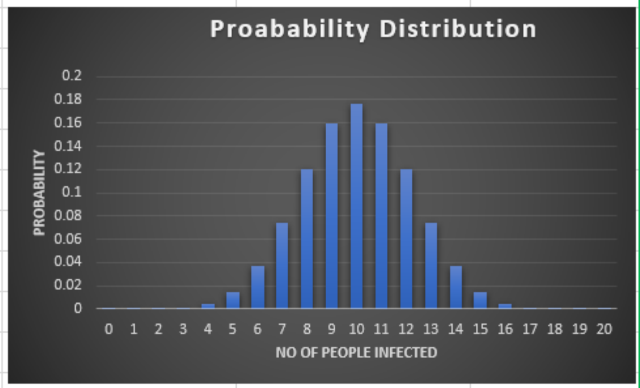
If X is a random variable with probability of success ***p*** and probability of failure ***(1-p)*** then Bernoulli distribution can be expressed as

Let's represent

$$\Pr(X = x) = \begin{cases} p & x = 1 \\ 1 - p & x = 0 \end{cases}$$

which gives $P(X) = p^x(1-p)^x$. For n trials it becomes binomial distribution as shown below.

No of people infected	combination	Proabability	1048576
0	1	9.53674E-07	
1	20	1.90735E-05	
2	190	0.000181198	
3	1140	0.001087189	
4	4845	0.004620552	
5	15504	0.014785767	
6	38760	0.036964417	
7	77520	0.073928833	
8	125970	0.120134354	
9	167960	0.160179138	
10	184756	0.176197052	
11	167960	0.160179138	
12	125970	0.120134354	
13	77520	0.073928833	
14	38760	0.036964417	
15	15504	0.014785767	
16	4845	0.004620552	
17	1140	0.001087189	
18	190	0.000181198	
19	20	1.90735E-05	
20	1	9.53674E-07	



Given that probability of infection is (p) = 0.02 and probability of non-infection = (1-0.02) = 0.98

Tommy can infect or not infect to 1 person 2 ways.

Tommy can infect or not infect to 2 person 2^2 ways.

Tommy can infect 20 people in 2^{20} ways.

Probability that Tommy can infect 0 person = $\frac{C(20,0)}{2^{20}}$ = 9.53E-07

Probability that Tommy can infect 1 person = $\frac{C(20,1)}{2^{20}}$ = 1.90E-05

The above graph is the required distribution.

(b) What is the expected number of kids that Tommy infects on Day 1?

The number of kids expected to be infected by Tommy is

$$E(x) = n * p = 20 * 0.02 = 0.4$$

(c) What is the expected number of kids that are infected by Day 2 (you can count Tommy if you want)?

Student come to school 5 days a week, So student contact 5/7

If student have met all days of week rate of transmission = $0.02 * \left(20 * \frac{7}{7}\right)$=0.4

Rate of transmission $(\beta) = 0.02 * \left(20 * \frac{5}{7}\right) = 0.29$ makes sense

Recovery rate $(\gamma) = 1\ out\ of\ 3\ days = \frac{1}{3}$ =0.33 given in the question

On day 1, 0.4 people infected. So, 20-0.4 =19.6 are healthy.

Therefore, expected people to be infected = 19.6*0.02 = 0.392 as usual.

Now total infected people = 1(Tommy) + 0.4 + 0.392 = 1.792 kids to be infected in day 2.

(d) Simulate the number of kids that are infected on Days 1,2, ... Do this many times. What are the (estimated) expected numbers of kids that are infected by Day i, i = 1, 2, . . .? Produce a histogram detailing how long the “epidemic” will last.

SIR model

Total Population(N)=	20	Recovery rate(γ)=	0.33
Infected(I)=	1		
Transmission rate(β)=	0.29		

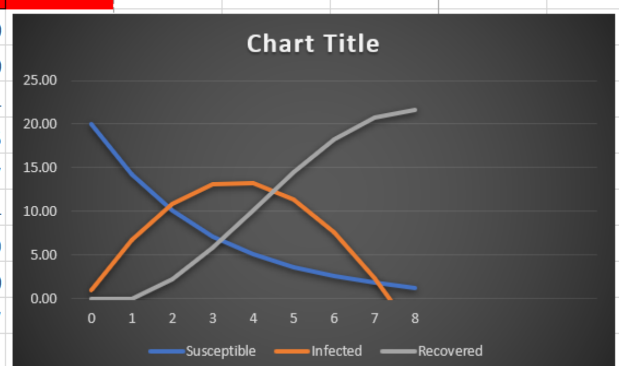
$S_{t+1} - S_t = -\beta S_t$..............................(8)

$I_{t+1} - I_t = \beta S_t - \gamma I_t$.........................(9)

$R_{t+1} - R_t = \gamma I_t$..............................(10)

$N_t = S_t + I_t + R_t$...........................(11)

Time	Susceptible	Infected	Recovered
0	20.00	1.000	0.00
1	14.20	6.800	0.00
2	10.08	10.918	2.24
3	7.16	13.101	5.85
4	5.08	13.248	10.17
5	3.61	11.365	14.54
6	2.56	7.613	18.29
7	1.82	2.319	20.80
8	1.29	-4.019	21.57



References

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2. Law, A. M. (2014). Simulation Modeling and Analysis. United Kingdom: McGraw-Hill Education.