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Problem Set 11

1. Optimal passive closed-loop admittance:

- Keeping Kenv = 200, as asked in the question, ensuring passivity, by keeping Bdes = 5.0625, Keeping the phase of robot admittance above pi/2,
- I changed the Kp and Kv value proportionally.
- I found the highest minimum in the interval to be 0.09.
- Recommended values Kp = 24000, Kv = 4800 and Bdes = 5.0625

```
Kenv= 200 %23 %100 %120 %200000 %environment model: a spring of this stiffness, N/m
%Kenv = a ;
%Kenv = b;
%desired impedance of Bdes and Kdes
%LOWER BDES IS BETTER, implies more responsive closed-loop admittance of the robot
a = 4.25; %13.6250 19.9375  23.0938

b = 5.8750;
Bdes = (a + b)/2 %damping of desired admittance  5.0625
%  Bdes = a ;
%   Bdes = b ;
Kdes = 0; %10; %20; %100 %stiffness of desired admittance


x= 1.2
%servo controller gains:
Kp = 20000*x%200 %200 %20000  %200
Kv = 4000*x %40 %3000  %40
```

```
Kenv =

   200

Bdes =

    5.0625

x =

    1.2000

Kp =

       24000

Kv =

        4800
```

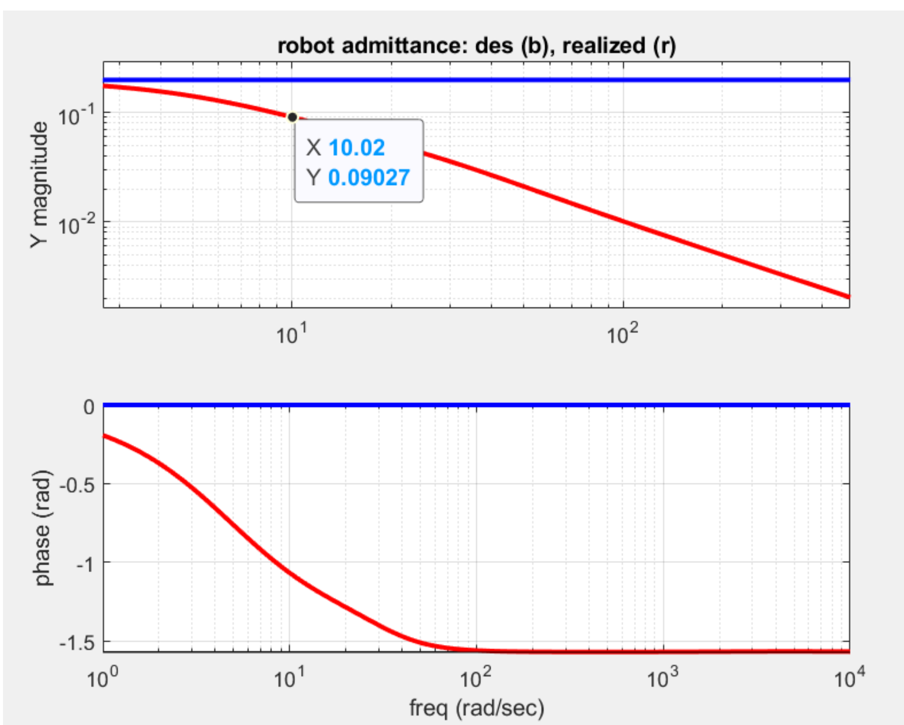

Figure 1

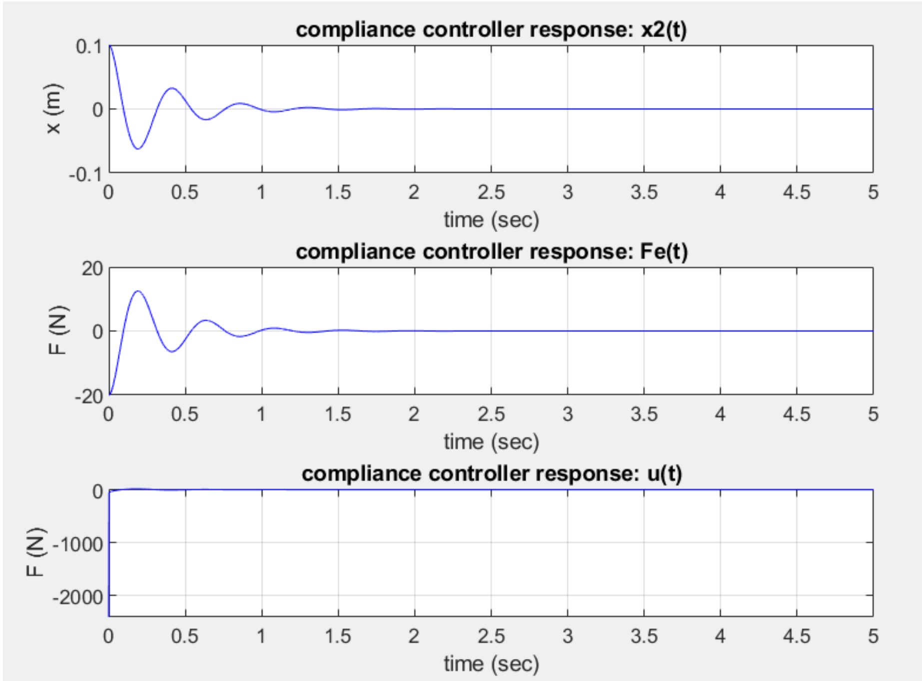
compliance controller response: x2(t)
x (m)
0.1
0
-0.1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
time (sec)
compliance controller response: Fe(t)
F (N)
20
0
-20
time (sec)
compliance controller response: u(t)
F (N)
0
-1000
-2000
time (sec)

