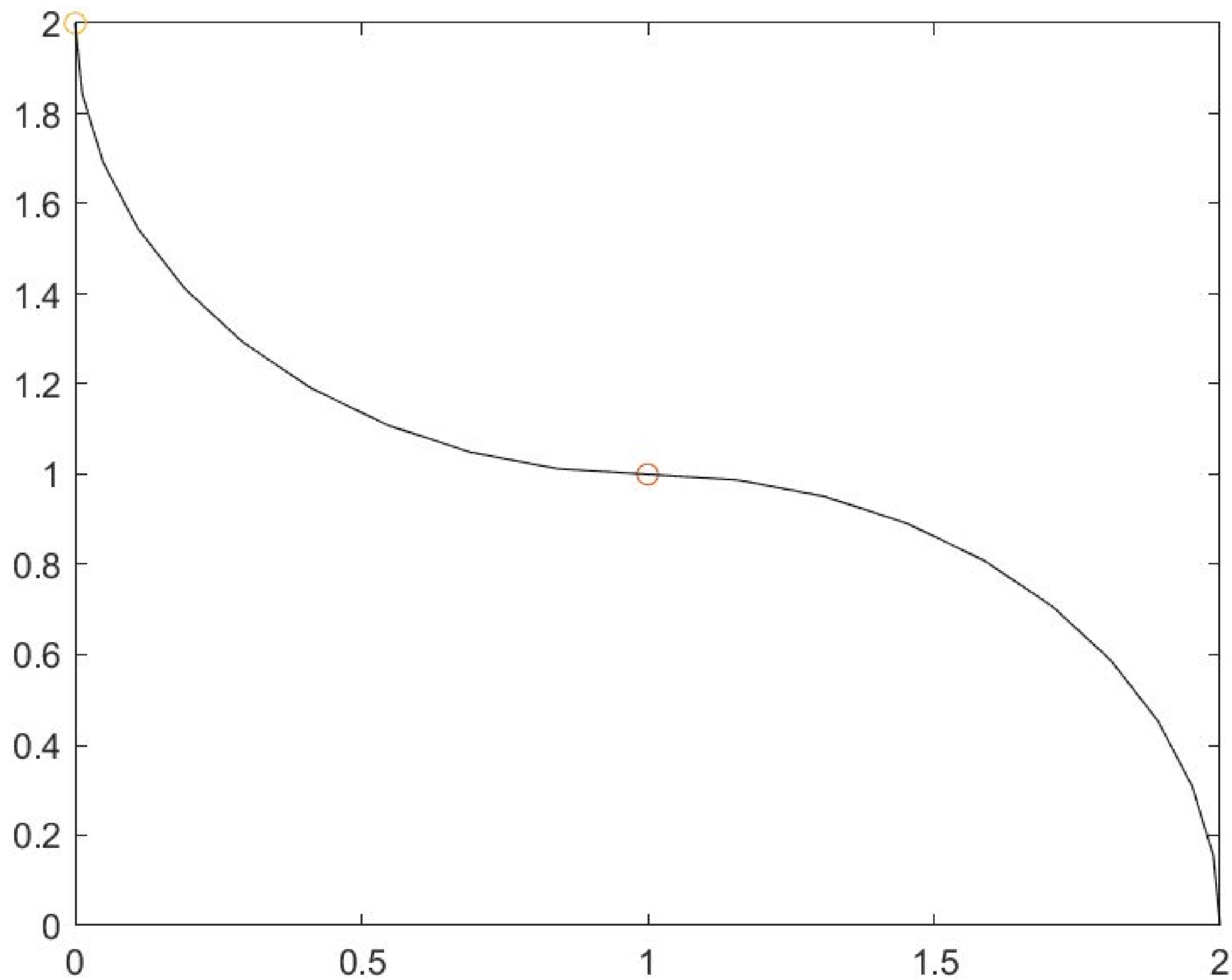
1 Velocity Motion Model

In this question, you are asked to implement the velocity motion model (Section 5.3 of PR) for a mobile robot with 3-DOF state space represented by $x_t = [x,\ y,\ \theta]^T$. The motion command applied to the robot in the time interval $[t_{i-1}, t_i)$ is represented by $u_{t_i} = [v_{t_i},\ \omega_{t_i}]^T$. The sampling period, $t_i - t_{i-1} = \Delta t = 1$ sec.

