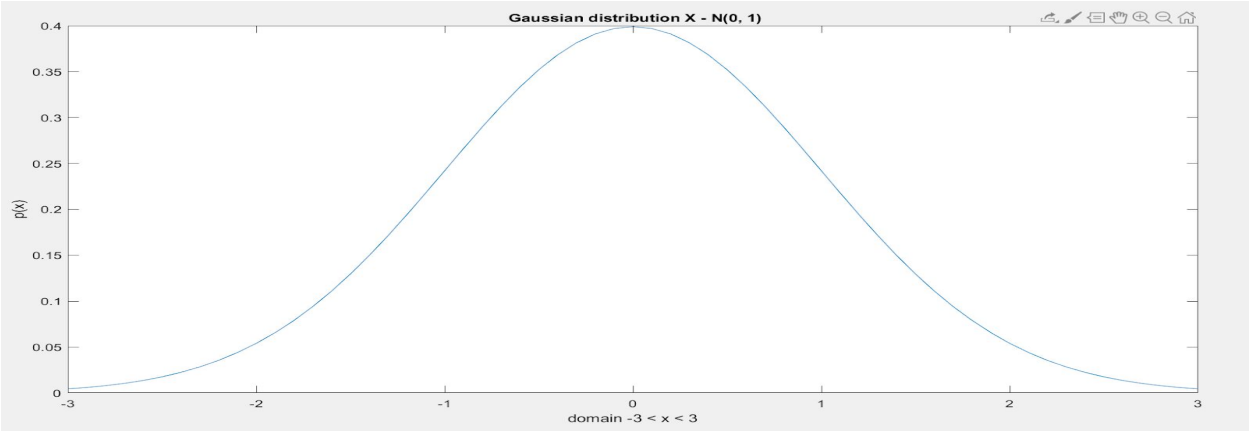
Homework 1 : Algorithmic robotics

1,2,3 - Written assignment

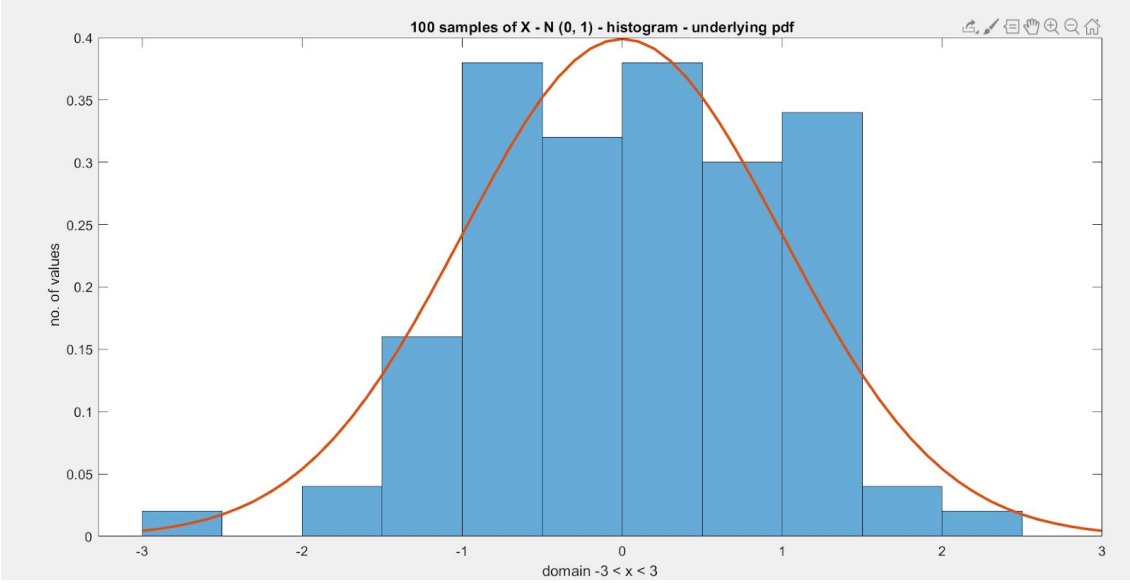
(P1)
Consider a Gaussian distribution X - N(0, 1).
Generate 3 different sets of samples from this distribution for each of the three sample sizes N = 100,N =1,000, and N =100,000,and construct the corresponding histograms. Plot the resulting histograms and the underlying probability density function.

Solution :
Gaussian distribution X - N(0, 1).