Figure 2

2. Bdes vs passivity:

- By slowly decreasing the value of the Bdes while keeping the value of Kp=200, Kv=40, and Kenv = 200, we find that the lowest value of Bdes that results in passive Ycl is:
    - Bdes = 5.073548
- For any value of Bdes that is larger than the above value, the system will remain to be passive, i.e. the phase of the closed-loop admittance never gets lower than -pi/2.
- For any value of Bdes that is smaller than the above value, the system will not be passive anymore. The phase of the closed-loop admittance, in this case, is guaranteed to cross over the -pi/2 mark.

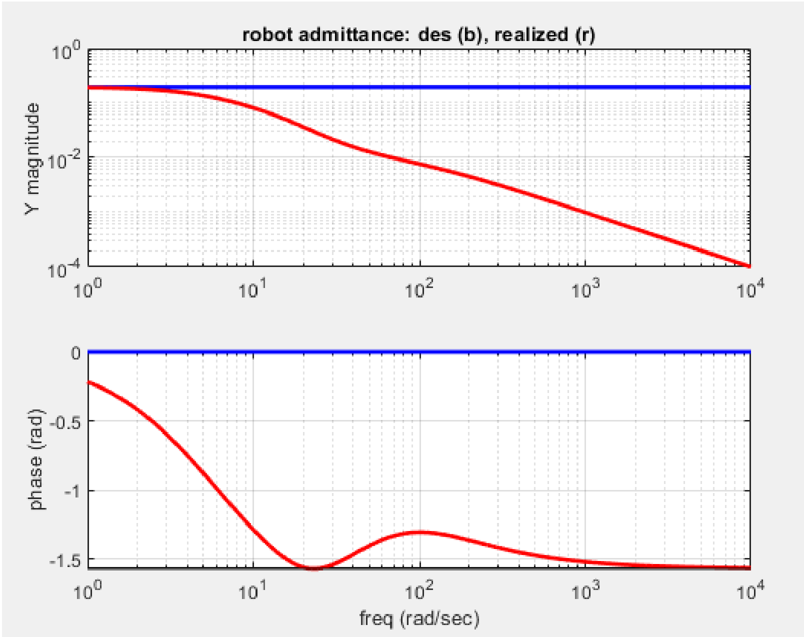

Figure 3

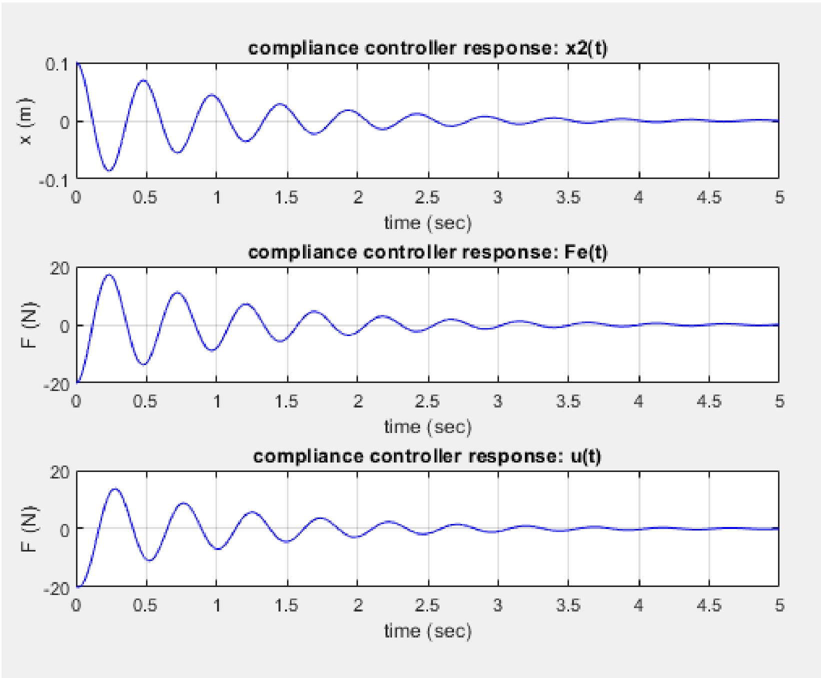

Figure 4

3. Influence of environment stiffness:

- Given parameters Bdes=1, Kp=200, and Kv = 40
- This represents a control system that is not passive
- This results in a crossover frequency at 10.1159 rad/s
- At this point, the gain must be less than 1
- If it is greater than 1, it is unstable
- By starting with soft environments and making them increasingly stiffer, we approach a gain of 1
- When we reached Kenv of 23.8 the gain was .9853 (figure 5)