(1.a) Consider the scenario where the robot starts its motion at state $x_0 = [2,\ 0,\ \frac{\pi}{2}]^T$, and is given the motion commands $u_1 = [\frac{\pi}{2},\ \frac{\pi}{2}]^T$ and $u_2 = [\frac{\pi}{2},\ -\frac{\pi}{2}]^T$. What would be the state of the robot at the two subsequent time steps, namely x_1 and x_2, if the robot perfectly executes the motion commands, i.e., without any motion uncertainties or errors?

Sol 1.a



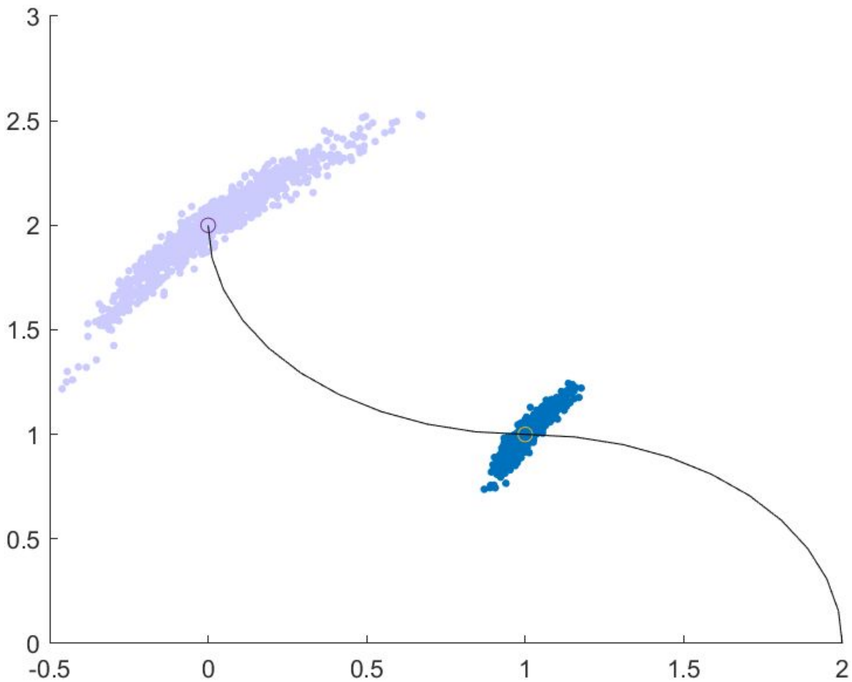
```
>> q1a

x1 =

    1.0000
    1.0000
    3.1416


x2 =

    0.0000
    2.0000
    1.5708
```

(1.b) Implement a Matlab function $x_t{}^i = \text{sample\_motion\_model\_velocity}(u_t,\ x_{t-1}, \alpha)$ which generates a sample $x_t{}^i$ of the robot state at time t, given the initial state x^i_{t-1}, motion command u_t, and motion uncertainty parameter vector $\alpha = [\alpha_1 ... \alpha_6]$, where the motion uncertainties are generated from a zero-mean normal distribution with standard deviations as defined in Equations 5.10 and 5.15 in PR.

