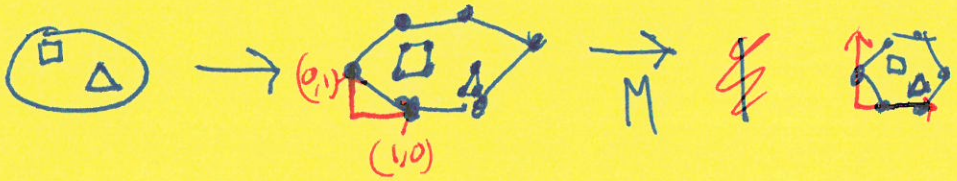
# Transformations

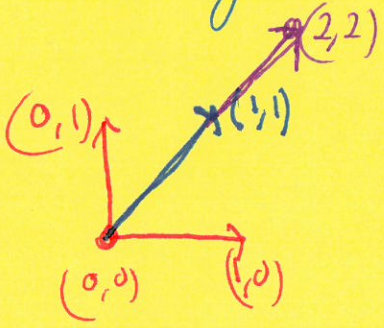
## Idea




## 2D Linear transformations

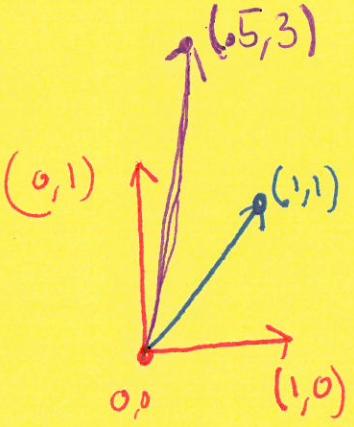
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix}$$

## Scaling



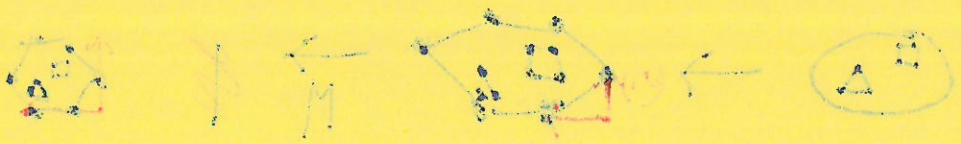

we want

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$




we want

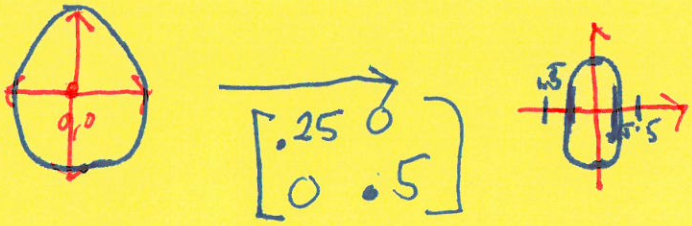
$$\begin{bmatrix} .5 \\ 3 \end{bmatrix} = \begin{bmatrix} .5 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



# Scaling Matrix

$$\text{Scale}(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

## Scaling a circle



# Rotation


$$r = \|a\| = \|b\|$$

$$= \sqrt{a_x^2 + a_y^2}$$

$$a_x = r\cos\theta$$
$$a_y = r\sin\theta$$

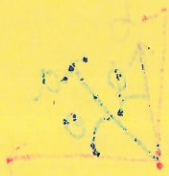
$$b_x = r\cos(\theta+\phi)$$
$$b_y = r\sin(\theta+\phi)$$

$$b_x = \underbrace{r\cos\theta}_{a_x}\cos\phi - \underbrace{r\sin\theta}_{a_y}\sin\phi$$

$$b_y = \underbrace{r\sin\theta}_{a_y}\cos\phi + \underbrace{r\cos\theta}_{a_x}\sin\phi$$

$$\begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} a_x\cos\phi - a_y\sin\phi \\ a_y\cos\phi + a_x\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} a_x\cos\phi - a_y\sin\phi \\ a_x\sin\phi + a_y\cos\phi \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$



$$\text{rotate}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$