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EMAE489

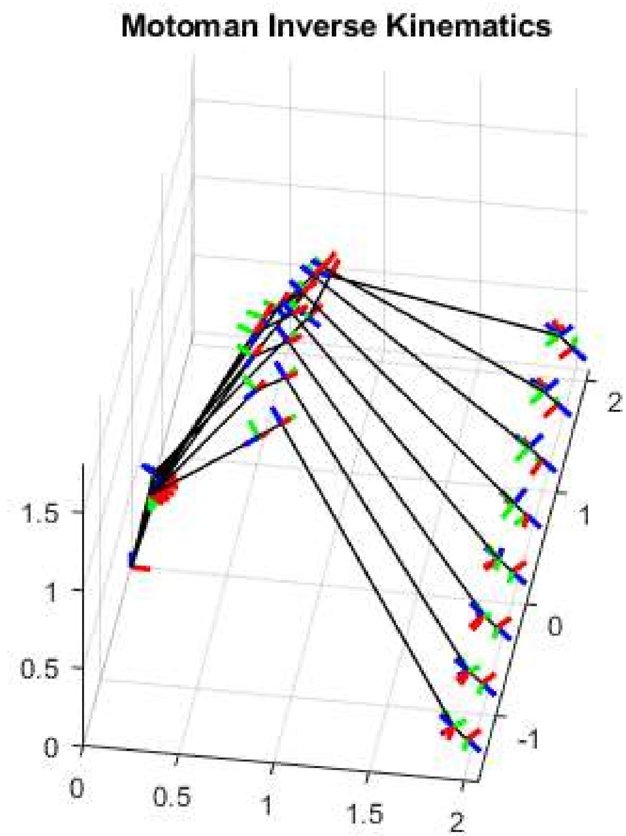
Problem Set 4

Contents

Motoman Inverse Kinematics

```
global motoman
% Symbolic variables for joint angles
q = sym('q%d', [6 1]);

% DH parameters:
a = [.145;1.15;.25;0;0;0];
d = [.540;0;0;-1.812;0;-0.1];
alpha = [3*pi/2;-pi;-pi/2;pi/2;3*pi/2;pi];
% i6 = eye(6);
% i6(3,3)=-1;
theta = q + [0;-pi/2;0;0;0;0];

R6_0 = [0.707107  0.0  0.707107;
        0.0      -1.0  0.0     ;
        0.707107  0.0 -0.707107];
qs = zeros(6,40);
poses = zeros(4,4,40);
for i = 1:40
    o6_0 = [2.0; -2.0+i*0.1; 0.2];
    motoman = kinchain(q,a,d,alpha,theta);
    qs(:,i) = ik(R6_0,o6_0);
    poses(:,:,i) = motoman.computePose(qs(:,i));
    if (mod(i,5)==0)
        motoman.visualizePose(qs(:,i));
        drawnow
    end
    hold on
end
% plot3([0,o6_0(1)],[0,o6_0(2)],[0,o6_0(3)],'p-','LineWidth',1.0);
for i = 1:40
    o6_0 = [2.0; -2.0+i*0.1; 0.2];
    motoman = kinchain(q,a,d,alpha,theta);
    qs(:,i) = ik(R6_0,o6_0);
    poses(:,:,i) = motoman.computePose(qs(:,i));
    if (mod(i,5)==0)
        motoman.visualizePose(qs(:,i));
        drawnow
    end
    hold on
end
title('Motoman Inverse Kinematics');
```