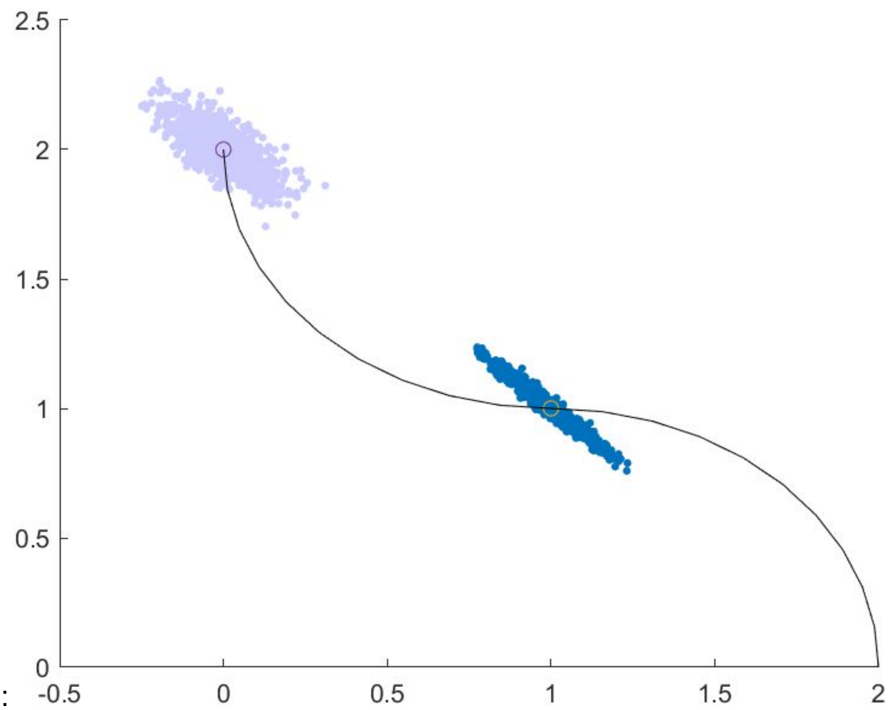
Generate a scatter plot of the samples of the states x_1 and x_2, for the nominal robot motion given in Part (1.a), for a sample size of N = 1000, and $\alpha = [0.0001,\ 0.0001,\ 0.01,\ 0.0001,\ 0.0001,\ 0.0001]$. Please also mark the nominal (noiseless) trajectory of the robot on your scatter plot. (See Figure 5.4 in PR for an example of a scatter plot).

Generate a second scatter plot for the case $\alpha = [0.005,\ 0.005,\ 0.0001,\ 0.0001,\ 0.0001,\ 0.0001]$.

Sol 1b



For alpha 1



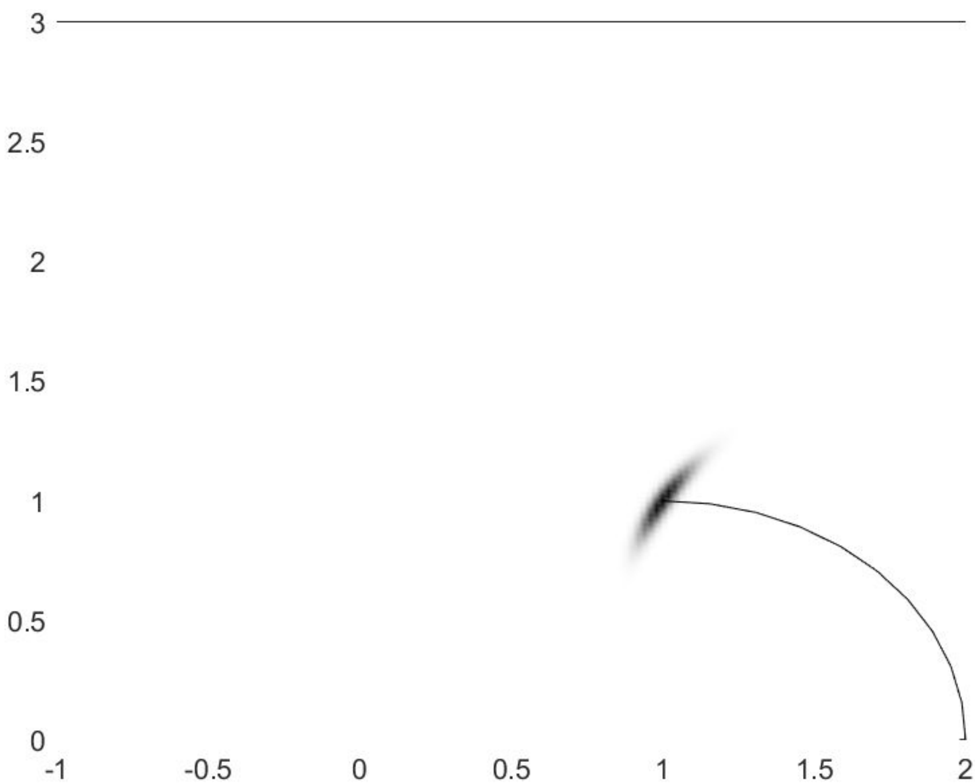
For alpha 2:

