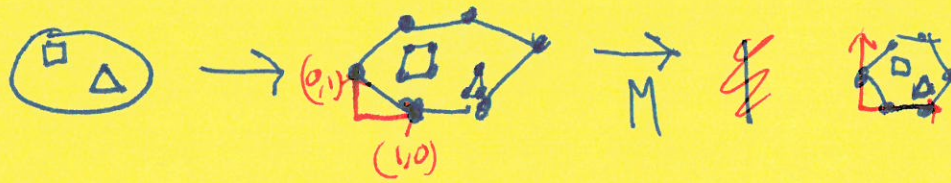


# Transformation S

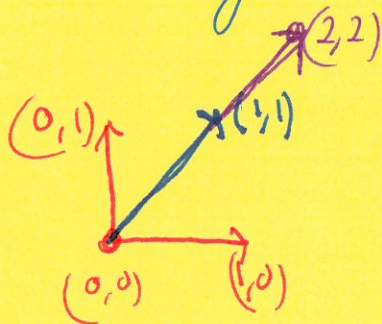
Idea



## 2D Linear transformations

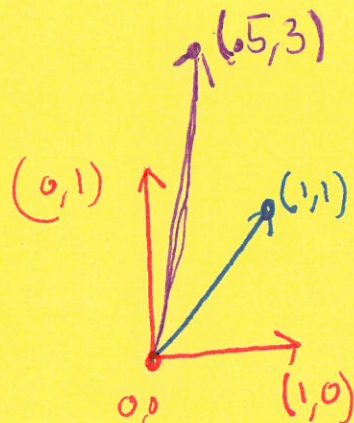
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix}$$

### Scaling



we want

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



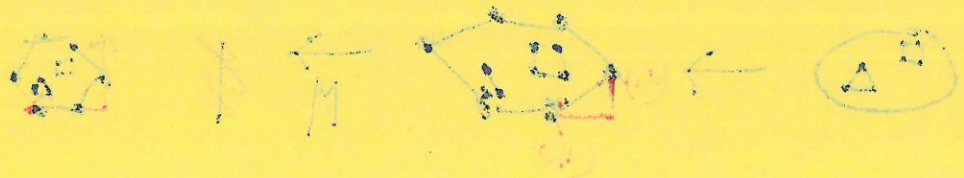
we want

$$\begin{bmatrix} 0.5 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Transformation 2

Iter



2D linear transformation

$$\begin{bmatrix} y_{2D} + x_{2D} \\ y_{2D} + x_{2D} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Scaling



translation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

translation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$



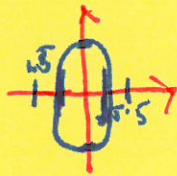
Scaling Matrix

$$\text{Scale}(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

Scaling a circle



$$\begin{bmatrix} .25 & 0 \\ 0 & .5 \end{bmatrix}$$





Scaling Matrix

$$\begin{bmatrix} 0 & x_2 \\ x_1 & 0 \end{bmatrix} = \begin{pmatrix} x_2 & x_1 \end{pmatrix}$$

Scaling a circle





# Rotation



$$r = \|a\| = \|b\| = \sqrt{a_x^2 + a_y^2}$$

$$a_x = r \cos \theta$$

$$a_y = r \sin \theta$$

$$b_x = r \cos(\theta + \phi)$$

$$b_y = r \sin(\theta + \phi)$$

$$b_x = \overbrace{r \cos \theta}^{a_x} \cos \phi - \overbrace{r \sin \theta}^{a_y} \sin \phi$$

$$b_y = \overbrace{r \sin \theta}^{a_y} \cos \phi + \overbrace{r \cos \theta}^{a_x} \sin \phi$$

$$\begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} a_x \cos \phi - a_y \sin \phi \\ a_y \cos \phi + a_x \sin \phi \end{bmatrix}$$

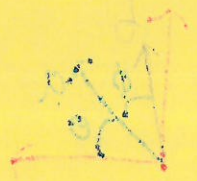
$$= \begin{bmatrix} a_x \cos \phi - a_y \sin \phi \\ a_x \sin \phi + a_y \cos \phi \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$



no. 10/07

$$\# \|x\| = \|y\| = 1$$

$$\sqrt{x^2 + y^2}$$



$$\Theta \cos \gamma = x_0$$

$$\Theta \sin \gamma = y_0$$

$$(\phi + \Theta) \cos \gamma = x_0$$

$$(\phi + \Theta) \sin \gamma = y_0$$

$$\phi \sin \Theta \sin \gamma - \phi \cos \Theta \cos \gamma = x_0$$

$$\phi \sin \Theta \cos \gamma + \phi \cos \Theta \sin \gamma = y_0$$

$$\begin{bmatrix} \phi \sin \Theta \sin \gamma - \phi \cos \Theta \cos \gamma \\ \phi \sin \Theta \cos \gamma + \phi \cos \Theta \sin \gamma \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \begin{bmatrix} \sin \Theta & -\cos \Theta \\ \cos \Theta & \sin \Theta \end{bmatrix} = \begin{bmatrix} \phi \sin \Theta \sin \gamma - \phi \cos \Theta \cos \gamma \\ \phi \sin \Theta \cos \gamma + \phi \cos \Theta \sin \gamma \end{bmatrix}$$



$$\text{rotate}(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$