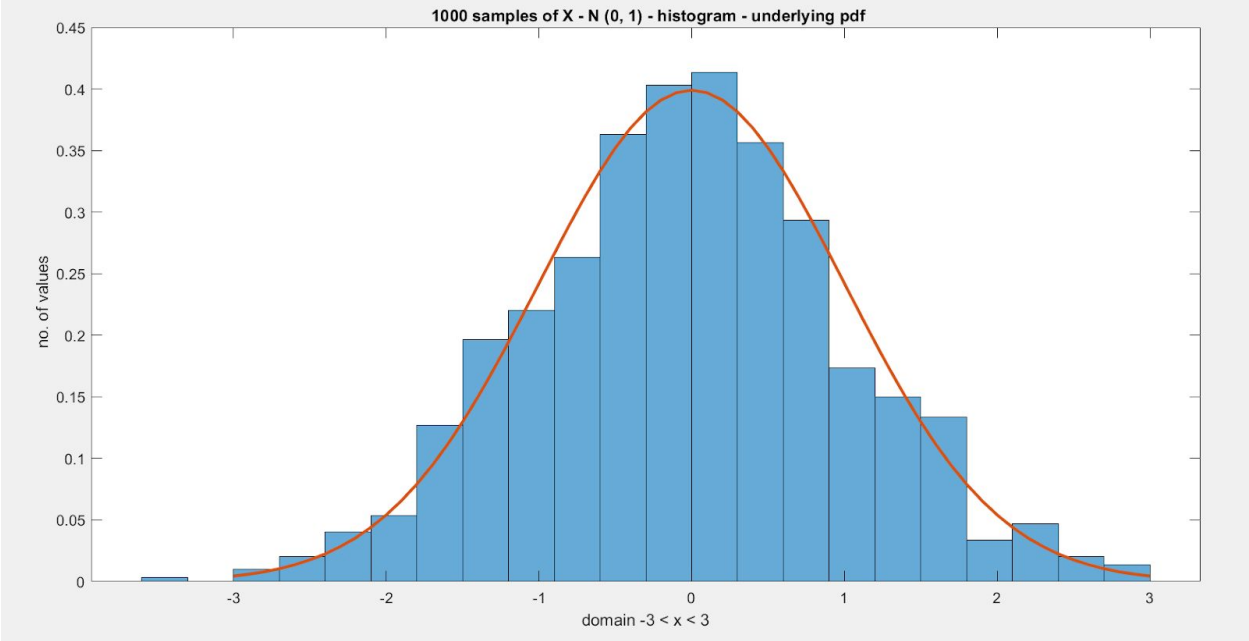


Generate 3 different sets of samples from this distribution for each of the three sample sizes N = 100,N =1,000, and N =100,000 ,and construct the corresponding histograms.

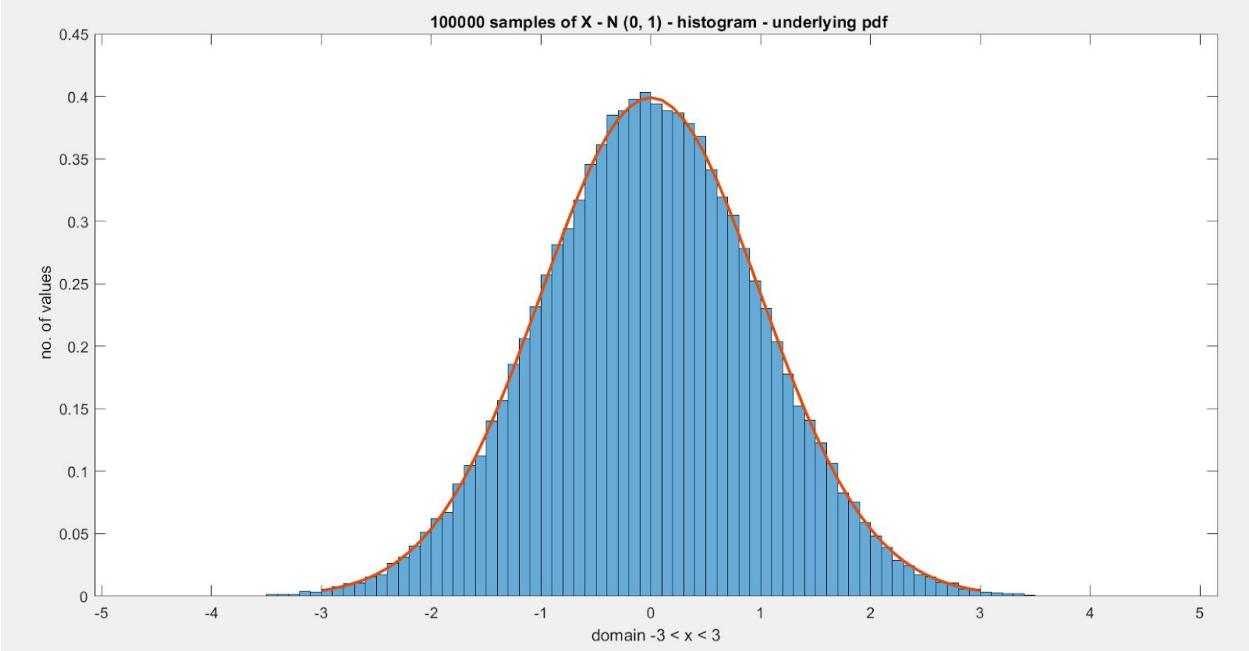
Histogram for hundred samples - underlying pdf in red