(**1.c**) Implement a Matlab function $p = \text{motion\_model\_velocity}(x_t,\ u_t,\ x_{t-1},\ \alpha)$ that evaluates the probability density function, $p(x_t \mid u_t,\ x_{t-1})$ of the velocity model, for a given vector $\alpha = [\alpha_1 ... \alpha_6]$, specifying the uncertainty standard deviations as defined in Equations 5.10 and 5.15 in PR.

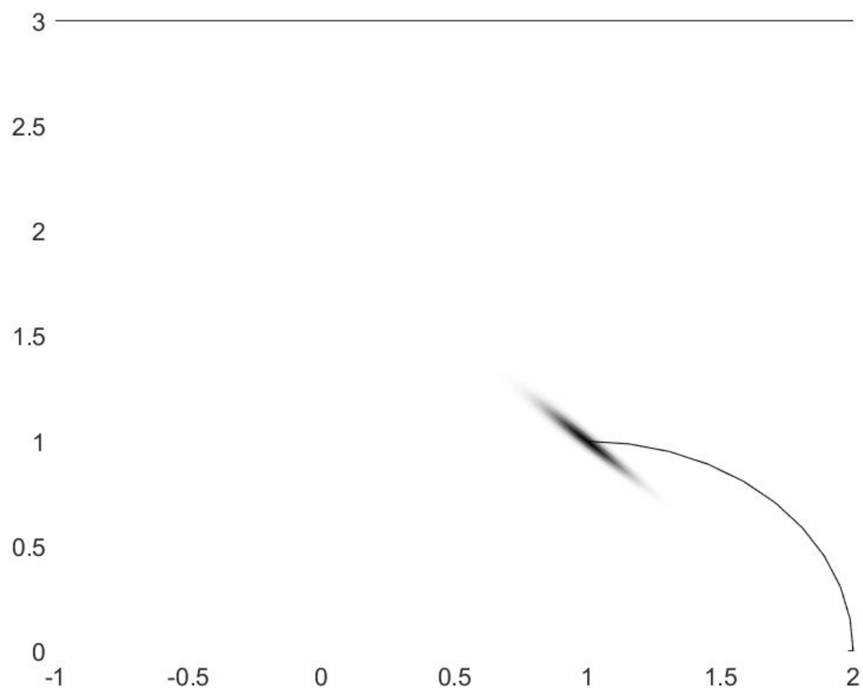
Plot the magnitude of the probability density function as a function of the $(x,\ y)$ coordinates (summing up the probabilities in θ dimension), for $\alpha = [0.0001,\ 0.0001,\ 0.01,\ 0.0001,\ 0.0001,\ 0.0001]$ and the motion command u_1 given in Part (1.b). Please also mark the nominal (noiseless) trajectory of the robot on your scatter plot. (See Figure 5.3 in PR for an example of a density plot).

Generate a second density plot for the case $\alpha = [0.005,\ 0.005,\ 0.0001,\ 0.0001,\ 0.0001,\ 0.0001]$, for the same robot motion.

