

Q1.

①
 $4R + 6G$

②
 $5R + 5G$

③
 $3R + 7G$

6 sided dice

$$1 \parallel 2 \Rightarrow \frac{2}{6} \rightarrow \textcircled{1}$$

$$3 \parallel 4 \Rightarrow \frac{2}{6} \rightarrow \textcircled{2}$$

$$5 \parallel 6 \Rightarrow \frac{2}{6} \rightarrow \textcircled{3}$$

$$P(R) = P(R)_{\textcircled{1}} + P(R)_{\textcircled{2}} + P(R)_{\textcircled{3}}$$

$$= \frac{2}{6} \times \frac{4}{10} + \frac{2}{6} \times \frac{5}{10} + \frac{2}{6} \times \frac{3}{10}$$

$$= \frac{1}{3} \left[\frac{4 + 5 + 3}{10} \right] = \frac{1}{3} \times \frac{12}{10} = \frac{4}{10} = \frac{2}{5}$$

W

②

4R , 10G , 6Y

→ 3 balls without replacement

→ ① all red

② 2-red & 1-green

$$P(\text{all Red}) = \frac{4}{20} \times \frac{3}{19} \times \frac{2}{18} =$$

$$= \frac{1}{5} \times \frac{1}{19} \times \frac{2}{9} = \frac{1}{19 \times 15} = \frac{1}{285}$$

$P[2 \text{ Red \& 1 green}]$.

R R G

G R R

R G R

$$\Rightarrow 3 \times \left[\frac{4}{20} \times \frac{3}{19} \times \frac{10}{18} \right]$$

$$= \frac{1}{19}$$

stamp plots
line

③

$$100 / 365$$

W3.

$$\frac{100}{365}$$

$P(R|B)$ ✓
 P

Let R be the event it rains
Let ~~the~~ B be the event that
it is predicted to rain.
Let \bar{B} be the event that it is
predicted not to rain

$$P(R|B) = \frac{P(B|R) \cdot P(R)}{P(B)}$$

$$P(B) = P(B|R)P(R) + P(B|\bar{R})P(\bar{R})$$

$$= \frac{0.9 \times 100/365}{0.9 \times 100/365 + 0.1 \times 265/365}$$

$$= \frac{0.9 \times 100}{0.9 \times 100 + 0.1 \times 265}$$

$$= \frac{90}{90 + 26.5} = 77.253\%$$

Homework 1 : Algorithmic robotics

1,2,3 - Written assignment

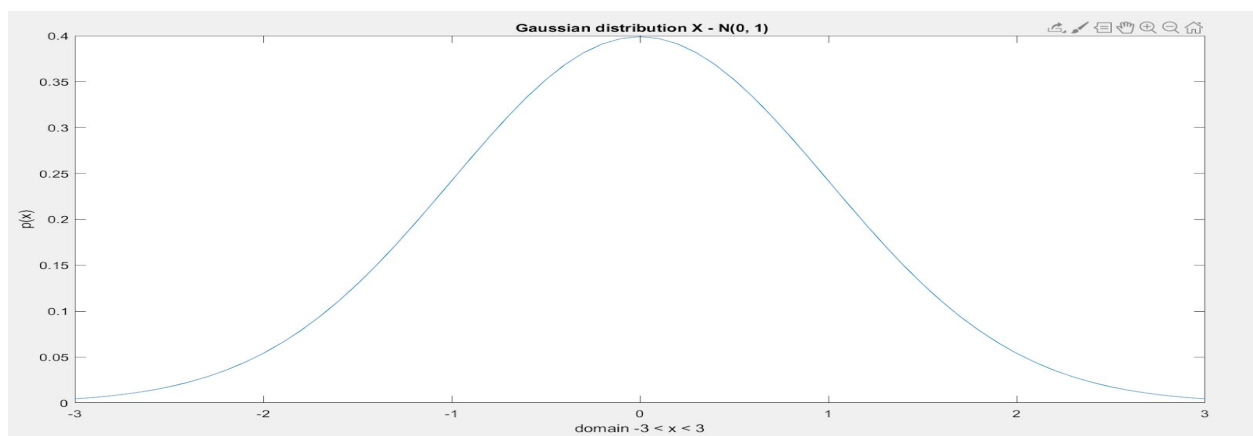
(P1)

Consider a Gaussian distribution $X \sim N(0, 1)$.

Generate 3 different sets of samples from this distribution for each of the three sample sizes $N = 100, N = 1,000$, and $N = 100,000$, and construct the corresponding histograms. Plot the resulting histograms and the underlying probability density function.