Histogram for thousand samples - underlying pdf in red



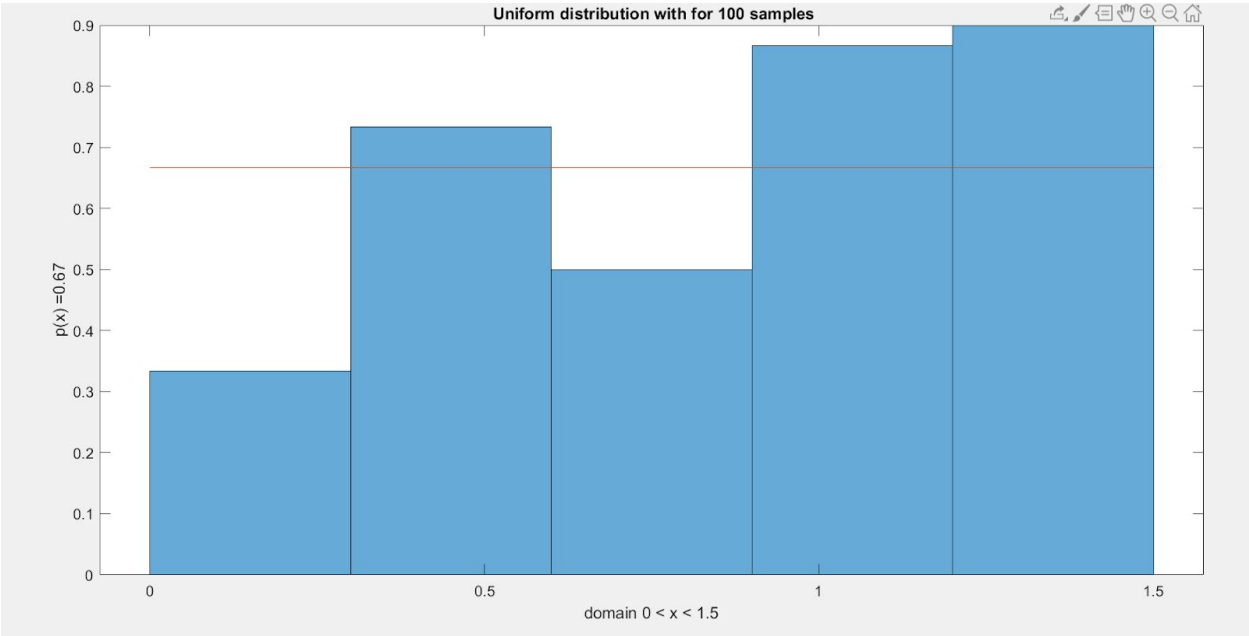
Histogram for hundred- thousand samples - underlying pdf in red



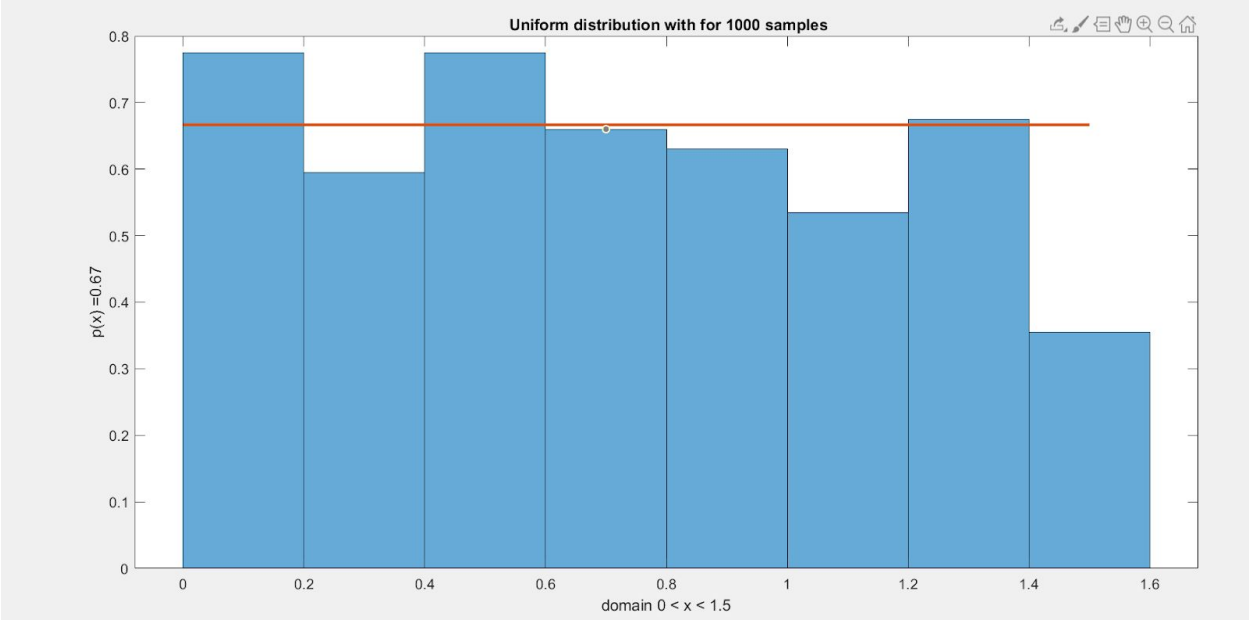
(P2)
Consider a Uniform distribution X - U (0,1.5). Generate 3 different sets of samples from this distribution for each of the three sample sizes N =100,N =1,000,and N =100,000,and construct the corresponding histograms. Plot the resulting histograms and the underlying probability density function.