For alpha 1:



For alpha 2:



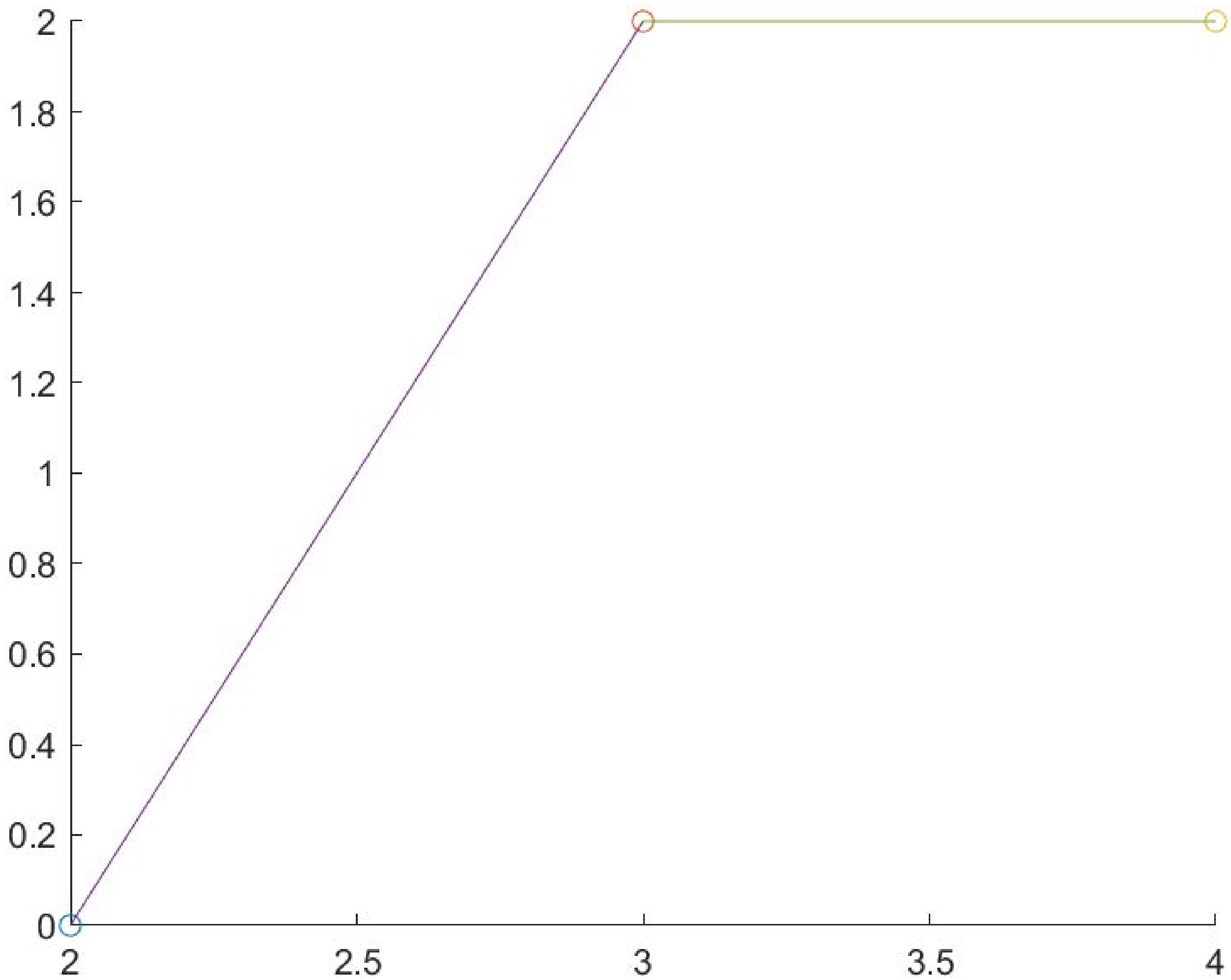
2 Odometry Motion Model

In this section, you will need to implement the odometry motion model of the mobile robot with state space represented by $(x,\ y,\ \theta)$ (Section 5.4 in PR). Similar to assignment 1, the initial state of the robot is $x_0 = [2,\ 0,\ \frac{\pi}{2}]^T$, and the sampling period is $t_i - t_{i-1} = \Delta t = 1$ sec.

