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Course: Robotics 1 - ECSE 489
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Problem Set 10

Question 1:

Open-loop control with computed torque:

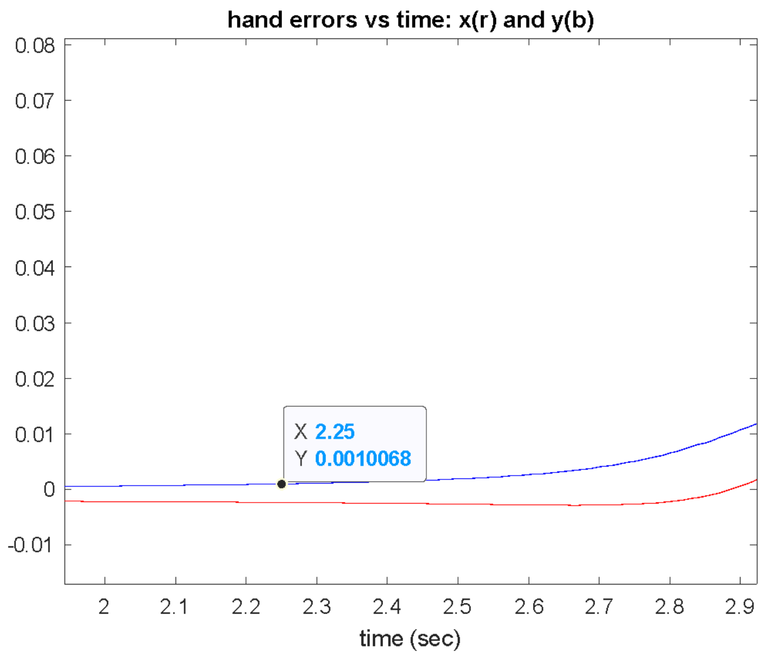

Figure 1: Hand error at 2.25 sec in y direction

The hand exceeds 1mm error at 2.25 sec in y direction.

Question 2:

Servo control, no feedforward:

Keeping the amax and vmax as in the previous question, and using even a simple control mechanism, the hand errors reduce significantly, and therefore the hand reaches the goal.

But the hand errors are significant, and need to be reduced.

Keeping the same configuration as the previous question, I noticed that changing Kp and Kv values in either direction does not reduce the hand errors to be in the bounds, therefore, I change the path trajectory i.e. amax and vmax

amax=0.07
vmax=0.04

Kp =

1800	0
0	1600

Kv =

42.4264 0
0 40.0000

wn1 = 39.1953 rad/s

wn2 = 69.2820 rad/s

zeta1 = 0.4619

zeta2 = 0.8660

Minimum time = 50s

This caused the hand- errors to move inside the bounds.