Generate 3 different sets of samples from this distribution for each of the three sample sizes N =100,N =1,000,and N =100,000

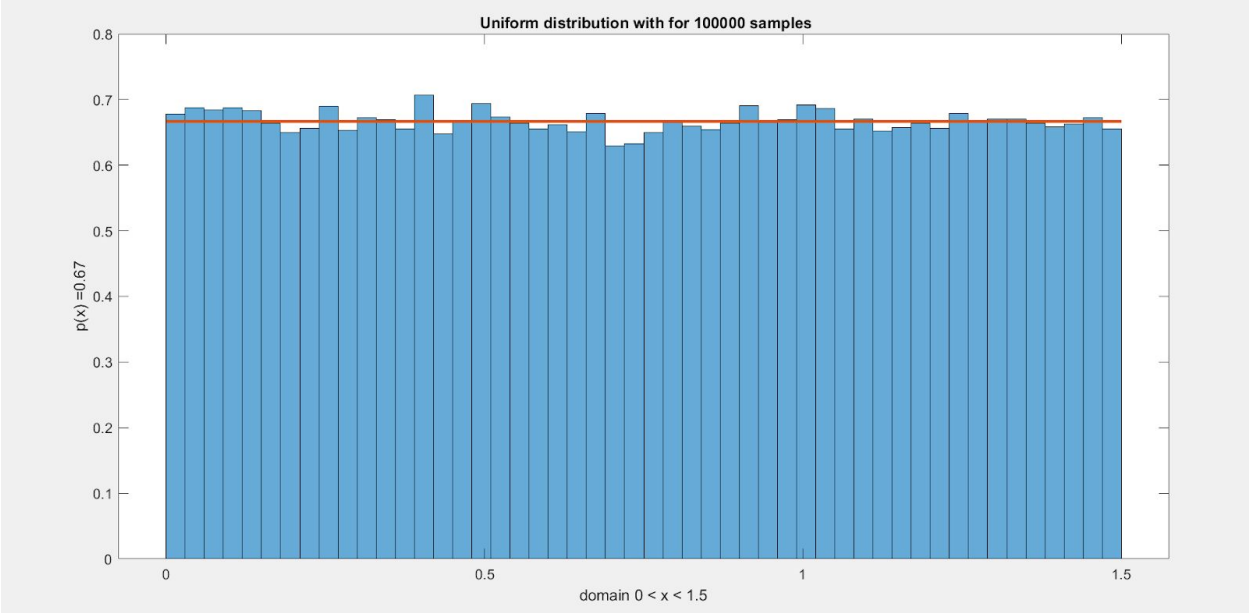
1- 100 samples - uniform distribution - histogram and underlying pdf