(2.a) Suppose the robot's odometry subsystem reports the following poses: $\bar{x}_0 = [1,\ 0,\ 0]^T, \bar{x}_1 = [3,\ -1,\ -1.571]^T$, and $\bar{x}_2 = [3,\ -2,\ 0]^T$. What would be the state of the robot at the two subsequent time steps, namely x_1 and x_2, if the robot's odometry did not have any errors measuring the relative motion of the robot?

Sol 2.a

```
Xf1 =

    3.0000    2.0000   -0.0002


Xf2 =

    4.0000    2.0000    1.5708
```

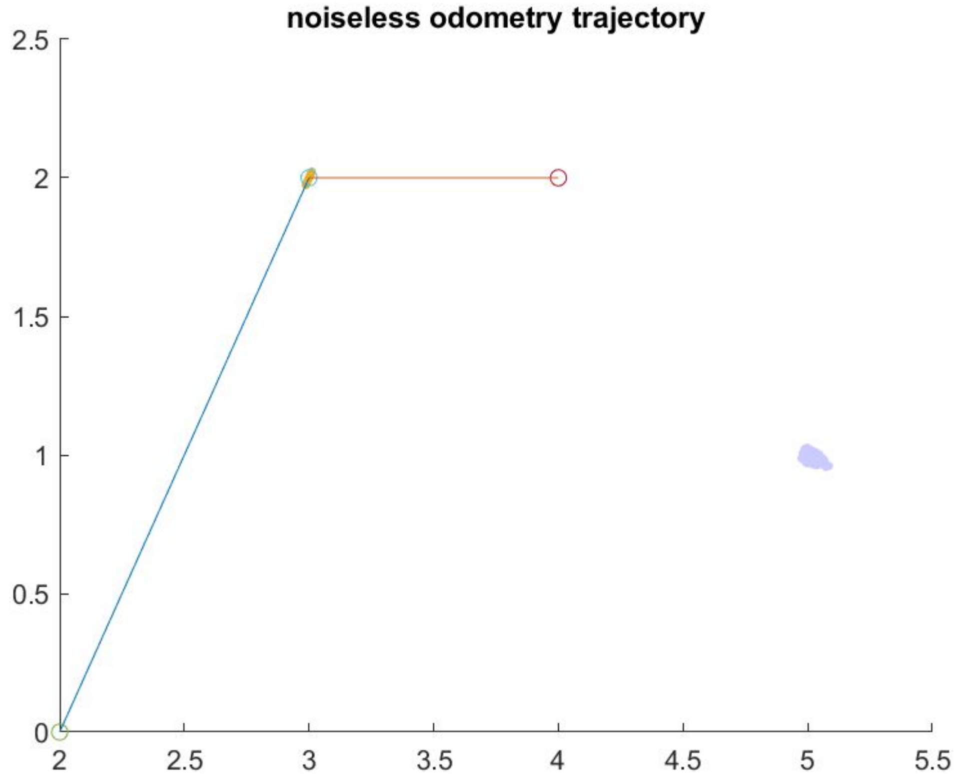


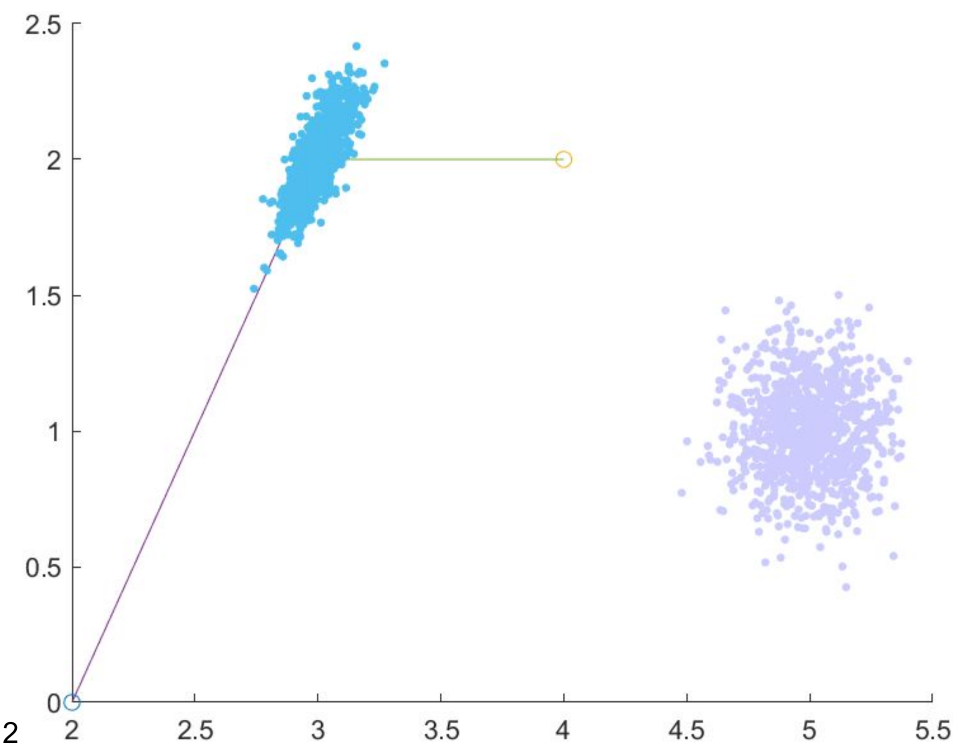
(2.b) Implement a Matlab function $x_t{}^i = \text{sample\_motion\_model\_odometry}(u_t,\ x_{t-1}, \alpha)$, which generates a sample $x_t{}^i$ of the robot state at time t, given the initial state x_{t-1}, motion command u_t, and motion uncertainty parameter vector $\alpha = [\alpha_1 ... \alpha_4]$, where the motion uncertainties are generated from a zero-mean normal distribution with standard deviations as defined in Equations 5.37-5.39 in PR, and the motion command $u_t = \begin{bmatrix} \bar{x}_{t-1} \\ \bar{x}_t \end{bmatrix}$, as defined in Equation 5.33 in PR.

