transformations

Feb 4

Composition: Eg scale & rotate object

Let: S be scale matrix

R be a rotation matrix

$$v_1 = Sv$$

$$v_2 = R v_1$$

$$\Rightarrow v_2 = R v_1 = R(Sv) = (RS)v$$

Warning 1: read transformations right to left

Warning 2: $AB \overset{?}{\neq} BA$

Example

$K = 45^\circ$

$S = (.25, .5)$

Write down 2 matrices and order (first rotate then scale)

$$\begin{bmatrix} .25 & 0 \\ 0 & .5 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$$

Problem: Scale X or Y e.g.  $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$

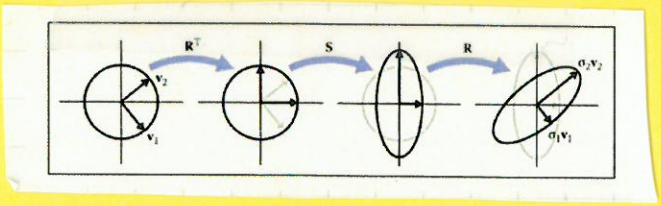
What if we want to scale diagonally 

$r_1 =$ rotate 45°

$S =$ scale (s_x, s_y)

$r_2 =$ rotate (-45°)

$T = r_2 S r_1$



$A = RSR^T$

Translation (Affine transforms)

+

so far

$$\begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

translate

$$b_x = x + t_x$$
$$b_y = y + t_y$$

Seq of transformations w/ translate as vectors

RTSRSRT

R T M T

(Matrix) Vector matrix vector

rewrite as a matrix

$$b_x = 1 \cdot a_{11}x + 0 \cdot a_{21}y + t_x \cdot 1$$
$$b_y = 0 \cdot a_{21}x + 1 \cdot a_{22}y + t_y \cdot 1$$

$$= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$Rot = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \qquad Scale = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

rewrite both matrices as

$$\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11}x + m_{12}y \\ m_{21}x + m_{22}y \\ 1 \end{bmatrix}$$

Translation ← what entries go in here?

$$\begin{bmatrix} & & \\ & & \\ & & \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$