2- 1000 samples - uniform distribution - histogram and underlying pdf

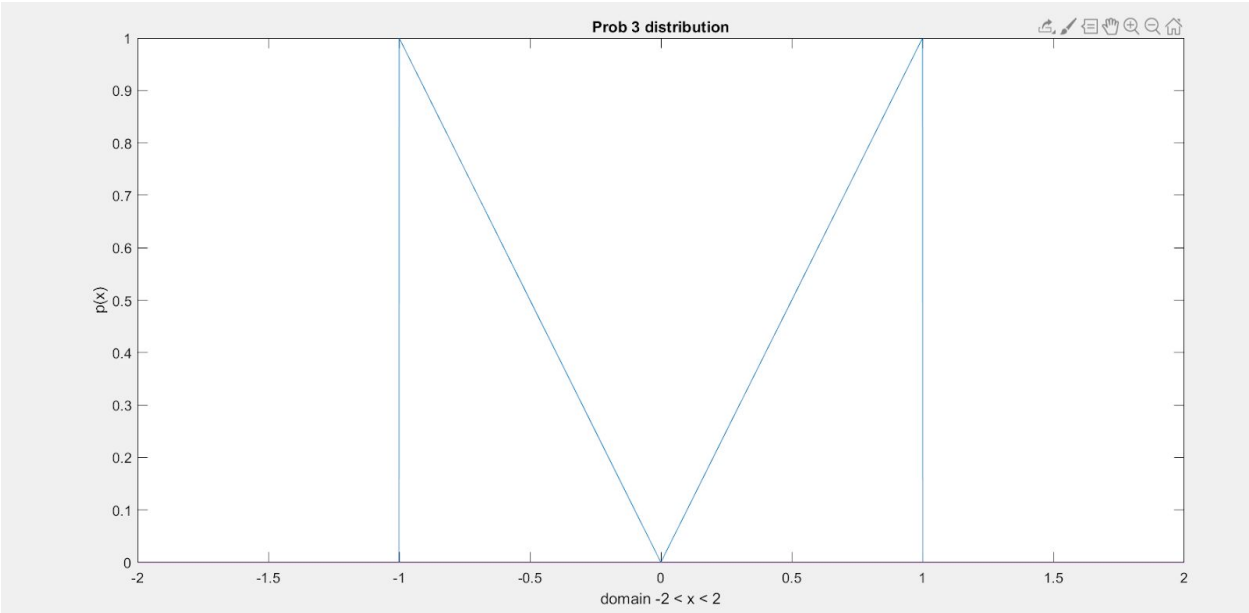


3- 100000 samples - uniform distribution - histogram and underlying pdf

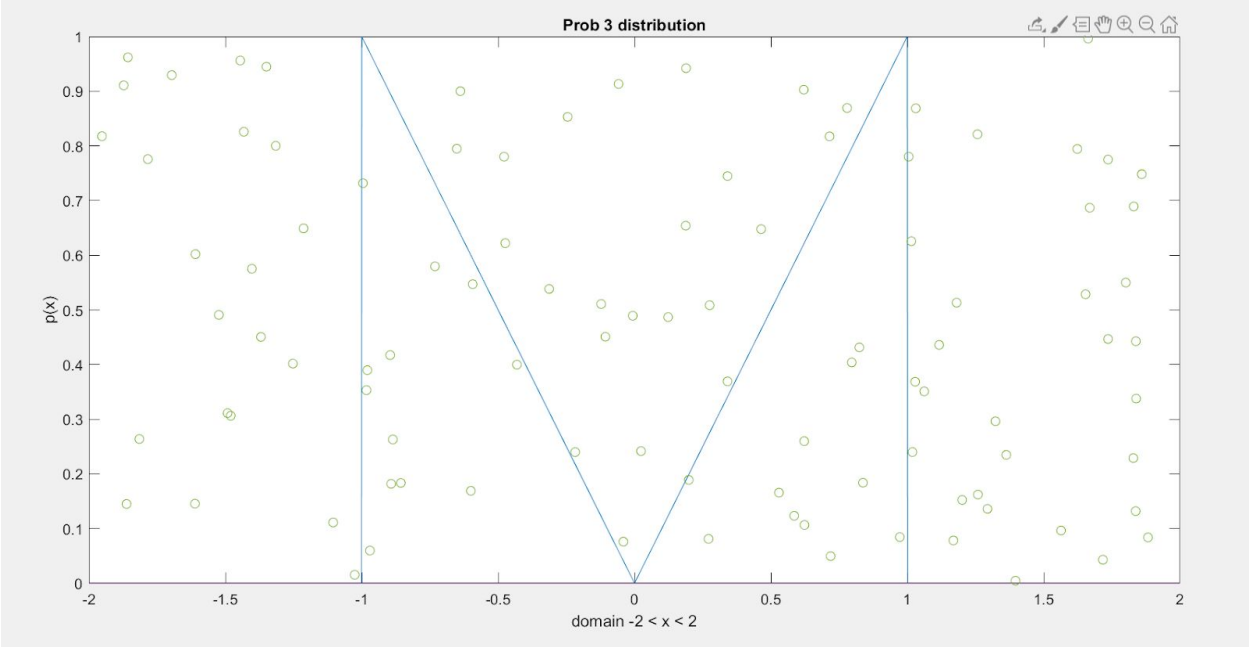


(P3)
Consider the probability distribution X ~ f(x) = |x| x∈[−1,1], 0 otherwise
With a domain of [−2, 2]. Generate a set of samples from this distribution for a sample size of N =100,000 using rejection sampling, and construct the corresponding histograms. Plot the resulting histogram and the underlying probability density function.