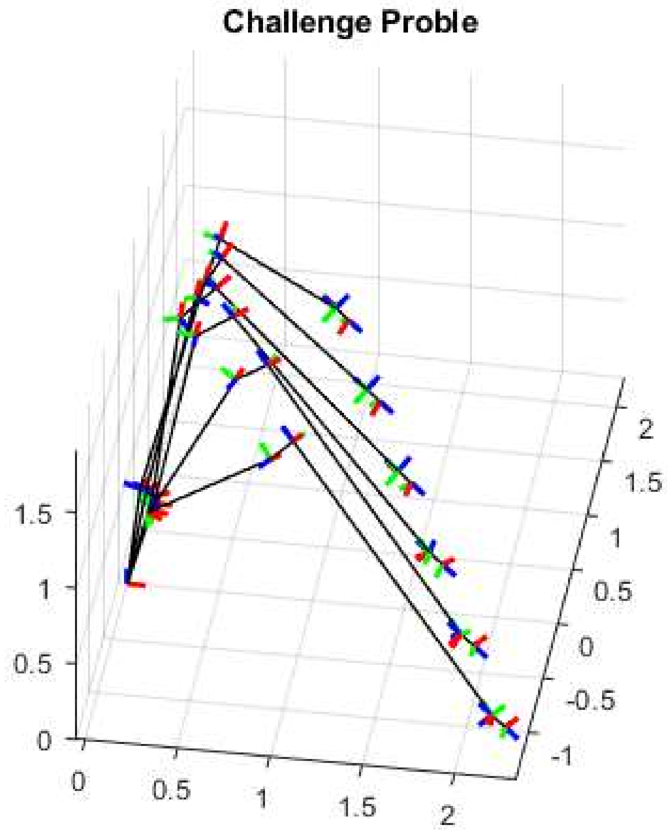
```
figure;
```



Challenge Problem

```
rmin = 0.8198; rmax = 2.9792; z = 0.2707;
xmax = sqrt((rmax^2)-((d(1)-z)^2))-a(1);
xmin = sqrt((rmin^2)-((d(1)-z)^2))-a(1);
y = linspace(sqrt((xmax^2)-(xmin^2)),-sqrt((xmax^2)-(xmin^2)),40);
x = linspace(xmin,xmax,40);
for i = 1:30
    o6_0 = [x(i); y(i); z];
    motoman = kinchain(q,a,d,alpha,theta);
    qs(:,i) = ik(R6_0,o6_0);
    poses(:,:,i) = motoman.computePose(qs(:,i));
    if (mod(i,5)==0)
        motoman.visualizePose(qs(:,i));
        drawnow
    end
    hold on
end
title('Challenge Proble');
function q = ik(R6_0,o6_0)
    global motoman
    d6 = motoman.d(6);
    o4_0 = o6_0 + d6*R6_0(:,3);
    x = o4_0(1);
    y = o4_0(2);

    %Simple arctangent
    q1 = atan2(y,x);
```

