2.10, 3.3 & 3.8.

Assignment - 2.

2.10. An e^- is described by a plane-wave wavefunction $\Psi(x,t) = Ae^{j(10x+3y-4t)}$. Calculate the expectation value of a function defined as $\{4p_x^2 + 2p_z^2 + 7mE\}$ where m is the mass of the electron, p_x and p_z are the x and z components of momentum and E is energy (give value in terms of Planck's constant)

(a) ~~f(x)~~ ~~f(x) dx p p(x) dx~~

Given :-

$$\psi(x,y,z,t) = A \cdot e^{j(10x+3y-4t)}$$

To find $\langle M \rangle$ where $M = 4p_x^2 + 2p_z^2 + 7mE$.

W.k.t.

$$\langle Q \rangle = \frac{\int_{-\infty}^{+\infty} \psi^* \cdot Q_{op}\, \psi \, dx\,dy\,dz}{\int_{-\infty}^{+\infty} \psi^* \psi}$$ for not normalised.

Q_{op} = Quantum operator.

$$Q_{op} \text{ for } p_x^2 = \left(\frac{h}{j}\frac{\partial}{\partial x}\right)^2$$

$$Q_{op} \text{ fo } E = \left(\frac{-h}{j}\frac{\partial}{\partial t}\right)$$

$$\langle p_z^2 \rangle = \frac{\int_{-\infty}^{+\infty} \psi^* \cdot \left(\frac{h}{j}\frac{\partial}{\partial z}\right)^2 \psi \; dz}{\int_{-\infty}^{+\infty} \psi^* \cdot \psi \; dz}$$

$= -\hbar^2$ since ψ is not a function of z.

$$\frac{\partial}{\partial z}[\psi] = 0.$$

$$\langle p_z^2 \rangle = 0.$$

$$\langle E \rangle = \frac{\int_{-\infty}^{+\infty} \psi^* \left(-\frac{h}{j}\frac{\partial}{\partial t}\right) \psi \; dt}{\int_{-\infty}^{+\infty} \psi^* \psi \; dt}.$$

$$= \frac{-\frac{h}{j}\int_{-\infty}^{+\infty} A \cdot e^{-j(10x+3y-4t)} \cdot \frac{\partial}{\partial t}\left(A \cdot e^{+j(10x+3y-4t)}\right) dt}{= \cancel{\frac{h}{j}A^2} \quad A^2 \int_{-\infty}^{+\infty} e^{-j(10x+3y-4t)} \cdot e^{+j(10x+3y-4t)} \, dt}$$

$$= \frac{-\frac{h}{j} A^2 \int_{-\infty}^{+\infty} e^{-j(\quad)} \cdot e^{+j(\quad)} \cdot (-4j) \, dt}{A^2 \int_{-\infty}^{+\infty} dt}$$

$$\langle E \rangle = \underline{4h}$$

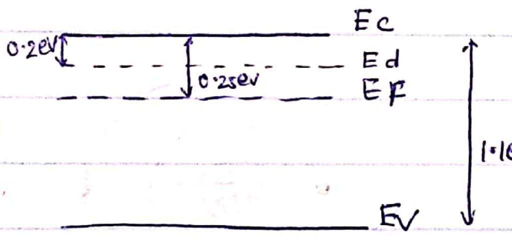
$\therefore \langle M \rangle = 4\langle p_x^2 \rangle + 2\langle p_z^2 \rangle + 7m\langle E \rangle$.

$= 4 \times 100 h^2 + 0 + 7 \times 4 \times 9.11 \times 10^{-31}$ kg.

$= 400h^2 + 28 \times [9.11 \times 10^{-31} \text{kg}]$

3.3 An unknown semiconductor has $E_g = 1.1eV$ and $N_c = N_v$. It is doped with $10^{15} cm^{-3}$ donors, where the donor level is 0.2 eV below E_c. Given that E_F is 0.25eV below E_c. calculate n_i and the concentration of electrons and holes in the semiconductor at 300 kV.

Solution→



$N = 10^{15}/cm^3$

~~$n_i^2 = \sqrt{n_o p_o}$ for intrinsic mat.~~ —— ③

$n_o = N_c \, e^{\frac{-(E_c - E_F)}{kT}}$ —— ①

$p_o = N_v \, e^{\frac{-(E_F - E_v)}{kT}}$ —— ②

from ①

$N_c = n_o \, e^{\frac{+(E_c - E_F)}{kT}}$ $\qquad k = 8.62 \times 10^{-5}$ eV/°K

$= \frac{10^{15}}{cm^3} e^{\frac{0.25eV}{8.62 \times 10^{-5} \times 300}}$

$= 10^{15} e^{\frac{0.25eV}{0.0259}}$

$= 10^{15} \times 1.556 \times 10^{4} = 1.556 \times 10^{19}$

As $N_c = N_v$

$N_v = 1.56 \times 10^{19} /cm^3$.

$p_0 = N_v\, e^{-\left(\frac{E_F - E_v}{kT}\right)} = 1.56 \times 10^{19}\, e^{-\left(\frac{0.85}{8.62 \times 10^{-5} \times 300}\right)}$

$E_F - E_v = E_g - (E_c - E_F) = 1.1 - 0.25 = 0.85$

$p = 1.56 \times 10^{19}\, e^{-\left(\frac{0.85}{0.0259}\right)}$

$p_0 = \frac{8.714 \times 10^4}{cm^3}$.

~~$n_i = \sqrt{n_0 p_0} = \sqrt{1.56 \times 10^{19} \times 8.714 \times 10^4}$~~

$n_i = \sqrt{N_c N_v}\; e^{-\frac{E_g}{2kT}}$.

$= \sqrt{1.556 \times 10^{19} \times 1.556 \times 10^{19}}\; e^{-\frac{1.1 ev}{2 \times 8.62 \times 10^{-5} \times 300}}$

$= 1.556 \times 10^{19} \times e^{-\frac{1.1 ev}{0.05172}}$

$n_i = \frac{9.046 \times 10^9}{cm^3}$.