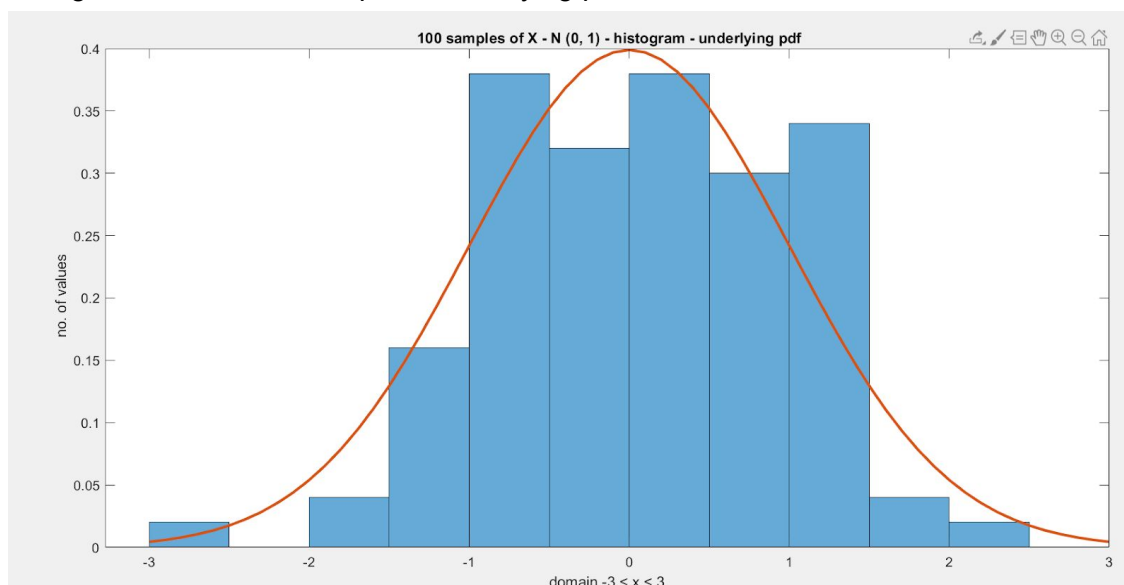
Solution :

Gaussian distribution $X \sim N(0, 1)$.

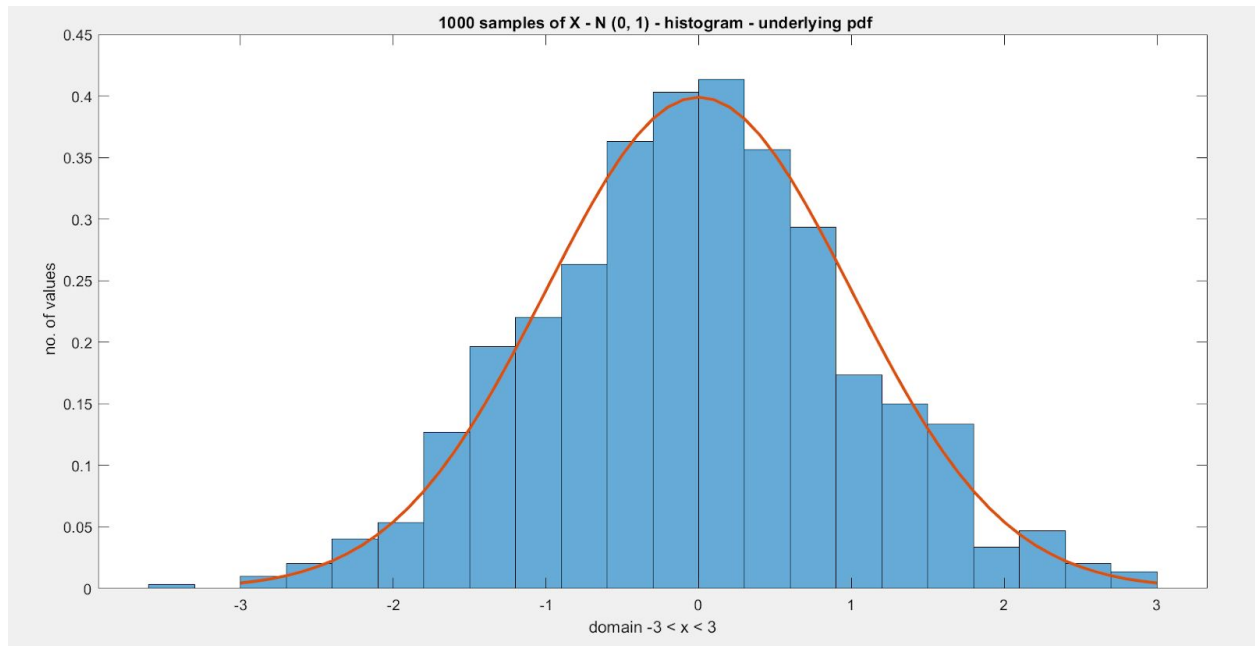


Generate 3 different sets of samples from this distribution for each of the three sample sizes $N = 100, N = 1,000$, and $N = 100,000$, and construct the corresponding histograms.