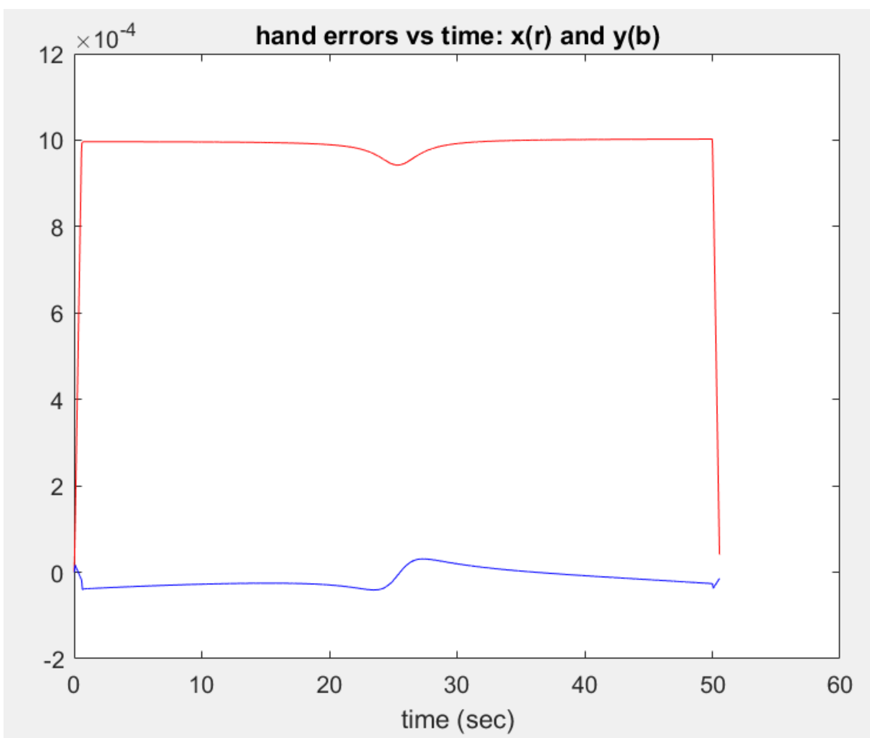

Figure 2: Hand error - no feedforward

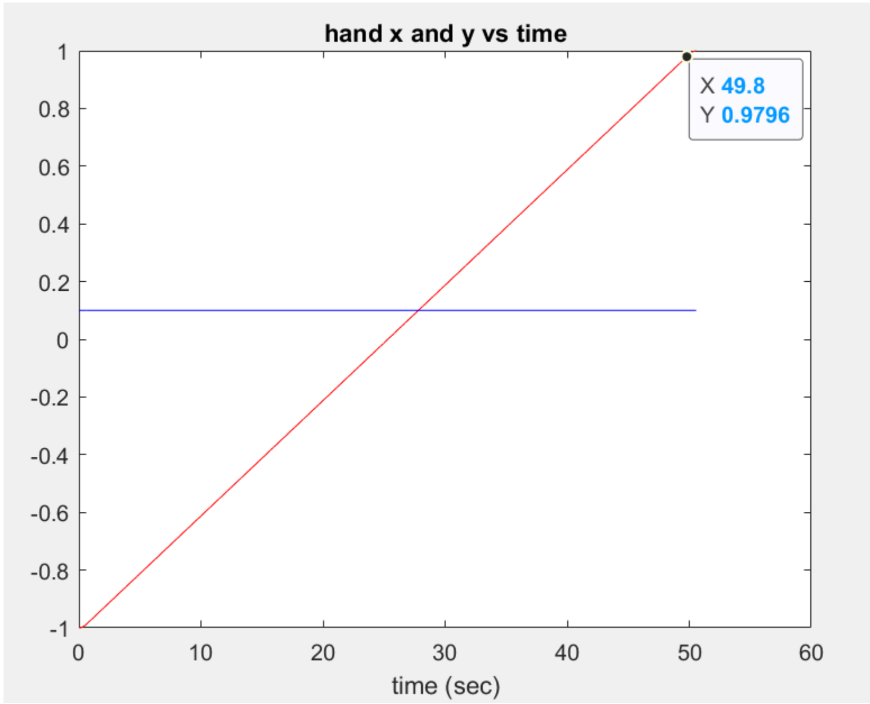

Figure 3: Time - No feed forward : 50s

Question 3:

Servo control with velocity feedforward:
Keeping the same parameters as the previous question, when we include velocity feed forward, the error drops significantly Fig 4.
Since the errors so significantly reduce, we can be aggressive in pushing the acceleration and velocity to reach the torque limits. Fig5 and Fig 6
amax=2.5
vmax=0.38