Solution:
Probability distribution X ~ f(x) = |x| x∈[−1,1], 0 , otherwise



Inserted randomly generated uniform distribution points

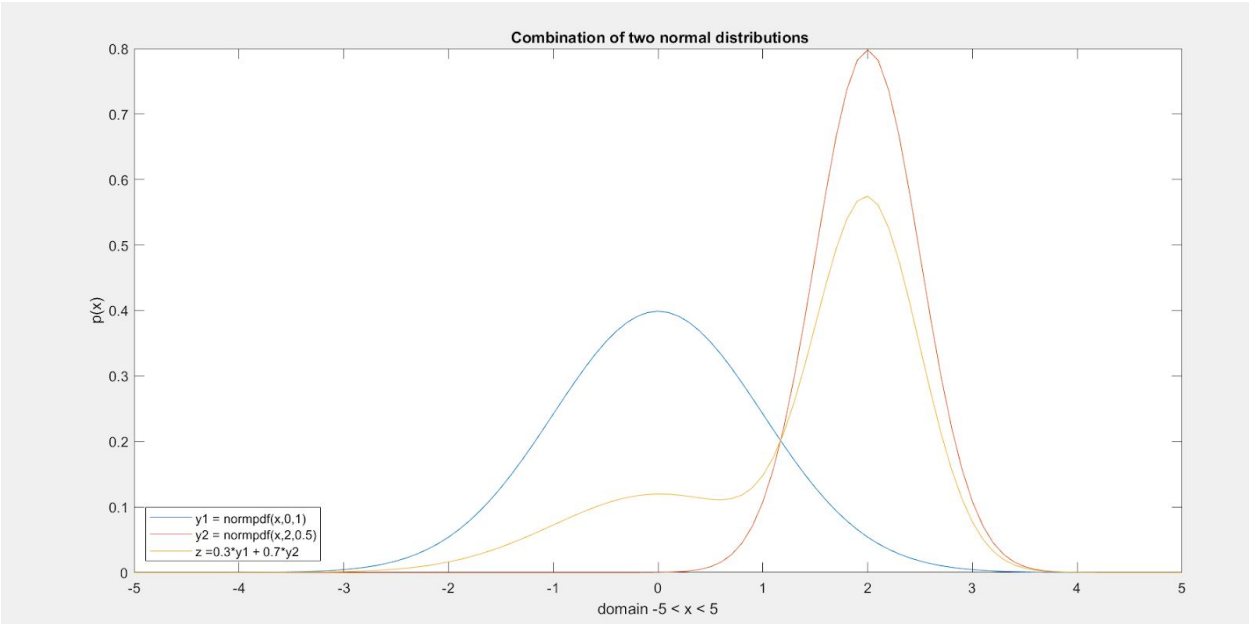


(P4)

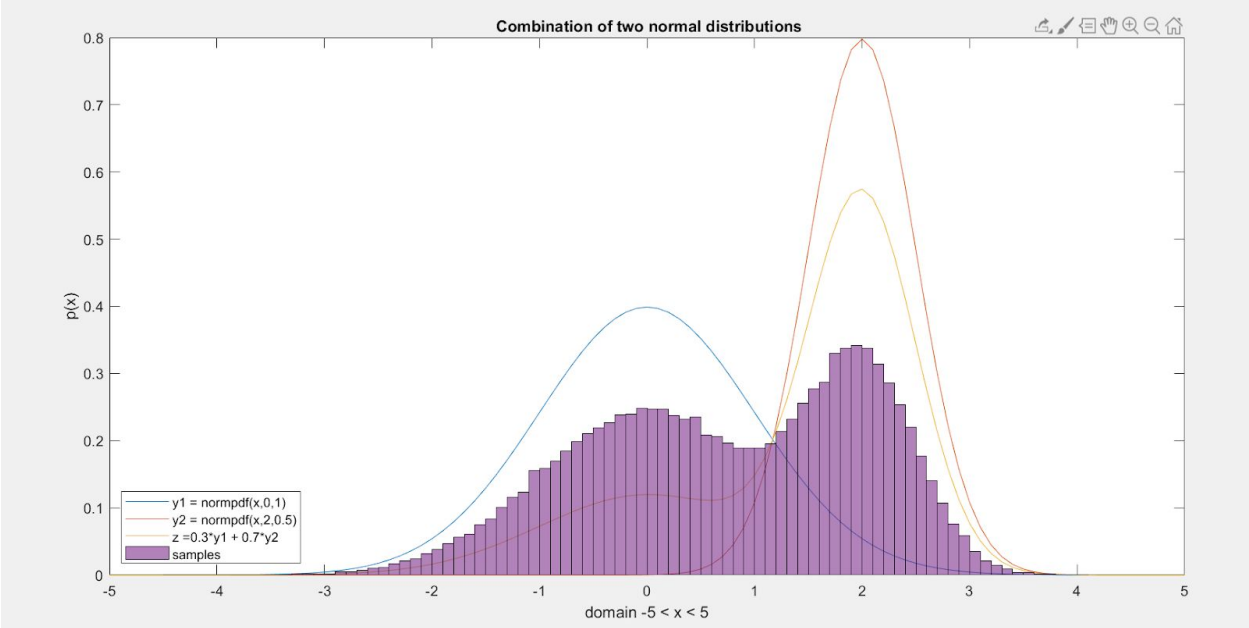
Consider a mixture of two Gaussians, f(x) = ∑2 i=1 pi * fi, where p1 = 0.3, p2 = 0.7 and f1 ~N(0, 1), f2 ~ N(2, 0.5), with the domain of [−5, 5]. Generate the samples from this distribution using the sample size of N = 100,000, then construct the resulting histogram. Plot the resulting histogram and the underlying probability density function.