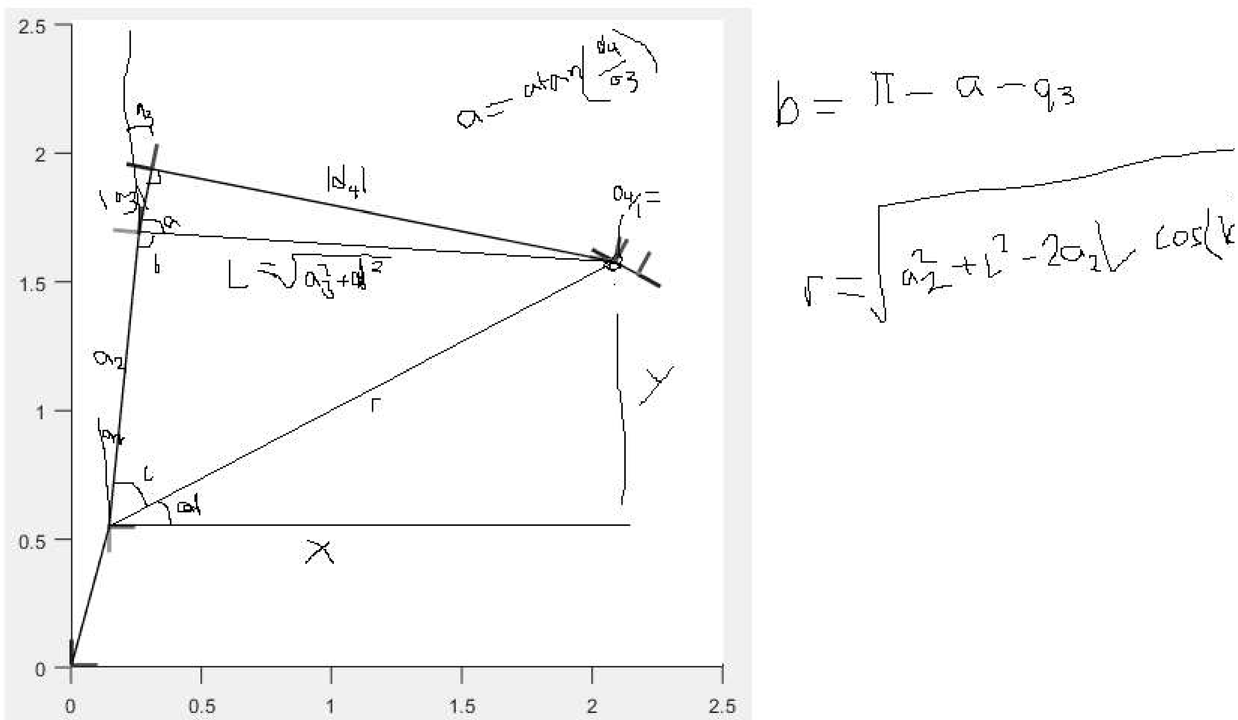
```
    A1_0 = double(subs(motoman.As(:,:,1),'q1',q1));

    % X and Y values wrt frame 1
    x = sqrt((o4_0(1)-A1_0(1,4))^2 + (o4_0(2)-A1_0(2,4))^2);
    y = o4_0(3)-A1_0(3,4);

    %Distance from wrist point to frame 1 origin
    r = sqrt(x^2+y^2);

    %Angle between r and horizontal
    beta = atan2(y,x);

    %DH parameters
    a3 = motoman.a(3);
    d4 = motoman.d(4);
    a2 = motoman.a(2);

    % angle between humerus and vertical = 90 - beta - law of cosines
    q2 = pi/2-beta-acos((a2^2+r^2-a3^2-d4^2)/(2*a2*r));

    % angle between humerus and a3 portion of the forearm link
    % 180 - fixed elbow angle - law of cosines
    q3 = -pi-atan2(d4,a3)+acos((a2^2+a3^2+d4^2-r^2)/(2*a2*sqrt(a3^2+d4^2)));

    % Compute transform to wrist point
    A3_0 = A1_0*double(subs(motoman.As(:,:,2),'q2',q2)*subs(motoman.As(:,:,3),'q3',q3));
    R3_0 = A3_0(1:3,1:3);

    % Rotation of end effector wrt frame 3
    R6_3 = transpose(R3_0)*R6_0;

    r33 = R6_3(3,3);
    r23 = R6_3(2,3);
    r13 = R6_3(1,3);
    r32 = R6_3(3,2);
    r31 = R6_3(3,1);

    % Simple inverse trig to compute qs from R matrix
    q5 = -pi+acos(r33);
    q4 = atan2(r23,r13)-pi;

    % Angle between desired frame 6 and frame 6 with q6=0
    A6_0_0q6 = motoman.computePose([q1;q2;q3;q4;q5;0]);
    R6_0_0q6 = A6_0_0q6(1:3,1:3);

    q6 = real(acos(dot(R6_0_0q6(:,2),R6_0(:,2))));

    q = [q1;q2;q3;q4;q5;q6];
    grid on;
    view(9,36);
end
```



Challenge Proble
2
1.5
1
0.5
0
-0.5
-1
1.5
1
0.5
0
0
0.5
1
1.5
2

Assignment No.4

Jdc183 - Joseph Cressman
Kxg360 - Krashagi Gupta
Wxh368 - Wei Huang



Challenge Problem:

Aim : For the same tool flange orientation and z-height, what is the longest continuous straight line path you can find from (x0,y0) to (xf,yf). A viable path must have no joint-angle discontinuities.

Sol.