Generate a scatter plot of the samples of the states x_1 and x_2, for the nominal robot motion given in Part (2.a), for a sample size of N = 1000, and $\alpha = [0.01,\ 0.002,\ 0.0001,\ 0.0001]$. Please also mark the nominal (noiseless) trajectory of the robot on your scatter plot. (See Figure 5.9 in PR for an example of a scatter plot).

Generate a second scatter plot for the case $\alpha = [0.0001,\ 0.0002,\ 0.01,\ 0.0001]$.

Sol 2b - For alpha 1





For alpha 2

(2.c) Implement a Matlab function $p = \text{motion\_model\_odometry}(x_t,\ u_t,\ x_{t-1},\ \alpha)$ that evaluates the probability density function, $p(x_t \mid u_t,\ x_{t-1})$ of the velocity model, for a given motion command u_t, and motion uncertainty parameter vector α, as defined in Part (2.b). Plot the magnitude of the probability density function as a function of the $(x,\ y)$ coordinates (summing up the probabilities in θ dimension), for the motion command u_1 given in Part (2.b), and $\alpha = [0.01,\ 0.002,\ 0.0001,\ 0.0001]$. Please also mark the nominal (noiseless) trajectory of the robot on your scatter plot. (See Figure 5.8 in PR for an example of a density plot).

Generate a second density plot for the case $\alpha = [0.0001,\ 0.0002,\ 0.01,\ 0.0001]$, for the same robot motion.

