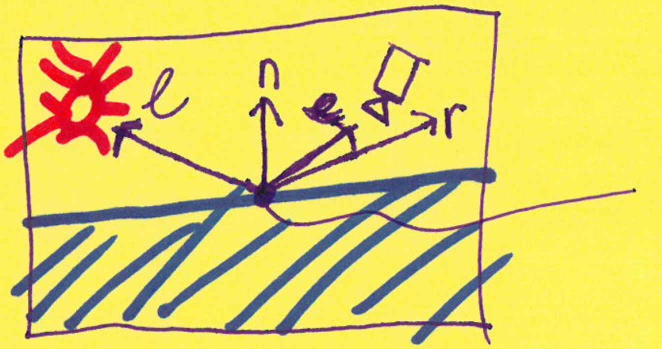
Phong Shading

02/20/2020



1st approx of modeling a highlite

color of light C_ℓ

$C = C_\ell (e \cdot r)$ — reflected (across normal) light vector

vector from point p to carea

Problems

1. $(e \cdot r)$ may be neg $\Rightarrow$ C could be neg

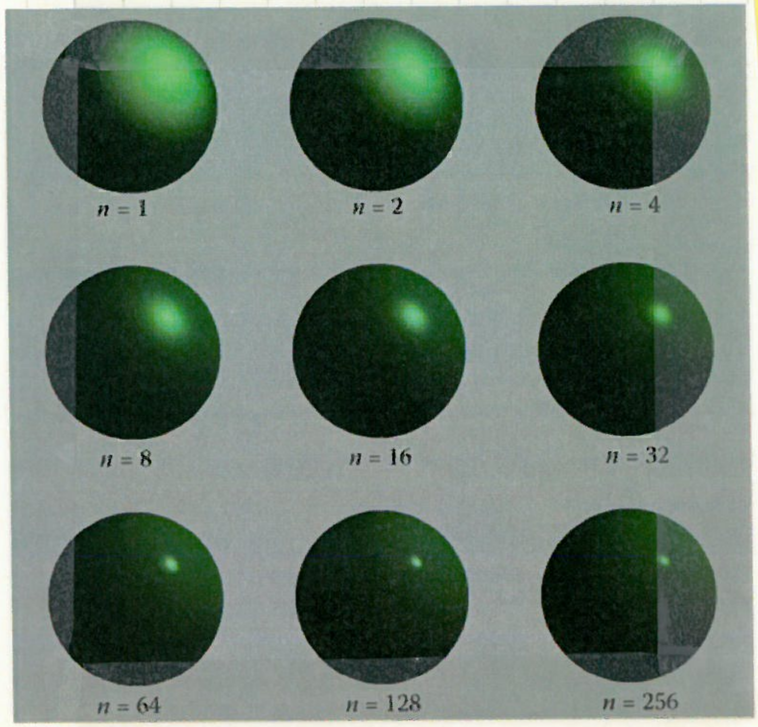
fix $C = C_\ell \max(e \cdot r, 0)$

2. highlite is much bigger than in reality

fix Phong exponent $p \in \mathbb{R}^+$

$$C = C_\ell * \left[\max(e \cdot r, 0)\right]^p$$

phong exponent



Put it together

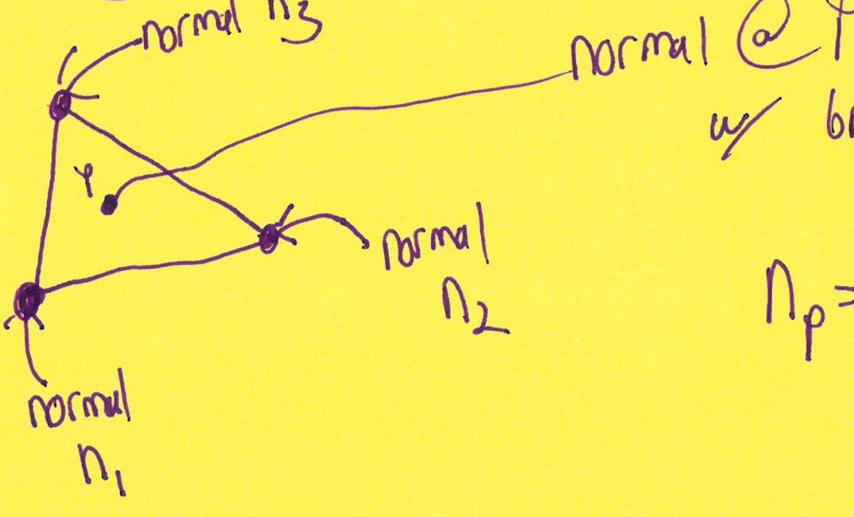
- compute r

$$r = -l + 2(l \cdot n)\, n$$

combine w/ diffuce & ambient

$$c = c_r\left(c_a + c_l * \max(0, n \cdot l)\right) + c_l \max(0, e \cdot r)^p$$

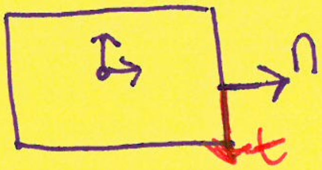
normals



w/ barycentric coord (α, β, γ)

$$n_p = \alpha n_1 + \beta n_2 + \gamma n_3$$

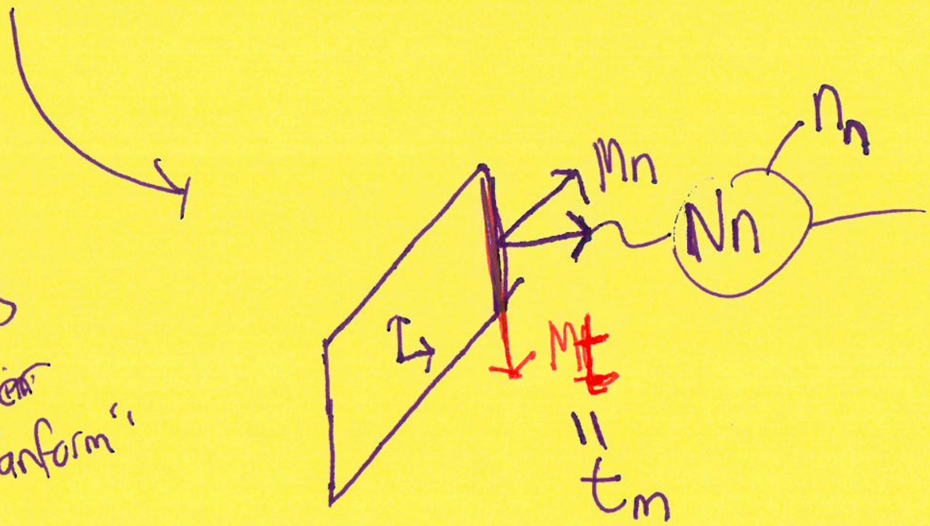
Transforming normals:



surface normal:
a vector that is $\perp$ to tangent plane

M

w/ M as a "shearing transform"



what is the N matrix that is our transform

Derive N

- surface normal n and tangent t are $\perp$ $\Rightarrow$ $n^T t = 0$

- want to find $n_n^T t_m = 0$

observe: ~~below I'll write~~ as

$$0 = n^T t = n^T I t = n^T (M^{-1} M) t = (n^T M^{-1})(Mt)$$

$$= (n^T M^{-1}) t_m = 0$$

$$n_n^T = n^T M^{-1}$$

$$\Leftrightarrow \quad n_n = \left(n^T M^{-1}\right)^T = \boxed{\left(M^{-1}\right)^T} n$$

|| looking for before

|| written as cofactor matrix

$$N = \begin{bmatrix} m_{11}^c & m_{12}^3 & m_{13}^c \\ m_{31}^c & m_{22}^c & m_{23}^c \\ m_{31}^c & m_{32}^c & m_{33}^c \end{bmatrix} = \begin{bmatrix} m_{22}m_{33} - m_{23}m_{32} & m_{23}m_{31} - m_{21}m_{33} & m_{21}m_{32} - m_{22}m_{31} \\ m_{13}m_{32} - m_{12}m_{33} & m_{11}m_{33} - m_{13}m_{31} & m_{12}m_{31} - m_{11}m_{32} \\ m_{12}m_{23} - m_{13}m_{22} & m_{13}m_{21} - m_{11}m_{23} & m_{11}m_{22} - m_{12}m_{21} \end{bmatrix}$$