We know that q3 determines the distance between the shoulder to the end-effector, Which means to find the closest distance that the robot can reach from origin, we have to use the smallest q3 possible.

```
>> rmin=sqrt(a2^2+L^2-2*a2*L*cos(pi-abs(atan(d4/a3))-1.39))
```

rmin =

0.8198

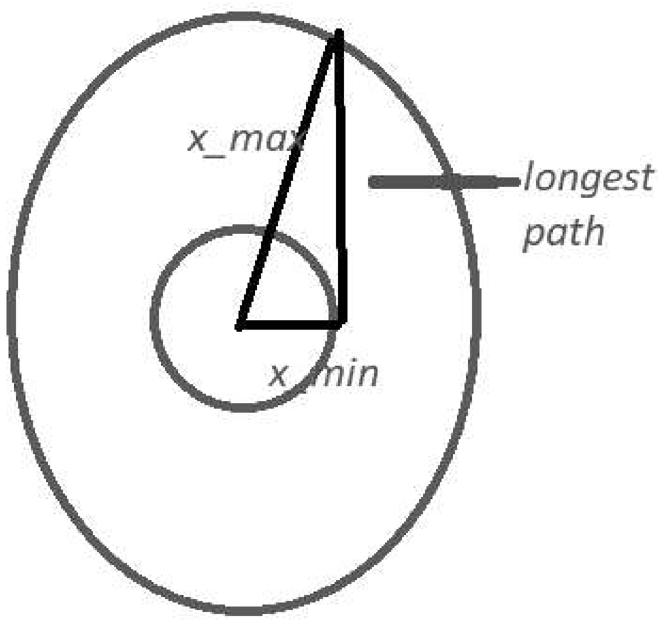
The longest distance that the robot can stretch is the full length provided by all its links.

```
>> rmax=a2+sqrt(a3^2+d4^2)
```

rmax =

2.9792

Therefore, in order to find the max length, we draw a tangent from the smaller circle to cut the outer circle.



We obtain x min :
Xmin = sqrt(rmin^2-(d1-z)^2)-a1

Xi = xf = xmin
Yi = -yf = sqrt(xmax^2 - xmin^2)

Ymax = 2.7509
Ymin = -2.7509

xmax =
   2.8220

xmin =
   0.6293

Longest path distance: rmax - rmin = 2.1594

We had also done hand calculations for all the q values :

The work is attached as below, we assumed w4/1 expression wrong as a result we couldn’t find the correct values, but followed the trigonometric methodology to find All the q values, as taught in class

$${}^{1}A_4 = {}^{1}A_2(q_2) \cdot {}^{2}A_3(q_3) \cdot {}^{3}A_4(q_4)$$ ①

$$W/_1 = \begin{bmatrix} S_{23}d_4 + a_2C_2 \\ -C_{23}d_4 + a_2S_2 \\ 0 \\ 1 \end{bmatrix} \qquad (S_{23}d_4 + a_2C_2)^2 + (-C_{23}d_4 + a_2S_2)^2$$

w.r.t shoulder. $\rightarrow$ length

$$(S_{23}d_4)^2 + (C_{23}d_4)^2 + (a_2C_2)^2 + (a_2S_2)^2$$

$$\Rightarrow \quad d_4^2 + a_2^2$$

$$+ 2(S_{23}d_4 \cdot a_2C_2) + 2(-C_{23}d_4 \cdot a_2S_2)$$

$$+ 2d_4a_2(S_{23} \cdot C_2) - 2(d_4a_2)(S_2 \cdot C_{23})$$

$$2d_4a_2\left[S_{23} \cdot C_2 - S_2 \cdot C_{23}\right].$$

if $A = \theta_2 + \theta_3$ $\qquad$ ∵ ~~sin(A+B) = cos(~~

$B = \theta_2$ $\qquad$ ∵ $\sin A \cdot \cos B - \cos A \cdot \sin B$.