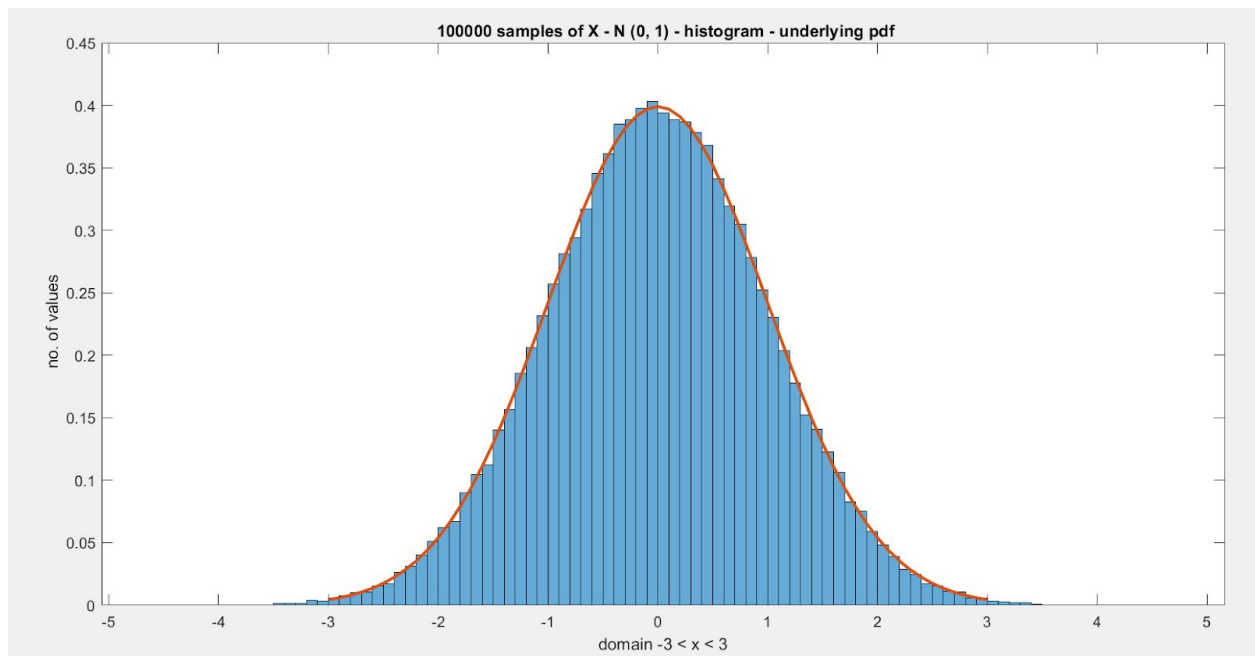
Histogram for hundred samples - underlying pdf in red



Histogram for thousand samples - underlying pdf in red



Histogram for hundred- thousand samples - underlying pdf in red

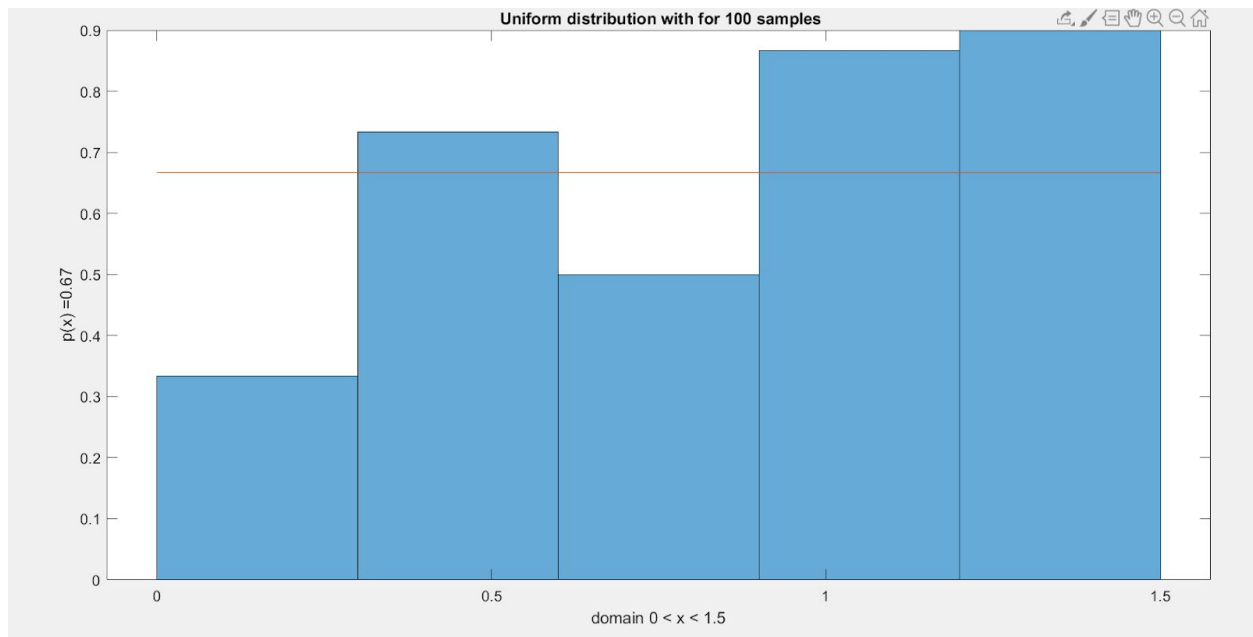


(P2)