3 Beam-Based Measurement Model

In this section, you will need to implement the beam-based measurement model (Section 6.3 of PR) of the mobile robot with state space represented by $(x,\ y,\ \theta)$. The initial state of the robot is given as $x_0 = [2, 0, \frac{\pi}{2}]^T$. Suppose there is a continuous wall at $m = \{(x,\ y) | x \in [0,3],\ y = 3\}$, and the robot has a beam-based range sensor with bearing angle equal to the heading of the robot and a maximum range of 5. The range sensor's noise parameters are given as: $\sigma^2_{\text{hit}} = 0.05$, $\lambda_{\text{short}} = 0.05$, $[z_{\text{hit}}, z_{\text{short}}, z_{\text{max}}, z_{\text{rand}}] = [0.8, 0.05, 0.05, 0.1]$.

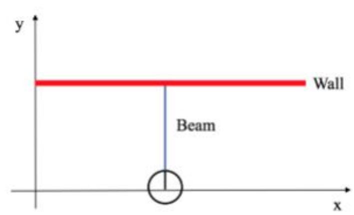


Figure 1: The initial configuration of the robot. A single beam is marked with a blue line, with global bearing angle $\phi = \frac{\pi}{2}$.

Using the velocity motion model with the motion command $u_1 = [v,\ w]^T = [0.5,\ 0]^T$, compute the "true" distance or range z_1^* of the wall measured by the beam z_{t_2} and generate a plot of the probability density function $p(z_1 \mid \bar{x}_1,\ m)$ of the proximity sensor measurement (you can refer to Figure 6.4). Given $u_2 = [0.2,\ 0.76]^T$, similarly compute the "true" range z_2^*, and plot $p(z_2 \mid \bar{x}_2,\ m)$.

4 Landmark-Based Measurement Model

In this section, you will need to implement the landmark-based measurement model (Section 6.6 of PR) for a mobile robot with state space represented by $(x,\ y,\ \theta)$. The initial state of the robot is given as $x_0 = [2, 0, \frac{\pi}{2}]^T$. Suppose the robot is equipped with *range and bearing sensors*. A set of discrete landmarks with coordinates marked as $m = [m_1,\ m_2,\ m_3,\ m_4]$ exist in the world. The global location of each landmark are given as $m_1 = [1,\ 1]^T$, $m_2 = [2,\ 1]^T$, $m_3 = [3,\ 2]^T$, $m_4 = [2,\ 4]^T$. The size of landmarks are small enough to be ignored. The landmarks are assumed to be uniquely identifiable, and as such, the correspondence between the landmarks and measurements are assumed to be known. The sensor reading report feature vectors of the form $z_t^i = (r_t^i,\ \phi_t^i)$, where r_t^i and ϕ_t^i are the range and bearing of the i-th landmark at time t. The standard deviations of the measurement uncertainty of the range and bearing measurements (as defined in Equation 6.40 of PR) are given as $\sigma_r = 0.3$ and $\sigma_\phi = 0.5$. The errors in bearing and range measurements and measurements for different landmarks are assumed be conditionally independent, given the state of the robot and the map.

At x_0, the sensor reports two valid readings $z_0 = \{z_0^1,\ z_0^2\} = \{\ (1.6213, 2.430),\ (1.1270, 1.4140)\ \}$. What is the likelihood, $p(z_0 \mid x_0,\ m)$, for this set of sensor readings?

During the next time step, the robot moves following the velocity motion model with the motion command $u_1 = [v,\ w]^T = [1.2,\ -1.0]^T$. Following this motion, at state x_1, the sensor reports two valid readings $z_1 = \{z_1^2,\ z_1^3\} = \{\ (0.5100, -2.6801),\ (1.0270, 1.405)\ \}$. What is the likelihood, $p(z_1 \mid \bar{x}_1,\ m)$, of this new set of sensor readings, z_1?

```
ans1 =

   6.6548e-05


ans2 =

    0.0351
```