$A - B = \theta_3$ $\qquad$ $= \sin(A - B)$.

$$\Rightarrow \|W/_1\|^2 = d_4^2 + a_2^2 + 2d_4a_2 \sin\theta_3$$

Scanned with CamScanner

$$A_1^0 = \begin{bmatrix} c_1 & 0 & -s_1 & a_1c_1 \\ s_1 & 0 & c_1 & a_1s_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow A_0^1 = \begin{bmatrix} c_1 & s_1 & 0 & \\ 0 & 0 & -1 & \\ -s_1 & c_1 & 0 & \\ 0 & 0 & 0 & \end{bmatrix}$$

$$-R^Tp \quad \begin{matrix} -c_1 \cdot a_1c_1 - a_1s_1^2 & -a_1c_1^2 - a_1s_1^2 & -a_1 \\ d_1 & d_1 & d_1 \\ a_1c_1s_1 - a_1s_1c_1 & 0 & 0 \end{matrix}$$

$$A_0^1 = \begin{bmatrix} c_1 & s_1 & 0 & -a_1 \\ 0 & 0 & -1 & d_1 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ 1 \end{bmatrix} \rightarrow \omega_z = 0.$$

$$\begin{bmatrix} c_1\omega_x + s_1\omega_y + 0 - a_1 \\ -\omega_z + d_1 \\ -s_1\omega_x + c_1\omega_y \\ 1 \end{bmatrix} = \omega_{4|1}$$

$$-S_1\omega x + C_1\omega y = 0$$

How ? $A\sin x + B\cos x = 0$

$A = r\cos\beta \qquad B = r\sin\beta$

$\omega x = r\cos\beta \qquad \omega y = r\sin\beta$

$-\sin\theta_1 \times r\cos\beta + \cos\theta_1 \times r\sin\beta$

$r[-\sin\theta_1\cos\beta + \cos\theta_1\sin\beta]$

$\cos A\sin B - \sin A\cos B$

$-(\sin A\cos B - \cos A\sin B)$

$-(\sin(A-B))$

$-\sin(\theta_1 - \beta) = 0$

$\theta_1 - \beta = 0$

$\theta_1 = \tan^{-1}(\omega y, \omega x)$

$\frac{\omega y}{\omega x} = \tan\beta \qquad \tan^{-1}(\omega y, \omega x)$

$$\therefore |\omega 4/1|^2 = C_1^2\omega x^2 + S_1^2\omega y^2 + a_1^2 + 2S_1C_1\omega x\omega y - 2a_1S_1\omega y - 2a_1C_1\omega x$$

$$- \cancel{\ } + d_1^2 + 1.$$

Ask them $\underline{\omega y \ \& \ \omega x} \Rightarrow 1 \Rightarrow 3 \Rightarrow 2$ .. using each value.

numerical expression

$$\|w/1\|^2 = d_4^2 + a_2^2 + 2d_4a_2\sin\theta_3$$

$$\sin\theta_3 = \frac{\|w_4/1\|^2 - d_4^2 + a_2^2}{2d_4a_2}$$

@ 12 after listening to lecture

$$\theta_3 = \sin^{-1}\left[\qquad\right]$$

⑥

w.k.t.

$$\begin{bmatrix} S23\,d_4 + a_2C_2 \\ -C23\,d_4 + a_2S_2 \\ 0 \end{bmatrix} = {}^{w}/_{1} = \begin{bmatrix} w_x/_1 \\ w_y/_1 \\ w_z=0 \end{bmatrix}.$$

$$w_x/_1 = S23\,d_4 + a_2C_2$$

$$\sin(\theta_2+\theta_3)\,d_4 + a_2\cos\theta_2 = w_x/_1.$$

using $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

$$d_4\left[\sin(\theta_2)\cdot\cos(\theta_3) + \cos(\theta_2)\cdot\sin(\theta_3)\right] + a_2\cos\theta_2 = \frac{w_x}{1}.$$

bring to form $A\sin\theta + B\cos\theta = C$

$$(\sin\theta_2)\cdot(d_4\cos\theta_3) + (\cos\theta_2)\left[d_4\sin\theta_3 + a_2\right] = w_x/_1$$

Restating.

$$\underbrace{(d_4\cos\theta_3)}_{A}\sin\theta_2 + \underbrace{(\{d_4\sin\theta_3\}+a_2)}_{B}\cos\theta_2 = w_x/_1.$$