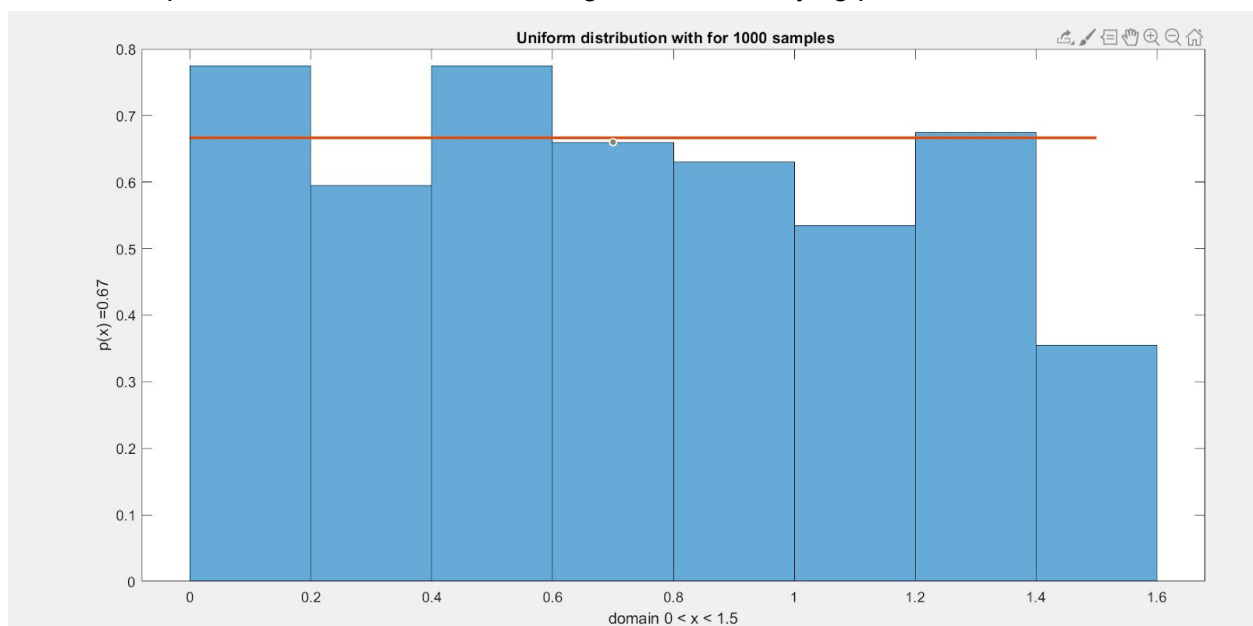
Consider a Uniform distribution $X \sim U(0, 1.5)$. Generate 3 different sets of samples from this distribution for each of the three sample sizes $N = 100, N = 1,000$, and $N = 100,000$, and construct the corresponding histograms. Plot the resulting histograms and the underlying probability density function.

Generate 3 different sets of samples from this distribution for each of the three sample sizes $N = 100, N = 1,000$, and $N = 100,000$

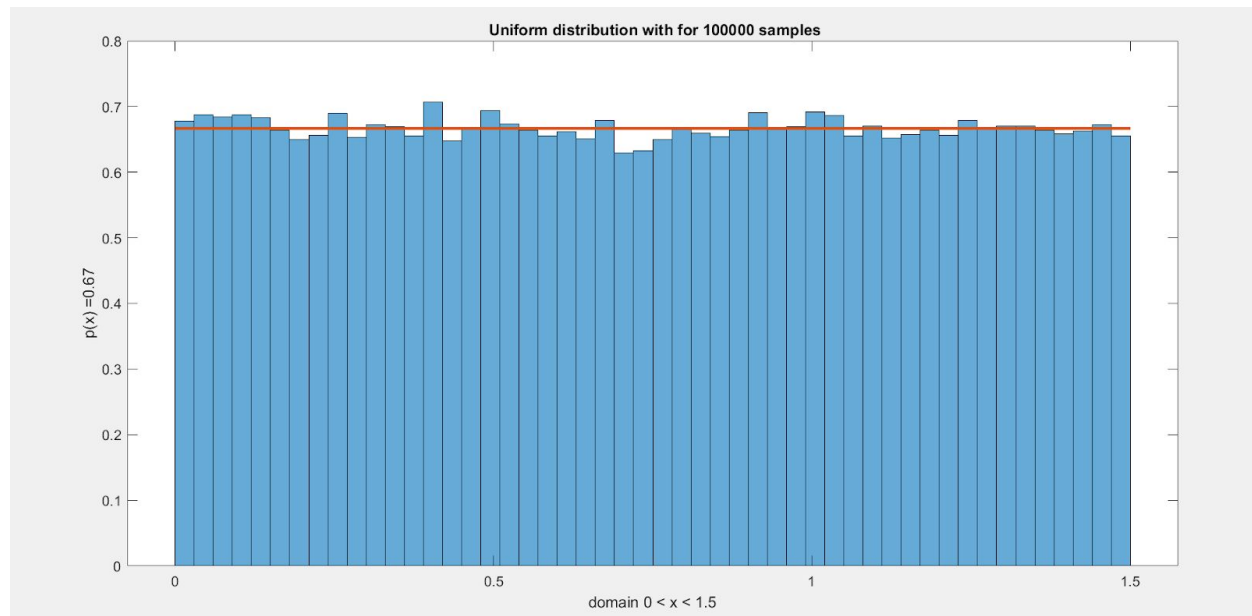
1- 100 samples - uniform distribution - histogram and underlying pdf