Kp =

1800	0
0	1600

Kv =

42.4264	0
0	40.0000

wn1 = 39.1953 rad/s

wn2 = 69.2820 rad/s

zeta1 = 0.4619

zeta2 = 0.8660

Minimum time : 5.3 s

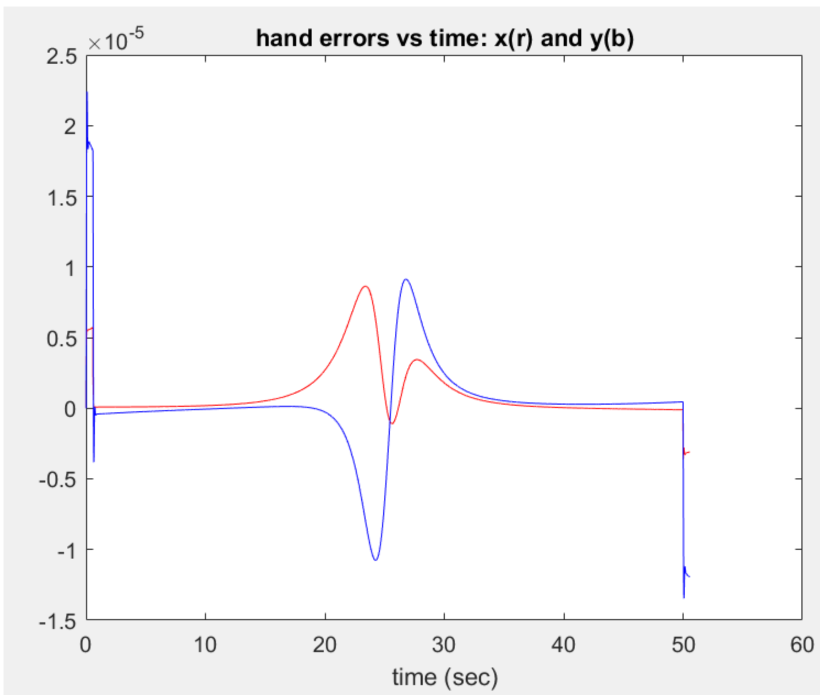

Figure 4: Error decreasing using velocity feedforward for previous example

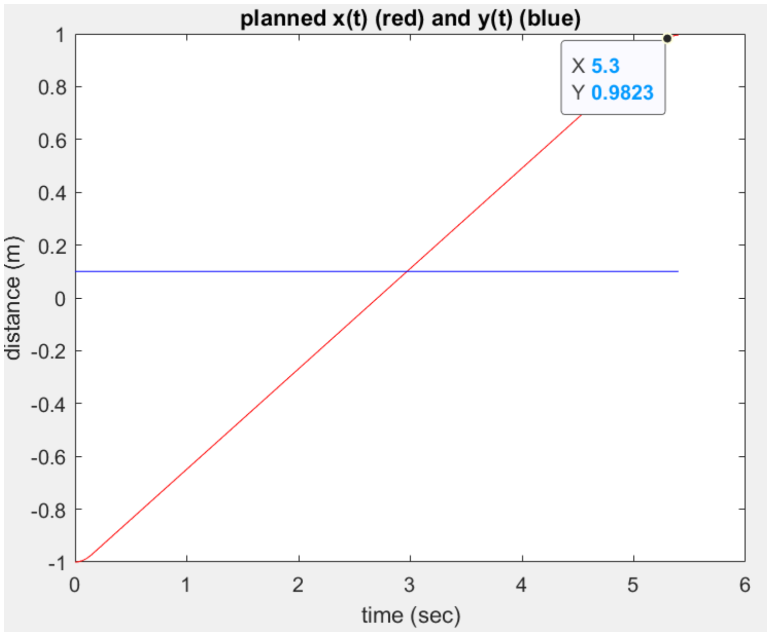

Figure 5: Time - with velocity feedforward 5.3

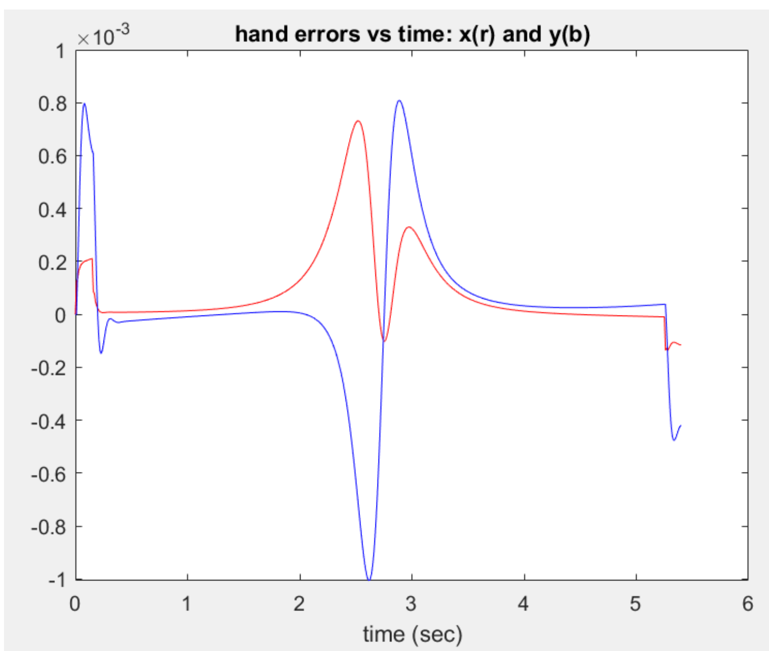

Figure 6: Hand error - with velocity feedforward

Question 4:

Servo control with computed torque feedforward

For this question computed torque feedforward was included in addition to the velocity feedforward of question 3. Our fastest trajectory subject to the 1mm error bound was with the following parameters:

Kp = 1250 0

0 550

Kv = 90 0

0 36

amax = 4.9

vmax = 0.5

zeta1=0.8764

zeta2=0.9909

wn1=32.6628 rad/s

wn2=54.4977 rad/s

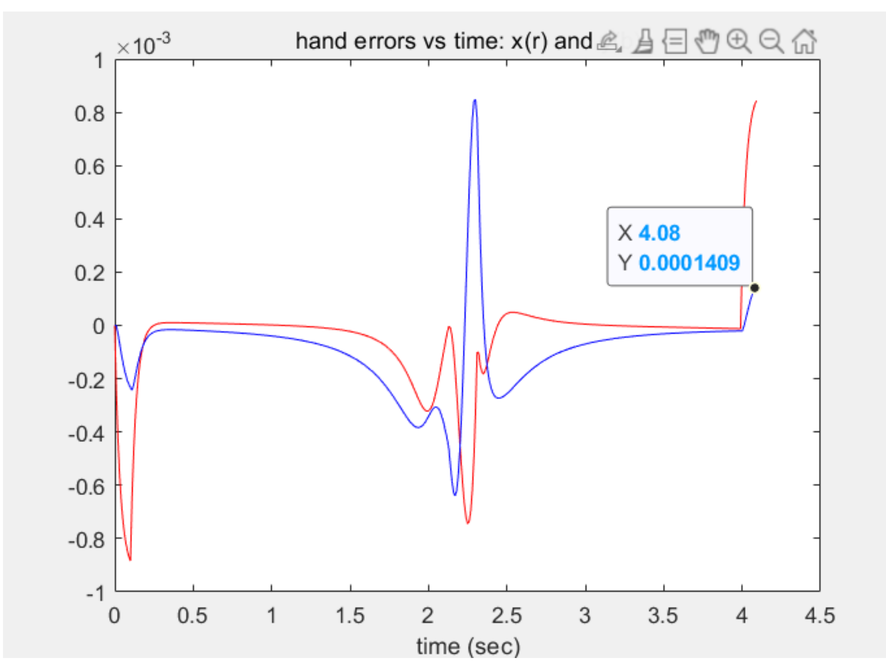

Figure 7: Servo control w/ torque feedforward using adjusted parameters

This trajectory had a time of 4.08 seconds which was the fastest of all questions. This is expected since this question incorporates both velocity and computed torque feedforward. Feedforward control allows better tracking compared to open loop control, like what we have in question 1.

Instability:

Instability can be achieved by the zeta values becoming negative, but more applicable to this problem set is the controller sampling rate. In general you want a sampling rate to be at least 10 times the natural frequencies of the system. A DT=0.01s corresponds to a samp_freq of 628.3 rad/s and wn = sqrt(Kp/H) so as the Kp values increase, wn values will increase until you reach a point of instability. A large Kv can also lead to instability in this system because it makes the response more overdamped, leading to a slower controller response, this slower response will not be able to keep up with the system. There are multiple combinations of Kp and Kv values can push the system to the brink of instability. So we will list examples of sets of parameters on the brink of instability for questions 2-4. For these examples if the Kp and Kv values were further increased a small amount the system will become definitively unstable. As an example of an unstable system for question 2, Figure 8 clearly shows an unstable system because the error is growing overtime and doesn't decrease. In contrast our examples for the brink of instability show the error come back down after climbing.

