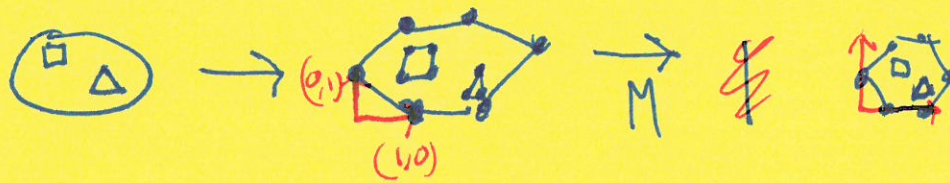


# Transformation S

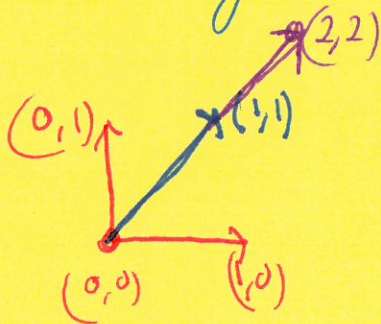
Idea



## 2D Linear transformations

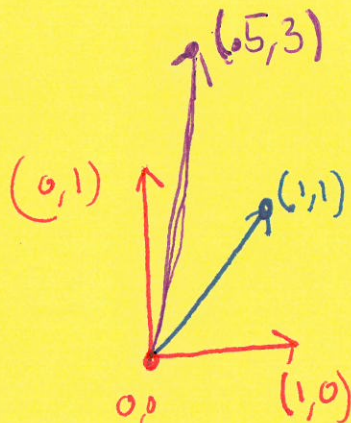
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix}$$

### Scaling



we want

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



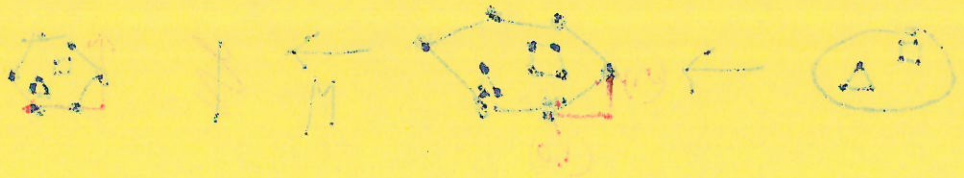
we want

$$\begin{bmatrix} 0.5 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Transformation

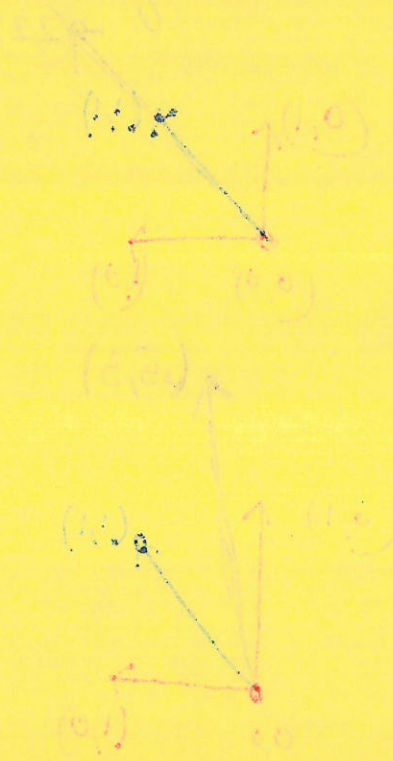
Then



2D linear transformation

$$\begin{bmatrix} y_1' & y_2' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix}$$

Scaling



Translation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$$

Rotation

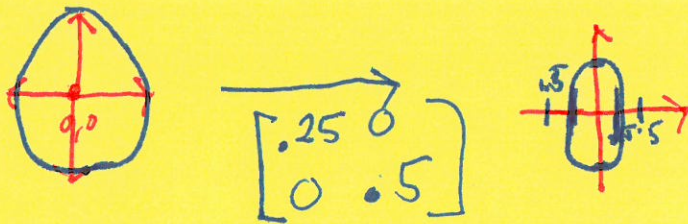
$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} x_1' & x_2' \end{bmatrix}$$



Scaling Matrix

$$\text{Scale}(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

Scaling a circle





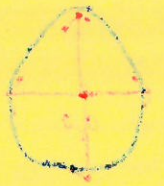
Scaling Matrix

$$\begin{bmatrix} 0 & x_2 \\ x_2 & 0 \end{bmatrix} = \begin{pmatrix} x_2 & x_2 \end{pmatrix}$$

Scaling a circle



$$\begin{bmatrix} 0 & x_2 \\ x_2 & 0 \end{bmatrix}$$





# Rotation



$$r = \|a\| = \|b\| = \sqrt{a_x^2 + a_y^2}$$

$$a_x = r \cos \theta$$

$$a_y = r \sin \theta$$

$$b_x = r \cos(\theta + \phi)$$

$$b_y = r \sin(\theta + \phi)$$

$$b_x = \overbrace{r \cos \theta}^{a_x} \cos \phi - \overbrace{r \sin \theta}^{a_y} \sin \phi$$

$$b_y = \overbrace{r \sin \theta}^{a_y} \cos \phi + \overbrace{r \cos \theta}^{a_x} \sin \phi$$

$$\begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} a_x \cos \phi - a_y \sin \phi \\ a_y \cos \phi + a_x \sin \phi \end{bmatrix}$$

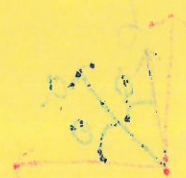
$$= \begin{bmatrix} a_x \cos \phi - a_y \sin \phi \\ a_x \sin \phi + a_y \cos \phi \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$



Option 10

$$\# \|d\| = \|a\| = r$$

$$\sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta}$$



$$\Theta \cos \gamma = x_d$$

$$\Theta \sin \gamma = y_d$$

$$(\phi + \theta) \cos \gamma = x_d$$

$$(\phi + \theta) \sin \gamma = y_d$$

$$\phi \sin \theta \cos \gamma - \phi \cos \theta \sin \gamma = x_d$$

$$\phi \sin \theta \sin \gamma + \phi \cos \theta \cos \gamma = y_d$$

$$\begin{bmatrix} \phi \sin \theta \cos \gamma - \phi \cos \theta \sin \gamma \\ \phi \sin \theta \sin \gamma + \phi \cos \theta \cos \gamma \end{bmatrix} = \begin{bmatrix} x_d \\ y_d \end{bmatrix}$$

$$\begin{bmatrix} x_d \\ y_d \end{bmatrix} \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} = \begin{bmatrix} \phi \sin \theta \cos \gamma - \phi \cos \theta \sin \gamma \\ \phi \sin \theta \sin \gamma + \phi \cos \theta \cos \gamma \end{bmatrix}$$



$$\text{rotate}(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$