2- 1000 samples - uniform distribution - histogram and underlying pdf



3- 100000 samples - uniform distribution - histogram and underlying pdf



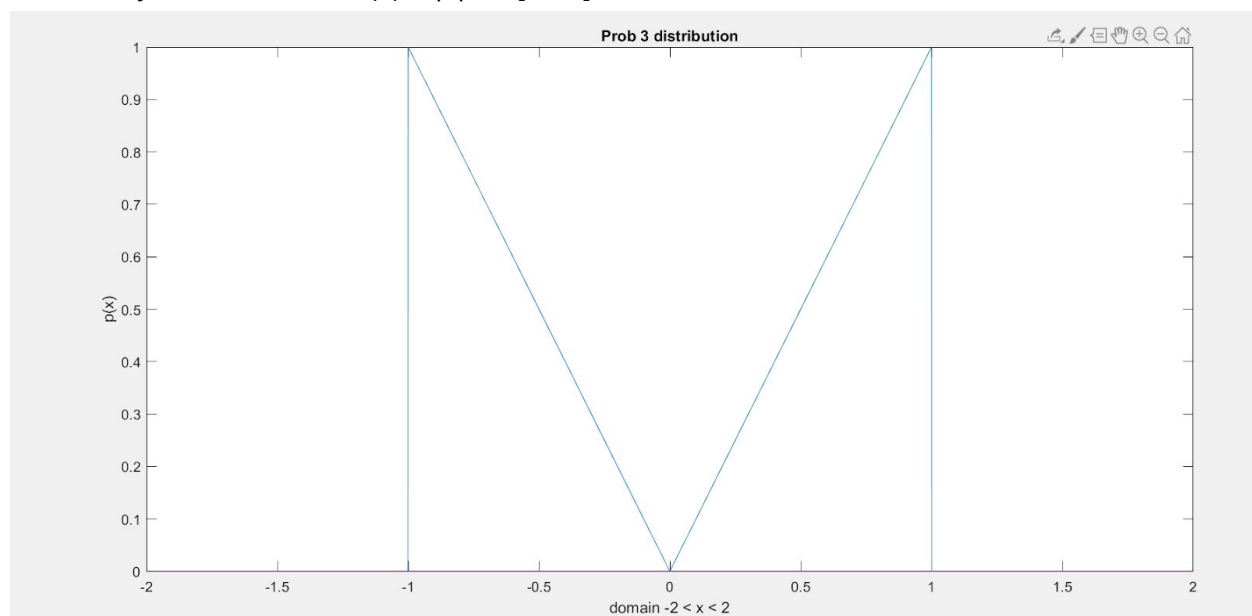
(P3)

Consider the probability distribution $X \sim f(x) = |x|$ $x \in [-1, 1]$, 0 otherwise

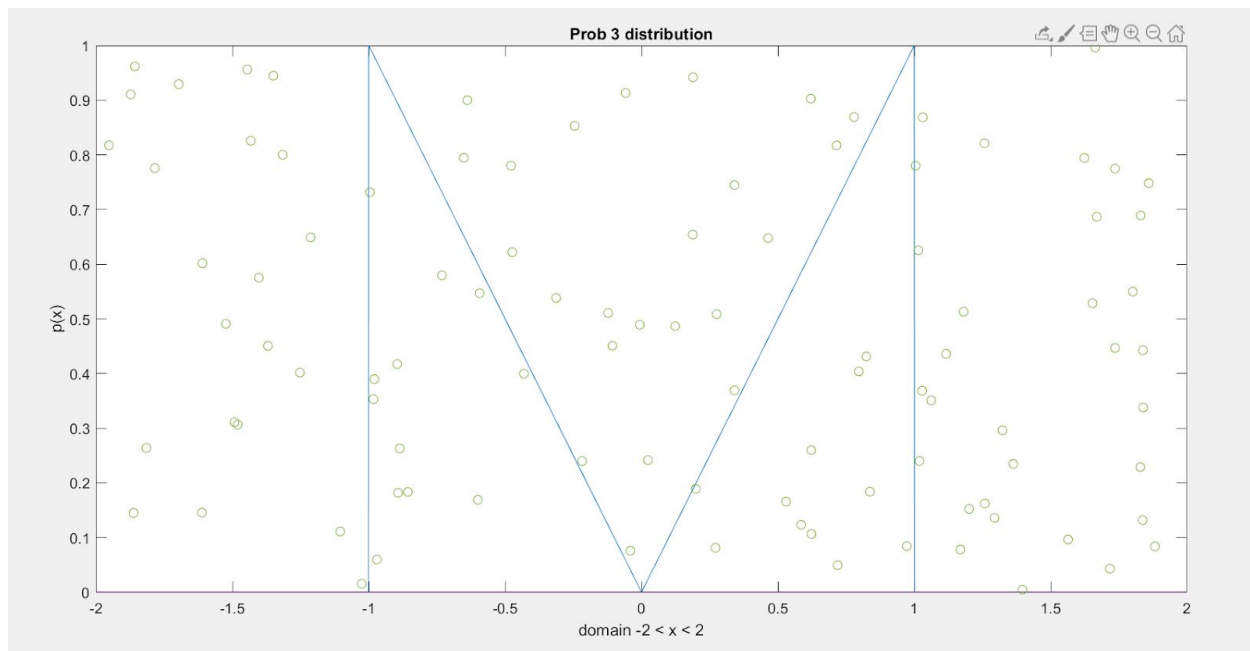
With a domain of $[-2, 2]$. Generate a set of samples from this distribution for a sample size of $N = 100,000$ using rejection sampling, and construct the corresponding histograms. Plot the resulting histogram and the underlying probability density function.