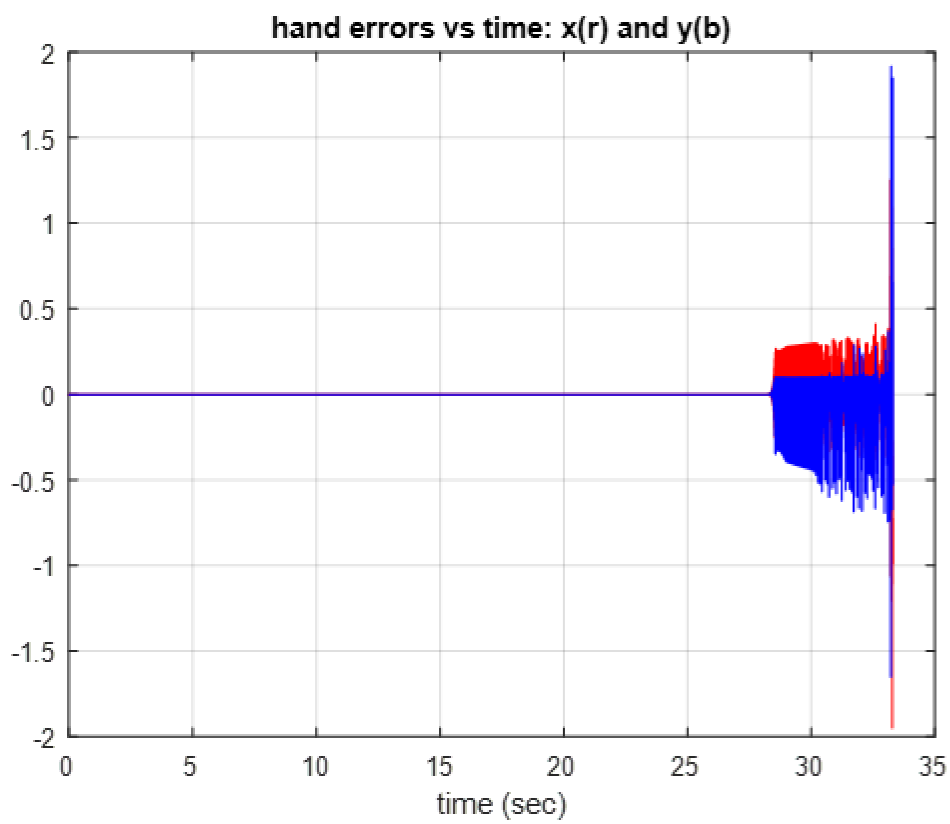

Figure 8: Question 2 unstable example

Kp = 7500 0

0 9200

Kv = 29 0

0 12

amax = 0.500

vmax = 0.250

zeta1=0.1547

zeta2=0.1083

wn1=80.0071 rad/s

wn2=166.1325 rad/s

Question 2:

An example of parameters for question 2 that brings the system to the brink of instability is:

Kp =

6000 0

0 9200

Kv =

29 0

0 12

Which have corresponding:

wn1 =71.5605

wn2 = 166.1325

samp_freq = 628.3185

zeta1 = 0.1729

zeta2 = 0.1083

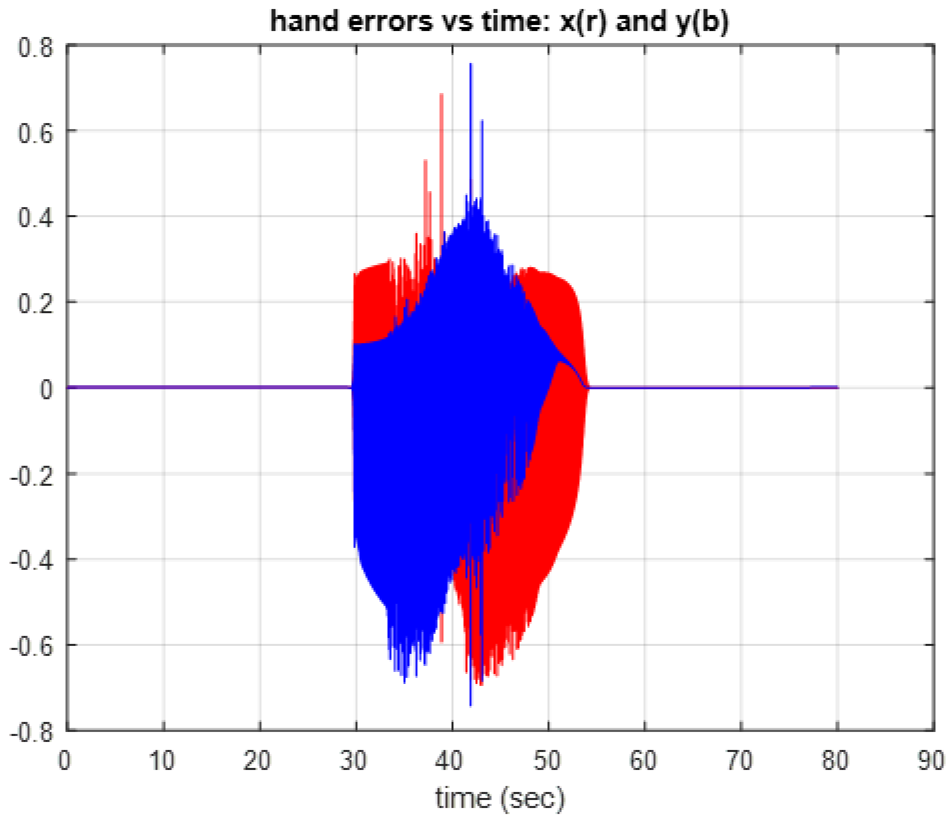

Figure 9: Question 2 almost unstable example

Question 3:

Compared to question 2 the brink of instability of question 3 seems to be able to be pushed a little farther. Despite increasing the max velocity, the inclusion of velocity feed-forward is generally making the system more stable. Here are the values, note that Kp1 is slightly larger than before.