- This is less than 1 and therefore stable.
- Figure 6 shows that the oscillations slowly decrease in size and the system will converge.
- Any Kenv less than 23.8 is stable and any environment greater than 23.8 is unstable.
- The open-loop transfer function is the admittance of the controller times impedance of the environment.

From the experience of finding the value, we found out that, with the open-loop transfer function of Ycl_robot * Z_env, if the resulting function's phase lower than -pi, that means the feedback output will add to the input signal (since we have the -1 multiplication on the feedback branch). Due to this, if the magnitude of the Ycl_robot * Z_env is larger than 1 when the crossover happened, the system output magnitude will diverge over time, as the output will be feedbacked and multiplied to a constant larger than 1. Hence, having the magnitude of the open-loop transfer function of Y_cl * Z_env to be less than 1 when phase crossing over -pi is required for the system to be stable.

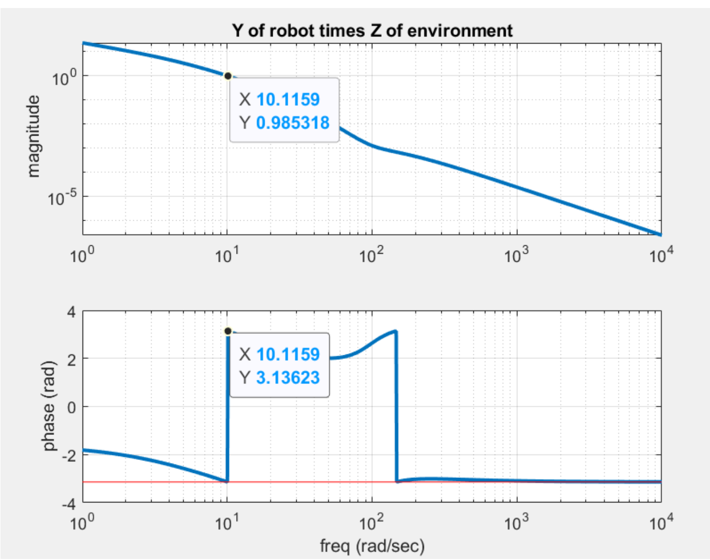

Figure 5

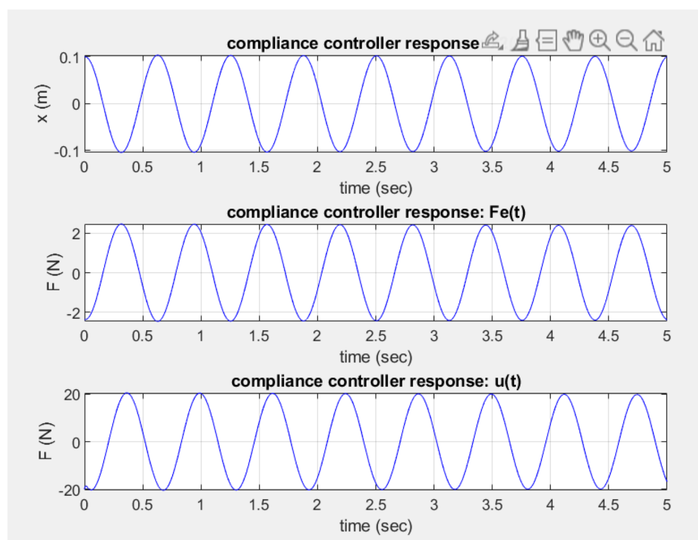

Figure 6