MCEN90018 Advanced Fluid Dynamics

Assignment 3 - Signal Analysis

Assignment to be completed in groups of two, and to be submitted by 11:59 pm on Sunday 28th May 2017. Total available marks are 112. The last 2 questions (Bonus questions) do not need to be attempted to obtain an H1 (a maximum of 92/112 are obtainable without attempting this question).

Part I - Signal analysis

A boundary layer traverse has been performed in a turbulent boundary layer at $Re_\tau = 14\,000$. A single-normal hotwire has been traversed at 40 positions between $y = 0.27$ mm and $y \approx 450$ mm. Simultaneously a flush mounted hotfilm sensor has also been sampled (which provides the fluctuating friction velocity - or the fluctuating velocity one wall unit from the surface). The experimental set-up can be viewed in slide 38 of lecture 13 (only the central hotfilm, immediately below the traversing hotwire is used here). The data are available in the following zipped directory `data` with the file names `u_hf_yposY.bin` for the hotfilm and `u_hw_yposY.bin` for the traversing hotwire probe, where $1 < Y < 40$ is an integer representing the y measurement station. The data are stored as floating point double precision velocities (all data have been calibrated already). Both signals were sampled at $f_s = 10$kHz for 30 seconds. The boundary layer thickness $\delta = 0.326$ m. The wall normal y locations (in mm) for the 40 measurement stations are given by the file `y.txt`

1. The hot-film sensor is unreliable for frequencies greater than 100Hz. Implement a spectral low-pass cut-off filter on the hotfilm for $Y = 3$. Plot a short time-series comparing the original and low-pass filtered (low frequency) hotfilm data for 1 s $< t <$ 2 s. **10 Marks**

2. Implement a spectral high-pass cut-off filter on the hotfilm for $Y = 3$. Plot a short time-series comparing the original (with mean subtracted) and high-pass filtered (high frequency) hotfilm data for 1 s $< t <$ 1.4 s. **2 Marks**

3. Compute the cross-correlation coefficient between the low pass hot-film signal and the similarly low pass filtered traversing hot-wire sensor at $Y = 3$. Use a long-hand approach initially,

$$R(\Delta t) = \frac{1}{N}\sum_{m=1}^{N} \frac{u_{hf}(t)u_{hw}(t + \Delta t)}{\sigma_{u_{hf}}\sigma_{u_{hw}}}$$

where u_{hf} and u_{hw} are the appropriately filtered fluctuating components from the hotfilm and hotwire respectively, and $\sigma_{u_{hf}}$ and $\sigma_{u_{hw}}$ are the standard deviation of the two signals. Taylors hypothesis can be used to convert from the time to the space domain,

$$x = -U_c t$$

where U_c is the convection velocity of turbulent fluctuations at the Yth position (taken to be the local mean as measured by the hotwire). Plot the cross-correlation coefficient R vs $\Delta x/\delta$ from $-3 < \Delta x/\delta < 3$.

10 Marks

4. Calculate the same cross-correlation spectrally (using FFTs) and show that you obtain the same result (do not use xcorr for this - I would like you to code your self). Note that the FFT method will return R for values from $\Delta t = -N\delta t/2 + 1 : N\delta t/2$, where N is the total length of the signal and δt is the sample interval $1/f_s$. Use the `tic` and `toc` commands in Matlab to compare the run-times for the long-hand and spectral approach. Comment on this. **10 Marks**

5. Now you are satisfied that your (faster) spectral code works, calculate the cross-correlation spectrally between the high pass filtered hot-film signal and the similarly high pass filtered traversing hot-wire sensor at $Y = 3$ (the two sensors are very close here, less than 0.5 mm seperating them vertically). Compare the maximum correlation values between the low-pass and high-pass filtered results, comment on the implications of this (and why we might choose to discard frequency content beyond 100Hz from the hot-film sensor). **5 Marks**

6. Now compute the cross-correlation coefficient for all Y positions for the low-pass filtered (low frequency) data, and plot isocontours of R as a function of Δx and y. Comment on the what we can learn from this figure in terms of the structural composition of the turbulent boundary layer. **10 Marks**