$A = r\cos\beta = d_4\cos\theta_3.$

$B = d_4\sin\theta_3 + a_2 = r\sin\beta.$

if $\theta_3 = 0.1656$ $\quad \cos\theta_3 = 0.9863.$ $\quad d_4 = -1.812$

$\sin\theta_3 = -0.1648.$ $\quad a_2 = 1.150$

$$(-1.812\times 0.9863)\sin\theta_2 + (-1.812\times \bar{0}.1648 + 1.150)\cos\theta_2$$

$$= 2.634$$

$$-1.7812\ \sin\theta_2 + \cancel{0.8514}\ 1.4486\ \cos\theta_2 = 2.634.$$

let $\Rightarrow$ $A = -1.7812 = r\cos\beta$

$B = \cancel{0.8514} = r\sin\beta$
1.4486

$\therefore$ $r\left[\cos\beta\cdot\sin\theta_2 + \sin\beta\cdot\cos\theta_2\right] = 2.634.$

$r\left[\sin(\beta+\theta_2)\right] = 2.634.$

$$\sin(\beta+\theta_2) = \frac{2.634}{r}$$

$$\beta+\theta_2 = \sin^{-1}\left(\frac{2.634}{r}\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{2.634}{r}\right) - \beta$$

$$\tan\beta = \frac{B}{A} \Rightarrow \beta = \tan^{-1}\left(\frac{B}{A}\right) = \beta\ \tan^{-1}\left(\frac{\overset{1.4486.}{\cancel{0.8514}}}{\underset{-1.7812}{\cancel{-1.7812}\ \cancel{0.8514}}}\right)$$

$A^2 + B^2 = r^2$

$r = \sqrt{A+B.} = 1.9742$ $\qquad = 2.6957$

$$\theta_2 = \sin^{-1}\left(\frac{2.634}{\cancel{1.9742}\ 2.30}\right) - 2.\overset{4605}{\cancel{6957}}$$

$\theta_2 = \cancel{1.125 - 0.7964i}$

$-0.8897 - 0.5320i$

To find A4, A5, A6 :-

$$\vec{R}_{des} = {}^0R_1(q_1) \cdot {}^1R_2(q_2) \cdot {}^2R_3(q_3) \cdot \ldots \cdot {}^5R_6(q_6).$$

$${}^2R_3^{-1} \cdot {}^1R_2^{-1} \cdot {}^0R_1^{-1} \cdot R_{des} = {}^3R_4(q_4) \cdot {}^4R_5(q_5) \cdot {}^5R_6(q_6)$$

$$({}^{i-1}A_i)^{-1} = {}^iA_{i-1} \ ; \quad {}^2R_3^{-1} = {}^2R_3^T = {}^3R_2$$

$${}^3R_2 \cdot {}^2R_1 \cdot {}^1R_0 \cdot {}^0R_{des(6)} = {}^3R_{(6)\,des}.$$

$\alpha = \pm \pi/2$.

$${}^3R_4 = \begin{bmatrix} C_4 & 0 & -S_4 \\ S_4 & 0 & C_4 \\ 0 & -1 & 0 \end{bmatrix} \quad {}^4R_5 = \begin{bmatrix} C_5 & 0 & S_5 \\ S_5 & 0 & -C_5 \\ 0 & 1 & 0 \end{bmatrix}$$

$${}^5R_6 = \begin{bmatrix} C_6 & -S_6 & 0 \\ S_6 & C_6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Choosing Z_6, Z_5 are coincident

$${}^3R_6 = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 \\ S_4S_5C_6 + C_4S_6 & -S_4S_5S_6 + C_4C_6 & S_4S_5 \\ -S_5C_6 & S_5S_6 & C_5 \end{bmatrix}$$

(known).

$$\underbrace{{}^3R_6(4,5,6)}_{uk} = {}^3R_2 \cdot {}^2R_1 \cdot {}^1R_0 \cdot {}^0R_{des(6)}$$

$${}^5R_{6\,des}(3,3) = \cos(q_5)$$
$$\Rightarrow \{q_{5\alpha}, q_{5,\beta}\}.$$

?? Why though?

unique. $q_6 = \text{atan2}(R(3,2), -R(3,1))$.

$q_4 = \text{atan2}(R(2,3), R(1,2))$

w.k.t $\|\omega/o\|^2$. A1/o