Kp =

6600	0
0	9200

Kv =

$$\begin{matrix} 29 & 0 \\ 0 & 12 \end{matrix}$$

Which have corresponding:

wn1 = 75.0533

wn2 = 166.1325

samp_freq = 628.3185

zeta1 = 0.1649

zeta2 = 0.1083

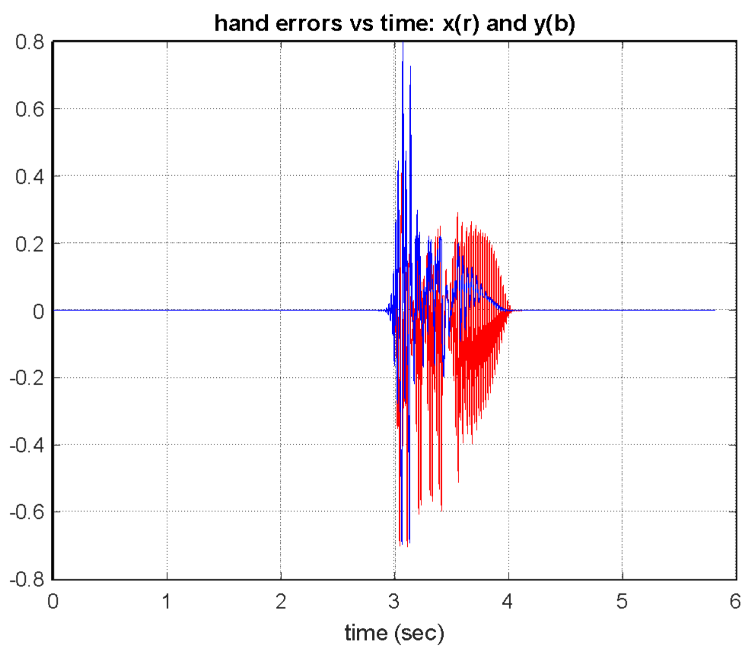

Figure 10: Question 3 almost unstable example

Question 4:

Compared to problem 3 the brink of instability for problem 4 seems to be about the same, I was able to increase Kp2 a small amount further. Here are the values, note that Kp2 is slightly larger than before.

Kp =

$$\begin{matrix} 6600 & 0 \\ 0 & 9400 \end{matrix}$$

Kv =

$$\begin{matrix} 29 & 0 \\ 0 & 12 \end{matrix}$$

Which have corresponding:

wn1 = 75.0533

wn2 = 167.9286

samp_freq = 628.3185

zeta1 = 0.1649

zeta2 = 0.1072

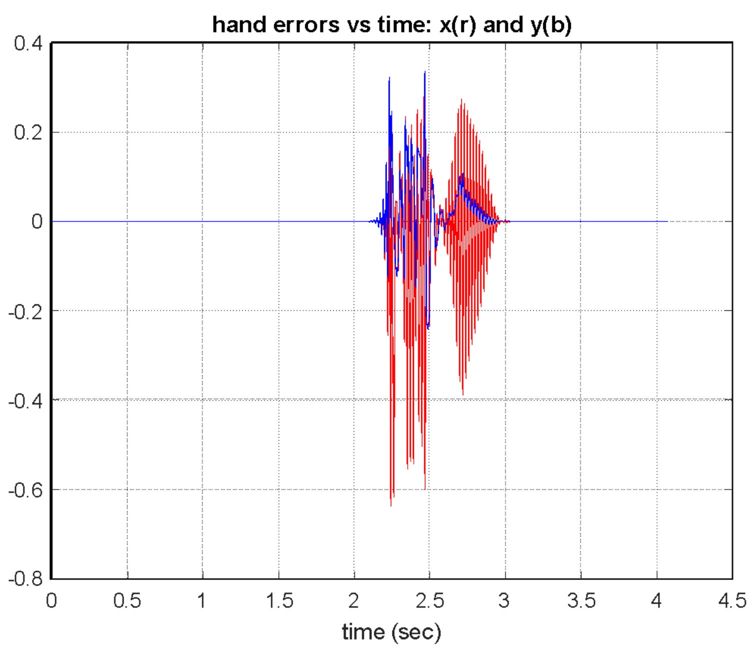

Figure 11. Question 4 almost unstable example