7. Produce a conditional average of the the hot-wire measured velocity for $Y = 20$ based on the occurrence of a positive temporal velocity gradient $du_\tau/dt > 0$ indicated by the spectrally low-pass filtered hot-film signal from question 1. **10 Marks**

8. Compute the power spectral density for the traversing hot-wire for position $Y = 20$ (using a single 30 s burst of data). Initially use no overlapping or intervals. Plot the pre-multiplied energy spectrum as a function of frequency and as a sanity check, show that,

$$\int_0^\infty \Phi(f)df = \frac{1}{N}\sum_{m=1}^{N} u_m^2$$

Comment on the sampling frequency - is it sufficient? **10 Marks**

9. Your spectrum is still hairy. There are 10 bursts of 30 second of data at $Y = 20$ this time sampled at 30kHz in the same directory (named `u_hw_yposY_burstB.bin`, where B is the burst number). Choose a suitable interval (such that you still capture most of the low frequency information) and attempt to calculate a more converged power spectral density. You can use over-lapping. **10 Marks**

Part II - Resistance of a developing smooth and rough-wall boundary layer

The mean velocity profile for a developing turbulent boundary layer on a smooth or rough flat plat can be written as,

$$\frac{u(y)}{U_\tau} = \frac{1}{\kappa}\log(\frac{yU_\tau}{\nu}) + A - \Delta U^+ - \frac{1}{3\kappa}\eta^3 + \frac{\Pi}{\kappa}2\eta^2(3 - 2\eta)$$

Where $\eta = y/\delta$, A = 5, $\kappa = 0.41$, ΔU^+ is the Hama roughness function which is 0 for the smooth wall or a function of the roughness length k_s for rough surfaces. Using this

equation we can obtain a prediction for the skin friction coefficient and the boundary layer thickness on the back of a submarine.
We know that the momentum thickness is given by,

$$\theta = \int_0^\delta \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) dz = \frac{\nu}{U_\tau}\int_0^{\delta^+} \left(\frac{u^+}{S} - \frac{{u^+}^2}{S^2}\right) dz^+$$

Where $S = U_\infty/U_\tau$ and is given for the above functional form (for $y = \delta$) by,

$$S = \frac{1}{\kappa}\log\delta^+ + A - \Delta U^+ - \frac{1}{3\kappa} + \frac{2\Pi}{\kappa},$$

Consider the example of a **smooth** submarine with a speed of 30 knots and a kinematic viscosity of 8.97×10^{-7} m^2 s^{-1}

1. For a range of logarithmically spaced δ^+ from 100 to 1000000, write a Matlab script to evaluate numerically the momentum thickness θ for a smooth surface at each δ^+. Plot $Re_\tau = \delta^+$ vs Re_θ. **10 Marks**

2. Using the relationship for the local skin friction coefficient,

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{2}{S^2}.$$

   Also plot Re_τ vs. C_f for the same smooth surface. **5 Marks**

3. **Bonus Questions** At the moment we lack the information about where along the length of the submarine a particular δ^+ and therefore C_f is attained. We need to relate the Re_θ to Re_x, which is very easy using von Kármán momentum integral equation,

$$\frac{d\theta}{dx} = \frac{dRe_\theta}{dRe_x} = \frac{C_f}{2},$$

   Thus we can calculate Re_x from

$$Re_x = \int \frac{2}{C_f} dRe_\theta$$

   Plot the skin friction coefficient as a function of Re_x, and hence estimate C_f and the boundary layer thickness (in m) towards the back of a smooth submarine at $x = 115$ m (you can assume that the flat plate approximates the boundary layer development on the submarine). **10 Marks**

4. **Bonus Questions** Assuming that for fully rough surfaces,

$$\Delta U^+ = \frac{1}{\kappa}\log k_s^+ + A - 8.5$$

   Plot (on top of the smooth wall result obtained above) the skin friction coefficient as a function of Re_x for the case where the submarine has a surface roughness with a $k_s = 325$ μm. Compare the C_f and the boundary layer thickness (in m) towards the back of a submarine at $x = 115$ m for the rough and smooth surfaces. **10 Marks**