Ex 2:

$\nabla f(x)$?

$$f(x) = \frac{1}{2} x^T A x + b^T x + c$$

$$f(x) = \frac{1}{2} \langle x, Ax \rangle + \langle b, x \rangle + c$$

$$df = d\left(\frac{1}{2} \langle x, Ax \rangle + \langle b, x \rangle + c\right) =$$

$$= \frac{1}{2} d(\langle x, Ax \rangle) + d(\langle b, x \rangle) = \cancel{\frac{1}{2}(\langle dx, A dx\rangle + \langle 1, dx \rangle)} =$$

$$\cancel{= \langle b, dx \rangle}$$

$$\frac{1}{2}(\langle dx, Ax \rangle + \langle x, d(Ax) \rangle) =$$

$$= \frac{1}{2}(\langle Ax, dx \rangle + \langle x, A\,dx \rangle) =$$

$$= \frac{1}{2}(\langle Ax, dx \rangle + \langle A^T x, dx \rangle)$$

По св-ву скалярного произв-я:

$$\langle a, dx \rangle + \langle b, dx \rangle = \langle a + b, dx \rangle$$

$$df = \left\langle \frac{1}{2} Ax + \frac{1}{2} A^T x + b, dx \right\rangle$$

$$\nabla f = \frac{1}{2}(A + A^T) x + b$$

$$df = \left\langle \frac{1}{2}(A + A^T) x + b, dx \right\rangle \to dx_1 = const$$

$$dg = d\left(\left\langle \frac{1}{2}(A + A^T) x + b, dx_1 \right\rangle\right) =$$

$$= \langle d\left(\frac{1}{2}(A+A^T)x+b\right), dx_1\rangle =$$

приводим к виду:

$$= \langle \frac{1}{2}(A+A^T)\,dx, dx_1\rangle = \qquad \left\{ dg = \langle (\ \ )\,dx_1, dx\rangle \right.$$

$$= \langle dx_1, \frac{1}{2}(A+A^T)\,dx\rangle = \qquad \left\{ \left(\frac{1}{2}(A+A^T)\right)^T = \frac{1}{2}(A^T+A) \right\}$$

$$= \langle \frac{1}{2}(A+A^T)\,dx_1, dx\rangle$$

гессиан

$d g$

g

$\cancel{dg}\quad g = \langle A^T(Ax-b), dx_1 \rangle$

$$dg = \langle A^TA\,dx, dx_1 \rangle = \langle \underbrace{A^TA}_{=H}\,dx_1, dx \rangle$$

Ex $\nabla f(x) - ?$

$$f(x) = \ln(1 + \exp\langle \alpha, x \rangle)$$

$$df = df(\ln(1 + \exp\langle \alpha, x \rangle)) =$$

$$= \frac{df(1 + \exp\langle \alpha, x \rangle)}{1 + \exp\langle \alpha, x \rangle} =$$

$$= \frac{df(\exp\langle \alpha, x \rangle)}{1 + \exp\langle \alpha, x \rangle} =$$

$$= \frac{\exp\langle \alpha, x \rangle * \langle \alpha, dx \rangle}{\underbrace{1 + \exp\langle \alpha, x \rangle}_{\text{сигмоида}}} =$$

$$\left\{ \frac{\exp\langle \alpha, x \rangle}{\exp\langle \alpha, x \rangle} \times \frac{1}{\frac{1}{\exp} + \exp} \right.$$

$$\left. \cancel{\exp\langle \alpha, x \rangle} \times \frac{1}{1 + \exp^{-1}} \right\}$$

$$= \frac{1}{1 + \exp(-\langle \alpha, x \rangle)} \langle \alpha, dx \rangle$$

$$\nabla f = \sigma(\langle \alpha, x \rangle) * \alpha$$

Ex $\quad f(x) = \log(x^T A x) = \ln(\langle x, Ax \rangle)$

$\nabla f_{(x)}$? $\quad$ ~~$df = d(\langle \log(x^T Ax) \rangle)$~~

вектор

$$\left\{ (\ln f(x))' = \frac{f'(x)}{f(x)} \quad , \quad d\ln \varphi(x) = \frac{d\varphi}{\varphi(x)} \right\}$$

$$\underset{\text{скаляр}}{df_{(x)}} = d(\ln(\langle x, Ax \rangle)) = \frac{d(\langle x, Ax \rangle)}{\langle x, Ax \rangle} =$$

$$= \frac{\langle x, A dx \rangle + \langle dx, Ax \rangle}{\langle x, Ax \rangle} = \frac{\langle A^T x, dx \rangle + \langle Ax, dx \rangle}{\langle x, Ax \rangle} =$$

$$= \frac{\langle (A + A^T) x, dx \rangle}{\langle x, Ax \rangle} = \left\langle \frac{(A + A^T) x}{\langle x + Ax \rangle}, dx \right\rangle$$

Gradient - ? $\quad \nabla f = \dfrac{(A + A^T) x}{\langle x + Ax \rangle}$

$$f(X) = \underset{\text{след}}{tr}(AXB)$$

$$f(x) = tr(AXB) = tr((A^T)^T \times XB) = \langle A^T, XB \rangle$$

$$d(\langle A^T, XB \rangle) = d(\langle A^T B^T, X \rangle)$$

$$f(x) = \frac{1}{2}\|Ax-b\|_2^2 = \frac{1}{2}\langle Ax-b, Ax-b\rangle$$

1. Ищем дифференциал скалярного произведения

$$df = d\left(\frac{1}{2}\langle Ax-b, Ax-b\rangle\right) =$$

$$= \underbrace{\underbrace{\frac{1}{2}\langle \underbrace{d(Ax-b)}_{\text{вектор}}, \underbrace{Ax-b}_{\text{вектор}}\rangle}_{\text{скал.}} + \underbrace{\frac{1}{2}\langle \underbrace{Ax-b}_{\text{век}}, \underbrace{d(Ax-b)}_{\text{век}}\rangle}_{\text{скаляр}}}_{\text{скаляр}} =$$

$$g = \langle A^T (Ax - b), dx \rangle$$

$$dg = d(\langle A^T Ax - A^T b, x_1 \rangle$$

$$dg = \langle A^T A\, dx, dx_1 \rangle$$

$$dg = \langle A^T A\, dx_1, dx \rangle$$