

transformations

Feb 4

Composition: Eg scale & rotate object

Let: S be scale matrix

R be a rotation matrix

$$v_1 = Sv$$

$$v_2 = Rv_1$$

$$\Rightarrow v_2 = Rv_1 = R(Sv) = (RS)v$$

Warning 1: read transformations right to left

Warning 2: $AB \neq BA$

Feb 4

from functions

Composition: 3rd scale 2nd scale object

Let: 2 be 2nd scale

R be a relation matrix

$$v_1 = 2v$$

$$v_2 = Rv$$

$$v(2g) = (v_2)g = v_1 = v =$$

Warning: 2nd scale transformation (light to left)

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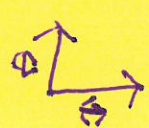
Example


$$K = 45^\circ$$

$$S = (.25, .5)$$

Write down 2 matrices and order (first rotate then scale)

$$\begin{bmatrix} .25 & 0 \\ 0 & .5 \end{bmatrix} \begin{bmatrix} \cos 45 & \sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$$

Problem: scale x or y e.g.  $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$

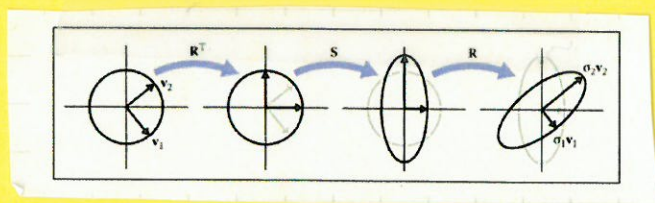
What if we want to scale diagonally 

$$r_1 = \text{rotate } 45^\circ$$

$$S = \text{scale } (s_x, s_y)$$

$$r_2 = \text{rotate } (-45^\circ)$$

$$T = r_2 S r_1$$



$$A = R S R^T$$

Example

$$K = 12^\circ$$

$$Z = (.72, .2)$$

With given 2 vectors and angle

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 12^\circ & \sin 12^\circ \\ -\sin 12^\circ & \cos 12^\circ \end{bmatrix}$$

Problem: scale X or Y



what if we want to scale differently

$$T = \text{rotate } 12^\circ$$

$$Z = \text{scale } (.72, .2)$$

$$T = \text{rotate } (-12^\circ)$$

$$T = T^T$$

$$A = R^T Z R$$

Translation (Affine transforms)

so far

$$\begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

translate

$$b_x = x + t_x$$

$$b_y = y + t_y$$

Seq of transformations w/ translate as vectors

$$\begin{array}{c} R T S R S R T \\ R T \quad M \quad T \\ \text{(M)} \text{ vector} \quad \text{matrix} \quad \text{vector} \end{array}$$

rewrite as a matrix

$$\begin{aligned} b_x &= \overset{1}{a_{11}}x + \overset{0}{a_{12}}y + t_x \cdot \underset{1}{1} \\ b_y &= \overset{0}{a_{21}}x + \overset{1}{a_{22}}y + t_y \cdot \underset{1}{1} \end{aligned} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation (Affine transform)
 +
 20 for

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.5x + 0.5y \\ 0.5x + 0.5y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

translation

$$x + x = x$$

$$y + y = y$$

set of transformations by translation of vectors

$$\begin{matrix} T & M & T \\ \text{Translation} & \text{Matrix} & \text{Translation} \end{matrix}$$

translation is 20 units

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.5x + 0.5y \\ 0.5x + 0.5y \end{bmatrix}$$

$$\text{Rot} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

$$\text{Scale} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

rewrite both matrices as

$$\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11}x + m_{12}y \\ m_{21}x + m_{22}y \\ 1 \end{bmatrix}$$

Translation

what entries go in here?

$$\begin{bmatrix} \\ \\ \end{bmatrix} - \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

1 1 1

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \text{swap}$$

$$\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} = \text{rot}$$

convert both matrices as

$$\begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & \sin \theta & \cos \theta \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \begin{bmatrix} 0 & \sin \theta & \cos \theta \\ 0 & \cos \theta & \sin \theta \\ 1 & 0 & 0 \end{bmatrix}$$

convert matrix to row

Translation

$$\begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$