Solution:

Probability distribution $X \sim f(x) = |x|$ $x \in [-1, 1]$, 0 , otherwise



Inserted randomly generated uniform distribution points

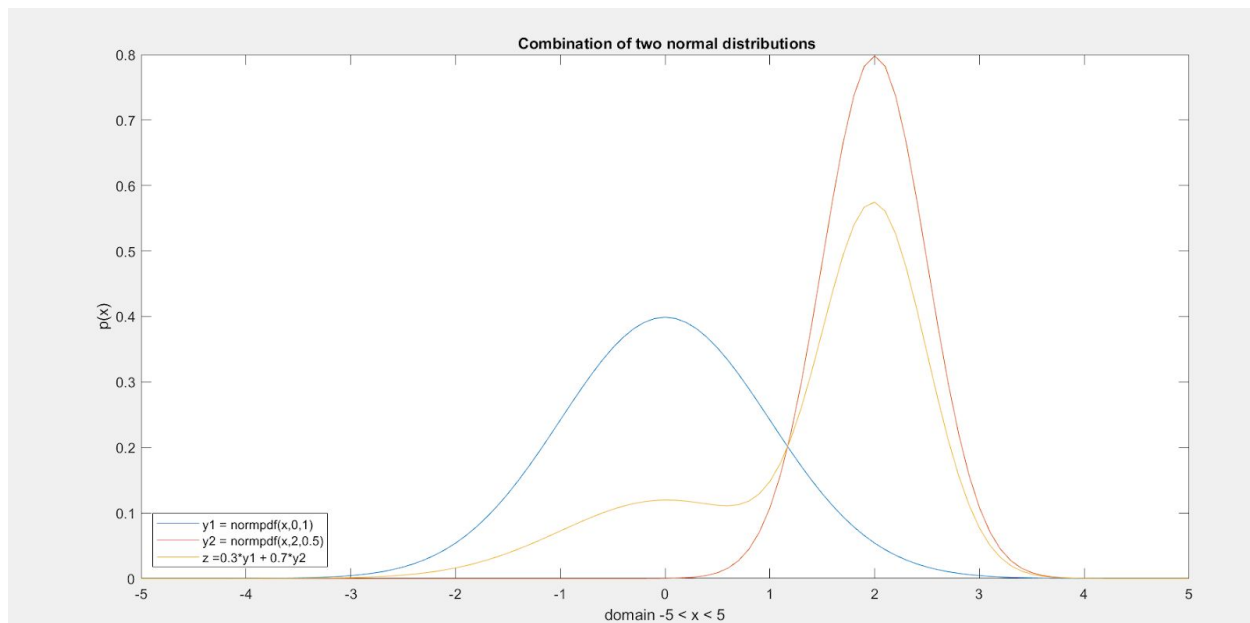


(P4)