Solutions:

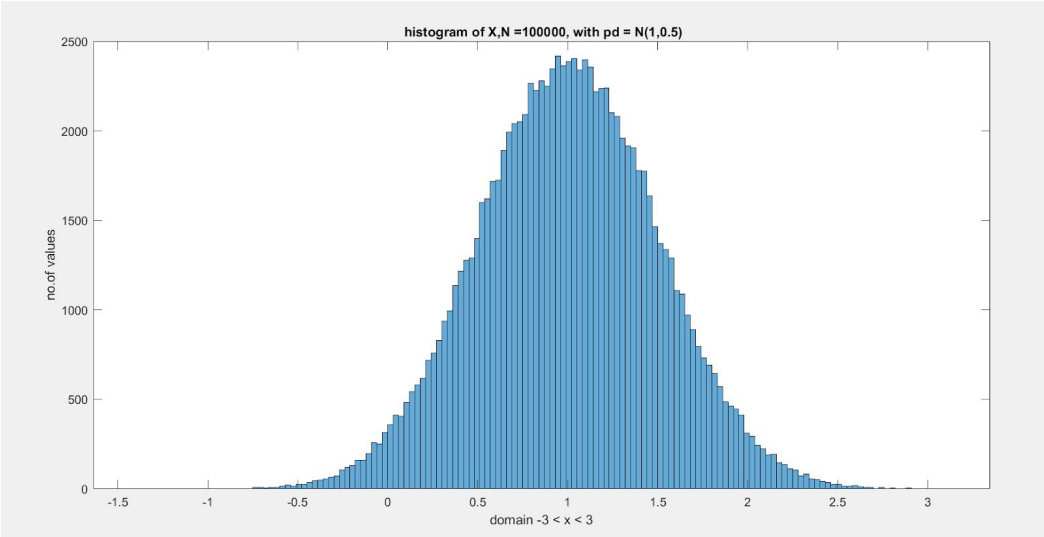
f(x) = ∑2 i=1 pi * fi, where p1 = 0.3, p2 = 0.7 and f1 ~N (0, 1), f2 ~ N (2, 0.5), with the domain of [−5, 5].



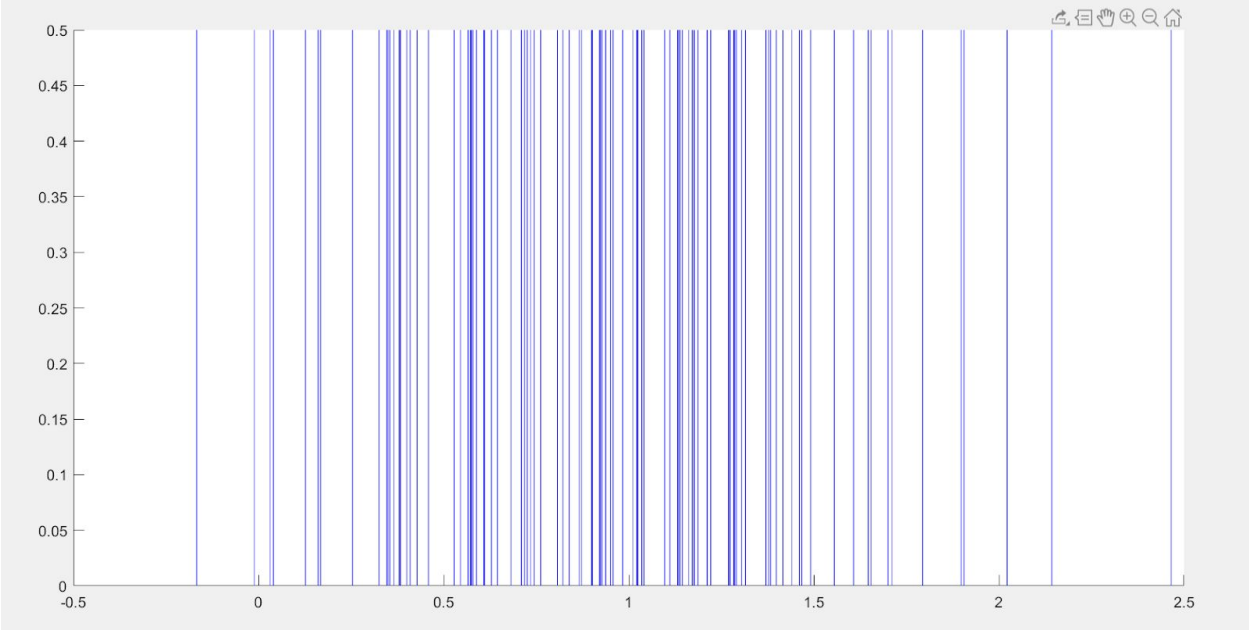
Generate 100000 samples from this distribution:



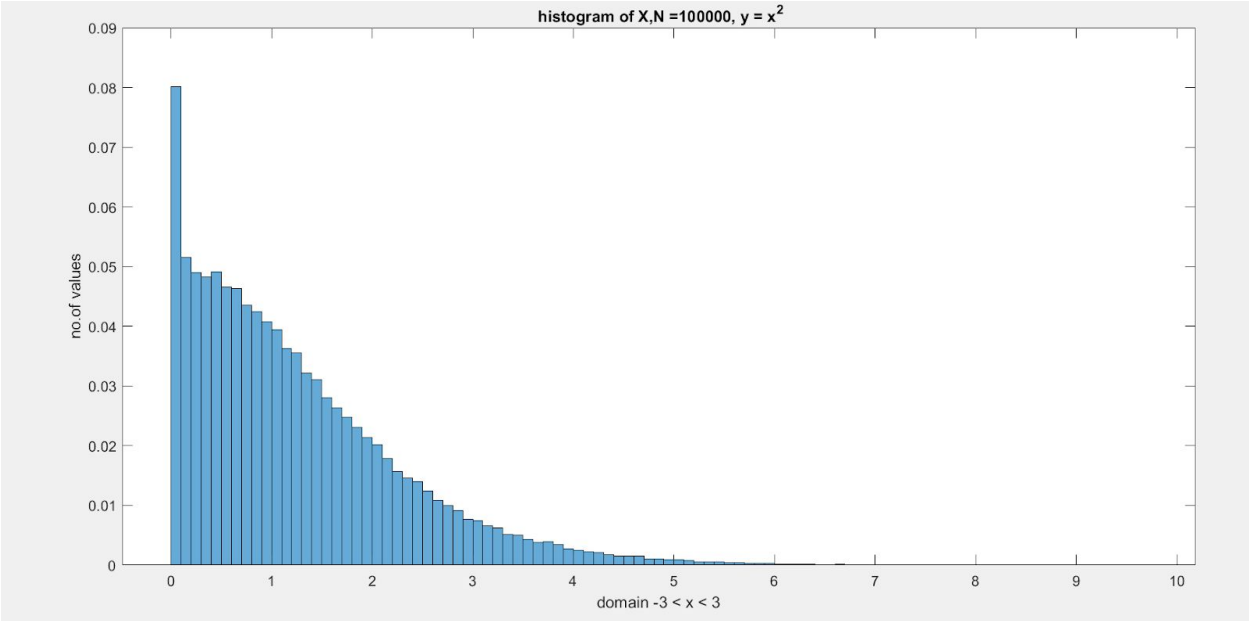
(P5) Consider a Gaussian distribution X~N(1, 0.5) with the domain of [−3, 3],generate its histogram h(X) using sample size of N = 100,000. For any x∈X, there is a nonlinear transformation y = x^2, what is the histogram of Y? Show your results of the histogram of X and the histogram ofY. Set the sample size N = 1,000, generate the histogram of X and the histogram ofY, then plot their corresponding density distribution based on the samples of X andY (you can use ”line” function to mark the density distribution, an example is given below).
Solution:
Generate X~N(1, 0.5)’s histogram h(X) using sample size of N = 100,000



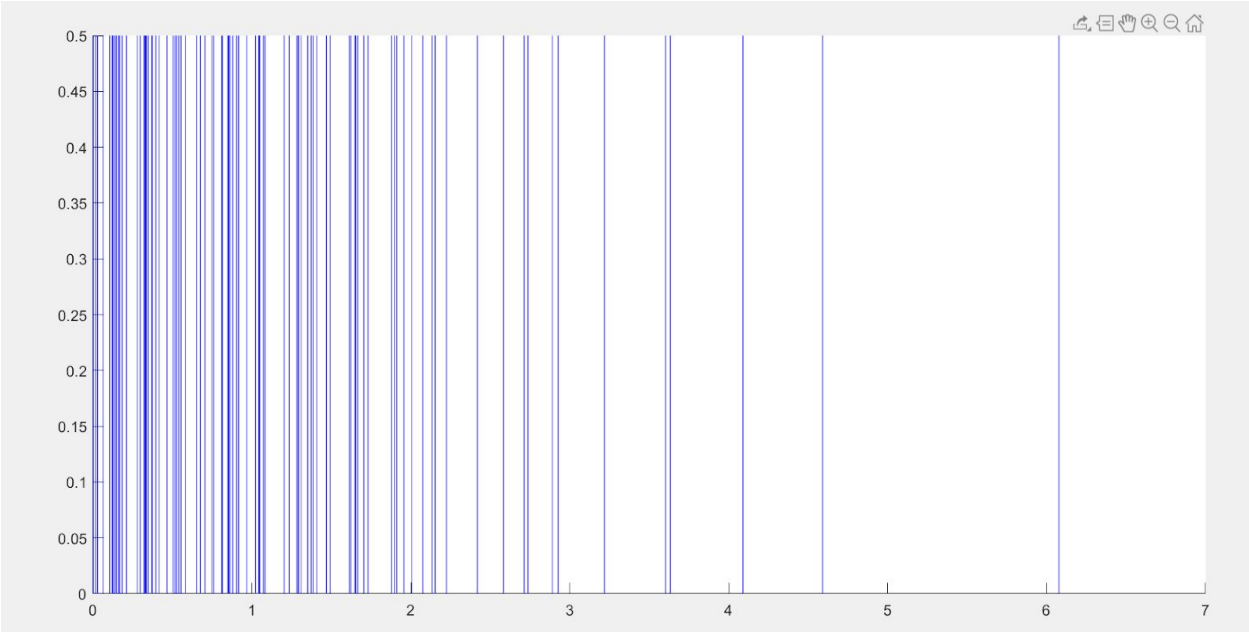
Density distribution of 100 samples of pd = N(1, 0.5)



y = x^2, what is the histogram of Y?



Density distribution for n= 100 samples


0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7