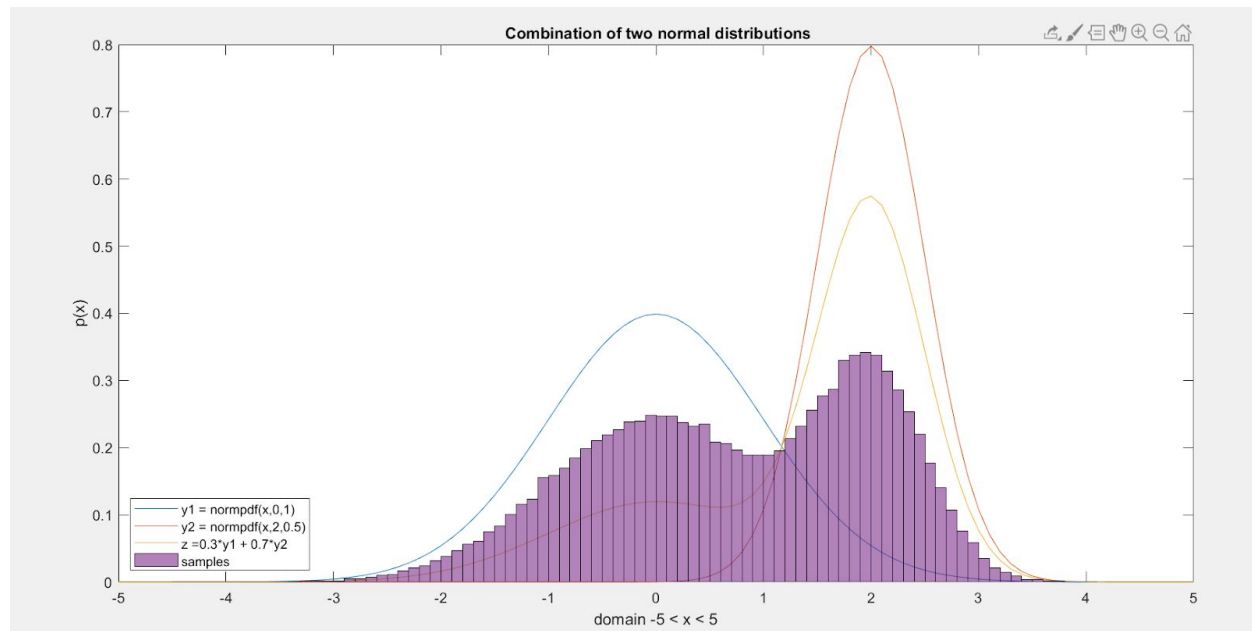
Consider a mixture of two Gaussians, $f(x) = \sum_{i=1}^2 p_i \cdot f_i$, where $p_1 = 0.3$, $p_2 = 0.7$ and $f_1 \sim N(0, 1)$, $f_2 \sim N(2, 0.5)$, with the domain of $[-5, 5]$. Generate the samples from this distribution using the sample size of $N = 100,000$, then construct the resulting histogram. Plot the resulting histogram and the underlying probability density function.

Solutions:

$f(x) = \sum_{i=1}^2 p_i \cdot f_i$, where $p_1 = 0.3$, $p_2 = 0.7$ and $f_1 \sim N(0, 1)$, $f_2 \sim N(2, 0.5)$, with the domain of $[-5, 5]$.



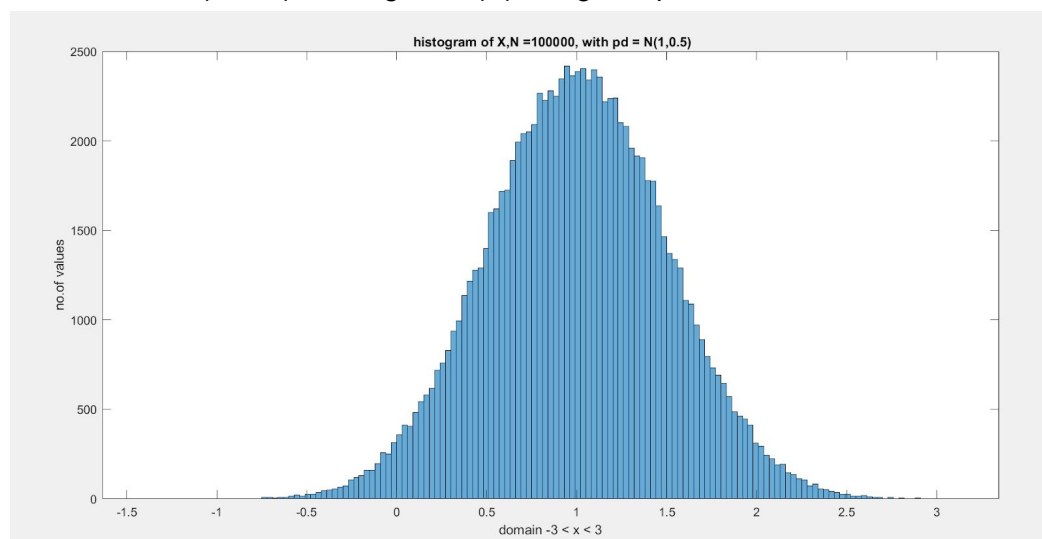
Generate 100000 samples from this distribution:



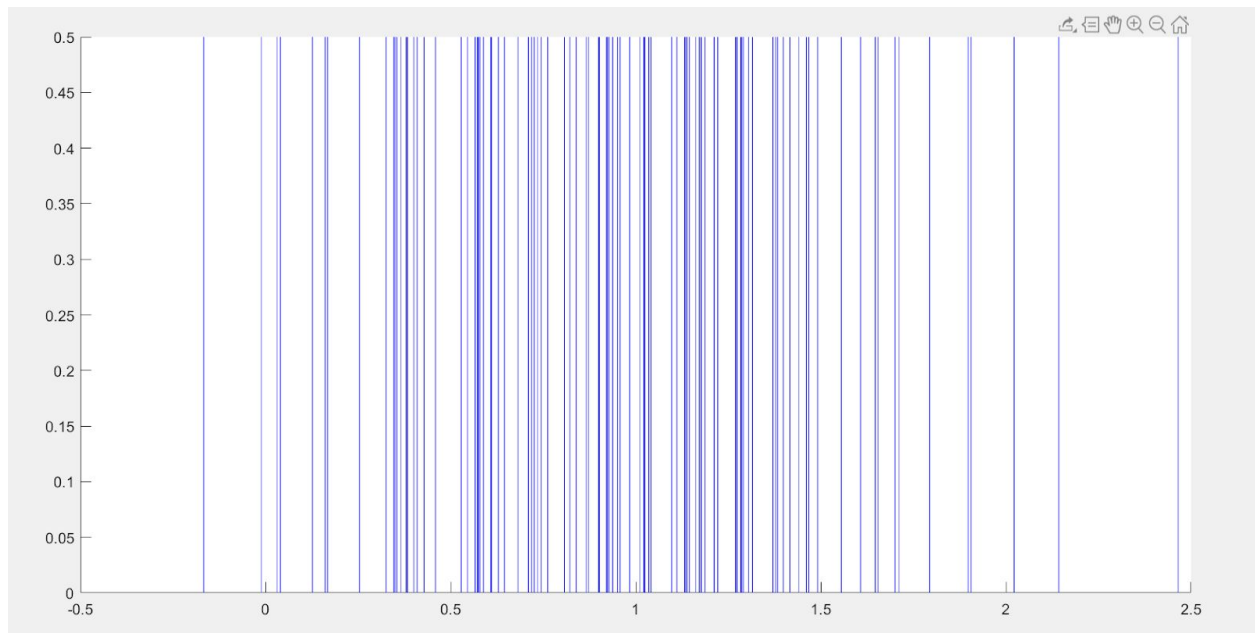
(P5) Consider a Gaussian distribution $X \sim N(1, 0.5)$ with the domain of $[-3, 3]$, generate its histogram $h(X)$ using sample size of $N = 100,000$. For any $x \in X$, there is a nonlinear transformation $y = x^2$, what is the histogram of Y ? Show your results of the histogram of X and the histogram of Y . Set the sample size $N = 1,000$, generate the histogram of X and the histogram of Y , then plot their corresponding density distribution based on the samples of X and Y (you can use "line" function to mark the density distribution, an example is given below).

Solution:

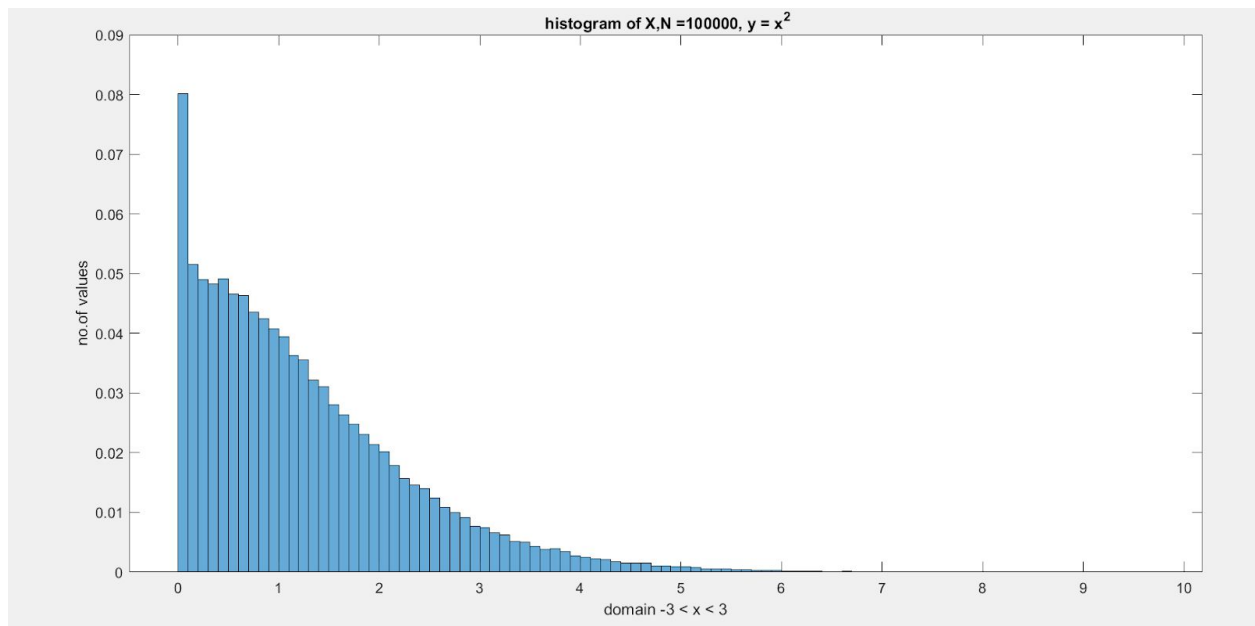
Generate $X \sim N(1, 0.5)$'s histogram $h(X)$ using sample size of $N = 100,000$



Density distribution of 100 samples of $pd = N(1, 0.5)$



$y = x^2$, what is the histogram of Y?



Density distribution for $n = 100$ samples

